## Final Report

## Master Thesis

## Geert Reuver

Evaluation of vibratory pile installation effects on adjacent buried pipe structures through a coupled analytic approach


# Final Report <br> Master Thesis 

by

## Geert Reuver

May 2016

# Evaluation of vibratory pile installation effects on adjacent buried pipe structures through a coupled analytic approach 

## Student:

Geert Reuver, 4226178
Klein Brembrood 29
4641 RH Ossendrecht
Tel. +31 (0)6 13172424
geert.reuver@gmail.com
Assessment committee:

## Chairman

Prof. dr. M.A. Hicks
TUDelft, Faculty of CITG
Department of Geo-Engineering
Tel. +31 (0) 152787433
M.A.Hicks@tudelft.nl

## TUDelft

Daily supervisor TUDelft
Dr. P.J. Vardon
TUDelft , Faculty of CITG
Department of Geo-Engineering
Tel. +31 (0) 152781456
P.J.Vardon@tudelft.nl

## TUDelft"

## External supervisor TUDelft

Ir. C. Kasbergen
TUDelft, Faculty of CITG
Department of Structural Engineering
Tel. +31 (0) 152782729
C.Kasbergen@tudelft.nl
TUDelft

Daily supervisor BT Geoconsult BV.
Ir. A.S. Ramkisoen
BT Geoconsult bv.
Department of Geo-Engineering
Tel. +31 (0) 683250693
A.S.Ramkisoen@btgeoconsult.nl


BT Geoconsult BV
ingenieursbureau

Daily supervisor TUWien
Univ.Prof. Dipl.-Ing. Dr.Techn. D. Adam
TUWien, Fakultät Grundbau Boden- und Felsmechanik
Institut für Geotechnik
Tel. +43 15880122100
dietmar.adam@tuwien.ac.at

## Preface

The final report is the second document contributing to the MSc Graduation Work (Master Thesis) that will be performed. This document consists also of a worked out literature review, which is used as a basis for the Master Thesis research.
This MSc project is mainly focused on the behavior of pipes and ducts, located in the subsurface, under the influence of vibration waves which occur as a result of vibratory sheet pile driving. Since my home is based in Vienna the investigation took mainly take place here. The research is supported by BT Geoconsult bv., located in the Hague (the Netherlands), the Technische Universität Wien (TU Wien) and Delft University of Technology (TU Delft).

Vienna,
May 2016
Geert Reuver

## Word of thank

I would like to thank all the persons that supported me during my study period and the constitution of of my Master Thesis project.

In particular I would like to thank Univ.Prof. Dipl.-Ing. Dr.techn. Dietmar Adam for the possibility to do my research project under his supervision and the ability to get my own desk next to his research team. The positive working environment and the collaborating colleagues inspired and motivated me to accomplish this thesis work.

I especially thank my daily super visor at the TU Wien, Dipl.-Ing. Peter Nagy, who always was open to questions, interested in my work and a good discussion partner. Your critical view on the problem statement made me think deeper into detail.

I want to thank BT Geoconsult for their financial support of the Master Thesis and for the subject of the Master Thesis project.

I not only want to thank my parents Anita and Carel Jan Reuver making it possible to study, but also for the support and good advise they gave when ever I needed it.

Then in particular I want to thank my girlfriend Hannah Unterluggauer for hare patience, positive attitude and support throughout my study and during the Master Thesis project.

At last I would like to thank my entire Master Thesis committee for their time and opinion on my work. In particular Ir. Cor Kasbergen for his effort and time to criticize my entire work extensively.

## Abstract

Urban areas in the Netherlands are expending rapidly and civilization is squeezed onto a decreasing space. As a result building projects intent to be located very close to adjacent constructions leading to potential risk damaging these structures. Especially structures that are constructed within the subsurface, like cables and ducts, require extra care. A fundamental challenge in engineering of vibratory pile installation methods is to quantify the vibration levels and wave propagation through the subsurface. Using empirical prediction methods, based on measured field data, the complexity of the problem lacks authenticity. No distinction between different wave types or layered soil is taken into consideration. These crude methods are good "first prediction methods", but lack the validly with respect to the impact on pipe structures present in the subsurface. Cost expensive precautionary measurements or time expensive 3D Finite Element calculations are in directly related to the complexity of the problem statement. The need for a reliable and simple to use prediction method is the fundamental basis for this Master Thesis research project.
Basic dynamics is mandatory to retrieve knowledge about the complexity of the problem spectrum. Various wave properties and behavior are therefore included in the investigation to obtain favorable judgment in modeling choice. Massarsch and Fellenius [80] proposed a simple model for the prediction of vibration levels at the surface induced by impact pile installation will be applied. This mothodology makes it possible to investigsted layer soil compositions and makes a clear distinction between different wave types. Investigation of vibratory pile installation problems, as carried out in this Master Thesis project, requires modification of this model. To prove the validity of the modified research methodology, the Eurocode 3 [2] and the application of an axis symmetric 2D Finite Element ABAQUS model is inserted as comparison method.
The pressure wave as modeled enforces the pipe to vibrate. Modeling the impact of this oscillations on pipe structures by means of a 1D Finite Element approach is in accordance with the aim for simplification and model authenticity. The constitutive behavior of the model is build on the Euler-Bernoulli beam theory and applied to the equation of motion. A realistic soil damping representation is incorporated by means of the Rayleigh-damping method.
A parameter sensitivity study is applied to test all individual model components on their behavior with respect to condition changes. From the study can be concluded that the coupled modified vibration estimation method of Massarsch and Fellenius [80] (named Wave Propagation Model in this research) to the proposed pipe structure representation (named Pipe Structure Model in this research) made it possible to accomplish the research goals of this Master Thesis project. The most important research goal achieved is the possibility to predict the oscillatory behavior of the pipe to enforced vibration waves from vibratory sheet pile installation. Although the comparison between the Wave Propagation Model and the ABAQUS model show complementary results, due to the the lack of observation regarding field measurement data the authenticity of the model is not yet confirmed.

## Contents

Preface ..... iii
Word of thank ..... v
Abstract ..... vii
List of Figures ..... xiii
List of Tables ..... xix
1 Introduction ..... 1
1.1 Problem statement ..... 1
1.2 Focus points literature review ..... 2
1.3 Hypothesis ..... 2
1.4 Research questions ..... 2
1.5 Research objectives ..... 2
1.6 Document outline ..... 3
2 General theory of vibrations ..... 5
2.1 Basic dynamics ..... 5
2.1.1 Harmonic waves ..... 5
2.1.2 Periodic waves ..... 6
2.1.3 Transient waves ..... 6
2.1.4 Random waves. ..... 6
2.2 General wave equation ..... 7
2.3 Wave types ..... 7
2.3.1 P-waves/cylindrical waves ..... 8
2.3.2 S-waves/spherical waves. ..... 9
2.3.3 R-waves ..... 9
2.3.4 Comparison waves. ..... 10
2.4 Damping of waves ..... 11
2.4.1 Geometric damping ..... 12
2.4.2 Material damping ..... 12
2.4.3 Total damping ..... 13
2.5 Peak Particle Velocity (PPV) ..... 14
2.6 Soil Impedance ..... 14
2.7 Reflection and refraction ..... 15
2.8 Soil resonance ..... 17
2.9 Working of vibratory-driven piles ..... 19
2.9.1 Type of vibrating systems ..... 19
2.9.2 Counter-rotating masses (Woods) ..... 20
2.9.3 The modern vibrating machines ..... 20
2.10 Conclusion ..... 24
3 Vibration estimation methods ..... 25
3.1 Empirical methods ..... 25
3.1.1 Attewell and Farmer method . ..... 25
3.1.2 Wiss method ..... 26
3.1.3 Handboek damwanden model (CUR 166) ..... 27
3.1.4 Attewell's renewed method. ..... 28
3.2 Engineering methods ..... 29
3.2.1 Massarsch and Fellenius method ..... 30
3.2.2 Application to vibratory pile driving ..... 35
3.3 Theoretical Methods ..... 36
3.3.1 Influence zone around a closed-ended pile during vibratory driving. ..... 36
3.3.2 Dynamic soil-structure interaction formulation (EDT-model) ..... 38
3.3.3 Nonlinear analysis of pile driving using the finite element method ..... 39
3.3.4 Finite Element Method (Plaxis) ..... 40
3.4 Conclusion ..... 41
4 Dynamics of the pipe structure ..... 45
4.1 System representations ..... 45
4.1.1 Single Degree Of Freedom system (SDOF) ..... 46
4.1.2 Multiple Degree Of Freedom system (MDOF) ..... 46
4.1.3 Continuous system (CS) ..... 46
4.2 Modeling of damping ..... 46
4.2.1 Viscous damping. ..... 46
4.2.2 Hysteretic Damping ..... 50
4.3 Models of continuous systems (CS) ..... 50
4.3.1 Euler-Bernoulli Beam model ..... 51
4.3.2 Kelvin-Voigt model. ..... 54
4.3.3 Timoshenko beam model ..... 54
4.4 Methods for linear systems ..... 55
4.4.1 Modal superposition approach ..... 55
4.4.2 Fourier transform ..... 56
4.5 Methods for non-linear systems ..... 56
4.5.1 Galerkin Method ..... 57
4.5.2 Numerical approach ..... 57
4.6 Soil structure interaction ..... 59
4.6.1 Direct force or displacement ..... 59
4.6.2 Indirect force or displacement ..... 59
4.7 Conclusion ..... 60
5 Conclusion literature review ..... 63
6 Final Model Choice ..... 65
6.1 Total program ..... 66
6.2 Wave propagation model (WPM) ..... 67
6.2.1 Model definition ..... 67
6.2.2 Model structure ..... 68
6.3 Verification with Finite Element Model ..... 75
6.3.1 Model definition ..... 76
6.3.2 Solution method for non-linear problems ..... 81
6.4 Comparison with codes of conduct ..... 83
6.5 Coupling function. ..... 83
6.5.1 Model structure ..... 83
6.6 Pipe structure model ..... 86
6.6.1 General model description. ..... 86
6.6.2 Model structure FEM with Rayleigh-damping ..... 87
7 Results ..... 95
7.1 WPM and ABAQUS ..... 95
7.1.1 Explanation of the result representation ..... 95
7.1.2 Different distances from pile, for R-waves WPM ..... 97
7.1.3 Different distances from pile, for P-waves WPM ..... 99
7.1.4 Influence Rayleigh-waves ABAQUS ..... 102
7.1.5 Variation with distance ABAQUS ..... 103
7.1.6 Variation with distance WPM ..... 104
7.1.7 Linear elastic versus linear elastic-perfect plastic ABAQUS model ..... 106
7.2 Coupled WPM - PSM ..... 107
7.2.1 Boundary condition ..... 107
7.2.2 Frequency ..... 108
7.2.3 Soil properties ..... 109
7.2.4 Pipe dimensions ..... 111
7.2.5 Calculation time ..... 113
7.2.6 Inaccuracy of calculation ..... 113
7.3 Conclusion ..... 114
8 Model limitations ..... 117
8.1 ABAQUS model ..... 117
8.2 Wave Propagation Model ..... 118
8.3 Pipe Structure Model ..... 118
9 Final Conclusion ..... 121
9.1 Wave propagation approaches ..... 121
9.2 Pipe Structure Model ..... 121
9.3 Research objectives and questions ..... 122
9.3.1 Research questions ..... 122
9.3.2 Research objectives ..... 122
9.3.3 Hypothesis ..... 123
9.4 Final comment ..... 123
10 Recommendations ..... 125
Bibliography ..... 127
A Derivations ..... 135
A. 1 Center of gravity pipe ..... 135
A. 2 Element matrices used in the FEM ..... 136
A.2.1 Components element stiffness matrix $\left[\mathrm{K}^{\text {Pipe }}\right]$ ..... 136
A.2.2 Components element mass matrix [M]. ..... 138
A.2.3 Components element force vector F ..... 140
B Plots ..... 143
C Matlab code ..... 147
C. 1 Coupled WPM - PSM ..... 147
C.1.1 Main code ..... 147
C.1.2 Wave propagation model function ..... 148
C.1.3 Stresses and strains function ..... 150
C.1.4 Animation plot function ..... 152
C.1.5 3D plot animation function (Not used). ..... 153
C.1.6 Model dimensions function ..... 154
C.1.7 Model parameters function ..... 156
C.1.8 Rayleigh-wave correction factor ..... 162
C.1.9 Force calculation function ..... 163
C.1.10 Variable dimensions function ..... 167
C.1.11 Variable parameters function ..... 168
C.1.12 Variable WPM function ..... 173
C.1.13 Element damping matrix $\left[C_{\text {Elem }}\right]$ function ..... 175
C.1.14 Element stiffness matrix [ $K_{\text {Elem }}$ ] function ..... 176
C.1.15 Element stiffness matrix boundary condition $\left[K_{B C, E l e m}\right]$ function ..... 177
C.1.16 Element matrix force vector $\left[F_{\text {Elem }}\right]$ function ..... 178
C.1.17 Element mass matrix [ $M_{\text {Elem }}$ ] function ..... 179
C.1.18 Global force function ..... 180
C.1.19 Global matrix function ..... 181
C.1.20 Global matrix assembly function ..... 182
C.1.21 Equation of motion function ..... 184
C. 2 Additional code suggestions ..... 185
C.2.1 Runga Kutta solver (RK4) function ..... 185
C.2.2 Stresses and strains, including Jacobian, function ..... 186
C.2.3 Jacobian ODE-solver function ..... 188
C.2.4 Jacobian estimation/assembly function ..... 189

