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A Platform for Benchmark Cases in Computational Acoustics

Maarten Hornikx[1], Manfred Kaltenbacher[2], Steffen Marburg[3]
[1] Eindhoven University of Technology, The Netherlands. m.c.j.horniks@tue.nl
[2] Vienna University of Technology, Austria
[3] Universität der Bundeswehr Munich, Germany

Summary
Solutions to the partial differential equations that describe acoustic problems can be found by analytical, numerical and experimental techniques. Within arbitrary domains and for arbitrary initial and boundary conditions, all solution techniques require certain assumptions and simplifications. It is difficult to estimate the precision of a solution technique. Due to the lack of a common process to quantify and report the performance of the solution technique, a variety of ways exists to discuss the results with the scientific community. Moreover, the absence of general reference results does hamper the validation of newly developed techniques. Over the years many researchers in the field of computational acoustics have therefore expressed the need and wish to have available common benchmark cases. This contribution is intended to be the start of a long term project, about deploying benchmarks in the entire field of computational acoustics. The platform is a web-based database, where cases and results can be submitted by all researchers and are openly available. Long-term maintenance of this platform is ensured. As an example of good practice, this paper presents a framework for the field of linear acoustic. Within this field, different categories are defined – as bounded or unbounded problems, scattering or radiating problems and time-domain as well as frequency-domain problems – and a structure is proposed how to describe a benchmark case. Furthermore, a way of reporting on the used solution technique and its result is suggested. Three problems have been defined that demonstrate how the benchmark cases are intended to be used.

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1. Introduction

1.1. Motivation

There are only a few problems in physical acoustics for which analytical solutions exist. Within arbitrary spatial domains, inhomogeneous and moving propagation media and arbitrary initial and boundary conditions, all solution techniques require certain assumptions and simplifications. Therefore, most of the technically relevant problems require numerical solutions or suitable measurements. Since the exact solution remains unknown for such problems, it is often quite difficult to evaluate the quality of the results obtained by numerical and experimental techniques. As a result, uncertainties exhibited by both simulation and experimental techniques can hardly be quantified.

The subject of computational acoustics is vast, as the nature of the acoustic problems can be very different, e.g. solution of boundary and initial value problems of the scalar wave equation for linear wave propagation in fluids, or the vector wave equation for linear wave propagation in structures, computation of transmission loss through walls, sound propagation in moving media, estimation of sound radiation from aeroacoustic and hydroacoustic sources, non-linear acoustic problems such as high–intensity ultrasound heating and sound radiation due to non-linear vibration effects such as squealing and squeaking noise. Consequently, a wealth of computational solution techniques has been developed over the years, tailored to solve the envisaged problem by simulation. The purpose of this paper is not to review these various techniques. Review papers of these simulation techniques can be found e.g. for time-harmonic acoustics problems [1, 2, 3] and for techniques dedicated to specific acoustic fields as e.g. outdoor sound propagation [4], room acoustics [5] and time-domain boundary impedances [6].

This paper is intended to initiate a long term voluntary project, about deploying benchmarks in the entire field of computational acoustics. The initiative has three main reasons, which will be detailed in the remainder of this section:

1. No clearly defined and widely accepted benchmark cases for computational acoustics exist;
2. No common way exists to report on the used solution technique for a physical acoustics problem and the obtained results;

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3. The absence of general reference results does hamper the development of validating new emerging techniques.

1.1.1. Absence of benchmark cases

Textbook examples exist for many fields in computational acoustics. These are often geometrical configurations with simple boundary conditions and possibly initial conditions for which analytical solutions exist. These examples can be used to benchmark computational techniques, but because of their simplicity, they are not always of too much value. For more complicated cases, researchers often define reference cases themselves or borrow cases from related scientific work. An example of this is a thin noise screen in the presence of a moving medium to demonstrate the accuracy of computational techniques in outdoor acoustics, e.g. [7, 8, 9, 10]. A framework of benchmark cases, specific to each field of computational acoustics, as well defined and easily accessible configurations for comparison of results from various techniques would therefore be highly valuable. In fact, benchmark cases have been defined in some physical acoustics fields. In the aeroacoustics community, NASA initiated 4 workshops on benchmark problems during the length of decade (1995–2003) [11]. During these workshops, researchers contributed with results from their methods applied to pre-defined benchmark cases. Another initiative concerns a benchmark platform on computational methods for architectural and environmental acoustics [12]. For this initiative, a website is maintained on which the benchmark cases have been posted as well as some of the computed results. A round robin on room acoustical computer simulation methods also comprises a good example of defining and using benchmark cases [13]. A recent benchmark initiative is the creation of a website where measurement results can be gathered and are openly available [14]. Thus, initiatives to set up benchmarks do exist, but from the several attempts undertaken in this direction, no initiative has covered a wide range of fields in computational acoustics. Also, it seems to be problematic to maintain a long term platform for benchmark cases.

