

Low-stiffness Dual Stage Actuator for Long Rage Positioning with Nanometer Resolution

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Abstract

This paper presents a dual stage actuator (DSA) capable of long-stroke positioning with nanometer resolution without an additional vibration isolation. The DSA system is composed of a linear motor and a compact Lorentz actuator used as the coarse and fine actuators, respectively. Since the fine actuator is constructed for a low-stiffness, the disturbances from the base are mechanically reduced and rejected by means of a feedback control. In addition, this system only requires one sensor measuring the fine actuator position, because the coarse actuator velocity and position can be estimated from the transmissibility of the fine actuator. Experimental results demonstrate that the DSA moves 100 mm and reaches the \pm 30 nm error band (peak-to-peak) in 0.41 s. At static positioning, the DSA achieves a precision of \pm 2.5nm (peak-to-peak).

Keywords: Nanopositioning, Dual stage actuator, Low stiffness actuator.

1. Introduction

In applications where high-precision positioning over a long range is required, a single actuator may not be able to achieve the desired performance. To satisfy the requirements, a long-stroke coarse actuator can be combined with a short-stroke fine actuator as a dual stage actuator (DSA). In nano-positioning DSA systems, only the fine actuator requires nanometer precision. Based on the choice of this fine actuator and the corresponding system design, DSAs can be broadly categorized by the spring constant of the fine actuator into high-stiffness, low-stiffness, and zero-stiffness systems.

High-stiffness systems can be realized by using piezoelectric materials as the fine actuator, which have a high intrinsic stiffness. Piezos have the advantages of easy miniaturization, high precision, high actuation force and fast response. They can be combined with a variety of coarse actuators in DSAs. For example, piezos are mounted onto Lorentz actuators (voice coil motor) and applied to hard disk drives (HDDs) [1] or magnetic tape recording systems [2]. Piezos' relatively high force and high stiffness are also used with linear motors in a feed drive system for machining [3]. However, due to the high stiffness of piezoelectric actuators, it is difficult to reject disturbances transmitting from their base, such as floor vibrations and coarse actuator oscillation [4]. While remedies have been taken, for example by adding passive damping for shock resistance [5], zero stiffness systems are best used to mechanically decouple the fine and coarse actuation.

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A zero-stiffness DSA can be constructed by installing Lorentz [4] or reluctance actuators [6] as the fine actuator, because its base and moving part can be mechanically decoupled by design. Particularly Lorentz actuators are commonly used due to their high linearity [7]. They are often combined with commutated motors [8–10] and applied to wafer scanners for lithography [11], as well as to robot arms for automatic assembly [12]. In these systems, the movable parts are suspended typically pneumatically or magnetically to ensure zero stiffness. Therefore, these setups tend to be heavy and bulky with air feet for pneumatic suspension or require several sensors and actuators for magnetic levitation [13]. In addition, the heavy moving parts cause a compromise between the achievable acceleration and the sensitivity to disturbances, limiting either the operational speed or the positioning accuracy, commonly referred to as the "mass dilemma" [4].

Low-stiffness actuators can be found in optical disk drives (ODDs) such as CD/DVD players, where the Lorentz actuators' moving part is loosely suspended by wires, thus mechanically coupling their moving part and the base with lowered stiffness [14]. These actuators have been mounted on a motor-worm-gear for the coarse actuation [15]. For the operation of the ODDs, the DSAs do not obtain the absolute position of the fine actuator, but the position error signal [15, 16]. Furthermore, there is no sensor for the coarse actuation. Instead, the coarse actuator is controlled based on the control input to the fine actuator [17].

In comparison with high-stiffness and zero-stiffness DSAs, low-stiffness systems have the advantages of avoiding complicated design to float the moving part, while the low spring constant of the fine actuator reduces the transmission of disturbances from the coarse actuator. However, the challenge remains in control design for the rejection of the residual disturbances to achieve high-precision positioning.

For control of DSA systems, multi-input multi-output (MIMO) control design [18–21] may be applied, since they are regarded as a system with multiple inputs. Another strategy to control DSAs is to design single-input single-output (SISO) controllers for the fine and coarse actuators in a certain configuration, such as the decoupling or the master-slave [22, 23]. By using the SISO control design, the order of controllers can be lower than that of MIMO [24], while the SISO design allows the freedom of selecting the control design techniques individually for the fine actuator and the coarse actuator. In fact, discrete SMC was successfully applied to the fine actuator, while proximate time-optimal control was used for the coarse actuator [25].

In general, the fine actuator and the coarse actuator of a DSA have different objectives to complement each other. While the fine actuator realizes the precision motion, the coarse actuator carries the fine actuator for the long range motion. To fulfill their own roles, the actuators have different types of control challenges. For the precision motion, the fine actuator typically requires a high control bandwidth and high disturbance rejection. In contrast, the coarse actuator needs to move fast enough to track the motion trajectory over a long range. To overcome the challenges, this paper proposes a SISO control design that utilizes the freedom to select control design techniques to individually solve the distinct control problems.