## List of Figures

1.1 Pile installation methods ..... 1
2.1 Harmonic motion according to Liden [71], Guillemet [47] ..... 6
2.2 Periodic wave/motion according to Liden [71] ..... 6
2.3 Transient wave/motion according to Liden [71] ..... 6
2.4 Random wave/motion according to Liden [71] ..... 7
2.5 Distinction of body and surface waves, according to soil medium characteristics, carried out by Deckner [34], Nordal [91] ..... 7
2.6 Wave propagation mechanisms within a homogeneous isotropic half-space a) P-wave, b) S- wave, c) R-wave and d) Love-wave according to Deckner [34], Woods [127], Kramer [69] ..... 8
2.7 Amplitude ratio vs. dimensionless depth for Rayleigh wave in a homogeneous halfspace, after Richart et al. [104] ..... 10
2.8 Relationship between propagation velocities of dynamic waves according to Whenham [124], Woods [127] ..... 10
2.9 Wave propagation by pile driving modified after Woods [127], Deckner [34], Attewell and Farmer [13], Martin [76] ..... 11
2.10 Geometric damping of R -waves ( $\mathrm{n}=0.5$, blue line), body waves ( $\mathrm{n}=1$, red line) and body waves at the surface ( $\mathrm{n}=2$, yellow line) ..... 12
2.11 Relationship between material damping, shear strain and plasticity index (PI), modified after IVA [58] and Whenham [124], after Vucetic and Dobry [121], Deckner [34] ..... 13
2.12 Material damping of waves with 40 Hz waves in soft soil ( $\alpha=0.15$, yellow line), competent soil ( $\mathrm{n}=0.045$, red line) and hard soil ( $\mathrm{n}=0.008$, blue line) ..... 13
2.13 Total damping of dynamic waves ..... 14
2.14 Influence of pile impedance on transmission of vibration energy from pile to soil according to Woods [127] ..... 16
2.15 Ray-path, ray and wavefront for a) isotropic material b) layered medium according to Kramer [69] ..... 16
2.16 Refracted and reflected Ray's as a result of incidental a) P-waves b) SV-waves c) SH-waves ac- cording to Kramer [69] ..... 16
2.17 Refraction of SH-wave Ray-path through series of successively softer (lower $v_{S}$ ) layers according to Kramer [69] ..... 17
2.18 Dependence of the reflection coefficient, of cylindrical waves at the free surface of a solid, on the angle of incidence for different values of Poisson's ratio according to Brekhovskikh [24] ..... 17
2.19 Dependence of the reflection coefficient, of spherical waves at the free surface of a solid, on the angle of incidence for different values of Poisson's ratio according to Brekhovskikh [24] ..... 18
2.20 Free-hanging vibrator system for installing (sheet)piles according to Deckner [34] ..... 19
2.21 Leader-mounted vibrator system for installing (sheet)piles according to Viking [120] ..... 20
2.22 Forces produced by two counter-rotating masses according to Woods [127], Deckner [34] ..... 20
2.23 Adjustments to the eccentric moment of a vibrator according to Guillemet [47] ..... 21
2.24 The main parts of modern hydraulic vibrators. Free-hanging vibrator (left), leader-mounted vibrator (right) according to Viking [120] ..... 21
2.25 Individual eccentric mass vibrator according to Viking [120] ..... 22
2.26 The unbalanced forces generated by the counter-rotating eccentric masses according to Viking [120] ..... 23
2.27 Vertical driving force amplitude as a function of the vibrator's static surcharge force, unbalanced moment, and angular frequency according to Viking [120] ..... 23
2.28 Definition of double amplitude, free-hanging displacement according to Viking [120] ..... 24
3.1 Focus of the chapter in conjunction to the total master thesis research model: Vibration estima- tion methods, modified after Deckner [34] ..... 25
3.2 Results obtained from field measurements for the empirical k -factor according to Massarsch [80] ..... 26
3.3 Comparison of ground vibrations at 3 different distances from the pile between measured ground vibrations and calculation of ground vibrations (according to Equation 3.1 with $\mathrm{k}=0.75$ ) accord- ing to Massarsch [80] ..... 26
3.4 Comparison of ground vibrations at 3 different distances from the pile between measured ground vibrations and calculation of ground vibrations (according to Equation 3.2 with $\mathrm{k}=0.75$ and $\mathrm{n}=$ 1.5) modified after Massarsch [80] ..... 27
3.5 Quadratic regression curve fitted for field data measurements of vibratory pile driven vibrations [14] ..... 29
3.6 Linear regression curve fitted for field data measurements of vibratory pile driven vibrations [14] ..... 29
3.7 Wave signals produced by pile driving methods, modified after Deckner [34] ..... 29
3.8 Schematic representation of hammer-pile and pile-soil interaction scheme according to Mas- sarsch and Fellenius [80] ..... 30
3.9 Strain softening factor/shear wave speed reduction factor, $R_{c}$ as function of plasticity index, $I_{P}$, for different conditions of penetrations resistance according to Massarsch and Fellenius [80] ..... 32
3.10 Illustration of vibrations emitted during pile driving at the pile toe and along the pile shaft as stated by Massarsch and Fellenius [80] ..... 33
3.11 Variation of vibration amplification factor $F_{\nu}$ with respect to the angle of incidence $\theta_{P}$ for P- waves modified after Massarsch and Fellenius [80] ..... 34
3.12 Variation of vibration amplification factor $F_{h}$ with respect to the angle of incidence $\theta_{P}$ for P - waves modified after Massarsch and Fellenius [80] ..... 35
3.13 ABAQUS model results in a visual format according to Ekanayake [41] ..... 36
3.14 Boundary conditions used in the ABAQUS model carried out by Ekanayaka [41], a) infinite el- ement principle according to Zienkiewicz [132], b) the viscous wave transmitting boundaries according to Lysmer [74] ..... 37
3.15 Boundary conditions used in the ABAQUS model carried out by Ekanayaka [41], the principle carried out by Deeks [36] a) Shear boundary, b) dilation boundary ..... 37
3.16 (a) Geometry of the sub-domains and (b) the scattered wave fields according to Masoumi [77] . ..... 39
3.17 The norm of the particle velocity in a homogeneous half space due to vibratory pile driving at 20 Hz for penetration depths (a) 2 m , (b) 5 m and (c) 10 m according to Masoumi [77] ..... 39
3.18 Peak particle velocity vs. scaled distance from the pile for different pile embedded lengths ac- cording to Serdaroglu [107] ..... 40
3.19 Peak vertical velocity vs. distance from the pile at different depths according to Serdaroglu [107] ..... 40
3.20 Plaxis model according to Whenham [124] ..... 41
3.21 Comparison between Plaxis model, EDT-model and field measurements at surface level accord- ing to Whenham [124], Masoumi [77] ..... 41
4.1 Focus of the chapter in conjunction to the total master thesis research model: Dynamics of the pipe structure, modified after Deckner [34] ..... 45
4.2 Categories of damping modeling according to Jia [60] ..... 47
4.3 Figurative representation of a viscous damping element (also named dashpot) modified after Jia [60] ..... 47
4.4 Responds of the system to viscous damping according to Metrikine [86] ..... 47
4.5 Variation of dynamic magnification factor with damping and frequency according to Clough [28] ..... 48
4.6 Response to resonant loading, $\beta=1$ for at-rest initial conditions after Clough [28] ..... 48
4.7 Variation of viscous damping as a function of period and frequency using Rayleigh damping formulation modified after Clough [28] ..... 48
4.8 Variation of viscous damping as a function of period and frequency using Rayleigh damping formulation modified by Youssef [129] after Clough [28] ..... 49
4.9 Extended Rayleigh damping after Clough [28] ..... 50
4.10 Figurative representation of a hysteretic damping element modified after Jia [60] ..... 50
4.11 Stress-strain hysteresis loop for reverse loading according to Tatsuoka [110], Benz [19] ..... 51
4.12 Sign convention of the prismatic beam modified after Spijkers [109] ..... 51
4.13 Sign convention of the prismatic cross-section modified after Spijkers [109] ..... 52
4.14 Deformation of a Timoshenko beam (blue) compared with that of an Euler-Bernoulli beam (red) after Banerje [16] ..... 54
4.15 Newmark method/trapezoidal rule, modified after Bathe [18] ..... 59
4.16 Direct force or displacement on pipe structure using Klevin-Voigt Foundation, modified after Metrikine [86] ..... 60
4.17 Indirect force or displacement on pipe structure using Klevin-Voigt Foundation, modified after Metrikine [86] ..... 60
6.1 Model flow for the total program ..... 66
6.2 Mohr-Coulomb's material model for soils, after Midas Information Technology Co., Ltd. [88] ..... 67
6.3 Model flow for the wave propagation model ..... 68
6.4 Representation of the force applied to the pile with the corresponding toe resistance force, mod- ified after Van Den Berghe [20] ..... 70
6.5 Interlocking effect of sheet pile walls during installation, after Van Den Berghe [20] and Viking [120] ..... 70
6.6 Cumulative distribution function of the input energy over time for one cycle ..... 72
6.7 Calculation method for the static displacement due to a surface plate load according to Boussi- nesq's methodology, according to Pistrol [98] ..... 72
6.8 Wave propagation behavior of Rayleigh-waves and P-waves in an elastic half-space, modified after Massarsch and Fellenius [80] ..... 75
6.9 Radial symmetric domain of the ABAQUS model: Soil domain existing of CAX4 elements; infi- nite domain existing CINAX4 elements ..... 76
6.10 ABAQUS elements for the Finite Element calculations ..... 77
6.11 Grid distribution for the axisymmetric ABAQUS model ..... 78
6.12 Variation of viscous damping, for medium dense sand, as a function of period and frequency for different values of damping using Rayleigh's damping formulation ..... 79
6.13 Variation of viscous damping, for medium packed clay, as a function of period and frequency for different values of damping using Rayleigh's damping formulation ..... 79
6.14 Conversion of surface area from sheet pile wall to round pile for the ABAQUS model ..... 80
6.15 Pressure distribution of the pile toe on the grid cells in the ABAQUS model with a sinusoidal pressure distribution ..... 81
6.16 Calculation of the first increment with the Newton-Raphson approach, according to Katzen- bach [64] ..... 81
6.17 Calculation of the first increment with the Newton-Raphson approach, according to Katzen- bach [64] ..... 82
6.18 Grid distribution of the pipe Finite Element Model dependent on the percentage of maximum wave velocity chosen at the outer boundary ..... 84
6.19 Displacement calculation method with the help of the cumulative Trapezoidal Rule and disre- garding linear fit curve ..... 85
6.20 Conversion from pressure to force on pipe structure ..... 85
6.21 Block function applied on the force ..... 86
6.22 Modified force on pipe structure by means of the application of the block function ..... 86
6.23 Finite Element Model schematic representation, modified after Metrikine [86] ..... 87
6.24 Model flow for the Finite Element approach of the pipe structure with Rayleigh-damping ..... 88
6.25 Element configuration pipe structure Finite Element Model ..... 88
6.26 Hermite cubic interpolation functions (or also known as cubic shape functions) used for the Galerkin Finite Element Model ..... 90
6.27 Deformed element after rotation $\theta_{1}$ and $\theta_{2}$, modified after Massachusetts Institute of Technol- ogy [92] ..... 93
7.1 The location representation of the graphical results regarding the WPM and ABAQUS calculations 96 ..... 96
7.2 WPM-PSM calculation, displacement, momentum, stress shown over the length of the pipe at t $=0.07 \mathrm{~s}$ ..... 97
7.3 WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths for Rayleigh- waves. Data positioning point is 5.0 meters from center pile at 1 meter below surface level. ..... 98
7.4 WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths for Rayleigh- waves. Data positioning point is 10.0 meters from center pile at 1 meter below surface level. ..... 98
7.5 WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths for Rayleigh- waves. Data positioning point is 20.0 meters from center pile at 1 meter below surface level. ..... 98
7.6 ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 0.50 meters from center pile at 1 meter below surface level. ..... 99
7.7 ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 5.0 meters from center pile at 1 meter below surface level. ..... 100
7.8 ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 10.0 meters from center pile at 1 meter below surface level. ..... 101
7.9 ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 20.0 meters from center pile at 1 meter below surface level. ..... 101
7.10 ABAQUS calculation, Peak Particle Velocity (PPV) as a result of a pile toe depth of $-3,-9,-15$ and -24 meters. Data positioning point is 10.0 meters from center pile at depths between surface and - 24 meters. ..... 102
7.11 ABAQUS calculation, Peak Particle Velocity (PPV) as a result of a pile toe depth of -1 meter. Data positioning point is between 0.5 and 20.0 meters from center pile at depths of $0,-1,-2,-3,-4$ and -5 meters. ..... 103
7.12 ABAQUS calculation, Peak Particle Velocity (PPV) as a result of a pile toe depth of -3 meter for different driving frequencies. Data positioning point is between 0.5 and 20.0 meters from center pile at -3 meter depth. ..... 104
7.13 ABAQUS calculation, Peak Particle Velocity (PPV) as a result of a pile toe depth of -3 meter for different soil stiffness and constant Poisson's ratio and soil density. Data positioning point is between 0.5 and 20.0 meters from center pile at -3 meter depth. ..... 105
7.14 WPM calculation for P-waves, Peak Particle Velocity (PPV) as a result of a pile toe depth of -1 meter, for different soil stiffness and constant Poisson's ratio and soil density. Data positioning point is between 0.5 and 20.0 meters from center pile at -1 meter depth. ..... 105
7.15 WPM calculation for R+P-waves, Peak Particle Velocity (PPV) as a result of a pile toe depth of -1 meter, for different soil stiffness and constant Poisson's ratio and soil density. Data positioning point is between 0.5 and 20.0 meters from center pile at -1 meter depth. ..... 106
7.16 WPM calculation, Peak Particle Velocity (PPV) as a result of a pile toe depth of -1 meter for different driving frequencies. Data positioning point is between 0.5 and 20.0 meters from center pile at -1 meter depth. ..... 107
7.17 WPM-PSM calculation, relation of the stress in the pipe structure - soil stiffness around the pipe structure. Comparison of different boundary conditions. ..... 108
7.18 WPM-PSM calculation, relation of the stress in the pipe structure - extended boundary length ..... 108
7.19 WPM-PSM calculation, stress in the pipe structure - frequency of excitation of the vibrating hammer ..... 109
7.20 WPM calculation, Peak Particle Velocity (PPV) as a result of a pile toe depth between 0.0 and -24 meters: for different soil stiffness and Poisson's ratio and a constant soil density. ..... 110
7.21 General wave velocity used for the WPM model ..... 111
7.22 WPM-PSM calculation, stress in the pipe structure - Rayleigh damping parameter ..... 111
7.23 WPM-PSM calculation, stress and momentum in the pipe structure in relation to the diameter of the pipe structure with constant pipe thickness of 1 mm . ..... 112
7.24 WPM-PSM calculation, stress and momentum in the pipe structure - thickness of the pipe struc- ture with constant pipe diameter of 500 mm . ..... 112
7.25 Cross-sectional area and moment of innersia of a pipe - thickness of a pipe structure with con- stant pipe diameter of 500 mm ..... 113
7.26 WPM-PSM calculation, calculation time - boundary length ..... 114
7.27 WPM-PSM calculation, displacement as a relation of the simulation time in the middle of the pipe ..... 114
7.28 WPM-PSM calculation, results obtained from the midpoint of the pipe ..... 115
A. 1 Center of gravity of half of a pipe cross-section ..... 135
B. 1 ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 0.50 meters from center pile at 2 meter below surface level. . . 143
B. 2 ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 5.0 meters from center pile at 2 meter below surface level. . . . 144
B. 3 ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 10.0 meters from center pile at 2 meter below surface level. . . 144
B. 4 ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 20.0 meters from center pile at 2 meter below surface level. . . 144
B. 5 ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 0.50 meters from center pile at 3 meter below surface level. . . 145
B. 6 ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 5.0 meters from center pile at 3 meter below surface level. . . . 145
B. 7 ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 10.0 meters from center pile at 3 meter below surface level. . . 145
B. 8 ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 20.0 meters from center pile at 3 meter below surface level. . . 146

## List of Tables

2.1 Range of velocity values P and S -waves according to Whenham [124] ..... 11
2.2 Absorption/attenuation coefficient according to the classification of soil materials, after Deck- ner [34], Woods [127] ..... 13
2.3 Eurocode 3: Maximum acceptable vibration levels to prevent structural damage according to Ekanayake [41] and Eurocode 3 [2] ..... 15
2.4 Richter's notation for wave impacts with layered interface according to Kramer [69] ..... 16
2.5 Natural frequencies of soils according to Deckner [34], Liden [71] ..... 18
2.6 Vibrator types according to Viking [120], Liden [71] ..... 20
$3.1 u_{0}, \alpha$ and $V_{0}$ obtained from field tests for hammers up to 350 kN according to CUR 166 [1] ..... 28
$3.2 \beta$ obtained from field tests and related to the probability of exceedance according to CUR 166 [1] ..... 28
3.3 Values of $x_{1}, x_{2}$ and $x_{3}$ for vibratory pile driving according to Attawell [14] ..... 28
6.1 Material properties for the ABAQUS Finite Element Model and the wave propagation model, Geotechdata [45] and Benz [19] ..... 67
6.2 Properties of an AZ32-750 sheet pile wall used for the model simulations, according to Arcelor Mittal [90] ..... 69
6.3 The properties of the PVE 55M vibrating hammer used for the model simulations, according to Diesko Groep [46] ..... 70
6.4 Eigenmodes with corresponding eigenfrequencies used for the Rayleigh-damping determina- tion of the system with Sand as material ..... 79
6.5 Eigenmodes with corresponding eigenfrequencies used for the Rayleigh-damping determina- tion of the system with Clay as material ..... 79
6.6 Eurocode 3: Maximum acceptable vibration levels to prevent structural damage, according to Ekanayake et al. [41] and Eurocode 3 [2] ..... 83
7.1 Material properties for the ABAQUS Finite Element Model and the wave propagation model, Geotechdata [45] and Benz [19] ..... 96
7.2 Material and model properties for the PSM ..... 96

## Introduction

Driving and vibrating are commonly used methods to install (sheet)piles (from now on named "piles") into the subsurface. With the help of heavy equipment like a vibrator or a hammering machine (shown in Figure 1.1) sheet pile walls can be installed in a subsurface consisting of soils. When installed they will provide a structure the necessary support by means of transferring the forces through the pile to the soil. Geo-technical engineers will provide the design for the piles according to the applied standards. The soil characteristics, type of construction, method of installation and the surroundings are the dominant factors studied in the design process of piles.

(a) Vibratory sheet pile installation by means of a free-hanging system

(b) Installation of prefabricated concrete piles by the process of (impact) driving/hammering

Figure 1.1: Pile installation methods
Vibrations induced by a pile installation will result in a propagating wave through the subsurface and further interact with structures present within the subsurface. The oscillating construction, generated by the interaction between the fluctuation pulse in the soil and the structure, may lead to damage.

### 1.1. Problem statement

Urban areas in the Netherlands are expending rapidly and civilization is squeezed onto a decreasing space. As a result building projects intent to be located very close to adjacent constructions leading to potential risk damaging these structures. Especially structures that are constructed within the subsurface, like cables and ducts, require extra care. Precautionary measurements and therefore an increase in costs are in direct relation to the required care. "If vibration levels are overestimated, this leads to selecting more expensive and time consuming construction methods than necessary. However, if vibrations levels are underestimated they result in damaged structures, disturbed occupants and suspensions to the construction work." Deckner [34]. Or as Massarsch [80] states: "Not having confidence in how to assess the risk of ground vibrations
during pile driving, regulatory authorities often feel compelled to impose restriction on the use of driven piles and sheet piles or to choose alternative foundation solutions". Good understanding of the potential risk is required. To fulfill this requirement a reliable model for the prediction of vibrations, induced by a pile installation methods, is required. This Master thesis project is concerned with the development of that prediction model and the influence of vibrations on cables and ducts in the subsurface.

### 1.2. Focus points literature review

Investigation of vibratory pile installation problems and their effect on adjacent structures requires extensive knowledge. For that reason comprehensive research on the individual aspects associated with the research question is carried out. The plan of work [103] states a list of all these individual aspects. Due to expended knowledge and a better insight into the problem definition adjustment are made.

- Basic theory of soil dynamics
- Pile-soil interaction
- Soil-structure interaction
- Dynamic properties of soil and dynamic soil behavior
- Modeling of soil dynamic problems
- Existing software to investigate dynamic wave properties
- Existing similar research projects
- Defining governing equations and theories for model
- Dynamics of slender structures


### 1.3. Hypothesis

Every research has its roots related to the outcome expectations. Prospects lead to an investigation outline and direction, since the aim is its prove. The following hypothesis is a guideline for this Master Thesis research project:

The ground vibrations induced by vibratory sheet pile driving can lead to failure of the pipe structure.

### 1.4. Research questions

Favorable outcome of the Master Thesis research project obligates not only excellent focus points of the literature review, but also adequate research questions. Objectives such as focus points serve and encourage the substantiation of the queries and vise-versa. Reuver [103] lists the main research questions. Modification as a result of increased knowledge leads to:

1. What are the additional stresses in the pipe caused by the vibration wave?
2. What are the factors influencing the wave propagation through the soil and how do they influence this process?
3. To what extend will damping influence the behavior of the pipe structure?
4. What will be the preferred method to solve the problem statement and to what extend do model assumptions influence the reliability of the end result?

### 1.5. Research objectives

The research questions are directly related to the research objectives. The objectives are the basis of the investigation and steer the direction of development. Below the most important research objectives are stated:

- Aim for simplification to obtain a fast result.
- Development of a model that describes pressure waves excited by vibratory sheet pile installation and their progression through a soil body.
- Predict and describe the behavior of an embedded pipe structure excited pressure waves.
- Incorporation of a layer soil body.


### 1.6. Document outline

This literature review consists of different chapters outlining the entire research project:

- General theory of vibrations: Parameters and soil behavior governing vibration motions and propagation of waves are outlined in detail throughout this chapter.
- Vibration estimation methods: Comparison and detailed description of individual prediction models for vibrations induced by pile installation are the fixation points of this chapter.
- Dynamics of the pipe structure: To acquire and comprehend the convoluted dynamics associated with the superposition principle driving the damping of waves within the pipe structure, extensive literature study is carried out in this chapter. Range of analytic modals are deliberated to retrieve a final suitable model approach.
- Conclusion of literature review: To finalize the literature review conclusions in relation to further study, carried out in conjunction with knowledge implemented in this analysis, are achieved. Reexamination of the plan of work [103] is mandatory and hence contained in the this chapter.
- Model description: To acquire research results, as carried out in Chapter 7, a model needed to be developed. The inner workings of the accomplished coupled Wave Propagation Model - Pipe Structure Model is outlined in detail throughout the content of this chapter.
- Results: Model results are presented and analyzed. The individual model components are tested by means of a parameter sensitivity study and compared with an ABAQUS radial symmetric model. Due to the lack of measurement data, no model validation is satisfied.
- Limitations: Chapter 7 outlined the results of the research. The model outcome displays the limitations of the developed approach. This chapter describes all the limitations in detail.
- Conclusion research: Judgment on the model results related to the research questions and objectives finalizes the Master Thesis research. Specific observations and assessment on the practical application of the developed method are considered important.
- Recommendations: Further development of the accomplished model is considered of high importance. Therefore suggestions for further research are carried out in this chapter.


## General theory of vibrations

A dynamic load on the surface or in the subsurface induces a dynamic wave to propagate in all directions through the body. The force acting on a soil particle is converted to motion conform Newton's second-law. The action of the moving soil particle transmits the kinetic energy to adjacent particles. This process continues over time and results in a motion wave traveling through the soil medium. According to Deckner [34] the wave propagation is the transportation of energy through a medium without the transportation of any materials. As a result of the dynamic wave, traveling through the medium, the particle undergoes deformation and movement. The motions of the particles can be separated into:

- Propagation velocity $c$, which relates to the velocity of the wave traveling through a body.
- Particle velocity $v$, that refers to the velocity of the particles oscillating around a reference position.


### 2.1. Basic dynamics

"Vibratory motion can be defined as an oscillatory movement around a state of equilibrium" [71]. According to Liden [71] the movement can be described in three different ways:

- Displacement: from a particle or body in time, $z(\mathrm{~m})$
- Velocity: from a particle, $\dot{z}(\mathrm{~m} / \mathrm{s})$
- Acceleration: from a particle, $\ddot{z}\left(\mathrm{~m} / \mathrm{s}^{2}\right)$

Energy transportation appears during the oscillatory movement around a state of equilibrium, but no movement of particles is involved in this process. The D'Alembert solution (1747), carried out by Equation 2.1 Metrikine [86], provides a good inside in the working of waves. Propagating waves are here represented by $f^{+}$and $f^{-} . f^{+}$refers to propagation waves in the positive x -direction and $f^{-}$the opposite. Both waves travel through the medium with the same wave speed $c(\mathrm{~mm} / \mathrm{s})$. The initial shape of the waves remain the same for both waves and therefore travel without distortion. The relation states that through excitation of a displacement $w(x, t)(\mathrm{mm})$ two waves in opposite direction are formed. The principle of symmetric excitation of waves, described by the D'Alambert solution, is used in many models by means of axis-symmetric model representations.

$$
\begin{equation*}
w(x, t)=f^{+}(x-c t)+f^{-}(x+c t) \tag{2.1}
\end{equation*}
$$

### 2.1.1. Harmonic waves

The most basic wave type can be described by a sine-function also known as a "Harmonic wave" (see Equation 2.2, Equation 2.3 and Equation 2.4). The relation that characterizes the harmonic motions correlates time $t(\mathrm{~s})$ and displacement $z(\mathrm{~m})$. Equation 2.3 is the first-order derivative of Equation 2.2 and Equation 2.4 the second-order according to Liden [71] and Woods [127].

$$
\begin{gather*}
z=A \sin (\omega t+\phi)  \tag{2.2}\\
\dot{z}=A \omega \cos (\omega t+\phi) \tag{2.3}
\end{gather*}
$$



Figure 2.1: Harmonic motion according to Liden [71], Guillemet [47]

$$
\begin{equation*}
\ddot{z}=-A \omega^{2} \sin (\omega t+\phi) \tag{2.4}
\end{equation*}
$$

The angular frequency $\omega(\mathrm{rad} / \mathrm{s})$ relates to the time period of a wave $T$ (s) applying $\omega=\frac{2 \pi}{T}(\mathrm{~Hz})$ as well as the frequency $f\left(\mathrm{~Hz} \mathrm{or} \mathrm{s}^{-}\right)$defined by $f=\frac{1}{T}$. In the relations $\phi(\mathrm{rad})$ is specified as the phase angle of the wave, $A(\mathrm{~m})$ the amplitude and $t$ the time. The factors governing the harmonic motion are also shown in Figure 2.1.