1.1.2. Common reporting procedure

Often, newly developed solution techniques are compared to existing methods. Partly due to the lack of a common way of comparing techniques, results and performance are differently compared between different methods, and not always in a fair manner. For example, a fair approach to a cost comparison between the finite element method (FEM) and the boundary element method (BEM) was presented by Harari and Hughes [15]. The paper started from the idea that both methods require a certain number of elements per wavelength to produce results of a certain, prescribed and equal accuracy. According to this reference level, Harari and Hughes argued that the degree of freedom N for a surface (boundary element) mesh behaves as $N = O((kl)^2)$ (k is the wavenumber, l is a representative length scale, e.g. the largest length scale of the model) whereas for a volume mesh in FEM, it is $N = O((kl)^3)$. The solution procedure for a conventional BEM (with iterative solution of the linear system of equations) requires $O(N^2)$ memory resources and its complexity (number of operations) shows the $O(N^2)$ behaviour too. In FEM, both memory and complexity show an $O(N)$ behaviour. This leads to memory requirements and complexity for BEM of $O((kl)^4)$ and for FEM of $O((kl)^3)$. This paper, although providing a similar conclusion, stood in contradiction to the article of Burnett [16] in which a new finite/infinite element formulation was presented and compared with a certain boundary element implementation. In that paper, FEM/IFEM and BEM were compared by measuring computation time per degree of freedom, a comparison that obviously favours FEM. Another reason for using different styles of comparing techniques origins from the various types of techniques with their different types of numerical errors. The above mentioned papers are from the early nineties. A few years later, it became well–known that FEM suffers from the so–called pollution effect which has been well described by Ihlenburg [17]. However, higher order finite elements (pFEM) may resolve the issue of pollution well and as analysed in [18], even a practical formula is available how to choose the order of the FE basis functions by a given spatial discretization. This effect has not been observed for boundary–element methods, cf. Marburg [19, 20]. Also, FEM solutions in the exterior domain require special treatment to fulfil the Sommerfeld radiation condition. This requires infinite elements and absorbing boundary conditions [3, 21, 22, 23] as well as absorbing layers, in particular perfectly matched layers (PML) techniques [24, 25, 26, 27, 28, 29]. At the other hand, the irregular eigenfrequencies make the boundary–element method more complicated, because each of the methods that are used to overcome this problem encounter new difficulties [30]. A problem like the irregular frequencies has not been reported for FEM.

Finally, when developers of a new method try to compare their development with another method, they often are not able to use modern implementations for the reference method as well. Therefore, these comparisons are quite often very beneficial with respect to the newly developed method, as in the paper which has been discussed above [16].

1.1.3. Absence of reference results

Numerical methods have been increasing in efficiency as well as variety, in parallel to the development of digital computers and their increasing processor power. The development of computational methods is more and more directed toward accurately solving large engineering problems. Accurate simulation of most engineering problems often implies domains that typically contain $10^2 - 10^3$ wavelengths per dimension for the highest frequency of interest. With the increasing computer power, larger domains can be solved. At the same time, researchers continue to develop efficient numerical techniques. In recent years, computational techniques have been implemented using the graphic processing unit (GPU). Each GPU is composed of hundreds of individual processors. GPU technology has evolved in the last decades, increasing both the number of individual processors per silicon and the complexity of code each of them can execute. Due to the increase in popularity and ongoing reductions in price, application of GPUs to accelerate scientific calculations have increased, e.g. [31, 32]. Also, methods are being developed that derive their efficiency from including a priori knowledge of the exact solution in their numerical approach. These solution methods are therefore either based, or enriched by, wave-based functions. Some examples are the wave BEM [33], the Wave Based Method (WBM) [34], which is a frameless Trefftz method, and plane waves as basis functions within ultra weak variational formulation [35, 36, 1]. In the near future, the development of new techniques is thus eminent, and reference results are essential for validation purposes. Especially for the complex engineering problems for which codes more and more have been developed, results of reference cases that extend the textbook examples are needed. As these cases cannot be solved analytically, reference results need to be provided by either suitable measurements or by other calculation methods. For the pace of developments of codes, it would be highly beneficial to have access to reference results on a publicly available platform.

1.2. Outline of this contribution

A platform of benchmark cases for computational acoustics is launched and is described in Section 2. Four fields of physical acoustics are defined to arrange the various benchmark cases. For the field of linear acoustics, Section 3 presents a systematic way of describing a benchmark case and reporting on the numerical results. It needs to be emphasized that this proposed definition of benchmark cases for linear acoustics should be seen as an example of good practice for creating benchmark cases. The minimum requirement for defining a benchmark case is similar to presenting results in a scientific paper, i.e. the necessary information to reproduce the results should be present.

For clarifying the platform, three benchmark cases in the field of linear acoustics are defined in Section 4: a long duct, the cat's eye radiator, an arbitrarily complex shaped radiatterer, i.e. radiator ∞ scatterer.

The paper ends with an outlook to establish a long term voluntary project out of this initiative in Section 5.