This paper proposes a low-stiffness DSA system capable of high-precision positioning over a long range. The disturbances reduced by the low stiffness are further rejected by feedback control. Due to the compact design, the moving part of the fine actuator is light and rigid enough to achieve a sufficient control bandwidth to reject the disturbances and to overcome the mass dilemma, retaining both operational speed and position accuracy. Furthermore, the

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DSA does not require any displacement sensors for the coarse actuation, because the necessary information can be estimated from the low-stiffness design and the residual transmissibility of the fine actuator. For control of the DSA, the control design techniques are individually selected for the fine and coarse actuators, based on the system analysis.

This paper is organized as follows. Section 2 introduces the low-stiffness DSA system. In Section 3 the system is modeled, and disturbances are analyzed. In Section 4 controllers and motion trajectories are designed with an observer. Section 5 presents experimental results, and Section 6 concludes the paper.

2. System description

Fig. 1 shows a DSA setup that is built on a pick-and-place machine (CAT(2a), Philips, Amsterdam, Netherlands), which is directly placed on the fifth floor of a building without external vibration isolation, such as an optical table [26]. This machine has six linear motors, and one of them is used as the long-stroke coarse actuator of the DSA. The linear motor is guided by roller bearings and operated by a serve driver in force control mode. For safety reasons, the actuation force is limited to about ± 250 N by software. As the fine actuator, a Lorentz actuator of a laser pickup (SF-HD65, Sanyo, Osaka, Japan) is used. The resistance and inductance of the coil are 4.8Ω and 71μ H, respectively, and the voltage over the coil is proportional to the current up to approximately 11 kHz. Because this is sufficiently higher than a target control bandwidth of 1 kHz in control design, a voltage amplifier is utilized to drive the actuator although the Lorentz force is proportional to the current. The actuation range of the fine actuator is limited to approximately $\pm 1 \,\mathrm{mm}$. To optically measure the fine actuator position, the objective lens is replaced by a cube-corner retroreflector (43-305, Edmund optics, Barrington, USA), which has a diameter of about 7 mm with a weight of about 0.35 g, and a single-path heterodyne Michelson interferometer (10899A, Agilent Technologies, San Francisco, USA) is mounted with a polarizing beam splitter on the platform for the real time control of the DSA. The interferometer has a resolution of 1.25 nm and detects movement up to 0.40 m/s. An additional retroreflector is mounted on the coarse actuator to measure its position with a second interferometric detector and a polarizing beam splitter; however, this second sensor serves only for evaluation and not for control of the DSA.

The servo driver, the voltage amplifier and the interferometers are all connected to a rapid prototyping control system, where controllers are implemented at a sampling frequency of $f_s = 20 \text{ kHz}$ by using the CPU (DS1005, dSpace GmbH, Paderborn, Germany). While the FPGA of the prototyping control system (DS5203, dSpace GmbH, Paderborn, Germany) is used to communicate with the interferometers, the DACs (DS2102, dSpace GmbH, Paderborn, Germany) are used to generate the inputs to the driver and the amplifier. Fig. 2 shows a block diagram of the system, where u_f and u_c are the input to the voltage amplifier and the servo driver, respectively. The force of the fine and coarse actuators is represented by F_f and F_c , and x_f and x_c are the position of the fine and coarse actuators. Since only x_f is used for control of the DSA, it can be regarded as a dual-input single-output system.





Figure 1: Photograph of the low-stiffness DSA.



Figure 2: Block diagram of DSA with input to the voltage amplifier u_f and to the serve driver u_c . The force F_f and position x_f of the Lorentz actuator correspond to the fine actuator, and the force F_c and the position x_c to the linear motor as the coarse actuator.

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3. System analysis

3.1. Dynamic model

Fig. 3 shows a lumped mass model of the DSA setup. Parameter k and c are the stiffness and damping of the fine actuator. The moving mass of the fine actuator with the retroreflector is m_f and is about 0.7 g. The total mass of the coarse actuator's moving part including the stator of the fine actuator is m_c , and it is approximately 8.5 kg. To relate the actuator forces with the actuator drivers, two constants K_{af} and K_{ac} are introduced as

$$F_f = K_{af} u_f, \qquad F_c = K_{ac} u_c, \tag{1}$$

where u_f and u_c are the inputs of the drivers for the fine and coarse actuation, respectively, and they are the control inputs of the DSA system. Using these inputs, the dynamic equations of the DSA are derived from the equation of motion about m_f and m_c as follows

$$X_{f}(s) = P_{f}(s)K_{af}U_{f}(s) + P_{f}(s)P_{d}(s)X_{c}(s),$$
(2)

$$X_{c}(s) = P_{c}(s) \left\{ K_{ac} U_{c}(s) + D_{c}(s) \right\} - \frac{m_{f}}{m_{c}} X_{f}(s),$$
(3)

where $X_f(s)$, $X_c(s)$, $U_f(s)$ and $U_c(s)$ are the Laplace transformations of x_f , x_c , u_f and u_c , respectively. Symbol $D_c(s)$ is the Laplace transformation of the friction force d_c due to the coarse actuator's roller bearings. Transfer function $P_f(s)$ and $P_c(s)$ show the resulting positions for given forces, and $P_d(s)$ represents the mechanical coupling of the two actuators. These transfer functions are given as

$$P_f(s) = \frac{X_f(s)}{F_f(s)} = (m_f s^2 + cs + k)^{-1},$$
(4)

$$P_c(s) = \frac{X_c(s)}{F_c(s)} = (m_c s^2)^{-1},$$
(5)

$$P_d(s) = cs + k. ag{6}$$

The second term in the right hand side of (3) results from the counter force of the fine actuator. Since m_c is about 12×10^3 times larger than m_f , the influence of the fine actuation on x_c is negligible and (3) can be simplified to

$$X_c(s) = P_c(s) \{ K_{ac} U_c(s) + D_c(s) \}.$$
(7)