### 2.1.2. Periodic waves

Waves composed of different sine waves and repeat itself after certain periods in time are called periodic waves. These waves can take all sorts of forms, but always contain the characteristic of repeating itself after certain time periods. Periodic waves are generated during as a result of vibratory sheet pile driving. "Vibratory sheet pile drivers produce harmonic vibrations, but which during the propagation through the soil may change in frequency, leading to periodic vibrations" [71]. An example of periodic waves is shown in Figure 2.2.


Figure 2.2: Periodic wave/motion according to Liden [71]

### 2.1.3. Transient waves

According to Liden [71] transient waves can be described by a sine starting with a high intensity (big amplitude) fading out quickly over time (decrease in amplitude). This type of wave motion induced by impact driven pile driving is presented in Figure 2.3.


Figure 2.3: Transient wave/motion according to Liden [71]

### 2.1.4. Random waves

Unlike periodic waves random waves do not follow a certain repeating wave pattern over time. These randomly created wave types (see Figure 2.4) are generated by for instance traffic induced vibrations (which do not follow any repeated pattern).


Figure 2.4: Random wave/motion according to Liden [71]

### 2.2. General wave equation

Whenham [124] describes the general wave equation for a compression wave in a pile generated by a vibrator. Modification of this equation, for the application of waves traveling through a homogeneous isotropic elastic solid medium, also named as 'Navier equations', is described by Equation 2.5, Equation 2.6 and Equation 2.7 according to Verruit [119] and Olsson [94]. In this relation $\lambda$ and $\mu$ represent the Lamé constants carried out by Equation 2.11 and Equation 2.12 respectively and $\rho\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ is representative for the soil density. These parameters relate to the displacement vector $u(\mathrm{~m})$ and time $t(\mathrm{~s})$. The volumetric strain $\epsilon(-)$ is defined by Equation 2.8.

$$
\begin{gather*}
(\lambda+\mu) \frac{\partial \epsilon}{\partial x}-\mu \nabla^{2} u_{x}-\rho \frac{\partial^{2} u_{x}}{\partial t^{2}}=0  \tag{2.5}\\
(\lambda+\mu) \frac{\partial \epsilon}{\partial y}-\mu \nabla^{2} u_{y}-\rho \frac{\partial^{2} u_{y}}{\partial t^{2}}=0  \tag{2.6}\\
(\lambda+\mu) \frac{\partial \epsilon}{\partial z}-\mu \nabla^{2} u_{z}-\rho \frac{\partial^{2} u_{z}}{\partial t^{2}}=0  \tag{2.7}\\
\epsilon=\frac{\partial u_{x}}{\partial x}+\frac{\partial u_{y}}{\partial y}+\frac{\partial u_{z}}{\partial z} \tag{2.8}
\end{gather*}
$$

In addition to the relation described above another representation of the wave propagation though elastic media, by means of the Helmholz's equation, can be conducted. Cornejo [29] specifies this theorem by the relation given in Equation 2.9. $\eta=\eta(\mathbf{x}, t)$ is defined by Cornejo [29] as the disturbance traveling with the speed of propagation $c=\sqrt{\frac{\lambda}{\rho}}$ and $\mathbf{x}$ is a vector with three orthogonal components.
[Note: that $\eta$ should not be confused with the viscosity term used to describe the viscous damping in a system.]

$$
\begin{equation*}
\nabla^{2} \eta=\frac{1}{c^{2}} \partial_{t t}^{2} \eta \tag{2.9}
\end{equation*}
$$

### 2.3. Wave types

For dynamic waves traveling through an elastic isotropic homogeneous half-space a variety of propagation mechanisms are caused by different motions of the particles within a soil body. The types of waves including their features, shown in Figure 2.6, are drawn out in this section. Distinction between propagation as well as surface interactive behavior of the wave type are illustrated.


Figure 2.5: Distinction of body and surface waves, according to soil medium characteristics, carried out by Deckner [34], Nordal [91]
a P-wave: A push-pull motion in the traveling direction of the wave
b S-wave: Oscillation perpendicular to the traveling direction of the wave
c R-wave: Surface wave, a combination of P-waves and S-waves resulting in a ellipsoidal motion of the particles
d Love-wave: A snake-like motion of the wave
The P-waves and S-waves can be categorized as so called body waves. These waves travel within a medium instead of on the surface. The P-waves and S-waves don't coincide with each other and propagate independently through the medium. These waves can be applied to infinite mediums (so without any boundaries) as shown in Figure 2.5. Davis [32] mentioned a combination between a compression wave in a fluid and in a soil, named a Biot Wave.
The interaction of the boundary conditions and the body waves result in Surface waves. The boundary condition is represented by the planar-free surface of the elastic half-space (= infinite medium including boundary condition at surface level as shown in Figure 2.5). The surface waves travel along the medium with the largest wave amplitudes at the surface level. According to Kramer [69] these wave amplitudes roughly decrease exponentially with depth.


Figure 2.6: Wave propagation mechanisms within a homogeneous isotropic half-space a) P-wave, b) S-wave, c) R-wave and d) Love-wave according to Deckner [34], Woods [127], Kramer [69]

### 2.3.1. P-waves/cylindrical waves

Deckner [34] states that P-waves are also known as primary, compression or longitudinal waves. Primary compression behavior makes it possible to transit through solids as well as fluids. The particles in the soil body undergo a compressive and dilative behavior, as the wave passes, resulting in a volume change within the specimen. Furthermore P-waves correspond with the highest wave velocity from all the wave types present in solids. The wave propagation velocity $c_{P}(\mathrm{~m} / \mathrm{s})$ can be calculated according to Kramer [69] and Verruit [119]. By specifically solving the general wave equation for longitudinal waves Equation 2.10 is obtained.

$$
\begin{equation*}
c_{P}=\sqrt{\frac{M}{\rho}}=\sqrt{\frac{\lambda+2 \mu}{\rho}}=\sqrt{\frac{G(2-2 v)}{\rho(1-2 v)}}=\sqrt{\frac{E(1-v)}{\rho(1-2 v)(1+v)}} \tag{2.10}
\end{equation*}
$$

To exemplify Equation 2.10 all individual components are stated: $G(\mathrm{~Pa})$ represents the shear modulus; $M(\mathrm{~Pa})$ is the deformation modulus or also called the oedometer modulus; $E(\mathrm{~Pa})$ is the elasticity modulus; $\rho\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ the material density and $v(-)$ the Poisson's ratio. The deformation modulus $M$ is in correspondence with the dilative and compressive behavior of the wave. The Lamé constants $\lambda$ and $\mu(=G)$ according to Verruit [119], Woods [127], Benz [19] can be resolved by Equation 2.11 and Equation 2.12, respectively.

$$
\begin{equation*}
\lambda=\frac{v E}{(1+v)(1-2 v)} \tag{2.11}
\end{equation*}
$$

$$
\begin{equation*}
\mu=G=\frac{E}{2(1+v)} \tag{2.12}
\end{equation*}
$$

Characterization of cylindrical waves, as accomplished by Verruit [119] is epitomized in Equation 2.13. Verruit aimed to investigate the propagation of these waves through a soil body in radial direction $r$ with respect to the cross-section of a pile.

$$
\begin{equation*}
\frac{\partial^{2} u}{\partial r^{2}}+\frac{1}{r} \frac{\partial u}{\partial r}-\frac{u}{r^{2}}-\frac{1}{c_{P}^{2}} \frac{\partial^{2} u}{\partial t^{2}}=0 \tag{2.13}
\end{equation*}
$$

Olsson [94] derived a solutions of Equation 2.5, where the propagation of waves through a isotropic elastic solid medium is given, Equation 2.14.

$$
\begin{equation*}
\frac{\partial \epsilon}{\partial t}=c_{P}^{2} \Delta^{2} \epsilon \tag{2.14}
\end{equation*}
$$

### 2.3.2. S-waves/spherical waves

The S-waves can be described as secondary shear waves (or transverse waves) according to Deckner [34]. Due to the fact that this wave results in a shearing motion within the specimen, it can only travel through solids. Fluids can not take shear forces and therefore not deport shear waves.
The particle motion is perpendicular to the wave direction and involves no volume change within the medium. According to Kramer [69] the horizontal and vertical oscillation motion of the particles can be described with the SH and the SV-wave, respectively. The vector sum of these wave types form the corresponding S-wave. The wave velocity of the S-wave can be calculated according Kramer [69] with Equation 2.15.

$$
\begin{equation*}
c_{S}=\sqrt{\frac{G}{\rho}}=\sqrt{\frac{E}{2 \rho(1+v)}} \tag{2.15}
\end{equation*}
$$

As for cylindrical waves, spherical waves can also be described by a wave equation. According to Verruit [119] this relation is described by Equation 2.16.

$$
\begin{equation*}
\frac{\partial^{2} u}{\partial r^{2}}+\frac{2}{r} \frac{\partial u}{\partial r}-\frac{2 u}{r^{2}}-\frac{1}{c_{P}^{2}} \frac{\partial^{2} u}{\partial t^{2}}=0 \tag{2.16}
\end{equation*}
$$

[Note: As shown the propagation velocity of compression/cylindrical waves is used, so not the propagation velocity of spherical waves described by Equation 2.15.]
Olsson [94] also described a more general form of spherical waves (Equation 2.17), which is the second solution of Equation 2.5. In this equation $\omega_{x}$ represents the rotation about the x -axis. Similar expressions exist for the $y$ and $z$-direction.

$$
\begin{equation*}
\frac{\partial \omega_{x}}{\partial t}=c_{S}^{2} \Delta^{2} \omega_{x} \tag{2.17}
\end{equation*}
$$

[NOTE: here the spherical propagation velocity is used.]

### 2.3.3. R-waves

Rayleigh-wave (R-wave), also known as surface wave, is a combination of P and S -waves interacting with the surface of the medium. According to Kramer [69] the R-waves consist of both horizontal as well as vertical retrograde ellipsoidal particle movement. Deckner [34] states that this movement changes in a pro-grade direction at a depth of about 0.2 times the wavelength $\lambda_{R}(\mathrm{~m})$.
The wave velocity of the R-waves carried out by Deckner [34], Bodare [22], Holmberg et al. [54] is described by the approximation formula according to Equation 2.18.

$$
\begin{equation*}
c_{R} \approx \frac{0.87+1.12 v}{1+v} c_{S} \tag{2.18}
\end{equation*}
$$

Kramer [69] affirms that the influence on particle motion depends on the length of the wave. Longer wave lengths relate to a lower frequency and vise-versa. Lower frequency waves have impact on particle motion at greater depths than high frequency (shorter wave lengths). According to Whenham [124] the wave attenuation and high percentage of vibration energy, conveyed by the R-wave, leads to a predominance of this wave type, with respect to an increase in distance from the wave source. Comparison of wave vibration energies are carried out later in this report.


Figure 2.7: Amplitude ratio vs. dimensionless depth for Rayleigh wave in a homogeneous halfspace, after Richart et al. [104]

The vibration amplitude of the Rayleigh wave is depth dependent according to the relation visualized in Figure 2.7. The decrease of the vertical amplitude relates to the right two lines and the horizontal amplitude to the left two (as indicated in the figure). The x-axis symbolizes the ratio depth over wavelength $(d / \lambda)(\mathrm{m} / \mathrm{m})$, whereas the y-axis represents the ratio amplitude at depth over horizontal surface amplitude. Depending on the Poisson's ratio $(v)(-)$ of the soil, different behavior is expected according to the figure. The vertical amplitude is bigger than the horizontal amplitude and it decreases more rapidly with depth. Furthermore a decrease in Poisson's ratio dominates the vertical component more than the horizontal one.

### 2.3.4. Comparison waves

Different types of waves lead to different properties and behavior within a soil body. This paragraph outlines the most decisive differences of these wave forms. Comparison of propagation velocity with respect to Poison's ratio for compression, spherical and surface waves is carried out in Figure 2.8.


Figure 2.8: Relationship between propagation velocities of dynamic waves according to Whenham [124], Woods [127]

The wave propagation velocity is dependent on the Poisson's ratio $(v)$ of a soil. Figure 2.8 presents the three different wave types ( $\mathrm{S}, \mathrm{P}$ and R -waves), taking into account the ratio between the wave propagation velocity $(V)(\mathrm{m} / \mathrm{s})$ and the Poisson's ratio $(v)$. The compressive-dilative P-wave propagation velocity in soils decreases rapidly with an increase of the Poisson's ratio. Table 2.1 conjugate a correlation between different soil materials and the corresponding estimated P-and S-wave propagation velocity. The difference in propagation velocity of the soil types can be declared by Figure 2.8 and relates to material behavior of soils. Soils react in a stiffer manner to compression than to shear resulting in faster traveling rates for P-waves (primary compression waves), through a soil, than S-waves (shear waves) [69]. A typical Poisson's ratio of clay is in the range of $0.4-0.5$ and for sand in the order of 0.2-0.4. This relates to a factor 1.0 and 1.9, obtained from Figure 2.8, for clay and sand, respectively.
Values for total input energy among the three elastic waves: " $c_{P}=7 \%, c_{S}=26 \%$ and $c_{R}=67 \%$ with Poisson's
ratio of $v=0.25$. Thus the Rayleigh wave is the primary cause of structural movement on the surface or for shallow foundations" Olsson [94] as stated by Miller and Pursey [89]. In contradiction to this finding, stating from 1955, Wolf [126] discovered that this only counts for low frequency rates. High frequency rates relate to a high input energy by P-waves.

| Material | $V_{P}[\mathrm{~m} / \mathrm{s}]$ | $V_{S}[\mathrm{~m} / \mathrm{s}]$ |
| :--- | :--- | :--- |
| Clay | 1500 | 150 |
| Sand | 480 | 250 |
| Gravel | 750 | 180 |

Table 2.1: Range of velocity values $P$ and S-waves according to Whenham [124]

Application of wave theory to pile driving results in Figure 2.9. Compression waves, that are a result of the kinetic energy in the downward direction as a result of the vibrating hammer or impact hammer, travel through the pile, to the tip and reflect back as a dilative wave. This process leads to downward movement of the pile together with the hammer. According to Ekanayake [41] a volume displacement at the pile tip, as a reaction to this instant movement of the pile, results in spherical waves traveling through the soil (also called P-Waves). "Shear waves are first generated from the upper contact point and propagated out in a conical shape with a very shallow angle" Ekanayake et al. [41]. The P-and S-waves are reflected at the surface and are partly reflected back as P -and S -waves. The rest of the wave energy is converted into R-waves, which as explained by Figure 2.7 decrease in amplitude with depth.


Figure 2.9: Wave propagation by pile driving modified after Woods [127], Deckner [34], Attewell and Farmer [13], Martin [76]

### 2.4. Damping of waves

The magnitude of wave strength, at a point within a specimen, dependents on the degree of damping that takes place. As mentioned by Reuver [103] and Dym [49] there are two different types of damping processes influencing the magnitude of the wave (plus a combination of the two), namely:

- Material damping
- Geometric damping
- Total damping

Kim [65] explains that the material damping is a phenomena compelled by physical parameters of the soil medium. Geometric damping involves the attenuation of waves through expansion of the wave front associated with the increasing distance from the source. Total damping involves both material and geometric damping (is the combination of both phenomena). Damping phenomena, as mentioned, are explained in further detail in the following sections.

### 2.4.1. Geometric damping

Geometric damping is related to the attenuation of a stress wave with distance. This means that the wave amplitude decreases proportional to the distance of the source. According to Deckner [34] this is on grounds of the energy spread of the wave, over an increasing surface area, as the distance from the source increases. Woods [127], Nordal [91] described the amplitude of the wave related to the distance of the source by Equation 2.19.

$$
\begin{equation*}
A_{2}=A_{1}\left(\frac{r_{1}}{r_{2}}\right)^{n} \tag{2.19}
\end{equation*}
$$

The relation adopts the amplitude of motion, $A_{1}(\mathrm{~m})$, at a distance $r_{1}(\mathrm{~m})$ from the source and $A_{2}(\mathrm{~m})$ at distance $r_{2}(\mathrm{~m})$, respectively. The $n(-)$ factor is related to the type of wave and holds a value of: 0.5 for R-wave; 1.0 for body-waves; 2.0 for body-waves at the surface. The phenomena of energy spread happens in the form of expanding rings. Surface waves have less area to spread their energy, therefore they experience less damping than body waves. On the other hand, body waves at the surface boundary experience an even higher damping than within the confines. Figure 2.10 illustrates how different wave types behave to geometric damping. The blue line in the figure symbolizes the geometric damping of R -waves, experiencing less damping than body waves expressed by the red and blue line.


Figure 2.10: Geometric damping of R -waves ( $\mathrm{n}=0.5$, blue line), body waves ( $\mathrm{n}=1$, red line) and body waves at the surface ( $\mathrm{n}=2$, yellow line)

### 2.4.2. Material damping

As a wave passes through a soil body, particle movement will take place as consequence. Internal friction of the particles, produce heat. The production of heat results in loss of kinetic energy (in the form of thermal energy). The loss of energy leads to a reduction in amplitude of the wave as the distance from the source increases. This type of damping mechanism is called material damping.
Kramer [69], Holmberg et al. [54], Attewell and Farmer [13] describe material damping as the loss of energy due to internal energy dissipation in the material as the soil particles are moved by the propagating wave. This type of behavior can be described, comprehensive with the distance of the source, in a similar way as geometric damping according to Dowding [39] by Equation 2.20.

$$
\begin{equation*}
A_{2}=A_{1} e^{-\alpha\left(r_{2}-r_{1}\right)} \tag{2.20}
\end{equation*}
$$

The material absorption parameter is $\alpha\left(\mathrm{m}^{-1}\right)$ and the distances 1 and 2 outlined in the equation are complementary to the distances used in Equation 2.19. On the other hand this equation is related to an exponential function describing the amplitude degradation. Material damping, exhibited in Figure 2.12, correlates to different damping behavior as geometrical damping. In Figure 2.12 distinction is made between soft and harder materials to outline the influence of the material absorption parameter $\alpha$.

$$
\begin{equation*}
\alpha=\frac{2 \pi D f}{c} \tag{2.21}
\end{equation*}
$$

Outlining the factors influencing the absorption parameter, the estimation formula is given by Equation 2.21. The bigger the value of $\alpha$ the greater the damping of the material is. Estimate ranges of the absorption parameter are demonstrated by Table 2.2. The ranges are sorted by weak, competent and hard soil. In agreement with Deckner [34], Equation 2.21 substantiates that a wave with a low frequency $f(\mathrm{~Hz})$ is damped less than

| Soil type | $\alpha\left[m^{-1}\right]$ |
| :--- | :--- |
| Weak or soft | $0.08-0.26$ |
| Competent | $0.026-0.08$ |
| Hard | $0.0026-0.026$ |

Table 2.2: Absorption/attenuation coefficient according to the classification of soil materials, after Deckner [34], Woods [127]
a wave with a high frequency. Other factors influencing the absorption factor (and therefore also material damping) are material damping parameter $D\left(\mathrm{~Hz}^{-1} \mathrm{~s}^{-1}\right)$ and wave propagation factor $c(\mathrm{~m} / \mathrm{s})$. A high velocity leads to a small attenuation coefficient (absorption parameter) and therefore also to a small magnitude of material damping. The graphical lines carried out in Figure 2.12 are based on frequency rates of 40 Hz waves. Since pile vibrators in the Netherlands in most cases work at approximately the same frequency ( 36 Hz ). Due to correspondence of frequency this methodology gives a good estimation of the material damping caused by the different soil types can be made. Not only does the frequency influence the absorption parameter $\alpha$,


Figure 2.11: Relationship between material damping, shear strain and plasticity index (PI), modified after IVA [58] and Whenham [124], after Vucetic and Dobry [121], Deckner [34]
also the size of deformation that results from the wave. Figure 2.11 shows that the magnitude of the strain, together with the Plastic Index (PI), influence the material damping parameter $D$ and therefore also the absorption parameter. Amick and Gendreau [8] stated that the magnitude of the material damping depends on vibration amplitude, soil type, moisture content and temperature.