2. Platform for benchmark cases

2.1. Fields of benchmark cases

Benchmark cases are of interest throughout the computational acoustics community, and can be defined for all fields of physical acoustics. The following fields are suggested:

1. **Linear acoustics**. Wave propagation as a solution of the wave equation, including scattering and radiation problems, interior and exterior problems, time-domain and frequency-domain problems.
2. **High frequency acoustics**. Higher frequency problems for which consideration of energy terms might be more interesting than deterministic evaluation of acoustic quantities such as pressure or particle velocity.
3. **Acoustics and vibration**. Fluid-structure interaction problems, including transmission loss through (layered) plates, shells and porous media.
4. **Acoustics involving heterogeneous and moving fluids**. Flow- and aeroacoustics (with coupling to structural vibrations), underwater acoustics.

2.2. Procedure of defining, discussing, submitting and utilizing benchmark cases

A platform for benchmark cases for computational acoustics is initiated to answer the shortcomings as pointed out in Section 1.1: the absence of benchmark cases, the absence of a common reporting procedure and the absence of reference results. A website is launched and maintained. It can be accessed via the European Acoustics Association (EAA) website, at the page of the technical committee (TC) of computational acoustics. At the website, benchmark cases can be found, as well as computed results to benchmark cases. The technical committee (TC) of computational acoustics of EAA will manage benchmark cases platform and act as initial Associate Editors (AE) of the benchmark cases. The following prodecure applies:

- **Defining and discussion benchmark cases**. The TC will organise, and encourage other scientists to organise, recurring benchmark case sessions at international conferences. Contributors are highly recommended and encouraged to submit a proposal of a benchmark case to such an upcoming event (conference, symposium). This gives the authors of new benchmark cases the possibility to adjust and optimise the defined benchmark case prior to submitting it to the website.
- **Submitting benchmark cases**. Three benchmark cases on linear acoustics are initially available at the website and serve as good practice examples for the submission of new benchmark cases. However, the minimum requirement for defining a benchmark case is similar to presenting results in a scientific paper, i.e. the necessary information to reproduce the results should be present. Also, a benchmark case should be valuable for its related technical field. Contributors of new benchmark cases or solutions to existing benchmark cases submit a case by e-mail to an AE of the benchmark cases.
- **Accepting benchmark cases**. Benchmark case Editors (BE) related to all fields of benchmark cases as defined above are involved. The Associate Editor will assign a Benchmark case Editor to handle the submitted benchmark case or solution to an existing benchmark case. The Benchmark case Editor will review the submitted matter, and/or request peer expert(s) to review the submission. A case will either be accepted, rejected, or a revision will be asked for.
- **Utilizing benchmark cases**. All benchmark cases and solutions to benchmark cases are openly available from

the website, free of charge. All researchers are encouraged to use these benchmark cases while working with computational techniques in all fields of acoustics.

3. Defining benchmark cases for linear acoustics

3.1. Categories and descriptions

The benchmark cases for the field of linear acoustics can be classified into categories, which will be detailed below. For every benchmark case, a description of the problem will be defined. The outline of such a description is described in Section 3.1.2. The way of categorising and presenting problems is quite similar to the benchmark platform on computational methods for architectural and environmental acoustics [12].

3.1.1. Categories

The benchmark cases are classified according to the categories below. The order of the categories is more or less hierarchical. In particular, the latter two categories can still be chosen after having defined the geometry of the configuration (and thereby categories 1 and 2).

1. **Bounded or Unbounded problems**. For bounded problems, the geometrical domain is completely circumfered by boundaries, whereas (part of) the boundaries for unbounded problems consist of non-reflecting boundaries. Bounded problems are often easier to solve than unbounded problems.
2. **Dimensionality of the case**. The dimension of the problems can either be 1D, 2D or 3D. Axisymmetric problems belong to the 3D category. Generally, the higher the dimension, the more complex the problem.
3. **Scattering or Radiation problem**. Problems without prescribed values of acoustic variables at boundaries belong to the scattering category. When acoustic variables are prescribed at boundaries, the problem falls in the radiation category. Combined scattering and radiation problems do also exist.
4. **Time–domain or Frequency–domain problem**. The time-domain problems cover broadband solutions, whereas frequency-domain solutions solve the time–harmonic wave equation (Helmholtz equation). A special case are the frequency-domain problems without source formulation, i.e. eigenvalue problems. Although both problems can be converted into each other, time-domain investigation provides insightful understanding of the governing physical phenomena, and can be useful for auralization purposes and when a broadband analysis is needed. When only single frequencies or a specific frequency range is of interest, frequency-domain methods are preferred.