The disturbances transmitted through the coarse actuator (e.g. floor vibrations) can be expressed as fluctuations of $X_c(s)$. Thus, (2) and (6) show the possibility to improve the disturbance rejection by decreasing k and c, which motivates the use of low-stiffness actuators. An increase of m_c is also effective since it reduces the fluctuation of $X_c(s)$ itself. However, this may not always be an option because the acceleration of the coarse actuation may be limited due to the maximum actuation force according to (5) (i.e. the mass dilemma).





Figure 3: Lumped mass model of DSA.

3.2. Parameter estimation

The system parameters are estimated by measuring the frequency responses from the amplifier inputs to the actuator positions and fitting them to the DSA model. The frequency responses are measured by using a network analyzer (3562A, Hewlett Packard, Palo Alto, USA), and the results of the fitted models (2) and (7) are shown in Fig. 4. For the estimation of the fine actuator properties, K_{af} , k and c are manually tuned, such that the magnitude and phase of the fine actuator model $K_{af}P_f(s)$ accord with those of the measured Bode plot from u_f to x_f . Fig. 4(a) shows a good fit up to about 2 kHz. In order to keep its order low, the fine actuator model does not capture the second mechanical resonance, occurring at round 6 kHz, which is beyond the targeted control bandwidth. Similarly, K_{ac} is tuned to have the gain and phase of the coarse actuator model $K_{ac}P_c(s)$ overlap those of the measured Bode plot from u_c to x_c as shown in Fig. 4(d). The measured Bode plot shows a mass line of -40 dB/dec below 100 Hz. To keep the model simple for control design, the mechanical resonance at about 240 Hz is not modeled. The estimated system parameters are listed in Table 1.

Although the Bode plot from u_c to x_f is not used for the fitting, the corresponding model shows a good fit up to 100 Hz in Fig. 4(c), which confirms the identified system parameters. The magnitude in Fig. 4(b) shows that the influence of u_f on x_c is extremely small, confirming that the assumption for simplifying (7) is valid.

Table 1: Estimated parameters			
Parameter	Value	Unit	
k	42	N/m	
c	$3.5 imes 10^{-2}$	N/(m/s)	
K_{af}	$2.59 imes 10^{-2}$	N/V	
K_{ac}	51	N/V	

3.3. Disturbance of fine actuation

To achieve high-precision positioning with nanometer resolution at static positions, the fine actuator needs to reject environmental vibrations. For the evaluation of the disturbances to be rejected by the fine actuator, its position x_f is measured while both actuators are





Figure 4: Frequency responses of the DSA system (blue solid line) and curve fitting result (red dashed line), based on the DSA system model (2) and (7). (a) and (b) show the response from u_f to x_f and x_c , respectively. (c) and (d) show the response from u_c to x_f and x_c , respectively.





Figure 5: Fine actuator position x_f (top) and its power spectrum (bottom) while both actuators are disabled.

turned off. The measured positional data in the time domain and the frequency domain are shown in Fig. 5. Analyzing the spectrum reveals that the major frequency components of the disturbances occur below 80 Hz.

3.4. Disturbance of coarse actuation

For high-precision positioning the fine actuator follows the reference position. Meanwhile, the coarse actuator carries the stator of the fine actuator, such that the coil of the fine actuator always stays within the uniform magnetic field (Fig. 3). Thus, the fine actuator always stays in its linear operational range over the full stroke of the DSA system. Therefore, the coarse actuator needs to move fast enough to achieve high-speed positioning, while its precision can be as poor as submillimeters just well below the fine actuator stroke (± 1 mm), but it does not require the high resolution of the DSA. For this reason, the major disturbance of the coarse actuator is friction due to the roller bearings, rather than the environmental vibrations.

Friction can be described as a combination of the static, Coulomb and viscous model [27, 28], and it is presumed that its viscous friction is not a major disturbance source, as Fig. 4(d) does not show obvious damping effects below 100 Hz. For the evaluation of the bearings, the linear motor is turned off, and its moving part is pushed while the applied force is measured by a sensor (FSG-15N1A, Honeywell, New Jersey, USA). Since the viscous friction is not dominant, the static force at the moment that the moving part starts sliding is recorded as the maximum friction. The measurement is conducted 200 times at different positions of the coarse actuator. The histogram of the measurement is shown in Fig 6. The friction varies between 7.7 N and 40.2 N, depending on the coarse actuator position. The mean value and the standard deviation of the 200 measurements are $\mu = 19.0$ N and $\sigma = 6.1$ N, respectively. To ensure stability under the influence of the friction, the measured maximum friction is considered at the controller design in Sec. 4.3.





Figure 6: Histogram of the coarse actuator friction at 200 different points. The friction is measured as the force required to start rolling the bearing balls.