Figure 2.12: Material damping of waves with 40 Hz waves in soft soil ( $\alpha=0.15$, yellow line), competent soil ( $\mathrm{n}=0.045$, red line) and hard soil ( $\mathrm{n}=0.008$, blue line)

### 2.4.3. Total damping

Both geometric and material damping occur at the same time, as why considering them independently is irrelevant when deliberating total damping of a wave. The change in amplitude can be considered according to Deckner [34] and Athanasopoulos [12] by Equation 2.22. Independent material and geometric damping (also named radiation) amplitudes are combined into one equation named the Bornitz equation.

$$
\begin{equation*}
A_{2}=A_{1}\left(\frac{r_{1}}{r_{2}}\right)^{n} e^{-\alpha\left(r_{2}-r_{1}\right)} \tag{2.22}
\end{equation*}
$$

Figure 2.13 gives an inside in the influence of material and geometric damping combined. The material damping is varied by the different lines in every figure. Each figure separately contains a different wave type so that every separate graph shows another wave behavior. From these graphs can be acknowledged that body waves at the surface, no matter what type of soil, are damped very fast compared to the other two wave types. Therefore body waves at the surface are not of interest for this thesis. Also can be expected that body waves have less impact on adjacent structures as R-waves unless the structure is placed very close to the source.


Figure 2.13: Total damping of dynamic waves

### 2.5. Peak Particle Velocity (PPV)

Terminology very often used in engineering to quantify the magnitude of the wave velocity is the Peak Particle Velocity (PPV). PPV quantifies the magnitude of the wave with respect to the damage cause to the structure. The definition of the PPV is of high importance. Below are four definitions of PPV stated, as most commonly used in engineering practice according to Deckner [34], Athanasopoulos [11], Hiller [52] and Head [51]:

1. SRSS: Square root of sum of squares defines the simulated vector sum of the peak particle velocities in three mutually perpendicular directions [34] given by Equation 2.23.
2. Uni directional peak: The peak value of a particle velocity in any mutually perpendicular direction described by Equation 2.24.
3. Vertical peak value: The peak value in the z-direction calculated with Equation 2.25.
4. Instantaneous (true) resultant: The true vector sum of all mutually perpendicular directions described by Equation 2.26.

$$
\begin{gather*}
v_{S R S S}=\sqrt{v_{x, \text { max }}^{2}+v_{y, \text { max }}^{2}+v_{z, \text { max }}^{2}}  \tag{2.23}\\
v_{x, \text { max }, y, \text { max } \text { or } z, \max }=v_{x, \max }, v_{y, \max }, v_{z, \max }  \tag{2.24}\\
v_{\max , z}  \tag{2.25}\\
v_{\max (t)}=\sqrt{v_{x(t)}^{2}+v_{y(t)}^{2}+v_{z(t)}^{2}} \tag{2.26}
\end{gather*}
$$

Ekanayake [41] states three codes of conduct with respect to the maximum Peak Particle Velocity, namely: the Eurocode 3 [2]; American Association of State Highways and Transportation Officials (AASHTO) [10]; the Swiss standard, SN640312 [59]. The most frequently applied standard in Europe is the Eurocode 3 and therefore stated in Table 2.3. For this thesis the buried structures are of importance (with a maximum allowable PPV of $25 \mathrm{~mm} / \mathrm{s}$ ).

### 2.6. Soil Impedance

Time dependent behavior is a dominant factor involved in the concept impedance. According to Massarsch [80] this phenomena is the ratio between the force applied and the acceleration integrated over the velocity. It is describes how force of an object is converted into wave energy and therefore describes the wave properties involved in this process. This means for instance that the response of, in case of this thesis, the pile penetration into the soil will be different (in this case stiffer) when it will be pushed with a high rate into the
agenieursat BV

| Building type | PPV (mm/s) |
| :--- | :--- |
| Architectural merit | 2 |
| Residential area | 5 |
| Light commercial | 10 |
| Heavy industrial | 15 |
| Buried structures | 25 |

Table 2.3: Eurocode 3: Maximum acceptable vibration levels to prevent structural damage according to Ekanayake [41] and Eurocode 3 [2]
soil compared to a slow rate, leading to a higher wave velocity within the material compared to the slower velocity. Low density ( $\rho$ ) material can lead to a lower transgression of energy from the pile to the soil compared to a high density material, where the opposite is true. The soil impedance $Z_{P}$ ( $\mathrm{KNs} / \mathrm{m}$ ) for P-waves is described by Equation 2.27, Massarsch [80]. This relation depends on the cross-sectional area of the pile tip $A_{\text {pile,tip }}\left(\mathrm{m}^{2}\right)$, the propagation velocity of the P-wave $c_{P}(\mathrm{~m} / \mathrm{s})$ and the density of the soil $\rho_{\text {soil }}\left(\mathrm{kg} / \mathrm{m}^{3}\right)$.

$$
\begin{equation*}
Z_{P}=A_{\text {pile,tip }} c_{P} \rho_{\text {soil }} \tag{2.27}
\end{equation*}
$$

In agreement with Massarsch [80] the absence of any pile resistance, when penetrated into the subsurface, the force $F$ equals the impedance $Z$ times pile velocity $c$. Since soil resistance is present, these two parts are not equal and more complex analysis is necessary. Impedance captures the pile-soil interaction and therefore is of influence on the wave propagation from the pile to the soil. As Massarsch [80] concludes: "The impedance of the pile and of the soil are the single most important parameters for calculating ground vibrations as these govern the transfer and propagation of vibrations in the pile, along the pile-soil interface, and in the surrounding soil". Deckner [34] describes the relation between the particle velocity of a propagating wave and the compressive stress. Product of wave velocity and material density describes this relation and is denoted as specific impedance $z\left(\mathrm{kNs} / \mathrm{m}^{3}\right)$ (also called $z$ lower case). The specific impedance can be calculated for P as well as S-waves and is described by Equation 2.28 and Equation 2.29, respectively.

$$
\begin{gather*}
z_{s p}=c_{P} \rho_{\text {soil }}=\frac{M}{c_{P}}=\sqrt{M \rho_{\text {soil }}}  \tag{2.28}\\
z_{s s}=c_{S} \rho_{\text {soil }}=\frac{G}{c_{S}}=\sqrt{G \rho_{\text {soil }}} \tag{2.29}
\end{gather*}
$$

Equation 2.27 has similarities to these equations, but does not take the pile into consideration, as it describes the impedance of a soil body itself. Dependency on deformation/oedometer modulus $M$ (MPa) for P-waves and shear modulus $G(\mathrm{MPa})$ for S-waves, can be worked out with help of the specific impedance relationships described. Pile impedance is an important factor in relation with pile induced vibrations. Woods [127] exerted ranges of pile impedance magnitudes with respect to the type of pile shown in Figure 2.14 and the influence on the k -factor (all of the parameter terminology not explained here is described in Chapter 3).

### 2.7. Reflection and refraction

In a layered medium the waves are conducted from one layer to the other searching for the path with the least resistance resulting in the least traveling time and is called the 'Ray-path'. According to Kramer [69] the direction of the so called 'Ray-path' is described by a vector named a 'Ray'. "A wavefront is defined as a surface of equal travel time" Kramer [69]. As a result an isotropic material (no layering) the 'Ray-path' is perpendicular to the 'wavefront'. For a layered body this 'Ray-path', for different waves including different wave velocities $v$, can be described according to Snell with the use of Fermat's principle (Equation 2.30). Both the isotropic wavefront and the layered wavefront are illustrated in Figure 2.15. In line with Kramer [69], in Equation $2.30 i$ (degrees) indicates the angle between the Ray-path and the normal to the interface; $v(\mathrm{~m} / \mathrm{s})$ is the wave velocity of the P or S -wave that is calculated.

$$
\begin{equation*}
\frac{\sin (i)}{v}=\text { constant } \tag{2.30}
\end{equation*}
$$

Kramer [69] explains that the relationship can be used for both reflected and refracted (transmitted) waves and when the wave propagation velocities of both layers differ the wave will be refracted (except when $i=$


Figure 2.14: Influence of pile impedance on transmission of vibration energy from pile to soil according to Woods [127]

(a)

(b)

Figure 2.15: Ray-path, ray and wavefront for a) isotropic material b) layered medium according to Kramer [69]

0 ). Particle motion perpendicular to the interface is not produced by SH-waves (horizontal component of Swave), but is involved for both P and SV-waves (vertical component of S-wave). The P and SV-waves produce, as a result of the impact with a layered interface, reflected and refracted $P$ and SV-waves, which is not the case for SH-waves (see Figure 2.16). Richter uses a notation, drawn out in Table 2.4 (and carried out by Kramer


Figure 2.16: Refracted and reflected Ray's as a result of incidental a) P-waves b) SV-waves c) SH-waves according to Kramer [69]
[69]), that is used to describe the relation between the incidental wave and the reflected and refracted waves from Figure 2.16. This notation is then applied to Equation 2.30 resulting in Equation 2.31. Direction of the incidental wave is related to the reflected and refracted waves resulting from the impact with the layered interface is resolved with this relation. From the equation it can be noticed that the angle of incident for both

| Wave type | Velocity | Amplitude | Angle with Normal |
| :--- | :---: | :---: | :---: |
| Incident P | U | A | a |
| Incident S | V | B | b |
| Reflected P | U | C | c |
| Reflected S | V | D | d |
| Refracted P | Y | E | e |
| Refracted S | Z | F | f |

Table 2.4: Richter's notation for wave impacts with layered interface according to Kramer [69]
P-and S-waves is equal to the angle of reflection ( $a=c$ and $b=d$ ), since the velocity of the material is equal. On the other hand the angle of refraction is different from the angle of incident, as there is a change in material
and therefore a change is material velocity $v$.

$$
\begin{equation*}
\frac{\sin (a)}{U}=\frac{\sin (b)}{V}=\frac{\sin (c)}{U}=\frac{\sin (d)}{V}=\frac{\sin (e)}{Y}=\frac{\sin (f)}{Z} \tag{2.31}
\end{equation*}
$$

According to Kramer [69] Snell's law, described in Equation 2.31, relates to the fact that waves traveling from a higher-velocity material to a lower-velocity material are refracted closer and closer with respect to the direction of the normal to the interface of the layers (Figure 2.17). "Note from the figure that the orientation of the Ray-path becomes closer and closer when ground surface is approached" Kramer [69]. In agreement with


Figure 2.17: Refraction of SH-wave Ray-path through series of successively softer (lower $v_{S}$ ) layers according to Kramer [69]
Brekhovskikh [24] there exists a relation between the angle of impact with the surface, the Poisson's ratio and the wave type. The influence factor for the magnitude of the reflective wave (so the percentage of reflected energy) is described by the relations given in Figure 2.18 and Figure 2.19 for cylindrical and spherical waves respectively.


Figure 2.18: Dependence of the reflection coefficient, of cylindrical waves at the free surface of a solid, on the angle of incidence for different values of Poisson's ratio according to Brekhovskikh [24]

### 2.8. Soil resonance

The vibrator hammers produce a steady-state vibration wave which does not depend on the soil conditions. According to Deckner [34] when the wave length produced by the vibrator is the same as the thickness of the body a standing wave is produced. Resonance (when vibrations waves are in 'phase') occurs only when the natural frequency is equal to the vibration frequency. "For optimum efficiency, the pile and the soil should not be vibrating 'in-phase', or no penetration occurs" Woods [127]. Ranges of natural frequencies are stated in Table 2.5, taking into account that Deckner [34] states that soils and rocks don't have natural frequencies, although vibrations that they transmit can be observed. Woods [127] states that there are three important frequencies affecting vibratory pile driving:


Figure 2.19: Dependence of the reflection coefficient, of spherical waves at the free surface of a solid, on the angle of incidence for different values of Poisson's ratio according to Brekhovskikh [24]

| Soil type | Typical 'natural' frequency |
| :--- | :---: |
| Very soft silt and clay | $5-20$ |
| Peat | $10-13$ |
| Clay | $10-25$ |
| Sand and gravel | $30-40$ |
| Weak rock | $30-80$ |
| Strong rock | $>50$ |

Table 2.5: Natural frequencies of soils according to Deckner [34], Liden [71]

- Driver-pile resonant frequency: This relates to the most optimum frequency for the pile-driver related to pile penetration rate. According to Woods if the mass of the driver and pile and the vibration frequency combine to produce a driver-pile resonance, then the process is most efficient. This phenomena is only applicable when the frequency of the driver-pile resonance is different from that of the soil or any nearby structure, since otherwise damage could occur.
- Soil-pile-driver system resonant frequency: This type of resonance frequency system will cause maximum ground vibrations near the pile and is depended on the type of soil that is penetrated. The magnitude of the vibrations, defined by the amplitude $A$ of the wave, are depended on the mass of the system, the force applied by the vibrator and the dynamic stiffness of the soil. This resonance frequency will cause the system to be 'in-phase' and is dangerous for large vibrations in the vicinity.
- Soil stratum resonant frequency: Large ground vibrations are the result of the soil stratum being in resonance. Woods states that at this resonance frequency, the stratum resonates, resulting in very large ground vibrations that can move very efficiently through the soil and effect nearby buildings. Determining the lowest natural frequency $f_{n}\left(\mathrm{~Hz} \mathrm{or} \mathrm{s}^{-1}\right)$ is important since this determines the frequency boundary at where the pile and the soil will vibrate 'in phase'. According to Woods [127] Equation 2.32 relates the lowest natural frequency to the height of the soil layer $H(\mathrm{~m})$ and the shear wave velocity $v_{S}$ $(\mathrm{m} / \mathrm{s})$ or the compression wave velocity $v_{P}(\mathrm{~m} / \mathrm{s})$.

$$
\begin{equation*}
f_{n}=\frac{v}{4 H} \tag{2.32}
\end{equation*}
$$

Liden [71] and Woods [127] state that it is important to fully understand the consequences related to this phenomena. Therefore Equation 2.32 can help to get a better understanding of the parameter influences and within what ranges the vibrations will be in resonance, which can lead to no penetration of the pile and very high vibrations in the surrounding area. According to Liden with a layer thickness of 1-5 meters and the fact that the shear wave velocities of soils are in the order of $120-600 \mathrm{~m} / \mathrm{s}$ there will be a danger for resonance (since the operating vibrator has a frequency in the order of $20-30 \mathrm{~Hz}$ ).

### 2.9. Working of vibratory-driven piles

A good understanding of the workings of a vibrator are necessary to make predictions of vibratory pile induced vibrations. Counter-rotating-mass vibrators driven by heavy electrical engines is an old-fashioned machine for installing (sheet)piles. Nowadays the machines work according to a similar principle, only driven by hydraulic-powered engines, which makes it possible to operate at a wide range of frequencies and amplitude.

### 2.9.1. Type of vibrating systems

According to Viking [120] there are two types of vibratory systems:

- Free-hanging: These systems are most commonly used in the Netherlands and Belgium since it is a cost beneficial method and due to the relative low weight of the system can be used on building sites containing soft soils. This system is shown in Figure 2.20 and build up out of:

1. Hydraulic pump
2. Power-transmission system
3. The vibrator itself
4. Hydraulic clamp or clamps
5. Carrier for the vibrator unit


Figure 2.20: Free-hanging vibrator system for installing (sheet)piles according to Deckner [34]

The hydraulic pump of the system is most commonly a diesel-powered hydraulic pump, but in some cases, where noise and environmental emission should be minimized on the site, electrical-powered hydraulic pump's are used. The vibrators itself can weigh within a range of $150-16000 \mathrm{~kg}$, which requires a mobile crane as a carrier. Due to the use of a mobile crane and the free-hanging system difficulties with the positioning of the object, regulating the surcharge load $F_{o}$ is simply only possible by putting more tension of the cable of the clamp, which in case lowers the surcharge load (the surcharge load is depended on the weight of the vibrator and can only be increased by choosing a higher weight of the vibrator).

- Leader-mounted: In Germany the most preferred method, since with this method surcharge loads can be regulated by the telescopic leader to the vibrator and therefore making it possible to vibrate (sheet)piles into hard soils. This method works according to the same principles as the free-hanging system with only the mounting system as a difference. Despite the higher cost with respect to the freehanging system, this method claims for better and more precise positioning of the (sheet)pile. Due to its high system weight application of this method on sites with soft soils could be problematic. Figure 2.21 gives a representation of a leader-mounted vibratory system for installing (sheet)piles.

Viking hands out a table where the most common 5 types of hydraulic-driven vibrators are drawn out. The 5 categories, shown in Table 2.6, are based on the driving frequency $f_{d}\left(\mathrm{~Hz} \mathrm{or} \mathrm{s}^{-1}\right)$ and results in their range in adjustment related to the unbalanced moment $M_{e}(\mathrm{kgm})$ (terminology is explained later in this section).


Figure 2.21: Leader-mounted vibrator system for installing (sheet)piles according to Viking [120]

| Type of vibrator | Range of $f_{d}(\mathrm{~Hz})$ | Range of $M_{e}(\mathrm{kgm})$ |
| :--- | :---: | :---: |
| Standard frequency | $21-30$ | $>230$ |
| High frequency | $30-42$ | $6-45$ |
| Variable eccentricity | 40 | $10-54$ |
| Excavator mounted | $30-50$ | $1-13$ |
| Resonant driver | $>100$ | 50 |

Table 2.6: Vibrator types according to Viking [120], Liden [71]

### 2.9.2. Counter-rotating masses (Woods)

Counter-rotating-masses used in vibrator systems is shown in Figure 2.22 and consists of a mass $m(\mathrm{~kg})$ which is placed at an eccentricity radius $r_{e}(\mathrm{~m})$ from the rotating axis. A product of the two results in a static moment $M(\mathrm{kNm})$ Equation 2.34 [127]. The peak centrifugal force $Q(\mathrm{kN})$ can then be determined from ?? [127], where $\omega(\mathrm{rad} / \mathrm{s})$ the velocity in radial direction represents.

$$
\begin{align*}
& M=m r_{e}  \tag{2.33}\\
& Q=M \omega^{2} \tag{2.34}
\end{align*}
$$



Figure 2.22: Forces produced by two counter-rotating masses according to Woods [127], Deckner [34]

### 2.9.3. The modern vibrating machines

According to Viking [120] the mechanical action of a modern vibrator is governed by the driving force $F_{d}(\mathrm{kN})$ and consist of two parts:

- Static surcharge force: Is the stationary part $F_{0}(\mathrm{kN})$
- Sinusoidal vertical force: Is the vibratory part $F_{\nu}(\mathrm{kN})$

Combined they form the driving force of the vibrator $\left(F_{d}=F_{0}+F_{\nu}\right)$ also named the theoretically-generated driving capacity. Viking [120] states the most important theoretical parameters that govern the behavior of the modern vibrators:

- Surcharge force $F_{0}(\mathrm{~N})$
- Excentric moment $M_{e}$ (kgm)
- Driving frequency $f_{d}(\mathrm{~Hz})$
- Dynamic mass $m_{d y n}(\mathrm{~kg})$
- Free-hanging (single) displacement amplitude $S_{0}$ (mm)
- Theoretical power $P_{t}(\mathrm{~kW})$

All of these individual parameters are explained in detail in the following sections. Relations between the parameter and the inner workings of the vibrator will be explained. Different techniques (explained later on in this section) can result in different driving forces $F_{d}$ applied by the vibrator to the (sheet) pile (see Figure 2.23).