3.1.2. Description

- **Partial differential equations**. The partial differential equations that govern the defined problem are stated. This could be linear or non-linear wave equations time-domain problems, or their frequency-domain counterpart (for linear cases) when a time-harmonic solution is sought for. The equations are homogeneous or inhomogenous, depending on the presence of acoustic sources.
- **Geometry and propagation medium**. If not stated otherwise, the properties of the propagation medium will be fixed for all cases, i.e. a constant value of the adiabatic speed of sound and an ambient density of air. The geometry is one of the components that determines the complexity of the problem. For problems with many edges and corners, some numerical methods need a special treatment, i.e. a local mesh refinement (and a possible smaller discrete time step in time-domain solution methods), a higher demand on the iterative solvers used, etc. Also, the dimension of the problem could cause trouble for solution techniques to perform well, as the size of the problem, related to the smallest wavelength of the interest, might lead to high computational costs. Another aspect that may lead to problems for some methodologies is the shape of the geometry, i.e. Cartesian-shaped boundaries are often more easy to solve than arbitrary shapes with oblique and curved boundaries. The geometry of the problem is documented in a form that includes the coordinates of all relevant nodes of the geometry and their connection.
- **Boundary conditions**. Conditions are assigned to the boundaries. For frequency-domain problems, a complex impedance or admittance value is given for the frequencies to be computed. For the time-domain problems, the time-domain function of a physical quantity at the boundary is specified. A special case concerns boundary media that can not be treated as locally reacting media. For such cases, the equations of the second medium will be formulated (e.g. the Biot model for porous media [37]) and a geometry file will be prescribed for it. For bounded problems, finite impedance or non-zero admittance boundary conditions could be meaningful, in particular if they are the only place to introduce damping into the system. In case the problem is (partly) a radiation problem, the values of the normal components of the acoustic velocity at the boundaries are specified. The values might be a function of frequency (for frequency-domain problems) or time (for time-domain problems).
- **Source**. In the presence of acoustics sources, their type (monopole, dipole, incident wave, etc.), strengths and location in Cartesian coordinates are prescribed analytically. If not stated otherwise, the acoustic source is a monopole and the strength corresponds to the volume velocity of the source in m^3/s. The source strength might be a function of frequency (for frequency-domain problems) or time (for time-domain problems). Radiation problems might be defined without a source separated from the boundary. Bounded scattering problems might be defined without a source, corresponding to an eigenvalue problem.
- **Receiver**. The solution of the problem is computed at predefined receiver positions. The receiver coordinates are prescribed in Cartesian coordinates.

- **Quantity to compute**. The solution at the receiver positions is requested for the complex acoustic pressure and possibly the acoustic velocity components as well. Other possible requested (global) quantities are the radiated sound power or eigenfrequencies. The results are requested for a given set of frequencies. For time-domain problems, a discrete Fourier transform (DFT) needs to be applied to obtain the results for the prescribed receiver positions. The sample frequency of the time signal prior to applying the DFT will be prescribed.

3.2. Evaluation of results

3.2.1. Error estimation

The accuracy of computed results of a benchmark case should be reported. Benchmark problems with prescribed values of acoustic variables at boundaries belong to the radiation category. For all radiation cases, an analytical solution as was proposed in two conference papers by Osetrov and Ochmann [38, 39] can be computed as reference result. This analytical solution is the free field solution of a point source. It can be computed by numerical methods when adopting the free field values of the sound pressure and (or) the normal component of the particle velocity at the surface of the radiating body, which is acoustically transparent. The error of computed results for the linear wave problems is defined as the following norms – related to the amplitude error and the phase error or actual error – for acoustic variable u, which can also be a component of the total field (e.g. the scattered field only)

$$|u|_a = \sqrt{\frac{\sum_{n=1}^{N}\sum_{m=1}^{M}\left(|u_{m,n}| - |u_{\mathrm{ref},m,n}|\right)^2}{\sum_{n=1}^{N}\sum_{m=1}^{M}|u_{\mathrm{ref},m,n}|^2}},$$

$$|u|_\phi = \sqrt{\frac{1}{NM}\sum_{n=1}^{N}\sum_{m=1}^{M}\left(\frac{|\phi[u_{m,n}] - \phi[u_{\mathrm{ref},m,n}]|}{\pi}\right)^2},$$

$$\text{or}\quad |u| = \sqrt{\frac{\sum_{n=1}^{N}\sum_{m=1}^{M}\left(|u_{m,n} - u_{\mathrm{ref},m,n}|\right)^2}{\sum_{n=1}^{N}\sum_{m=1}^{M}|u_{\mathrm{ref},m,n}|^2}}, \qquad (1)$$

with $u_{\mathrm{ref,m,n}}$ the reference solution, M the number of frequencies to be computed and N the number of receiver positions. Accuracy classes of error can be defined for the benchmark cases, and they might be different depending on the nature of the problem, i.e an academic case or a case with high practical relevance.

Furthermore, in the case of experimental data, the same error computation as defined in (1) can be applied. Here, $u_{\mathrm{ref},m,n}$ will be the measured values at a defined position according to the the size of the sensor. Therefore, it will be an spatially averaged quantity, which also has to be considered for the computed values $u_{m,n}$.