4. Control design

After an observer is introduced, feedback controllers are designed, followed by motion trajectory design in this section. The control design techniques are individually selected for the fine and coarse actuators by considering the disturbances analyzed in the previous section. Also, this section presents classical control of the DSA, i.e. without the freedom to select individual control techniques, for benchmarking.

4.1. Observer

As mentioned in Sec. 2, only the fine actuator position x_f is measured for the real time control. To regulate the coarse actuator, its velocity and position are estimated by a Kalman filter.

The system inputs of the model for the design are the command to the fine actuator driver u_f and the coarse actuator driver u_c , while the system output is the fine actuator position x_f . From (2) and (7), the DSA model can be described for analysis and Kalman filter design in the form of a state-space with the process noise w and the measurement noise v as follows

$$\dot{x}_{ss} = Ax_{ss} + Bu_{ss} + Bw, \qquad y = Cx_{ss} + v, \tag{8}$$

using

$$x_{ss} = \begin{bmatrix} x_f & \dot{x}_f & x_c & \dot{x}_c \end{bmatrix}^T, \qquad u_{ss} = \begin{bmatrix} u_f & u_c \end{bmatrix}^T, A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k/m_f & -c/m_f & k/m_f & c/m_f \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ K_{af}/m_f & 0 \\ 0 & K_{ac}/m_c \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix},$$
(9)

where y is the system output. In (8), the friction force $D_c(s)$ in (7) is included in the process noise w. The system's observability can be checked by the observability matrix U_o . In the



case that the system is observable, U_o has full rank, and its determinant in the following equation is non-zero

$$|U_o| = \begin{vmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{vmatrix} = \frac{k^2}{m_f^2}.$$
(10)

In the case of both low and high stiffness DSAs, the above equation is non-zero and the system is observable. Thus, the position and velocity of the coarse actuator can be estimated by an observer based on the actuation of the fine actuator. However, since k is zero in the case of a zero-stiffness system, it is unobservable and requires an additional sensor for the DSA operation [10].

For the Kalman filter design, the covariance matrices of the measurement and process noise are given as follows

$$E[ww^{T}] = \begin{bmatrix} q_{1} & 0\\ 0 & q_{2} \end{bmatrix}, \qquad E[vv^{T}] = n,$$
(11)

where n is the measurement noise variance and set at 1 nm^2 for the interferometer. Parameter q_1 and q_2 are the variance of noise injected at the control input u_f and u_c , respectively. From (8) all the disturbances are considered as the noise added to u_f or u_c . Thus, the value of q_1 is estimated as 1.53 mV^2 from the control input u_f , while the fine actuator stays at a static point with feedback control, by assuming that the process noise is canceled by the control. Similarly, q_2 is roughly estimated from the control input u_c while the coarse actuator moves at slow speed with feedback control, and it is fine tuned to 0.465 V^2 at the implementation. The large variance of the coarse actuator results from the friction and cogging force. In contrast, the fine actuator does not have these disturbances, because the motion guide is the wire suspension and the moving part does not need a ferromagnetic core. With these variance setting and the model (8), the Kalman filter for the low-stiffness DSA is obtained by using MATLAB command kalmd.

The designed Kalman filter estimates the state vector x_{ss} , which includes the position and velocity of the coarse actuator, from the control inputs u_f and u_c , as well as the measured position x_f . The large value of q_2 implies that the Kalman filter mainly relies on the fine actuation (using u_f and x_f) to estimate the state vector. Note that the Kalman filter is used because a controller to be presented later is state feedback for the coarse actuation. In the case of an output feedback controller requiring only the coarse actuator position, it can also be estimated by simply filtering the control input to the fine actuator [29].

4.2. Feedback control of fine actuator

For good disturbance rejection over the identified spectrum (Fig.5), the feedback controller of the fine actuator needs a bandwidth of about 1 kHz. Since the fine actuator shows a phase of -180° from the first resonance at a low frequency to high frequencies in Fig. 4(a), the feedback controller has to provide a phase lead around the cross-over frequency for stability. To reduce the computational complexity, a simple PID controller with a low pass filter (tamed PID) is selected for the fine actuation [4]. This controller is tuned based on the fine actuator model $K_{af}P_f(s)$, such that its open-loop crossover frequency is 1 kHz with a sufficient phase





Figure 7: Simulated open-loop transfer function of fine actuator model $K_{af}P_f(s)$ with tamed PID.

margin of 48°, as shown in Fig.7. The resulting tamed PID controller $C_{PID}(s)$ can be described in the zero-pole-gain form as follows

$$C_{PID}(s) = 3.2 \times 10^6 \frac{(s + 2\pi \cdot 100)(s + 2\pi \cdot 333)}{s(s + 2\pi \cdot 3000)}$$
(12)

For the validation of the control design and implementation, Fig. 8 compares the simulated and measured sensitivity function of the closed-loop fine actuation, showing that disturbances are rejected by more than 60 dB up to 40 Hz and by more than 40 dB up to 80 Hz. Since the fine actuator deviation due to the environmental disturbances is between ± 130 nm (Fig. 5), the position error using the feedback controller is expected to be less than ± 1.3 nm, which is close to the interferometer resolution. Since the feedback control sufficiently rejects the disturbances, the low-stiffness DSA is no longer trapped by the mass dilemma.