Figure 2.23: Adjustments to the eccentric moment of a vibrator according to Guillemet [47]

## Surcharge force

The roots of the force are in case of a leader-mounted method related to the surcharge force applied and in case of a free-hanging system the weight of the bias mass (vibrator as a whole). "The stationary action (static surcharge force) is applied to the vibrators exciter block via the elastomer dampers that couple and isolate the suppressor housing to and from the exciter block" [120] (see Figure 2.24 for explanation of the terminology). The static surcharge force $F_{0}(\mathrm{~N})$ for free-hanging vibrators is depending on the weight of the bias mass $m_{0}$


Figure 2.24: The main parts of modern hydraulic vibrators. Free-hanging vibrator (left), leader-mounted vibrator (right) according to Viking [120]
$(\mathrm{kg})$ related to the gravitational acceleration coefficient $g\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ and the suspension force of the carrier $T(\mathrm{~N})$ (crane) and is described by Equation 2.35 [120]. The suspension force of the crane has a negative contribution to the surcharge load, since the crane can only lower the magnitude and not apply a positive load onto the vibrator through the flexible cable.

$$
\begin{equation*}
F_{0}=m_{0} g-T \tag{2.35}
\end{equation*}
$$

For the leader-mounted surcharge load is a similar relation shown in Equation 2.36 [120]. The difference here is that the weight is different due to the fact that a different housing for the vibrator is used (suppression housing), named the suppression housing weight $m_{s p}(\mathrm{~kg})$. The force applied to the suppression housing to the leader mast is generated by hydraulic-pre-stress pressure $P_{0}\left(\mathrm{~N} / \mathrm{m}^{2}\right)$. Together with the area of the
hydraulic cilinder $A_{c y l}\left(\mathrm{~m}^{2}\right)$ this generates a force onto the vibrator, which gives a positive contribution to the surcharge load.

$$
\begin{equation*}
F_{0}=m_{s p} g+P_{0} A_{c y l} \tag{2.36}
\end{equation*}
$$

## Eccentric moment

The eccentric moment is defined by Viking [120] in Equation 2.37 a similar way to the method described by Woods [127] in Equation 2.34, only Viking defines the eccentric moment with the symbol $M_{e i}$ (kgm), the weight of the eccentric mass as $m_{e i}(\mathrm{~kg})$ and the eccentric radius as $r_{e i}(\mathrm{~m})$.

$$
\begin{equation*}
M_{e i}=m_{e i} r_{e i} \tag{2.37}
\end{equation*}
$$

The modern vibrator uses more eccentric masses resulting in a specified eccentric moment $M_{e}$ (kgm) (see Equation 2.38), which is the sum of each individual eccentric moment $M_{e i}$ (see Figure 2.25). In this relation $N$ indicates the number of individual eccentric moments applied for the vibrator.

$$
\begin{equation*}
M_{e}=\sum_{i=1}^{N} m_{e i} r_{e i} \tag{2.38}
\end{equation*}
$$



Figure 2.25: Individual eccentric mass vibrator according to Viking [120]

## Driving frequency

According to Viking [120] the number of revolutions of the eccentric masses per second, sometimes also expressed as rotations per minute (rpm) or expressed as the angular frequency $\omega(\mathrm{rad} / \mathrm{s})$ is specified as the driving frequency $f_{d}(\mathrm{~Hz})$. The three are terminologies combined in Equation 2.39 relating the angular frequency $\omega$ to the rotations per minute $n$ (rpm) or to the driving frequency $f_{d}(\mathrm{~Hz})$.

$$
\begin{equation*}
\omega=2 \pi f_{d}=\frac{2 \pi n}{60} \tag{2.39}
\end{equation*}
$$

## Unbalanced force

Within the exciter block the individual counter-rotating-eccentric masses cause centrifugal forces $F_{c}(\mathrm{~N})$. The vertical component of these forces $F_{v}(\mathrm{~N})$ causes the vibratory action. The centrifugal acceleration $a_{c i}\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ of a single eccentric mass can be found by application of Newtons Second Law $\left(\sum F=m a\right)$ and is drawn out in Figure 2.26 and described by the relation in Equation 2.40.

$$
\begin{equation*}
a_{c i}=r_{e i} \omega^{2} \tag{2.40}
\end{equation*}
$$

When the angular frequency is kept constant, there will be no acceleration and therefore the tangential component of the centrifugal acceleration $a_{t i}\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ will be zero. According to Viking [120] the radially-directed centrifugal force $F_{c}(\mathrm{~N})$ (also called the maximum unbalanced centrifugal force of the vibrator) can be calculated applying Newtons Second law and results the relation described by Equation 2.41.

$$
\begin{equation*}
F_{c}=m_{e i} a_{c i}=m_{e i} r_{e i} \omega^{2}=M_{e i} \omega^{2} \tag{2.41}
\end{equation*}
$$

The vibrator won't move in horizontal direction since both horizontal forces $F_{h}(\mathrm{~N})$, resulting from the centrifugal force (of the two eccentric rotating masses) are in balance with each other (taking into account that


Figure 2.26: The unbalanced forces generated by the counter-rotating eccentric masses according to Viking [120]
both masses are equal). The vertical component $F_{\nu, v i b r}(\mathrm{~N})$ of the centrifugal force can be calculated according to Equation 2.42. $\theta$ (degrees) in the equation relates to the angle between the centrifugal force and the vertical force. The vertical force can be adjusted by the crane operator by changing the maximum eccentric moment $M_{e}$ or/and the driving frequency $f_{d}$ of the vibrator. These two parameters change the eccentric force $F_{c}$ and therefore also the vertical component $F_{v, v i b r}$ (also named the vertical unbalanced driving force).

$$
\begin{equation*}
F_{\nu, v i b r}=F_{c} \sin (\theta)=M_{e} \omega^{2} \sin (\theta) \tag{2.42}
\end{equation*}
$$

## Driving capacity

According to Viking [120] the theoretical driving force applied to the head of the vibrator $F_{d}(\mathrm{~N})$, described by the relation in Equation 2.43, is the sum of the static surcharge load $F_{0}$ and the unbalanced vertical load $F_{\nu, v i b r}$.

$$
\begin{equation*}
F_{d}=F_{0}+F_{\nu, v i b r} \tag{2.43}
\end{equation*}
$$

The theoretical driving force $F_{d}$ in relation to the angular frequency $\omega$ is drawn out in Figure 2.27. Lines for different maximum percentages of the driving force of the vibrator are shown, crossing the $y$-axis at a height of $F_{0}$.


Figure 2.27: Vertical driving force amplitude as a function of the vibrator's static surcharge force, unbalanced moment, and angular frequency according to Viking [120]

## Dynamic mass

The dynamic mass $m_{d y n}$ is the mass of the vibrator that will contribute to the dynamic force $F_{d}$. This dynamic mass is simply the sum of the masses that contribute: the weight of the exciter block $m_{e b}$ (kg); the weight of the clamping device $m_{c l}(\mathrm{~kg})$; the weight of the (sheet)pile $m_{p}(\mathrm{~kg})$. This relation is shown in Equation 2.44.

$$
\begin{equation*}
m_{d y n}=m_{e b}+m_{c l}+m_{p} \tag{2.44}
\end{equation*}
$$

## Free-hanging (double) displacement amplitude

The maximum specified displacement amplitude $S_{s p}(\mathrm{~mm})$ (see Figure 2.28) is often listed in the specification of the vibrator and corresponds, according Viking [120], to the total (double) amplitude of the movement of a free-hanging vibrator $S_{0}(\mathrm{~mm})$. Viking states that the relation of $S_{s p}$ is described by the quotient of the
maximum specified unbalanced moment $M_{e}(\mathrm{kgm})$ and the dynamic mass $m_{d y n}(\mathrm{~kg})$ and is described in Equation 2.45. In this equation the specific single displacement amplitude is defined as $s_{0}(\mathrm{~mm})$.

$$
\begin{equation*}
S_{s p}=2 s_{0}=2 \frac{M_{e}}{\left(m_{e b}+m_{c l}\right)} \tag{2.45}
\end{equation*}
$$

The actual double-displacement amplitude $S_{0}(\mathrm{~mm})$ is decreased by means of an increase in weight (the


Figure 2.28: Definition of double amplitude, free-hanging displacement according to Viking [120]
(sheet)pile) and due to soil and clutch resistance that take place during driving compared to the calculated double-free-hanging-displacement amplitude $S_{0} P(\mathrm{~mm})$ (see Equation 2.46).

$$
\begin{equation*}
S_{0 P}=\frac{m_{e b}+m_{c l}}{m_{e b}+m_{c l}+m_{p}} S_{s p} \tag{2.46}
\end{equation*}
$$

## Theoretical power

According to Viking [120] the power applied by the hydraulic motor to the vibrator is depending on the driving condition and therefore also the power consumption. Viking however describes the theoretical power $P_{t}(\mathrm{~kW})$ by means of Equation 2.47. In this relation the amplitude of the dynamic motion of the vibrator $Z(\mathrm{~mm})$ is described by Equation 2.48.

$$
\begin{array}{r}
P_{t}=\frac{1}{T} \int_{0}^{T} F(t) v(t) d t= \\
\frac{1}{T} \int_{0}^{T}\left[F_{0}+\left(M_{e} \omega^{2}+M Z \omega^{2}\right)\right] Z \omega \cos (\omega t) d t=  \tag{2.47}\\
Z\left[4 F_{0}+2\left(M_{e} \omega^{2}+M Z \omega^{2}\right)\right]
\end{array}
$$

Other parameters used in Equation 2.47 and Equation 2.48 are: period of time $T$ (s); net force acting on the vibrator $F(t)(\mathrm{kN})$; vertical velocity of the vibrator $v(t)(\mathrm{mm} / \mathrm{s})$; time $t(\mathrm{~s})$; static surcharge force $F_{0}(\mathrm{kN})$; unbalanced moment $M_{e}(\mathrm{kgm})$; angular frequency $\omega(\mathrm{rad} / \mathrm{s})$; weight of the exciter block $M(\mathrm{~kg})$; the natural angular frequency $\omega_{n}$.

$$
\begin{equation*}
Z=\frac{\omega^{2} M_{e}}{M\left(\omega_{n}^{2}+\omega^{2}\right)} \tag{2.48}
\end{equation*}
$$

### 2.10. Conclusion

Essential wave behavior in elasto-dynamics is delineated by means of consequence and origin assessment. As main topic "pile installation effects on adjacent pipe structures in the elasto-dynamics", distinctive literature investigation is worked out to a chapter of the report. Finally advise from the collected literature review can be contrived.
All wave movements can find their base in the D'Alambert solution. P-waves contain the most input energy (of P, S-and R-waves), according to Wolf [126], and exist of the highest propagation velocity. As both factors have great impact on the nearby pipe construction, great care needs to be taken when P-waves are the dominant wave types. Situations where body waves progress near the surface, their wave energy is quickly paved out making them irrelevant to consider in this thesis. Circumstances where pipe constructions place further than the critical distance, where emergence of Rayleigh waves takes place, application of the graphical relation proposed by Richard et al. [104] should be applied (over the depth).
Total damping of waves arises from both a materialistic role as well as a geometric damping process. Restrictions of buried constructions, by the Eurocode 3 [2] with a maximum allowable PPV of $25 \mathrm{~mm} / \mathrm{s}$, are strongly correlated to the quantity of the damping process. Supplementary components that control this process are: the method of installation; soil properties; layering; whether or not resonance occurs.

## Vibration estimation methods

Development of estimation methods for vibrations induced by pile driving, with the aim to get a better insight into the problem, are practiced for centuries (as presented by by Deckner et al. [35]). The complexity of the problem is associated with the difficulty to include all aspects of the situation into a model, what can lead to uncertainties. According to Waarts [122] aspects like model parameters as well as the mechanism of wave propagation lead to these uncertainties with factors of uncertainty for the models differing between 3-15 for (sheet)pile driving. This chapter outlines the most relevant models (related to this thesis) developed for the prediction of waves induced by vibratory pile driving (see Figure 3.1) leading to the model choice.


Figure 3.1: Focus of the chapter in conjunction to the total master thesis research model: Vibration estimation methods, modified after Deckner [34]

### 3.1. Empirical methods

Organizationally the energy-based empirical methods were developed to predict vibrations caused by blasting. They were modified to predict the vibration pollution caused by pile installations. This section outlines empirical methods most commonly used in practice.

### 3.1.1. Attewell and Farmer method

The method developed by Attewell and Farmer [13], mentioned in Massarsch and Fellenius [80], is a very conservative energy-based method that relates the vibration velocity $v(\mathrm{~mm} / \mathrm{s})$ to an empirical factor $k\left(\mathrm{~m}^{2} / \mathrm{s}\right.$ $\sqrt{J}$ ) obtained from field results and is described by Equation 3.1. The results of the field test for the $k$-factor
are shown in Figure 3.2. This figure shows a very scattered plot of the measuring results and therefore leads to no direct correlation.

$$
\begin{equation*}
v=k \frac{\sqrt{W}}{r} \tag{3.1}
\end{equation*}
$$

In the relation described by Equation $3.1 W(\mathrm{~J})$ relates to the energy input at the source and $r(\mathrm{~m})$ the distance from the pile. One of the limitations of this method is that the distance $r$ used in Equation 3.1 is not defined properly. Meaning that it can be either the distance from the pile toe to the receiver or the direct distance from the pile to the measuring equipment. Most used in practice is the direct distance from the pile to the receiver


Figure 3.2: Results obtained from field measurements for the empirical k-factor according to Massarsch [80]
leading to very conservative results. Visualized by Massarsch and Fellenius [80] in Figure 3.3 is the distance from the pile toe (the source) to the receiver giving a more realistic representation of the method. Massarsch and Fellenius state that: "Consequently, the horizontal distance at the ground surface is often chosen for the predictions, neglecting the fact that in most cases the source of vibrations is either located along the pile shaft and/or at the pile toe" Massarsch and Fellenius [80]. From Figure 3.3 can be concluded that the method gives consistently lower results than the measured data, but shows a more or less similar line shape. Note that the data used for the comparison are driven piles and therefore NOT vibratory driven piles. A downside of this method is then also that it does not make a distinction for the method of pile installation or pile type. Attewell [14] showed that this method relates to a $31 \%$ chance of exceeding the predicted vibration level, requiring an additional safety factor to be applied on the predicted level moving the prediction towards the upper limit.


Figure 3.3: Comparison of ground vibrations at 3 different distances from the pile between measured ground vibrations and calculation of ground vibrations (according to Equation 3.1 with $\mathrm{k}=0.75$ ) according to Massarsch [80]

### 3.1.2. Wiss method

A revised version of the relation drawn out in Equation 3.1, Woods [127] is made by Wiss [125]. The relation shown in Equation 3.2 proposed by Wiss relates the wave velocity $v(\mathrm{~mm} / \mathrm{s})$ the same empirical factor $K\left(\mathrm{~m}^{2} / \mathrm{s}\right.$ $\sqrt{J}$ ) as used in Equation 3.1, but an extra attenuation rate factor $n(-)$ is added. In the equation E is equal to W (of Equation 3.1) and D equal to r. The lack of description of the driving energy $E$ and the definition of the distance $D$ are also present in this relation. "Equation 3.2 is ambiguous as the value of the exponent, n , affects K. This makes the comparison of the K-values impossible for different attenuation rates" Massarsch
and Fellenius [80]. Another downside of this method, which also holds for the method of Attewell and Farmer [13], is that it only takes account for one source for the wave development, whereas real wave development comes from the toe of the pile as well as the shaft.

$$
\begin{equation*}
v=K\left(\frac{D}{\sqrt{E}}\right)^{-n} \tag{3.2}
\end{equation*}
$$

The results of the calculation are compared with the same measurements as drawn out in Figure 3.3 and are shown in Figure 3.4. From these results can be concluded that the vibrations near the pile are better predicted than with the method used by Attawell and Farmer [13] (shown in Equation 3.1). The other two locations, 20 meters and 40 meters from the pile respectively, show a similar performance. [Note that for both Figure 3.3 and Figure 3.4 the shortest distance from the pile TOE is taken and so not the shortest distance on surface level (this would only give a strait line and is not the correct distance in comparison with the measured data)].