3.2.2. Reporting details

The method that is applied to the benchmark case should be reported in detail. The following details are relevant:

- **Computational technique**. The name of the technique and a short description, e.g. how are the boundary conditions imposed, what technique is used to evaluate in space and time.
- **Computed results**. The requested results at the receiver positions for the specific benchmark case.
- **Programming details**. The used language of the code and the libraries used for the computations, e.g. preconditioners. Description of the complementary software needed to process the results, e.g. mesh generators.
- **Code accessibility**. Description on the origin of the code, e.g. commercial code or in-house code, the authors of the code and whether the code is accessible for others.
- **Processing details**. Description whether the code is running on CPU(s), a combination of CPU-GPU or solely on GPUs. Describe the machines used for the computations – processor type, number of processors, memory, cache – and describe the computational cluster (if any).
- **Computational complexity**. Memory use in GB, number of floating point operations (flops), total computation time, and computation time versus total degrees of freedom.
- **Notes**. Any comments on the performed calculations, the used technique, convergence behaviour of the method (e.g. as a function of space discretization or basis function), a discussion on the obtained results.
- **References**. References to the used technique (if any).
- **Contributing institute**. Name(s) and affiliation(s) of the contributing researcher(s).

4. Description of examples

The framework of benchmark cases is now demonstrated by defining three benchmark cases including some results, which partly can be found in literature. For material data of air, density $\rho = 1.3\,\mathrm{kg/m^3}$ and speed of sound $c = 340\,\mathrm{m/s}$ have been assumed. An overview of the three benchmark cases is given in Table I and at the benchmark case website.

4.1. The duct problem

The first benchmark is the same example as in reference [19, 20], see Figure 1. The 3D model consists of an air–filled duct of length $l = 3.4$ m with a 0.2m×0.2m square cross section. Since the solutions of the three–dimensional method will be compared with the analytical solution of the corresponding one-dimensional problem, it is necessary to apply zero boundary conditions for the boundary admittance, i.e. $Y = 0$, and the particle velocity on the entire surface, with the exception of $Y(l) = (\rho c)^{-1}$. Furthermore, for the particle velocity at $x = 0$, $v_x(0) = 1$ m/s has been used. The exact solution of the corresponding one-dimensional problem is given by

$$p(x) = -v_x(0)\,\rho c\,\mathrm{e}^{\mathrm{i}kx}, \qquad (2)$$

Table I. Overview of the three benchmark cases: categories and description.

Name	Duct	Cat's eye	Radiatterer
Code	BA3–1|SF	UN3–1|RF UN3–1|SF	UN3–2|SRF
Categories	Bounded 3D Scattering Frequency-domain	Unbounded 3D Scattering or Radiating Frequency-domain	Unbounded 3D Scattering and/or Radiating Frequency-domain
PDE	Helmholtz Equation	Helmholtz Equation	Helmholtz Equation
Geometry	Figure 1	Figure 2	Figure 3
BCs	$Z = \infty$ at walls $Z(l, y, z) = \rho c$	$Z = \infty$ at all boundaries	$Z = \infty$ at all boundaries
Source	$v_x = 1$ m/s at $x = 0$	$v_n = 1$ m/s at spherical surface or, incident plane wave	$v_n = 1$ m/s at all surfaces
Receiver	$(x, y, z) = (l, 0, 0)$	located on sphere around object	located on sphere around object
Quantity	pressure	pressure	pressure particle velocity sound power

with $v_x(0)$ the acoustic particle velocity. The sound pressure magnitude is constant in the duct and over the entire frequency range. The solution may be considered as waves traveling through the duct. The boundary condition at $x = l$ ensures that the wave is fully absorbed. Although a smooth solution is expected over the entire frequency range, the numerical solution may be unstable if modes perpendicular to the traveling waves occur. For the above given cross section, these modes occur for frequencies of 850, 1700, 2550, 3400 Hz and higher. This benchmark case has been used in reference [40], which provides an analytical solution of the 1D duct problem including the solution of the eigenvalue problem with arbitrary admittance boundary conditions at both ends. Furthermore, it gives the 1D finite element system matrices, studies the eigenvalue distribution of the finite element solution with respect to Shannon's sampling theorem and critically discusses the accuracy of mode superposition for reconstruction of the solution in frequency-domain.

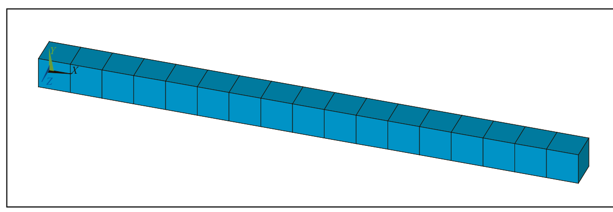

Figure 1. Benchmark case 'Duct', coded BA3–1|RF, see Table I.

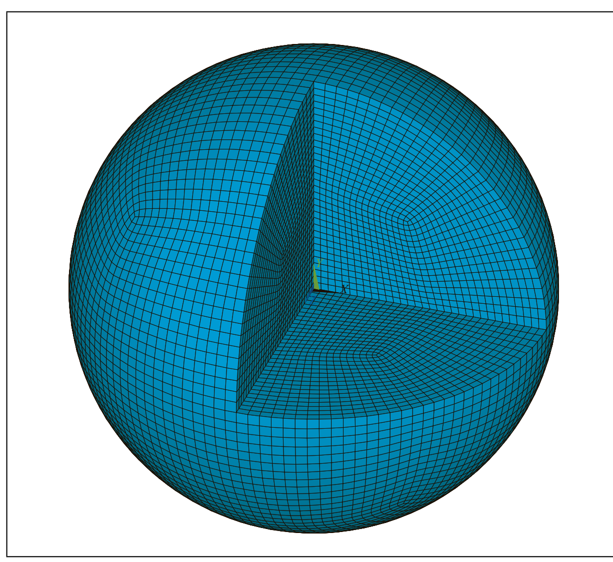

Figure 2. Benchmark case 'Cat's eye', coded UN3–1|RF or UN3–1|SF, see Table I.