Saturation can be a problem with short-stroke actuators, where anti-windup control can improve stability [30]. For precision motion control, however, the saturation has to be avoided by keeping the fine actuator within its linear operation range, which requires an according control of the coarse actuator as presented in the next section.

4.3. Feedback control of coarse actuator

Sliding mode control (SMC) [31] is used as the feedback controller for the coarse actuation to compensate for the friction. SMC is nonlinear control and ideally switches the control input to react to errors, realizing an infinite gain [32]. As a result, SMC has an excellent performance on disturbance rejection. Particularly when the matching condition is satisfied, the closed-loop system can be theoretically invariant to the disturbances [31]. Such property is used to compensate for nonlinear uncertainties and disturbances, including friction [33]. Moreover, the controller design requires only the upper bound of the disturbances for stability analysis. In this section, the SMC design is divided into three steps: augmentation of the





Figure 8: Measured (blue solid line) and simulated (red dashed line) sensitivity function of fine actuation.

coarse actuator model, sliding surface design based on the behavior of the system in the sliding mode, and control input design to enforce the sliding mode.

The design of SMC is discussed based on Fig. 9, which shows the coarse actuator model and the SMC structure. First of all, the coarse actuator model $P_c(s)$ with K_{ac} is described in the form of a state-space model with the friction force as disturbance d_c :

$$\dot{x}_{c_ss} = A_c x_{c_ss} + B_{cu} u_c + B_{cd} d_c, \tag{13}$$

using

$$x_{c_ss} = \begin{bmatrix} x_c & \dot{x}_c \end{bmatrix}^T,$$

$$A_c = \begin{bmatrix} 0 & 1\\ 0 & 0 \end{bmatrix}, B_{cu} = \begin{bmatrix} 0\\ K_{ac}m_c^{-1} \end{bmatrix}, B_{cd} = \begin{bmatrix} 0\\ m_c^{-1} \end{bmatrix},$$
(14)

For the SMC design, this state-space model is augmented with an integrator and a low pass filter (LPF). The integrator integrates the position error e_c of the coarse actuation, so that the coarse actuator is able to track its reference r_c without a steady-state error. The second order LPF is added as a prefilter of the coarse actuator to mitigate the chattering phenomenon by attenuating unmodeled mechanical resonances [34]. The cutoff frequency is set at 70 Hz, which is sufficiently lower than the first unmodeled mechanical resonance at 240 Hz seen in Fig. 4(d). The LPF is implemented in the form of a second-order state-space model

$$\dot{x}_{lpf} = A_{lpf} x_{lpf} + B_{lpf} u_a, \qquad u_c = C_{lpf} x_{lpf}, \tag{15}$$

using

$$A_{lpf} = \begin{bmatrix} -284 & -370 \\ 370 & 0 \end{bmatrix}, B_{lpf} = \begin{bmatrix} 440 \\ 0 \end{bmatrix}, C_{lpf} = \begin{bmatrix} 0 & 0.596 \end{bmatrix},$$
(16)

where u_a is the input to the filter. With the integrator and the LPF, (13) is augmented as follows

$$\dot{x}_{a_ss} = A_a x_{a_ss} + B_{a_u} u_a + B_{a_r} r_c + B_{a_d} d_c,$$
(17)

using

$$x_{a_ss} = \begin{bmatrix} e_i \\ x_{c_ss} \\ x_{lpf} \end{bmatrix}, A_a = \begin{bmatrix} 0 & -C_c & 0 \\ 0 & A_c & B_{cu}C_{lpf} \\ 0 & 0 & A_{lpf} \end{bmatrix}, B_{a_u} = \begin{bmatrix} 0 \\ 0 \\ B_{lpf} \end{bmatrix},$$

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$$B_{a_r} = \begin{bmatrix} 1\\0\\0 \end{bmatrix}, B_{a_d} = \begin{bmatrix} 0\\B_{cd}\\0 \end{bmatrix},$$
(18)

where C_c is a vector of [1 0] and e_i is the integration of e_c (i.e. $\int (r_c - x_c) dt$).

As the second step of the design, a linear sliding surface is introduced as the following manifold

$$\sigma(x_{a,ss}) = Sx_{a,ss} = 0,\tag{19}$$

where S is a 1×5 vector. While the system is ideally restricted to the sliding mode, $\sigma = 0$ and $\dot{\sigma} = 0$ are satisfied, and the disturbance d_c does not influence the system behavior. Thus, using $\dot{\sigma} = 0$ and (17) with $d_c = 0$, the equivalent control u_{eq} can be obtained to find the system motion during the sliding mode:

$$u_{eq} = -(SB_{a_u})^{-1}(SA_a x_{a_ss} + SB_{a_r} r_c).$$
⁽²⁰⁾

Substitution of (20) for the control input of (17) with $d_c = 0$ yields the system motion restricted to the sliding surface:

$$\dot{x}_{a_ss} = (I - B_{a_u}(SB_{a_u})^{-1}S)(A_a x_{a_ss} + B_{a_r} r_c).$$
(21)