Figure 3.4: Comparison of ground vibrations at 3 different distances from the pile between measured ground vibrations and calculation of ground vibrations (according to Equation 3.2 with $\mathrm{k}=0.75$ and $\mathrm{n}=1.5$ ) modified after Massarsch [80]

### 3.1.3. Handboek damwanden model (CUR 166)

The "Sheet pile handbook" CUR publication 166 presents a method based on 250 vibration measurements located all over the Netherlands. By definition of 7 zones across the Netherlands all different soil profiles are categorized. The method is valid for vibratory pile driving as impact driven pile driving and includes adjustments for the eccentric driving force $F(\mathrm{kN})$ of the vibrating hammer.

$$
\begin{equation*}
u(r)=u_{0} \sqrt{\frac{r_{0}}{r}} e^{-\alpha\left(r-r_{0}\right)} e^{0.7 \beta V_{0}} \tag{3.3}
\end{equation*}
$$

The empirical relation is shown in Equation 3.3 and relates the vibration velocity $u(\mathrm{~mm} / \mathrm{s})$ (in the rest of this thesis defined as $v$ ) to the reference distance $r_{0}(\mathrm{~m})$ set at 5 meters, the measurement distance from the source $r(\mathrm{~m})$, the reference velocity $u_{0}$ described by Equation 3.4 and empirical factors $\alpha(\mathrm{m}), \beta(-)$ and $V_{0}(-)$.

$$
\begin{equation*}
u_{0}=u_{0.350}+0.002(F-350) \tag{3.4}
\end{equation*}
$$

The vibration measurements taken at 250 sites in the Netherlands resulted in the values for the empirical factors used in the equations and are shown in Table 3.1 and Table 3.2.
Advantages of this method are the application to vibratory pile driving (instead of just installation by means of driving) and the fact that the vibration predictions are based on practice (in the form of vibration measurements gather from 250 sites across the Netherlands). On the other hand this also enlightens the downside of the method regarding the application to sites outside the Netherlands. This is simply not possible (unless prove to similar ground conditions can be made, what still leads to a low safety guarantee/high risk). According to the extensive study applied by Ramkisoen [100] this method of vibrating prediction is a very conservative method for vibration predictions (in the case of the study applied by Ramkisoen [100], predictions were a factor 8 higher than the measured data).

| Soil Profile | $u_{0}(\mathbf{m m} / \mathbf{s})$ |  | $\alpha(\mathbf{m})$ |  | $V_{0}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Vert | Hor | Vert | Hor | Vert | Hor |
| 1. (Amsterdam) | 1.1 | 1.6 | 0 | 0 | 0.9 | 1.5 |
| 2. (Eindhoven) | 1.9 | 2.6 | 0 | 0 | 1.1 | 0.8 |
| 3. (Groningen) | 1.7 | 0.9 | 0 | 0 | 1.8 | 0.5 |
| 4. (The Hague) | 1.9 | 2.6 | 0 | 0 | 1.1 | 0.8 |
| 5. (Maasvlakte) | - | - | - | - | - | - |
| 6. (Rotterdam) | 1.1 | 1.6 | 0 | 0 | 0.9 | 1.5 |
| 7. (Tiel) | 1.1 | 1.6 | 0 | 0 | 0.9 | 1.5 |

Table 3.1: $u_{0}, \alpha$ and $V_{0}$ obtained from field tests for hammers up to 350 kN according to CUR 166 [1]

| Probability of exceedance | $\beta$-value |
| :---: | :---: |
| 0.5 | 0.0 |
| 0.1 | 1.18 |
| 0.05 | 1.64 |
| 0.01 | 2.32 |
| 0.005 | 2.57 |
| 0.001 | 3.09 |

Table 3.2: $\beta$ obtained from field tests and related to the probability of exceedance according to CUR 166 [1]

### 3.1.4. Attewell's renewed method

In the paper published in 1992 by Attewell [14] a new method describing an empirical method based on field measurements is made. Here with the help of a statistical analysis a quadratic regression curve is made to describe vibrations as a results of vibratory pile driving. According to Attewell [14] a quadratic regression curve rather than a linear regression curve provides a better visual fit to the several data sets. The linear regression curve Attewell [14] is referring to, is the method described by Attewell and Farmer [13] and also explained in detail in the paper by Massarsch [80]. "By assuming a normal distribution of the data points this reduction of risk can be quantified in terms of a probability of occurrence. The distribution of substantial sets of data for both impact hammers and vibro-drivers has shown quite clearly the shortcomings of a linear regression fit over a range of scaled distance" Attewell [14]. Due to the shape of the scattered field data measured of vibrations caused by vibratory pile driving Attewell [14] suggests to use the quadratic regression curve to make a good estimation of the vibrations at particular distance from the pile. The three lines shown in Figure 3.5 and Figure 3.6 represent the mean regression curve (the middle line) fitting to the data, the lower limit (the bottom line) and the upper limit (the upper line). The two most outward lines are to be defined by the contractor since they define the limits of risk to be taken in a prediction. The middle line on the other hand has $31 \%$ of the data higher then the line and resulting data are lower (this is for both the quadratic and linear regression curve). This implies that the half-standard deviation line is used. For the so called 'best fit line' (relating to the line that fits the data best) $50 \%$ lies above and below the line and with the use of the one-standard deviation line $16 \%$ lies above the line. Attewell [14] states that the one-standard deviation line should be used with construction works implying high risk, the half-standard deviation line should be used for normal construction works.

$$
\begin{equation*}
\log (v)=x_{1}+x_{2} \log \left(\frac{\sqrt{W_{0}}}{r}\right)+x_{3} \log ^{2}\left(\frac{\sqrt{W_{0}}}{r}\right) \tag{3.5}
\end{equation*}
$$

The quadratic regression curve can be expressed by a relation drawn out in Equation 3.5. This formula relates the vibration velocity $v(\mathrm{~mm} / \mathrm{s})$ to the input energy $W_{0}(\mathrm{~J})$, the distance between the source and an arbitrary point $r(\mathrm{~m})$ and three constants of proportionality $x_{1}, x_{2}$ and $x_{3}(-)$ expressed in Table 3.3. This method

| Curve fit | $x_{1}$ | $x_{2}$ | $x_{3}$ |
| :---: | :---: | :---: | :---: |
| Best fit | -0.464 | 1.64 | -0.334 |
| Half a standard deviation | -0.213 | 1.64 | -0.334 |
| One standard deviation | 0.038 | 1.64 | -0.334 |

Table 3.3: Values of $x_{1}, x_{2}$ and $x_{3}$ for vibratory pile driving according to Attawell [14]
implies not to contribute in lower risk, since this is to be determined by the contractor, but assumes to be a better estimation for the prediction of vibrations caused by vibratory pile driving.


Figure 3.5: Quadratic regression curve fitted for field data measurements of vibratory pile driven vibrations [14]


Figure 3.6: Linear regression curve fitted for field data measurements of vibratory pile driven vibrations [14]

### 3.2. Engineering methods

Empirical methods are based on measured data and build up on that to predict the result. This method on the other hand, developed by Massarsch and Fellenius [80], uses an analytic approach to describe the contributing factors in wave propagation by means of impact pile driving. Hammer-pile as well as pile-soil interaction are taken into account resulting in nearly the only method who accounts for hammer impact forces on the pile (related to wave propagation within the pile). Other methods, like the method described by Equation 3.1, use only one wave origin (namely the pile toe or just the pile at surface level). According to Massarsch and Fellenius [80] waves originate from the pile toe as well as the pile shaft. Therefore they included the wave contribution from the pile toe as well as the pile shaft in their method by means of separatism (meaning that at the source the origin of the wave can be determined). In this way a good understanding of the wave propagation through the pile to the soil body as well as the wave propagation through the soil body itself is possible. This method on the other hand is only applicable for impact pile driving. Whenham [124], Deckner [34] described the same method but then also applicable for vibratory pile driving. One other downside of this method is that reflection and refraction in between layer interfaces as well as amplification effects (superposition of waves or frequency cancellation of waves) are not taken into account. To underline the importance


Figure 3.7: Wave signals produced by pile driving methods, modified after Deckner [34]
of the differences between wave signals resulting from impact and vibratory pile driving Figure 3.7 is given. Clearly the signals produced by impact pile driving differ very much from the repetitive cycle produced by vibratory pile driving.

### 3.2.1. Massarsch and Fellenius method

"It is shown that the energy-based, empirical approach, which is still widely used by practicing engineers, is too crude for reliable analysis of ground vibrations and can even be misleading" Massarsch and Fellenius [80]. According to Massarsch and Fellenius [80] the development of a new method based on the three wave types produced by piles when installed by impact hammering gives a better and more reliable estimation on vibrations.
The three wave types produced by pile installation are:

- Spherical waves: Also called P-waves, produced by the pile toe.
- Cylindrical waves: Also known as S-waves, occur by means of the pile shaft friction with the soil.
- Surface waves: Also known as R-waves, when body waves come in contact with the surface refraction will produce R-waves at a critical distance from the pile (see further on for more detailed explanation).

According to Massarsch and Fellenius [80] impedance, for each component involving wave propagation, is the most important factor in the analysis of ground vibration related problems. The impedance of the hammer, pile and soil are therefore included in the model and interact with each other to describe the wave propagation from the hammer to the pile into the soil. Figure 3.8 gives a schematic representation of the hammer-pile and pile-soil interaction scheme used as a guideline for the model buildup.


Figure 3.8: Schematic representation of hammer-pile and pile-soil interaction scheme according to Massarsch and Fellenius [80]

## Hammer impact force on pile

According to Massarsch and Fellenius [80] the impact force of the hammer $F_{i}$ on the pile is describe by Equation 3.6 and represents the relation between the pile impedance $Z^{P}$ (see Equation 3.7) and the pile velocity $v^{P}(\mathrm{~mm} / \mathrm{s})$ at the time of impact. The cross-sectional area $A^{P}\left(\mathrm{~m}^{2}\right)$, use in Equation 3.7, relates to the speed of stress wave in the pile $c^{P}(\mathrm{~m} / \mathrm{s})$ and the material density of the pile $\rho^{P}\left(\mathrm{~kg} / \mathrm{m}^{3}\right)$.

$$
\begin{gather*}
F_{i}=Z^{P} v^{P}  \tag{3.6}\\
Z^{P}=A^{P} c^{P} \rho^{P} \tag{3.7}
\end{gather*}
$$

The impact velocity $v_{0}$ of the hammer with the pile is described by Equation 3.8 [80] and depending on the height $h(\mathrm{~m})$ it is dropped from and the gravitational acceleration $g\left(\mathrm{~m} / \mathrm{s}^{2}\right)$.

$$
\begin{equation*}
v_{0}=\sqrt{2 g h} \tag{3.8}
\end{equation*}
$$

At the moment of impact a stress wave is generated in the pile and in the hammer itself. The hammer velocity $v^{H}$ will decrease as the pile will accelerate resulting in a velocity $v^{P}$. The forces at the moment of impact are in equilibrium meaning that the force in the hammer, described in a similar way as Equation 3.6, equals the force in the pile. Equation 3.9 [80] describes the equilibrium relation between the hammer and the pile.

$$
\begin{equation*}
Z^{H} v^{H}=Z^{P} v^{P} \tag{3.9}
\end{equation*}
$$

The impedance of the hammer $Z^{H}$ is different for every hammering machine and is known by the producers of the machine. $v^{H}$ represents the particle velocity moving back up the hammer on the moment of impact and is used to describe the particle velocity within the pile as a result of the impact described by Equation 3.10. In agreement with Massarsch and Fellenius [80] this relation results from the fact that the decrease of velocity of the hammer is equal to the increase in velocity of of the pile.

$$
\begin{equation*}
v_{0}-v^{H}=v^{P} \tag{3.10}
\end{equation*}
$$

When substitution of Equation 3.9 and Equation 3.10 are performed the pile velocity $v^{P}$ can be expressed in terms of the pile impedance $Z^{P}$, the hammer impedance $Z^{H}$ and the hammer velocity at impact $v_{0}$ and is described by Equation 3.11.

$$
\begin{equation*}
v^{P}=\frac{v_{0}}{1+\frac{Z^{P}}{Z^{H}}} \tag{3.11}
\end{equation*}
$$

The length of the wave is influenced by the impact time of two objects. In this case the impact time of the hammer $t^{H}$ (s) with the pile (Equation 3.12). According to Massarsch and Fellenius [80] the impact time of the hammer equals the time the wave travels from the top of the hammer to the bottom and back up (meaning the wave travels a distance of two times the length of the hammer $L^{H}(\mathrm{~m})$ ) and is therefore also dependent on the stress-wave velocity in the hammer $c^{H}(\mathrm{~m} / \mathrm{s})$. When the wave is reflected at the bottom of the hammer it will be converted to a tension wave, resulting in an upward movement of the hammer.

$$
\begin{equation*}
t^{H}=\frac{2 L^{H}}{c^{H}} \tag{3.12}
\end{equation*}
$$

## Pile wave propagation

The pile, that acts as an elastic rod where the longitudinal stress wave can pass through, reacts to the impulse given by the hammer. According to Deckner [34] depending on the boundaries of the pile i.e. pile head, pile toe and pile shaft behavior and the cross-sectional area the stress wave is influenced. The wave will travel as a compression wave down the pile where it is reflected at the pile toe and converted to a tension wave upwards. The travel time $t^{P}$ (s) of the wave through the pile and back up is exerted in Equation 3.13. This relation is similar to Equation 3.12 describing the travel time $t^{H}(\mathrm{~s})$ of a wave trough the hammer. In this equation the stress-wave velocity $c^{P}(\mathrm{~m} / \mathrm{s})$ within the pile relates to the length of the pile $L^{P}(\mathrm{~m})$.

$$
\begin{equation*}
t^{P}=\frac{2 L^{P}}{c^{P}} \tag{3.13}
\end{equation*}
$$

## Pile-soil wave transitivity

The soil will absorb wave energy from the hammer impact as result of the vibration wave traveling through the pile. Deckner [34] explains that the energy from the hammer impact is partially converted into kinetic energy, resulting in pile movement/penetration and dissipation of energy to the boundaries of the pile (meaning the soil body). Whenham [124] states that the energy dissipation to the surrounding soil is in the order of 50 to $60 \%$. The energy dissipation of pile installation is depending on:

- Shaft and toe vibration transmission efficacy: In agreement with Massarsch and Fellenius [80] the dynamic soil resistance of the soil, depending on the soil impedance adjacent to the pile shaft, governs a big part of the energy transmitted to the surrounding soil. The ratio of the dynamic soil resistance to the force applied by the hammer is called 'vibration transmission efficacy' $E_{S}(-)$ (Equation 3.17) for the shaft and $E_{T}(-)$ (Equation 3.18) for the toe of the pile.
- The hammer: A longer hammer will result in a longer wave propagating through the pile. The length of the wave, so the contact surface from the pile to the soil $S^{P}$ (Equation 3.15), governs by itself the transmission efficacy of vibration waves from the pile shaft to the surrounding soil (see Equation 3.17 for the relation describing this phenomena).


Figure 3.9: Strain softening factor/shear wave speed reduction factor, $R_{C}$ as function of plasticity index, $I_{P}$, for different conditions of penetrations resistance according to Massarsch and Fellenius [80]

Dynamic behavior of soil, as a result of pile installations with an impact or vibration hammer result in a dynamic reaction force of the soil governed by the dynamic soil resistance along the shaft $R_{S}(\mathrm{kN})$ described by Equation 3.14 and the dynamic soil resistance $R_{T}(\mathrm{kN})$ at the toe of the pile (see also Figure 3.8).

$$
\begin{equation*}
R_{S}=R_{c} R_{R} z_{s} v^{P} S^{P} \tag{3.14}
\end{equation*}
$$

This soil property is dependent on the specific soil impedance for shear waves $z_{s}\left(\mathrm{kNs} / \mathrm{m}^{3}\right)$ (Equation 2.29), the particle velocity of the pile $v^{P}(\mathrm{~m} / \mathrm{s})$ (Equation 3.11) and the contact area between the pile and the shaft $S^{P}\left(\mathrm{~m}^{2}\right)$ (Equation 3.15), which depends on the length of the stress wave propagating through the pile $L^{W}(\mathrm{~m})$ (Equation 3.16) and the pile diameter $b^{P}(\mathrm{~m})$. Both $R_{c}(-)$ (see Figure 3.9) and $R_{R}(-)$ are reduction factors related to soil behavior, representing the reduction factor accounting for strain softening and the remoldiation/disturbance of the soil (the disturbance of the soil due to pile installation), respectively. "A typical upper range of reduction factor for disturbance or remolding (which will provide conservative estimates of ground vibrations) is in the range 0.2 to $0.4^{\prime \prime}$ Massarsch and Fellenius [80].

$$
\begin{gather*}
S^{P}=\pi b^{P} L^{W}  \tag{3.15}\\
L^{W}=t^{H} c^{P} \tag{3.16}
\end{gather*}
$$

The travel time $t^{H}(\mathrm{~s})$ for the wave through the hammer relates to the wave length in the pile $L^{W}(\mathrm{~m})$ through Equation 3.16, since the contact time of the hammer with the pile is equal to the travel time $t^{H}$ and governs the wave length $L^{W}$.

$$
\begin{equation*}
E_{S}=\frac{R_{S}}{F_{i}}=2 R_{c} R_{R} \frac{c_{s}}{c^{P}} \frac{\rho_{\text {soil }}}{\rho^{P}} \frac{L^{W}}{b^{P}}=2 R_{c} R_{R} \frac{z_{s}}{z^{P}} \frac{L^{W}}{b^{P}} \tag{3.17}
\end{equation*}
$$

The pile-soil energy transmission ratio at the pile toe is described by Equation 3.18, also known as 'vibration transmission efficacy' of the toe $E_{T}(-)$ and for the shaft $E_{S}(-)$ (described by Equation 3.17.

$$
\begin{equation*}
E_{T}=\frac{R_{T}}{F_{i}}=2 R_{R} \frac{c_{P}}{c^{P}} \frac{\rho_{\text {soil }}}{\rho^{P}}=2 R_{R} \frac{z_{P}}{z^{P}} \tag{3.18}
\end{equation*}
$$

The dynamic resistance of the pile toe $R_{T}(\mathrm{kN})$ (Equation 3.21) is governed by a remoldiation/disturbance factor $R_{R}(-)$, just like the pile shaft is, but different values are assigned to this factor (course-grained soils undergo compaction disturbance and relate to $R_{R}=2$, in over-consolidated clay the stiffness will gradually decrease and therefore $R_{R}=0.2-0.5$ is applied).

$$
\begin{equation*}
J_{c}=2 \frac{Z_{P}}{Z^{P}}=2 \frac{z_{P}}{z^{P}} \tag{3.19}
\end{equation*}
$$

Here the soil impedance for P-waves at the pile toe $Z_{P}(\mathrm{kNs} / \mathrm{m})$ is calculated with the use of Equation 3.20. The cross-sectional area of the pile toe $A^{P}\left(\mathrm{~m}^{2}\right)$ relates to the velocity of the P-wave in the soil $c_{P}(\mathrm{~m} / \mathrm{s})$ and the specific weight of the soil $\rho_{\text {soil }}\left(\mathrm{kg} / \mathrm{m}^{3}\right)$.

$$
\begin{equation*}
Z_{P}=A^{P} c_{P} \rho_{\text {soil }} \tag{3.20}
\end{equation*}
$$

Furthermore the dynamic resistance of the pile toe is dependent on a damping factor $J_{c}(-)$ described by Equation 3.19 relating the pile and (specific) soil impedance. An other factor of influence is the particle velocity of the pile $v^{P}(\mathrm{~m} / \mathrm{s})$ drawn out in Equation 3.11.

$$
\begin{equation*}
R_{T}=R_{R} J_{c} Z^{P} v^{P} \tag{3.21}
\end{equation*}
$$

" [Note, the two impedance symbols, $Z^{P}$ and $Z_{P}$, the pile impedance and the soil impedance, respectively, can easily be confused with each other]" Massarsch and Fellenius [80].