4.2. The cat's eye as a radiator and scatterer

The cat's eye geometry has been analysed in a number of papers in recent years, cf. [41, 42, 43, 44, 45, 46]. As shown in Figure 2, it is a sphere of radius $R = 1.0$ m with the positive octant cut out. Herein, we investigate the radiation problem of the vibrating surface where it coincides with the spherical one. The plain surfaces of the missing octant are assigned a zero admittance. The idea of investigation of a cat's eye structure is essentially based on three considerations:

- This radiator allows construction of a smooth solution that will make it easy to identify solution failures caused by the ill–conditioning of the integral operator for techniques that solve the Helmholtz equation in an integral formulation, i.e. the irregular frequencies.
- The cat's eye structure is a more complicated shape than a sphere. Hence, the solution is expected to expose more irregular frequencies in a BEM solution than the sphere. This is clearly shown in the papers [30, 46].
- For finite and boundary element solutions, sound pressure evaluation at the centre point can be challenging.

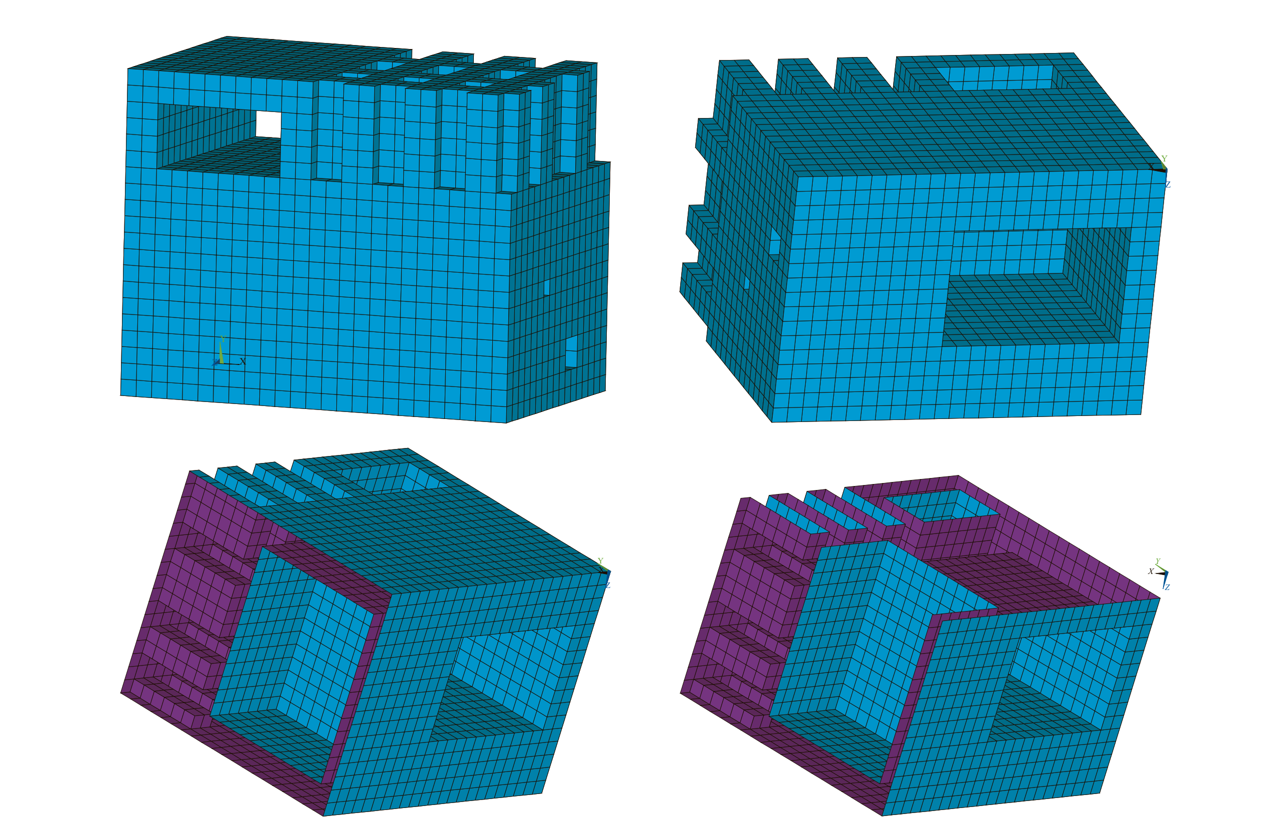

Figure 3. Benchmark case 'Radiatterer', coded UN3–2|SRF, see Table I.