By selecting x_c as the output of (21), the response to the reference r_c during the sliding mode can be simulated. For a fair comparison of the control design, the sliding surface is designed, such that its frequency response starts to attenuate at around 15 Hz, which is the maximum open-loop cross-over frequency achieved by the lead-lag compensator of the benchmarking system (cf. Sec. 4.5). To have such a desired behavior, S is determined by solving the Riccati equation [35] to place the poles of (21) in a region that their real parts are smaller than $-2\pi \cdot 10$. Afterward S is manually fine-tuned, such that the response during the sliding mode is similar to that of the lead-lag compensator, as shown in Fig. 10 with the resulting vector of

$$S = \begin{bmatrix} -1.70 \times 10^6, & 2.28 \times 10^4, & 225, & 1.91, & 2.71 \end{bmatrix}.$$
 (22)

As the third step of the design, the control input u_a is selected as a combination of the equivalent control without the disturbance (20) and a switching control u_{sw} :

$$u_a = u_{eq} + u_{sw}.\tag{23}$$

Input u_{sw} enforces the sliding mode to the system. As described below, it is composed of a relay as well as a linear feedback that gives an extra force to approach the sliding surface faster when the state $x_{a,ss}$ is far.

$$u_{sw}(\sigma(x_{a_ss})) = -k_{sw1}\operatorname{sgn}(\sigma(x_{a_ss})) - k_{sw2}\sigma(x_{a_ss}),$$
(24)

where k_{sw1} and k_{sw2} are constants. To determine their values, a Lyapunov function $V = \sigma^2/2$ is considered for the existence and the stability of the sliding mode, and the following equation needs to be satisfied

$$V = \sigma(x_{a_ss})\dot{\sigma}(x_{a_ss}) = \sigma(x_{a_ss})S\dot{x}_{a_ss} < 0.$$
⁽²⁵⁾

Using (17)(23)(24), (25) leads to the following satisfactory condition

$$k_{sw1} + |\sigma(x_{a_ss})|k_{sw2} > |(SB_{a_u})^{-1}SB_{a_d}||d_{c_max}|,$$
(26)





Figure 9: Coarse actuator model and SMC structure for control design. The colored blocks are expressed as the "SMC" block in Fig.12(a) and are implemented in the prototyping control system.

where $d_{c.max}$ is the maximum friction measured in Sec. 3.4.

Finally, the relay function in (24) is smoothed by a sigmoid function, which prevents chattering by restricting the system not to the sliding surface (19) but to its neighborhood [31]. The sigmoid parameter, k_{sw1} and k_{sw2} are fine-tuned at the controller implementation, satisfying (26). Fig. 11 shows the switching control before and after the smoothing, where the horizontal axis virtually shows the distance to the sliding surface. The slope of the smoothed curve at the origin results from the sigmoid function. While a gentle slope is necessary to prevent chattering, a steeper slope is preferred to achieve better disturbance rejection.

4.4. Overall control of DSA

Fig. 12(a) shows overall control blocks of the DSA system. The observer block is the Kalman filter to estimate the position and velocity of the coarse actuator in Sec. 4.1. Since the fine actuation and the coarse actuation are mechanically coupled, they are separated by a prefilter using the inverse of the mechanical coupling $(K_{af}^{-1}P_d(s)^{-1})$. With the prefilter, the overall stability of the DSA can be guaranteed by stabilizing each actuator. For the decoupled fine and coarse actuation, the tamed PID (12) in Sec. 4.2 and the SMC in Sec. 4.3 are applied for feedback control, respectively. To achieve high-speed positioning for tracking the reference r, the inverse of the fine actuator dynamics $K_{af}^{-1}P_f(s)^{-1}$ is added as feedforward control of the fine actuation. For feedforward of the coarse actuation a transfer function $G_{ffc}(s)$ is used as an input shaping filter, because a signal added to the control input would be regarded as disturbances and rejected by the SMC. The filter $G_{ffc}(s)$ is designed such that its inverse transfer function is close to the coarse actuator response in the frequency domain (Fig. 10). Finally the coarse actuation reference r_c is generated with a parameter α that can be tuned between 0 and 1. By setting α at zero the coarse actuator tracks r. This is the decoupling configuration and often used for high-stiffness DSA systems, such as HDDs [25]. With $\alpha = 1$ the coarse actuator follows the fine actuator, which is the master-slave configuration. Such configuration is often seen in zero-stiffness DSA systems [10, 11]. For the proposed low-stiffness DSA α is set at 0.5 for equal weight.

4.5. Classical control for benchmarking

A classical control of the DSA is designed for comparison, as shown in Fig. 12(b). Although it uses the same tamed PID for the fine actuator, a lead-lag compensator is used for





Figure 10: Simulated frequency response from coarse actuator reference r_c to its position x_c during sliding mode (blue solid line), compared with complementary sensitivity function of lead-lag compensator in benchmarking system (red dashed line).



Figure 11: Simulated switching control, showing the influence of the smoothing by the sigmoid function.





