Inverse Scheme for Acoustic Source Localization based on Microphone Array Measurements

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Abstract

In the last years, considerable improvements have been achieved in acoustic source localization using microphone arrays. However, main restrictions include simplified source models and using Green's function for free radiation as the transfer function between source and microphone signal. To overcome these limitations, we aim to solve the corresponding partial differential equation (Helmholtz equation) with the actual boundary conditions as given in the measurement setup.

Keywords: Acoustic source localization, inverse method, Helmholtz equation

1 Introduction

Acoustic beamforming is used to determine source locations and distributions, measure acoustic spectra for complete models and subcomponents, and project results from the array to far field points. The fundamental processing method, Frequency Domain Beamforming (FDBF) [?] is robust, fast, and renders continuous source distributions as continuous images. Here, the beamforming map is computed by

$$a(\boldsymbol{g}) = \boldsymbol{g}^* \boldsymbol{C} \boldsymbol{g} \tag{1}$$

with $\boldsymbol{g}$ the steering vector, * the complex conjugate, and $\boldsymbol{C}$ the cross spectral matrix (CSM) of the array. The CSM is computed out of the measured microphone signals and is modeled by

$$\boldsymbol{C} = \sum_{j=1}^{M} \sigma_j \boldsymbol{g}\boldsymbol{g}^* \tag{2}$$

with M the number of assumed sound sources. Assuming that only the source σ_k is nonzero, we obtain

$$\boldsymbol{C} = \sigma_k \boldsymbol{g}_k \boldsymbol{g}_k^* \tag{3}$$

and the source map results in

$$a(\boldsymbol{g}) = \sigma_k \boldsymbol{g}_k^* \boldsymbol{g}_k \boldsymbol{g}_k^* \boldsymbol{g}_k \,. \tag{4}$$

By forcing $\boldsymbol{g}_j^* \boldsymbol{g}_j = 1$, we obtain the correct source location with strength σ_k. The resolution is limited by the Rayleigh limit [?], and the dynamic range to about 20 dB by the finite aperture of the microphone array and further reduced to 7-12 dB by the sparse array design that is necessary for high frequency operation with a limited number of microphones. A second processing step is required to convert a raw FDBF map into a source density map. This can be done by application of an overall scaling factor, known as the integration technique [?] or deconvolution by, e.g., CLEAN [?], DAMAS [?] or CLEANSC [?]. A main restriction is currently that the sources are modeled as monopoles or/and dipoles and the steering vector $\boldsymbol{g}$ describing the transfer function between source and microphone signal is modeled by Green's function for free radiation. To overcome these limitations, we plan to solve the corresponding PDE with the actual boundary conditions as given, e.g., in aeroacoustic wind tunnels, where such measurements are often performed.

2 Physical Model

We assume that we have the original geometry of the setup and Fourier-transformed acoustic pressure signals $p_{\mathrm{m}i}(\omega)$ (ω being the angular frequency, $i = 1, ..., M$) measured by microphones at positions $\boldsymbol{x}_i$. Therefore, our physical model is the Helmholtz equation

$$\Delta p + k^2 p = \sigma \tag{5}$$

with the wave number k and the searched for acoustic sources $\sigma(\boldsymbol{x}, \omega)$. Since we will do the identification separately for each frequency ω_j, we will neglect dependence on ω in the following. For the acoustic sources, we may write

$$\sigma(\boldsymbol{x}) = \sum_{j=1}^{N} a_j e^{i\varphi_j} \delta_{\boldsymbol{x}_j} \tag{6}$$

with delta pulses $\delta_{\boldsymbol{x}_j}$ located at N grid points $\boldsymbol{x}_j$, the searched for amplitudes $a_1, a_2, ..., a_N \in$