Originally, it was the basic idea to test boundary element techniques with respect to the occurrence of irregular frequencies. The cat's eye exposes more of them then a radiating sphere. At the same time, the sound pressure at a backside point, i.e. $(x, y, z) = (-R/\sqrt{3}, -R/\sqrt{3}, -R/\sqrt{3})$ of the radiator (assuming the cut-out octant being the one with positive x, y, and z coordinates) is a smooth function asymptotically approaching the solution of a pulsating sphere [30]. This smooth function makes it easy to identify frequencies where the solution fails.

Since the cat's eye as a scatterer shows multiple reflections of an incoming wave [41, 42], it may be suited for methods which encounter problems in such cases. Furthermore, the sound pressure solution in the centre of the cut-out octant may require mesh refinement due to the large gradient in this region. The cat's eye radiator may be interesting when preparing an analytical solution as discussed in Section 3.2.1. Mechel [45] has presented an analytical solution for the cat's eye model. It will be interesting to compare his results which are based on Fourier series and the results based on FEM, BEM and other numerical methods.

4.3. A complex radiator and scatterer (radiatterer)

The third case is called a radiatterer as the defined geometry both acts as a radiator of sound as well as it scatterers the radiated sound waves, see Figure 3. The geometry is a single cuboid of size $(x, y, z) = (2.5, 2.0, 1.7)$ m. From this cuboid 21 smaller cuboids have been cut out, cf. Table II. This introduces open cavities, including a Helmholtz resonator, and many corners and edges. From this complex structure, multiple diffraction and resonance effects are to be expected. All surfaces are in parallel to the axes of the global cartesian coordinate system, and all angles are consequently an integer multitude of $\pi/2$ radians. The particle velocity normal to all surfaces, v_n equals 1 m/s.

For this contribution, the radiatterer problem has been solved in the frequency range up to 1.0 kHz using a boundary element collocation method for the Kirchhoff–Helmholtz integral equation without and with treatment of the irregular frequency phenomenon.

In the case of no treatment of irregular frequencies, continuous quadratic elements are used. The sound pressure at two arbitrarily chosen field points is plotted in Figure 4. It is easy to identify numerous resonances starting at the frequency of approximately 20 Hz for which the Helmholtz resonator with two openings at the $x = 2.5$ m face exhibits its lowest resonance. A quick check of eigenfrequencies of the associated interior Dirichlet problem reveals that irregular frequencies can be expected above 370 Hz. There are only approximately 20 expected up to 600 Hz, but approximately 300 of them are expected up to 1 kHz. This high density of resonances, either physical or spurious, is confirmed by the sound pressure curves in Figure 4, in particular in the frequency range above 600 Hz. The sound pressure level curve in the upper subfigure is somewhat

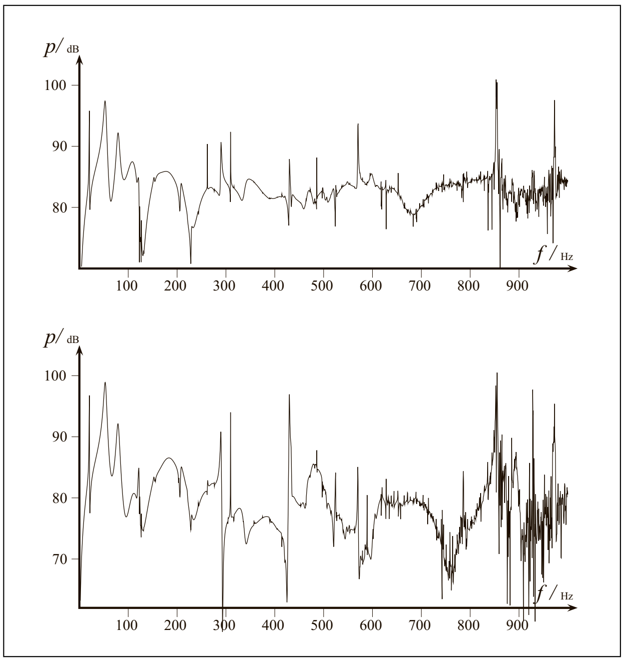

Figure 4. Benchmark case 'Radiatterer', coded UN3–2|SRF: Sound pressure level at points $(-0.3, 0.3, 0.0)$ (top) and $(0.0, 0.0, -0.3)$ (bottom) in frequency range up to 1 kHz. Solution by BEM without treatment of irregular frequencies.