Figure 12: Overall control block diagram of DSA: (a) individual control design techniques are selected for each actuator, and (b) both feedback controllers are a second-order filter for benchmarking. Transfer function P_f , P_c and P_d with K_{af} and K_{ac} represent the DSA model. "SMC" and "PID" are the feedback controllers for the coarse and fine actuators (i.e. Fig. 9 and (12), respectively). "Observer" is the Kalman filter for the coarse actuation. Filter G_{ffc} and the blocks with α as a tuning parameter generate the reference for the coarse actuation.

the coarse actuation (cf. [29]). The compensator is designed based on the coarse actuator model $K_{ac}P_c(s)$, and it is fine-tuned to maximize its open-loop cross-over frequency at the implementation, which is about 15 Hz (cf. [29]). The DSA configuration is slightly different in that the master-slave configuration is used (α =1) with the feedforward signal added to the control input of the coarse actuation. Note that both feedback controllers are a second-order filter in this benchmarking system.

4.6. Motion trajectory design

As the reference r of the DSA, three trajectories are designed for the positioning over 100 mm. The first trajectory is a linear acceleration curve. The designed trajectory Tr_1 has a travel time of 400 ms. It takes 100 ms for acceleration and deceleration, respectively and spends 200 ms to move at a velocity of 0.33 m/s, which is determined by considering the interferometer's detectable speed (0.40 m/s) with a margin. In general, linear acceleration curves can achieve a shortest travel time with a bang-bang control [36]. On the other hand, they include infinite jerks and can excite unmodeled mechanical resonances, extending the settling time.

To avoid the excitation of mechanical resonances, the second trajectory for the repositioning is a minimum jerk trajectory Tr_2 . Its reference position r(t) at time t is given by the following equation [36]

$$r(t) = L \left\{ 6(t/T)^5 - 15(t/T)^4 + 10(t/T)^3 \right\},$$
(27)

where L is the travel distance (100 mm). Parameter T is the travel time and tuned to 550 ms by considering that its maximum velocity (0.34 m/s) corresponds to 85% of the detectable speed of the interferometers (0.40 m/s). The last trajectory is for slow repositioning to compare the two control systems and is a minimum jerk trajectory Tr_3 by setting the travel time T to 5 s.



5. Experimental results

To evaluate the low-stiffness DSA system, the control in Sec. 4 is implemented in the hardware described in Sec. 2, and two experiments are conducted. The first experiment is slow positioning with the trajectory Tr_3 to compare the two control systems. The second experiment is fast positioning to investigate the performance of the low-stiffness DSA system in details, comparing the linear acceleration curve Tr_1 and the minimum jerk trajectory Tr_2 .

5.1. Control comparison

Fig. 13 shows repositioning with the slow minimum jerk trajectory Tr_3 , to compare the proposed control system, consisting of the tamed PID and the SMC individually selected for the fine and coarse actuators, with the benchmarking system composed of the tamed PID and the lead-lag compensator. The DSA position error $r - x_f$ shown in Fig. 13(b)(c) is less than $\pm 100 nm$ even during motion for the high disturbance rejection of the tamed PID, and it does not show major difference between the two control systems. However, the relative distance between the actuators measured by the two interferometers (i.e. $x_f - x_c$) is reduced by almost 80% by the use of SMC in Fig. 13(d). The graph also shows that the relative distance of the benchmarking system (lead-lag) is close to the stroke limit of the fine actuator ($\pm 1 \text{ mm}$). This denotes that the lead-lag controlled system cannot move much faster without hitting the stroke limits of the short actuator, while the DSA can be still operated in the linear range of the Lorentz actuator when controlled by the PID and SMC.

5.2. Static performance and validation

Fig. 14 shows the fast positioning, comparing the linear acceleration curve and the minimum jerk trajectory. Because the fine actuator reaches its stroke limit with the benchmark control, this experiment is conducted only for the DSA with the tamed PID and the SMC. As seen in the magnified position error $r - x_f$ of Fig. 14(g)(h), the DSA can achieve a precision of $\pm 2.5 \text{ nm}_{pp}$ at a static position without an additional vibration isolation. Since x_f fluctuates between $\pm 130 \text{ nm}$ without the control (Fig. 5), the DSA is able to reduce the influence of the environmental disturbances by 98%. Similar to the data shown in Fig. 14(g)(h), repositioning of 100 mm to the origin (i.e. from r = 100 mm to r = 0 mm) also results in a position error within $\pm 2.5 \text{ nm}_{pp}$ (data not shown). This value corresponds to two times the least significant bit of the interferometer resolution given by 1.25 nm, which is the current limitation of the positioning system in both the achievable velocity and the positioning precision.

The relative distance between the fine and coarse actuators is measured by the two interferometers and shown in Fig. 14(i)(j), where the peaks are sufficiently smaller than the fine actuator stroke ($\pm 1 \text{ mm}$). As a result, the DSA is stable, and the position error is up to about $\pm 500 \text{ nm}$ even during motion of 0.33 m/s for Fig. 14(c) and 0.34 m/s for Fig. 14(d). The control input to the coarse actuator u_c in Fig. 14(k)(l) shows smooth and flat voltage before (0-0.5 s) and after the repositioning (1.2-2.5 s). This confirms that the sigmoid function of the SMC successfully prevents chattering.

Fig. 15 shows additional signals simultaneously recorded with the data in Fig. 14, comparing the coarse actuator position estimated by the Kalman filter \hat{x}_c and measured by the interferometer x_c . While the DSA stays at a static point, the Kalman filter estimates the coarse actuator position with (sub) micrometer accuracy. The steady-state error might be





Figure 13: Slow positioning of DSA, comparing control designed with and without using the freedom to select control techniques for each actuator: (a) reference r, (b)(c) position error $r - x_f$, and (d) distance between the fine and coarse actuators $x_f - x_c$ measured by two interferometers.