## Propagation of waves through the soil

As indicated earlier in this chapter three type of waves propagate through the soil specimen: spherical waves (P-waves) emitted from the toe of the pile, cylindrical waves (S-waves) developed by the pile shaft and surface waves (R-waves) due to refraction of P and S-waves at the surface level. Massarsch and Fellenius [80] state


Figure 3.10: Illustration of vibrations emitted during pile driving at the pile toe and along the pile shaft as stated by Massarsch and Fellenius [80]
that there is a critical distance $d_{\text {crit }}(\mathrm{m})$ (Equation 3.23) from the pile where the spherical waves developed by the pile toe will be transformed into surface waves due to refraction.

$$
\begin{equation*}
\Theta_{c r i t}=\arcsin \left(\frac{c_{S}}{c_{P}}\right) \tag{3.22}
\end{equation*}
$$

This critical distance is depending on the critical angle $\Theta_{\text {crit }}$ (degrees) (Equation 3.22) and the penetration depth of the pile $D$ and is visualized in Figure 3.10. The factors $c_{S}(\mathrm{~m} / \mathrm{s})$ and $c_{P}(\mathrm{~m} / \mathrm{s})$ drawn out in Equation 3.22 represent the S-wave and P-wave velocities respectively.

$$
\begin{equation*}
d_{c r i t}=\tan \left(\Theta_{c r i t}\right) D \tag{3.23}
\end{equation*}
$$

As stated earlier in this chapter the material coefficient $k$, used in Equation 3.1 influences the particle velocity of the waves. This parameter can be worked out for spherical waves $k_{S}\left(\sqrt{\frac{m^{2}}{k g}}\right)$, relating to Equation 3.24 and cylindrical waves $k_{c}\left(\sqrt{\frac{m}{k g}}\right)$ worked out in Equation 3.25.

$$
\begin{align*}
& k_{S}=\frac{1}{\sqrt{2 \pi \rho \lambda}}  \tag{3.24}\\
& k_{c}=\frac{1}{\sqrt{\pi \rho \lambda h_{c}}} \tag{3.25}
\end{align*}
$$

The difference in units between the parameters, making it not entirely possible to compare them not one-to-one, is related to the source of the wave. The spherical waves are related to a 'point-source' (the pile toe), whereas the cylindrical waves are produced by a 'line-source' (the pile shaft). Therefore the parameter indicating the cylinder height $h_{c}(\mathrm{~m})$ is not included in the formula determining the k -factor for spherical waves. In both relations describing the k -factor $\rho\left(\mathrm{m}^{3} / \mathrm{kg}\right)$ relates to the unit weight of the soil.

$$
\begin{equation*}
\lambda=\frac{c}{f} \tag{3.26}
\end{equation*}
$$

The length of the wave $\lambda(\mathrm{m})$, described by Equation 3.26 according to Wersall [123], is incorporated in both equations as well and relates to the frequency of the wave $f(\mathrm{~Hz})$ and the wave propagation speed $c(\mathrm{~m} / \mathrm{s})$. As explained earlier in this literature review, when waves impinging with a free surface (in most cases the ground surface) they will be reflected and refracted. Massarsch and Fellenius [80] describe a simplified method for P-waves (coming from the pile toe), after Bodare (2005) where the amplification factor in vertical direction $F_{v}$ $(-)$ as well as in horizontal direction $F_{h}(-)$ can be determined according to Equation 3.27 and Equation 3.28 respectively. "When waves are reflected at a surface, vibration amplification depends on dynamic characteristics of the wave, angle of reflection and shape of the surface. On a plane, free surface, the vibration amplitude is doubled if the incident angle is perpendicular to the surface" Wersall [123].

$$
\begin{align*}
& F_{\nu}=2 \frac{\cos \left(\theta_{P}\right) \cos \left(2 \theta_{S}\right)}{s^{2} \sin \left(2 \theta_{P}\right) \sin \left(2 \theta_{S}\right)+\cos ^{2}\left(2 \theta_{S}\right)}  \tag{3.27}\\
& F_{h}=2 \frac{\cos \left(\theta_{P}\right) \sin \left(2 \theta_{S}\right)}{s^{2} \sin \left(2 \theta_{P}\right) \sin \left(2 \theta_{S}\right)+\cos ^{2}\left(2 \theta_{S}\right)} \tag{3.28}
\end{align*}
$$

These relations are related to the angle of incidence for the P-wave $\theta_{P}$ (degrees) as well as the S -wave $\theta_{S}$ (degrees), since the angle of incidence influences the reflection and refraction process at the surface interface. The incident angle is the angle with respect to the vertical. The factor $s(-)$ is the ratio of sines for angles of incidence of the P-wave and the S-wave and is described by Equation 3.29. This ratio can either be described by the ratio between the angles of incidence or by a relation according to the Poison's ratio of the soil $v(-)$ (which by itself influences the angle of incidence of the waves). Both the vertical and horizontal amplification factors are shown in Figure 3.11 and Figure 3.12 with a variable incident angle for the P-waves and Poisson's ratios $0.25,0.30,0.35$ and 0.49 respectively.

$$
\begin{equation*}
s=\frac{\sin \left(\theta_{S}\right)}{\sin \left(\theta_{P}\right)}=\sqrt{\frac{1-2 v}{2(1-v)}} \tag{3.29}
\end{equation*}
$$

From these figures can be concluded that the Poisson's ratio $v$ has a significant influence on the horizontal


Figure 3.11: Variation of vibration amplification factor $F_{\nu}$ with respect to the angle of incidence $\theta_{P}$ for P-waves modified after Massarsch and Fellenius [80]
amplification factor, but not so much on the vertical. From this fact can then also be concluded that for clays with a Poisson's ration of about $v=0.50$ the horizontal amplification factor can be neglected. Further more the horizontal amplification factor only accounts for impact angles $\theta_{P}$ between 30-85 degrees and for the vertical component this is in the range of 0-60 degrees.
Finally the vertical components of the wave velocities for spherical $v_{S v}(\mathrm{~mm} / \mathrm{s})$ as well as cylindrical $v_{C v}$ ( $\mathrm{mm} / \mathrm{s}$ ) waves can be determined according to Equation 3.30 and Equation 3.31 respectively.

$$
\begin{gather*}
v_{S \nu}=k_{s} F_{\nu} E_{T} \frac{\sqrt{F^{H} W_{0}}}{r_{r}} \cos \left(\theta_{P}\right)  \tag{3.30}\\
v_{C \nu}=k_{c} E_{S} \frac{\sqrt{F^{H} W_{0}}}{\sqrt{r_{c}}} \tag{3.31}
\end{gather*}
$$



Figure 3.12: Variation of vibration amplification factor $F_{h}$ with respect to the angle of incidence $\theta_{P}$ for P-waves modified after Massarsch and Fellenius [80]

In these formulations the material coefficients $k_{S}\left(\sqrt{\frac{m^{2}}{k g}}\right)$ and $k_{c}\left(\sqrt{\frac{m}{k g}}\right)$ are determined according to Equation 3.24 and Equation 3.25 respectively; the vertical amplification factor $F_{\nu}(-)$ with use of Equation 3.27; the vibration transmission efficacy for the toe $E_{T}(-)$ and for the shaft $E_{S}(-)$ are determined by Equation 3.18 and Equation 3.17; the hammer efficiency factor $F^{H}$ can be determined according to table 7 of Massarsch and Fellenius [80]; the total energy $W_{0}$ (J) can be calculated with Equation 3.33; the radial distance from the pile $r_{r}(\mathrm{~m})$ as well as the horizontal distance $r_{c}(\mathrm{~m})$ both have a big impact on the outcome of the wave velocities calculated.

$$
\begin{equation*}
W_{0}(t)=\rho\left(v_{0} \sin (\omega t)\right)^{2} \tag{3.32}
\end{equation*}
$$

Equation 3.33 hands out a energy distribution over time, $t(\mathrm{~s})$. As the importance of the time aspect is not relevant when considering the maximum energy applied, a different approach method should be enforced. As the total energy consists of a combination of potential and kinetic energy, both equal to $W_{k i n}=W_{p o t}=$ $0.5 m v^{2}(\mathrm{~J})$ (where the mass is indicated by $m(\mathrm{~kg})$ and the velocity with $v(\mathrm{~m} / \mathrm{s})$ ), the total energy will be calculated according to ??. The mass used in this equation relates to the mass of the impact hammer, $m_{H}$ (kg).

$$
\begin{equation*}
W_{0}=0.5 m_{H} v_{0}^{2} \tag{3.33}
\end{equation*}
$$

### 3.2.2. Application to vibratory pile driving

"During vibratory driving, the whole system of vibrator and pile moves simultaneously up and down with the same displacement amplitude and acceleration. This means that the vibrator-pile system can be assumed to be a rigid body and that the wave propagation in a vibratory driven pile/sheet pile can be neglected" Viking [120]. The pile plus vibrator move simultaneously up and down, resulting in the same frequency and no need to integrate hammer-pile impedance analogy. Due to this fact Equation 3.17 and Equation 3.18, for the calculation of the vibration transmission efficacy of the shaft and the toe of the pile $E_{S}(-)$ and $E_{T}^{(-)}$ respectively, will be adjusted to Equation 3.34 and Equation 3.35 respectively, with the use of the vertical component of the centrifugal force $F_{\nu, v i b r}(\mathrm{~N})$ calculated with Equation 2.42.
[NOTE that when the relation stated in Equation 3.36 and Equation 3.37 is not true (resulting in no rigid body) the impact force to the pile and therefore also the vibration transmission efficacy's are calculated according to the method of Massarsch and Fellenius [80]. Furthermore care needs to be taken to the choice of disturbance factor $R_{R}(-)$ and the factor taking account for strain-softening $R_{c}(-)$. The method of installation is different (different vibration frequencies used) so that exes-pore water pressure can play a role in soil behavior and therefore also in the wave propagation behavior.]

$$
\begin{align*}
& E_{S, v i b r}=\frac{R_{S}}{F_{v, v i b r}}  \tag{3.34}\\
& E_{T, v i b r}=\frac{R_{T}}{F_{v, v i b r}} \tag{3.35}
\end{align*}
$$

The wave length and amplitude can be determined from all the formulations presented in Chapter 2. Deckner [34] and Whenham [124] present two more relations regarding the definition of a rigid body (see Equation 3.36
and Equation 3.37). Since earlier in this section is stated that wave propagation within the (sheet)pile as a result of this can be neglected.

$$
\begin{equation*}
\frac{T}{4}=\frac{1}{4 f_{d}} \geq t=\frac{4 L_{P}}{c_{P}} \tag{3.36}
\end{equation*}
$$

"The rule of thumb is that one-fourth of the time period $T(\mathrm{~s})$ for the chosen driving frequency, $f_{d}(\mathrm{~Hz})$, should be equal to or greater than the time, $t$, it takes for the stress wave to travel $4 L_{p}(\mathrm{~m})$ of the pile" Deckner [34]. A similar relation is given by Whenham [124], where $f_{n}(\mathrm{~Hz})$ is the longitudinal natural frequency of a free slender bar and $c_{P}(\mathrm{~m} / \mathrm{s})$ the longitudinal wave velocity in the pile.

$$
\begin{equation*}
f_{d} \leq 0.1 f_{n}=\frac{c_{P}}{20 L_{p}} \tag{3.37}
\end{equation*}
$$

### 3.3. Theoretical Methods

The aim for a better insight in the prediction of vibrations has led to the development of theoretical models. These models use advanced computer software to solve dynamic problems related to wave propagation and all sorts of other dynamic related problems. This section outlines the most relevant models developed for the estimation of wave induced vibrations by pile driving.

### 3.3.1. Influence zone around a closed-ended pile during vibratory driving

Ekanayake [41] investigated the effect of wave propagation on the surrounding soil when installing closedended steel piles with a vibratory hammer. The aim of the investigation was to underline the soil and vibrator properties influencing the wave propagation. The research was performed using an elastic-perfectlyplastic soil model and performed by the commercial software ABAQUS/Explicit finite element program [3] (see model in Figure 3.13). In this program an explicit central difference time integration is used to model a dynamic analysis of the vibratory pile installation. With the use of Hermite cubic shape functions the constitutive material behavior is applied to the entire grid and integrated in the general equation of motion (see Equation 4.53). "The numerical modeling technique adopted for the analysis takes into account the large soil


Figure 3.13: ABAQUS model results in a visual format according to Ekanayake [41]
deformations around the pile during driving and is based on the Arbitrary Lagrangian Eulerian technique" [41]. This method preserves the quality of the mesh during pile driving by the so called adaptive meshing technique. According to Ekanayaka [41] and ABAQUS Inc. [3] this techniques makes sure that the same number of elements in the initial mesh is preserved throughout the analysis but elements are readjusted to avoid the distortions otherwise inherent in small strain finite element procedures. Linear four-node axisymmetric elements with reduced integration (CAX4R) in the ABAQUS/Explicit are used to model the soil around the pile.
A combination of the two makes it possible to model dynamic problems without having to model a sufficient large domain to overcome the disturbance caused, at the point of interest, by the reflection of the waves at the boundaries. Ekanayaka [41] explains that the boundaries of the model are according to the principle carried out by Zienkiewicz [132] and Lysmer [74] as shown in Figure 3.14. This boundary condition was compared with the boundary condition carried out by Deeks [36]. Due to the fact that the boundary condition proposed by Deeks does not show rigid body motion and therefore makes it possible to reduce the ground settlements at the boundary to zero making this the condition choice for the final model. Du and Zhao [40] proposed a


Figure 3.14: Boundary conditions used in the ABAQUS model carried out by Ekanayaka [41], a) infinite element principle according to Zienkiewicz [132], b) the viscous wave transmitting boundaries according to Lysmer [74]


Figure 3.15: Boundary conditions used in the ABAQUS model carried out by Ekanayaka [41], the principle carried out by Deeks [36] a) Shear boundary, b) dilation boundary
wave transmittive boundary like Deeks, but then with the use of negative masses for the dilative boundary. Since ABAQUS can't handle negative boundaries for explicit calculations, this boundary is not an option. An extensive parameter study was carried out, listed below, to investigate the different factors influencing the wave propagation.

- The driving force: investigate the influence in the far field by varying the amplitude and frequency of the waves applied by the vibrator.
- Rigidity index: Changing the shearing stiffness $G$ of the soil with no change in void ratio $v$ (rigidity index $G / s_{u}$ ).
- Material damping: Verification of the magnitude of the material damping of the soil by comparison with field data carried out by [11], [65] and the effect on wave propagation.

Two different types of vibratory pile driving were used in this case study. Vibratory pile driving with frequencies of 28 and 40 Hz respectively named (normal) vibratory pile driving and driving frequencies of 80 and 150 Hz also known as resonant pile driving (according to the specifications given by ABI GMBH [5] and Resonance Technology International [56] respectively). Resonance pile driving is used when adjacent structures are highly sensitive to vibrations. This advanced form of vibratory pile driving uses high frequencies and reduced operating forces. These high frequencies, with low amplitudes, cause less damage in certain situations to adjacent structures. To give good predictions of these magnitudes and to investigate in what situations these vibratory rigs should be used, this study is performed.
From the study could be concluded that for (normal) vibratory pile driving the more critical situation is when driving the pile tip close to surface level compared to situations where the pile tip is deeper located within the soil body. Resonance vibratory pile driving relates to a higher influence zone compared to the (normal) vibratory pile driving, but with a lower amplitude of the Peak Particle Velocity (PPV). Ekanayaka [41] therefore concludes that resonance vibratory pile driving is a better suited method when dealing with vibration sensitive adjacent structures.

Since the stiffness of the soil varies with location a good insight into the influence, governed by this soil property, is mandatory. Ekanayaka [41] concludes that at lower frequencies, radial influence zone is governed by the lower rigidity indexes giving larger influence zones but for higher frequencies, radial influence zone is governed by PPVs extracted for higher rigidity indexes. For lower frequencies a high soil rigidity index relates to high PVVs near the pile and the attenuation of the PVVs is high with respect to the distance of the pile and vise-versa for low rigidity indexes of the soil. For the high frequencies there is no clear correlation discovered with respect to the influence zone.
"Rayleigh damping is applied to the soil domain in order to introduce material damping, as described in ABAQUS Inc. [3]" [41]. The model deals with both low and high frequency waves, induced by the vibratory rigs, and therefore the Rayleigh wave damping is used. In agreement with Ekanayaka [41] its properties consist of both stiffness (dealing with forces generated by strain rates of the model) and mass proportional damping (dealing with forces generated by velocities of the model). For the model carried out by Ekanayaka [41] only material damping was used, applied as damping stress $\sigma_{d}(\mathrm{kPa})$ calculated according to Equation 3.38 and Equation 3.39.

$$
\begin{equation*}
\sigma_{d}=\beta_{R} D^{e l} \dot{\epsilon} \tag{3.38}
\end{equation*}
$$

The damping stress is proportional to the viscous damping parameter $\beta_{R}$ (s) (see Equation 3.39), the elastic stiffness matrix $D^{e l}$ and the strain rate $\dot{\epsilon}$. A parametric study was carried out varying $2-20 \%$ of the critical damping, described by the fraction of the critical damping $\xi$ (\%).

$$
\begin{equation*}
\beta_{R}=\frac{2 \xi}{\omega_{1}} \tag{3.39}
\end{equation*}
$$

Equation 3.39 describes the viscous damping parameter relating the fraction of the critical damping $\xi$ (\%) to the first frequency mode of vibration $\omega_{1}(\mathrm{~Hz})$ (explained further on in this report). In comparison with field measurement data Ekanayaka [41] concluded that $2 \%$ material damping was a sufficient amount. Relations for the prediction of the Peak Particle Velocities (PPVs) were drawn out for different frequency rates described in Equation 3.40, Equation 3.41 and Equation 3.42 accounting for 40,80 and 150 Hz respectively. Here $r$ (m) describes the distance from the pile and $D(\mathrm{~m})$ the diameter of the pile. These equations are according to Athanasopoulos [11] and modified for the high frequency rates ( 80 and 150 Hz ) by Ekanayaka [41].

$$
\begin{gather*}
P V V=32 r^{-1.5}  \tag{3.40}\\
P V V=34\left(\frac{r}{D}\right)^{-1.5}  \tag{3.41}\\
P V V=25\left(\frac{r}{D}\right)^{-1.5} \tag{3.42}
\end{gather*}
$$

### 3.3.2. Dynamic soil-structure interaction formulation (EDT-model)

Masoumi [77], [78], [79] and [106] developed a method for the prediction of free-field vibrations (in the far field) due to impact and vibratory pile driving. A coupled sub-domain made it possible to simulate plastic strains developed around the pile and linear elastic behavior for the soil domain further from the pile. "During pile driving, the transmitted energy through the soil is very high and causes plastic deformations in the near field. In the far field, however, reported data show that the induced vibrations cause deformations in the elastic range" Kim [65]. A sub-domain formulation for dynamic soil-structure interaction, developed by Aubry [15], is used for the model build up. The model is build up by a coupled Boundary Element Method (BEM) model (see also Kirup [66], Pyl [99], Francois [43], Costabel [30]) Finite Element Method (FEM) model (see also Mackerele [75], Smith [108], Serarogle [107]) representing each a part of the sub-domain. The BEM model is used for the soil domain (also named far field) and the FEM model is used to simulate the pile plus the soil near the pile (also named near field) (see Figure 3.16). In the program the general equation of motion is solved to obtain the solution spectrum (see Equation 4.53). Masoumi [77] states three general points causing ground motions due to pile driving:

1. The source parameters: The method of driving governs the energy released plus the depth of the pile is of influence.
2. Interaction: The interaction between the driving equipment, the pile and the soil (this is also what Massarsch and Fellenius [80] used in their approach).


Figure 3.16: (a) Geometry of the sub-domains and (b) the scattered wave fields according to Masoumi [77]
3. Propagation: The propagation of the wave through the pile and the soil respectively.

The soil and pile impedance govern the dynamic soil-structure behavior and are therefore very important factors in the determination of the vibration response. According to Masoumi [77] the dynamic impedance of the soil is calculated by means of a boundary element formulation based on the Green's function of a horizontally stratified soil. The impedance of the pile is incorporated in the determination of the soil impedance by means of the program MISS (see [77]). Masoumi [77] made some model hypothetical assumptions as stated below:

- The soil medium is elastic with frequency independent material damping (hysteresis damping).
- No separation is allowed between the pile and soil medium.
- All displacements and strains remain sufficiently small.
- The soil stratum, used for the calculation model is horizontally layered.

From the results of the model, shown in Figure 3.17, could be concluded that the highest intensity of the waves (marked with the black color) is due to Rayleigh waves, when the wave propagates further from the pile. This is what could be expected, since Rayleigh waves experience the least damping as shown in Figure 2.13 in Chapter 2. The shear waves generated by the shaft of the pile propagate in radial direction and the cylindrical waves caused by the pile toe propagate in spherical direction what could be expected from the theory carried out by Woods [127] and shown in Figure 2.9 from Chapter 2.


Figure 3.17: The norm of the particle velocity in a homogeneous half space due to vibratory pile driving at 20 Hz for penetration depths (a) 2 m , (b) 5 m and (c) 10 m according to Masoumi [77]

### 3.3.3. Nonlinear analysis of pile driving using the finite element method

Serdaroglu [107] investigates the effect of soil plasticity pile penetration length and hammer energy on ground induced vibrations due to pile installation by means of a numerical Finite Element Method (FEM) analysis with the use of ABAQUS software [3]. By including factors like stated below, he claims to improve the precision of the model simulation with respect to the real vibrations. With the use of Hermite cubic shape functions the constitutive material behavior is applied to the entire grid and integrated in the general equation of motion (see Equation 4.53).

- Geostatic stresses prior to the dynamic analysis
- Elasto-plastic behavior of soil
- Shear slip at the pile-soil interface

Serdaroglu [107] uses the same boundary elements as proposed by Ekanayaka [41] to minimize the calculation time needed and to convert the boundaries into non-reflective boundary elements. Also modification of the boundary conditions proposed by Liu [72] are investigated (and used as the final choice).
From the study can be concluded that the embedded length of the pile has a big influence on the area close to the pile, but no influence in the far field (as shown in Figure 3.18). The study concludes that the amplitudes


Figure 3.18: Peak particle velocity vs. scaled distance from the pile for different pile embedded lengths according to Serdaroglu [107]
in softer soils (lower stiffness) are higher than in soils with higher stiffness's, but the Peak Particle Velocity is lower for a low stiffness soil compared to high stiffness soils.
The energy transmitted from the hammer has a big impact on the PPVs of the waves generated. The higher the wave energy the bigger the magnitude of the PPV is (higher the force the higher the wave energy).
Figure 3.19 shows a graph including the magnitude of the PPVs at different depth related to different distances from the pile. This graph is of interest related to this master thesis, since it can give an insight into the magnitude of the waves at depth (from the pipe structure). It can be concluded from this figure that the PPV is highest near the pile for all depths and that the deeper you go, with respect to the toe of the pile, the higher the impact is for this particular case (where the pile toe is embedded at -18 m from surface level).