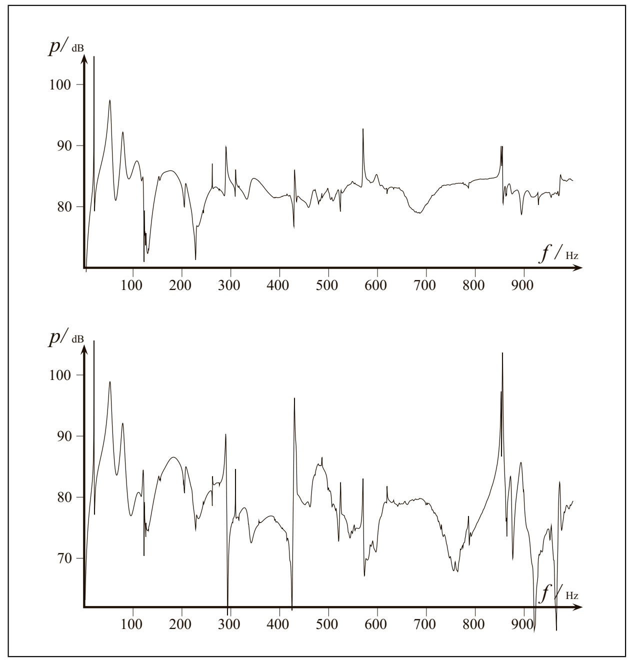

Figure 5. Benchmark case 'Radiatterer', coded UN3–2|SRF: Sound pressure level at points $(-0.3, 0.3, 0.0)$ (top) and $(0.0, 0.0, -0.3)$ (bottom) in frequency range up to 1 kHz. Solution by BEM using the Burton and Miller method.

Table II. Geometry of the complex radiatterer, blocks no 2–22 subtracted from block 1.

Block No.	$[x_{\min}, x_{\max}]$	$[y_{\min}, y_{\max}]$	$[z_{\min}, z_{\max}]$
1 (basis)	[0.0, 2.5]	[0.0, 2.0]	[0.0, 1.7]
2	[0.2, 1.4]	[0.0, 1.2]	[0.4, 1.2]
3	[2.3, 2.5]	[0.2, 0.4]	[0.5, 0.7]
4	[2.3, 2.5]	[0.7, 0.8]	[1.0, 1.1]
5	[1.6, 2.3]	[0.1, 1.2]	[0.2, 1.5]
6	[0.2, 1.0]	[1.4, 1.8]	[0.0, 1.7]
7	[1.2, 1.4]	[1.4, 2.0]	[1.6, 1.7]
8	[1.6, 1.8]	[1.4, 2.0]	[1.6, 1.7]
9	[2.0, 2.2]	[1.4, 2.0]	[1.6, 1.7]
10	[2.4, 2.5]	[1.4, 2.0]	[1.6, 1.7]
11	[1.2, 1.4]	[1.4, 2.0]	[1.2, 1.4]
12	[1.6, 1.8]	[1.4, 2.0]	[1.2, 1.4]
13	[2.0, 2.2]	[1.4, 2.0]	[1.2, 1.4]
14	[2.4, 2.5]	[1.4, 2.0]	[1.2, 1.4]
15	[1.2, 1.4]	[1.4, 2.0]	[0.6, 1.0]
16	[1.6, 1.8]	[1.4, 2.0]	[0.6, 1.0]
17	[2.0, 2.2]	[1.4, 2.0]	[0.6, 1.0]
18	[2.4, 2.5]	[1.4, 2.0]	[0.6, 1.0]
19	[1.2, 1.4]	[1.4, 2.0]	[0.0, 0.4]
20	[1.6, 1.8]	[1.4, 2.0]	[0.0, 0.4]
21	[2.0, 2.2]	[1.4, 2.0]	[0.0, 0.4]
22	[2.4, 2.5]	[1.4, 2.0]	[0.0, 0.4]

smoother and does not seem to be as polluted by resonances as the one in the lower subfigure, at least in the frequency range up to 830 Hz. However, these are only two slightly different appearances of the same phenomenon.

Irregular frequencies are suppressed by the Burton and Miller method [47, 46]. For this method, discontinuous linear elements are used to fulfill the smoothness requirements at the collocation points. Figure 5 presents the sound pressure at the same positions as in Figure 4. However, this time the curves are much smoother which is due to the Burton and Miller formulation. This example and the coupling parameter of the Burton and Miller method are further discussed in [46].

The complex radiator and scatterer, i.e. the radiatterer, is certainly worth to be investigated in much more detail. In addition to the problem of irregular frequencies, it seems worth to compare resonance amplitudes yielded by different methods and different types of finite and boundary elements. It might be worth to investigate whether mesh refinement in regions with large gradients is necessary or can be neglected. Similar to the cat's eye, it may be interesting to create an analytical solution as discussed in Section 3.2.1 and compare with numerical solutions to estimate the error.

5. Outlook

A platform for benchmark cases for computational acoustics in initiated. It is at least of equal importance to establish this initiative as a long lasting contribution. The commitment of the technical committee (TC) of computational acoustics of EAA to coordinate the benchmark cases will ensure this. The TC will undertake the following activities in this respect:

- The TC will act as initial Associate Editors of the benchmark cases.

- The TC will recruit the Benchmark Editors for handling the cases of the various fields of acoustics.
- The TC will discuss the progress of the benchmark cases during its TC meetings at annual EAA events.
- The TC will organize sessions at conferences and symposia around the benchmark cases.
- The TC will encourage scientists in all fields of computational acoustics to submit and use the benchmark cases, and to volunteer as a reviewer of benchmark cases for a specific field. Also, the TC will encourage young scientists to deploy the benchmark cases, e.g. by including benchmark cases at summer and winter school in acoustics.
- The TC will promote the benchmark cases to become a product of the EAA.

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