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due to the fine actuator's hysteresis [37] and the alignment of the interferometers (i.e. the two lasers are not perfectly parallel). During motion of the DSA, the error increases to several tens of micrometers. The possible cause of the error is that the coarse actuation excites mechanical resonances, as analyzed in the next section, and the induced vibrations deteriorate the estimation. Although the estimation error during motion may seem large, it is still significantly less than the fine actuation range of about $\pm 1 \text{ mm}$.

5.3. Influence of trajectories on dynamic performance

Fig. 14 compares the positioning performance with the minimum jerk and the linear acceleration trajectories. The key parameters from the figure are summarized in Table 2. Settling time shows how long the actuator takes to settle down within a given error band after the reference reaches the target point. The positioning time is the sum of the settling time and the traveling time of the trajectory, showing how long the actuator takes to move from the origin to the target position at 100 mm.

Since the low stiffness of the fine actuator decreases the transmission of the coarse actuator motion, the DSA quickly settles down within the \pm 30 nm range for both trajectories. Particularly with the minimum jerk trajectory, the DSA reaches the \pm 30 nm error band when the reference arrives at the target position. For this reason, its settling time is represented as zero in Table 2. As a result of short settling time, the travel time of the references dominates the positioning time, and the linear acceleration curve completes the repositioning in a shorter time of 411 ms. In the case of the \pm 10 nm error band, however, the minimum jerk trajectory can achieve a shorter positioning time of 595 ms, because the position error with the linear acceleration curve takes a longer time to converge (Fig. 14(e)). For the analysis in the frequency domain, the power spectral density (PSD) of the position error between 1.1 s and 1.8 s in Fig. 14(e)(f) is calculated as shown in Fig. 16. The error with the linear acceleration curve shows higher PSD after 100 Hz. Since the parasitic dynamics of the coarse actuator exist in this frequency range (Fig. 4(d)), it is considered that these dynamics are excited by the linear acceleration curve, and the DSA takes more time to settle down due to the induced vibrations.

As shown above and validated experimentally, the low-stiffness DSA can achieve fast positioning with proper feedback control and trajectory design, since the fine actuator with the lowered stiffness reduces the transmission of the vibrations while the SMC-controlled coarse actuator keeps the relative position error between fine and coarse actuation small. This enables fast point-to-point positioning of the system with nanometer resolution without external vibration isolation.

6. Conclusion

This paper proposes a low-stiffness DSA capable of positioning with nanometer resolution over a long range. The fine actuator reduces the transmission of the disturbances by the low stiffness and achieves the high control bandwidth to reject the residual disturbances. The experimental results show that the position error of the system stays within a range of ± 2.5 nm at a static position without an additional vibration isolation. The DSA additionally has a good performance on dynamic responses by bypassing the mass dilemma and using the lowered stiffness. The demonstration shows that repositioning over 100 mm takes only 411 ms





Figure 14: Fast positioning of the DSA with the linear acceleration curve (left column) and the minimum jerk trajectory (right column): (a)(b) reference r, (c)(d) position error $r - x_f$, (e)-(h) magnified position error $r - x_f$, (i)(j) relative distance between the fine and coarse actuators $x_f - x_c$ measured by the two interferometers, and (k)(l) control input to the coarse actuator u_c .

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Figure 15: Coarse actuator position estimated by the Kalman filter and measured by the interferometer (a)(b), as well as their difference (i.e. estimation error $\hat{x}_c - x_c$) (c)(d). The estimation error is magnified in (e)(f). The data is recorded simultaneously during the repositioning in Fig. 14.

Table 2: Key parameters of trajectories and response			
Parameter	Error band	Linear acceleration	Minimum jerk
Maximum velocity of r	-	$0.33\mathrm{m/s}$	$0.34\mathrm{m/s}$
Traveling time of r	-	$400\mathrm{ms}$	$550\mathrm{ms}$
Settling time	$\pm 30\mathrm{nm}$	$11\mathrm{ms}$	$0\mathrm{ms}$
Positioning time		$411\mathrm{ms}$	$550\mathrm{ms}$
Settling time	$\pm10\mathrm{nm}$	$571\mathrm{ms}$	$45\mathrm{ms}$
Positioning time		$971\mathrm{ms}$	$595\mathrm{ms}$

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Figure 16: Power spectral density (PSD) of position error $r - x_f$, calculated from data between 1.1s and 1.8s in Fig. 14(e)(f). The blue solid line shows the PSD in the case of the linear acceleration curve Tr_1 , and the red dashed is in the case of the minimum jerk trajectory Tr_2 .

to reach the ± 30 nm error band. On top of that, the low stiffness makes the DSA system observable, and positioning without a sensor for the coarse actuation has been demonstrated based on the state vector estimation by a Kalman filter. By using the low-stiffness actuator as the fine actuator and selecting proper control techniques for each actuator, high-speed positioning is achieved over a long range with nanometer resolution without an additional vibration isolation.

Future work includes mechanical and electric design of a low-stiffness DSA system for precision in-line metrology, such as optical 3D inspection.

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