Figure 3.19: Peak vertical velocity vs. distance from the pile at different depths according to Serdaroglu [107]

### 3.3.4. Finite Element Method (Plaxis)

Whenham [124] made a comparison between measured data, he obtained from field tests for vibratory sheet pile driving and calculations performed with the commercial FEM software Plaxis [25]. This model is shown in Figure 3.20 and consists of non reflective boundaries on the bottom and right hand side of the model. These boundaries can only deal with compression waves and not with shear waves (which will be reflected back). Therefore Whenham [124] used large model dimensions, with respect to the problem geometry, so that reflection of waves would have no significant influence on the model results. Assumed is that the force is equally distributed along the pile length. The hysteretic material behavior is used to solve the general equation of motion (see Equation 4.53). In the model, plastic material behavior, with the incorporation of the hysteresis, leads to energy loss within the soil body. Due to this phenomena, non-linear material behavior is included within the constitutive behavior of the material model. This assumption is made since the main focus of the study relates to the attenuation profile of the wave in the soil. "For this kind of study, it may be expected that the error made by simulating the shaft friction by point loads is smaller than the error made by neglecting the nonlinear degrading behavior of soil at the pile-soil interface" Whenham [124]. The calculation is not only compared with the measurements carried out, but also with the model proposed by Masoumi [77] (see Figure 3.21) and therefore also consists of linear elastic soil behavior. Damping of the system is drawn out by the use of Rayleigh damping properties of the soil. From Figure 3.21 could be concluded that both models have a good agreement with the measured field data. Both numerical models show a similar model


Figure 3.20: Plaxis model according to Whenham [124]
performance with respect to the field data. A down point of the Plaxis calculation is the mesh size needed to perform the calculations without reflection disturbance in the calculation. [NOTE that all results are drawn out in a dimensionless manner. This is performed to graphically compare the results with each other]


Figure 3.21: Comparison between Plaxis model, EDT-model and field measurements at surface level according to Whenham [124], Masoumi [77]

### 3.4. Conclusion

In this chapter numerous models for the evaluation of vibrations induced by pile installations are carried out. The models are divided into three categories: 1) Empirical models; 2) Engineering models; 3) Theoretical models. Empirical models exist of relations between empirical factors obtained by laboratory or field tests. These methods mostly require a good engineering judgment and experience for application. Engineering models are developed for the application by engineers. They apply physical laws and translate them into a simplified model, which is easy to use (necessary for a quick engineering judgment).
The method carried out by Attewell and Farmer [13] describes the relation between the energy $W$ as an input of the system to the wave velocity $v$ as a result of the energy. Although this is a very quick method for the estimation of the wave velocity the method is very inaccurate (there is a $31 \%$ of under-estimating the wave velocity according to Attewell [13]) and can only be applied for impact pile driving. Also no distinction is made related to the distance $r$ that need to be used in the relation. This will lead to inaccuracies as well since the vibration estimations would not be related to the depth of the pile toe with respect to the surface level. Most commonly used in the direct distance from the pile to the source (but application of the distance between the pile toe and the source is also be a correct approach).
The method proposed by Wiss [125] shows a similar relation as model proposed by Attewell and Farmer [13]. By addition of an extra attenuation factor $n$ a better model performance was obtained with respect to field measurements. On the other hand, the down points connected to the Attewell and Farmer model [13] are of similar manner, since addition of an extra parameter does not change to limitations of the original model.
CUR 166 [1] used field measurements from all over the Netherlands to develop an empirical method for vibration predictions. This method is only applicable for the Netherlands, since the empirical factors, used in the descriptive relation, only relate to regions in the Netherlands with similar soil conditions. Another imperfection of this method is its conservative manner. Due distribution of the Netherlands in regions, safe empirical factors needed to be considered to ensure low risk.
Two decades after the proposed method by Attewell (1973) [13] a renewed method was proposed by Attewell [14]. This method build up on the existing model, but modifications by means of quadratic regression curves (instead of linear regression curves) were implemented to overcome the shortcomings of the linear regression curves used in the old model. Implementation of risk assessment led to three model regression curves related to levels of probability:

1. Best fit: The best fit/mean of measured field data. $50 \%$ of the data lie below/above this regression curve.
2. Half-standard deviation: Used for normal construction works and includes $69 \%$ of the field data.
3. One standard deviation: Regression curve involves $84 \%$ of the field data and is used when high risk structures are involved.

Model limitations are identical to the model proposed by Attewell [13] in 1973. Model improvement on the other hand is obtained by implementation of risk related probability factors.
By the use of physical laws Massarsch and Fellenius [80] achieved a method capable of handling spherical waves produced by the pile tip; cylindrical waves arising from the pile shaft friction with the soil and surface waves originating when body waves come in contact with the surface. Integration of vibrator-pile and pile-soil interaction schemes in combination with formulation of the wave propagation within the soil led to realistic model results. The use of simple physical laws including known model parameters make it possible to obtain model results without the use of empirical factors (accept the use of the remoldiation factor, which is a limitation of the model). The implementation of the critical distance, which describes the distance from the pile where Rayleigh/surface waves are developed translates into pragmatic model outcomes. "Another uncertainty is the superposition of ground vibrations during pile penetration, as the wave propagation process from different depths and sources (at different frequencies) can lead to superposition or canceling of vibration amplitudes" Massarsch and Fellenius [80]. Furthermore the effect of reflection and refraction within the soil layers is not incorporated into the model, what effects the outcome. Although this model represents a simplified approach, it gains its strength in usability and reliability due to the possibility to recognize the effect of different input parameters and their response on ground vibrations. Impact pile driving is the only application for which this model is applicable, making it not possible to use it for vibratory pile driving without modifications.
Application of the method proposed by Massarsch [80] to vibratory pile driving is suggested by Whenham [124]. The modification is applied to the vibration transmission efficacy of the shaft and the toe of the pile respectively. Care need to be taken in the choice of remoldiation/strain-softening factor since exes-pore water pressure build-up plays a role in the soil behavior related to vibratory pile driving (longer duration of the applied force leads to less dissipation of exes-pore water pressure, especially in soils with low permeability properties like clays and peaty soils). Furthermore two boundaries are stated correlating the length of the pile to the rule of a rigid body and the application to the vibration transmission efficacy. Similar limitations can be expected from this model as the method proposed by Massarsch [80].
Theoretical models use numerical techniques, applied to a geometric representation of the problem, to approximate the solution. Reliability of these models can be low since approximation of the solution with numerical methods leads to a truncation error (which in most cases is very small and does not play a role). The number of required unknown parameters can lead to unreliable model outcomes as well. Therefore a good understanding of the meaning of the parameters and their consequences on the model behavior is required to sustain reliable solutions.
Wave propagation effects on the surrounding soil caused by installation of closed-ended steel piles by means of vibratory hammering was investigated by Ekanayaka [41]. With the help of the commercial Finite Element Software ABAQUS/Explicit [3] an elastic-perfectly plastic soil model could be incorporated. Hardening and softening laws lead to a better representation of soil behavior, but are not incorporated in the model due to complexity. The application of non-reflective boundaries led to a reduction of the disturbance from reflective waves and their effect on the model results. Extensive parameter studies were carried out to investigate the sensitivity of the model to different factors (the driving force; rigidity index; material damping). The study carried out entails good understanding of the individual parameters and their sensitivity concerning the model outcome. Relations for the Peak Particle Velocity (PPV) are obtained correlated to the direct distance between the pile and the point of interest.
A coupled Boundary Element Model (BEM) Finite Element Model (FEM) approach for the investigation of wave propagation as a result of vibratory pile driving was proposed by Masoumi [77]. By using a sub-domain formulation for the dynamic soil-structure interaction both linear-elastic soil behavior in the far field and elasto-plastic soil behavior around the pile was incorporated. Model results showed the development of body waves near the pile and domination of Rayleigh wave in the far field near the surface. Furthermore the essence of the method of driving, interaction between the soil and the pile; depth of the pile and propagation of the wave through the soil were outlined as the most important factors governing the vibration emission due to pile driving.

Serdaroglu [107] developed a similar model in ABAQUS as Ekanayaka [41]. By incorporating geo-static stresses prior to the dynamic analysis and shear slip at the pile-soil interface he claims to improve the precision of the model simulation. Comparison with field data show a good similarity between the modeled and measured data. The study showed that the embedded length of the pile as well as the energy transmitted by the hammer have a big impact on the magnitude of the PVV at the point of interest.
Comparison of a non-linear material behavior of the Plaxis model and the linear material behavior of the EDTmodel with field data was carried out by Whenham [124]. The study showed that both the Plaxis and EDTmodel fit the measured data well and have a similar model performance related to the field data (although the Plaxis model includes non-linear material behavior). Due to the conclusions made by Whenham [124], the more simple linear material behavior can be used as constitutive material behavior in this Master Thesis research project.

## Dynamics of the pipe structure

Simplification of a model without reduction of performance is what every scientific technical research is aiming for. Awareness of the consequences linked to the model reductions is of great importance. In this Master Thesis project the effects of propagating pressure waves on buried pipe structures are investigated by means of analytic and numerical methods. Finding a suitable method for modeling the pipe structure is aimed for in this chapter. Important is that the pipe structure model should be developed in such a way that direct coupling with the model calculating the Peak Particle Velocity is possible. The pipe structure model will be represented in accordance with the general equation of motion as described by Equation 4.1. This chapter applies the simplification principle on the build-up of a model for a pipe in the subsurface exposed to ground vibrations. The vibrations are induced by pile installation what leads to a coupling of a vibration estimation model with the proposed models for the pipe summarized in this chapter as visualized in Figure 4.1 and Figure 3.1


Figure 4.1: Focus of the chapter in conjunction to the total master thesis research model: Dynamics of the pipe structure, modified after Deckner [34]

### 4.1. System representations

The general equation of motion, according to Clough [28], Metrikine [86], Dijk [117], Spijkers [109], Hilster [53] and de Brabander [33] is (including viscous damping) described by Equation 4.1. Single Degree Of Freedom systems (SDOF), Multiple Degree Of Freedom systems (MDOF) as well as a Continuous System (CS),
explained in further detail in this chapter, use this relation as the basis.

$$
\begin{equation*}
m \ddot{w}(t)+c \dot{w}(t)+k w(t)=q(w, t) \tag{4.1}
\end{equation*}
$$

In the solution spectrum, of the system considered, the imaginary part of the eigenvalues corresponds to the frequency of the vibration and the real part to the decrements/increments of the vibration. Instability of a linear system of vibrations is reached if at least one of the eigenvalues has a positive real part.

### 4.1.1. Single Degree Of Freedom system (SDOF)

The most straightforward model representation is in the form of a Single Degree of freedom system (SDOF). This type of model representation quantifies its characteristic behavior with the use of Equation 4.1. As the name states, only one degree of freedom (related to the movement of the system) is possible. Clough [28] states that a SDOF system can be characterized with a single lumped mass that is constrained so that it can move only in a single fixed direction and its response may be expressed in terms of a single displacement quantity $w(t)$.

### 4.1.2. Multiple Degree Of Freedom system (MDOF)

Multiple Degree Of Freedom systems (MDOF) is a system representation existing of N-degrees of freedom. This system can exist of a coupled or an uncoupled structure.

- Coupled system structure: When an element is coupled to another element; an element of the system influences more than one degree of freedom. The matrices are not symmetric and therefore solving the system as a whole is mandatory.
- Uncoupled system structure: Every element of the system refers to one degree of freedom. The system can be solved as multiple ( N x) SDOF systems, since there is no coupling between the structures present.

Exhibition of the general representation of the equation of motion for MDOF systems is ensured by Equation 4.2. Here the displacement vector $\{w(t)\}$ is influenced by individual system matrices $[M],[C]$ and $[K]$.

$$
\begin{equation*}
[M]\{\ddot{w}(t)\}+[C]\{\dot{w}(t)\}+[K]\{w(t)\}=\{q(w, t)\} \tag{4.2}
\end{equation*}
$$

### 4.1.3. Continuous system (CS)

Embodiment of a 1D-structure scheme with continuous properties throughout the entire system is entitled as a Continuous System (CS). Due to the correspondence between the system properties and their linked internal activity patterns, simplification in the form of one system of equations can be accomplished. The solution spectrum, in the form of displacement, depends both on space and time. The spacial part is linked to the position in the system affiliated to the requested answer (see Figure 4.12). Accomplishment of these type of system depictions is achieved further on in this chapter.

### 4.2. Modeling of damping

Kinetic and potential energy are involved in the process of an oscillatory deformation according to Jia [60]. This process will in any case lead to dissipation of energy in the form of thermal energy, also known as the process of damping. "Damping is the energy dissipating property of materials and members undergoing time dependent deformations and or displacements" [44]. Material behavior involves elastic and plastic behavior depending on the force applied. The proposed damping mechanisms in this section coincide with elastic material behavior used for the pipe structure itself as well as the support material, the soil. Plastic material behavior is not taken into account throughout this thesis. Dissipation of energy is the main process involved in damping and is therefore the main driving force for the models proposed in this section. Jia [60] categorized the main damping mechanism by means of physical processes involved (see Figure 4.2).

### 4.2.1. Viscous damping

"This form of damper dissipates energy by applying a resisting force over a finite displacement through the action of a piston forced through a fluid-filled chamber for a completely viscous, linear behavior" Kareem [63]. Damping with the use of the viscosity principle, named viscous damping, is conducted in case of purely elastic deformation limits. Kramer [69] and Jia [60] state that in case in-elastic material behavior, reached after


Figure 4.2: Categories of damping modeling according to Jia [60]


Figure 4.3: Figurative representation of a viscous damping element (also named dashpot) modified after Jia [60]
larger deformation has occurred, the determination of a variation in damping coefficient, which depends on the deformation amplitude, can be established by approximating a damping value corresponding to the expected deformation amplitude level at the deformation close to the linear elastic limit. Viscous damping can be used to model the response behavior of soil undergoing cyclic loading.


Figure 4.4: Responds of the system to viscous damping according to Metrikine [86]

## Linear viscous damping

Linear relation for the viscosity term $c$, established by Clough [28], is expressed in terms of the specification procedure of Equation 4.3. The expression relates the mass $m(\mathrm{~kg})$, damping coefficient $\xi(\%)$ and the natural frequency $\omega_{n}(\mathrm{~Hz})$ to the viscosity term $c$.

$$
\begin{equation*}
c=2 m \xi \omega_{n} \tag{4.3}
\end{equation*}
$$

Conclusions related to linear damping can be made by the parameter sensitivity of the model projected by means of Figure 4.5. This figure shows that without damping $\zeta=0$ the frequency ratio $\beta=\frac{\bar{\omega}}{\omega_{n}}$ (where $\bar{\omega}$ is the frequency of the load working on the system) will tend to infinity when $\beta=1$ (so when the system reaches the resonance frequency $\bar{\omega}=\omega_{n}$ ). "The ratio of the resultant harmonic response amplitude to the static displacement which would be produced by the force is called the dynamic magnification factor $D^{\prime \prime}$ Clough [28]. It is impossible to reach an infinitive large increase of frequency by resonance when applying viscous damping in the system, since the system will tend to converge to a certain equilibrium situation (see Figure 4.6).

## Rayleigh viscous damping

Determination of the viscosity term $c$, related to the dashpot principle, can be applied with the use of the Rayleigh viscous damping method. Lanzo [70], Youssef [129], Jia [60], Oller [93], Clough [28], Rao [101] state all that the definition of Rayleigh damping is the linear combined effort of mass proportional $\alpha_{R}[M]$ and


Figure 4.5: Variation of dynamic magnification factor with damping and frequency according to Clough [28]


Figure 4.6: Response to resonant loading, $\beta=1$ for at-rest initial conditions after Clough [28]
stiffness proportional $\beta_{R}[K]$ damping described by Equation 4.4.

$$
\begin{equation*}
[C]=\alpha_{R}[M]+\beta_{R}[K] \tag{4.4}
\end{equation*}
$$

The relation multiplies the Rayleigh damping parameters $\alpha_{R}$ and $\beta_{R}$ with the mass-matrix [ $M$ ] and the stiffnessmatrix $[K]$ to obtain the Rayleigh damping-matrix $[C]$. The physical definition of the Rayleigh dampingmatrix is implemented in graphical form displayed in Figure 4.7.


Figure 4.7: Variation of viscous damping as a function of period and frequency using Rayleigh damping formulation modified after Clough [28]
[Note that in Figure 4.7 the stiffness proportional part is frequency depended but follows a linear relation proportional to the frequency, whereas the mass proportional part does not.]
According to Youssef [129] both the mass $\alpha_{R}$ and stiffness proportional $\beta_{R}$ Rayleigh damping parameters can be obtained by solving the system of equations described by Equation 4.5 . (when the damping ratio $\xi$ is frequency depended). In these relations different frequencies (modes) are used, characterized by $m$ and $n$.

$$
\frac{1}{2}\left[\begin{array}{cc}
\frac{1}{\omega_{(m)}} & \omega_{(m)}  \tag{4.5}\\
\frac{1}{\omega_{(n)}} & \omega_{(n)}
\end{array}\right]\left\{\begin{array}{l}
\alpha_{R} \\
\beta_{R}
\end{array}\right\}=\left[\begin{array}{l}
\xi_{(m)} \\
\xi_{(n)}
\end{array}\right]
$$

Figure 4.8 outlines the capability of the Rayleigh damping method and delineates the frequency depended inner workings of the system described in Equation 4.4. Important to underline is, that here not only the mass proportional part is frequency dependent but also the stiffness proportional part. Youssef [129] explains that care needs to be taken in the choice of the frequency depended modes. "The Rayleigh viscous damping formulation represents an approximate solution and has some important features and limitations" [129], [28]. Caution is required since frequency values outside the domain chosen (in the case of Figure 4.8 is this ( $\mathbb{D} \mid 0.1 \leq \omega \leq 1.0$ )) will result in significantly high damping and therefore ground motion content can be filtered out. The lower limit, in the case of pile driving, corresponds with the natural frequency of the soil $\omega_{n, \text { soil }}(\mathrm{Hz})$ and the upper limit of the domain to the excitation frequency $\omega_{\text {ex }}(\mathrm{Hz})$ resulting in $\left(\mathbb{D} \mid \omega_{n, \text { soil }} \leq\right.$ $\omega \leq \omega_{e x}$ ). Higher frequencies than the excitation (as a result of resonance in the system) and lower values than the natural frequency may occur in the system, but are not significant since they will be damped out due to the corresponding high value for $\xi$. Clough [28] states that information of the frequency dependent damping


Figure 4.8: Variation of viscous damping as a function of period and frequency using Rayleigh damping formulation modified by Youssef [129] after Clough [28]
ratio is seldom available, the frequency dependent damping ratio's $\xi_{(m)}$ and $\xi_{(n)}$ are assumed to have the same control sequence leading to: $\xi_{(m)}=\xi_{(n)} \equiv \xi$. Assuming the assumed simplification the resulting frequency independent Rayleigh damping parameters can be obtained by the system described in Equation 4.6.

$$
\left\{\begin{array}{l}
\alpha_{R}  \tag{4.6}\\
\beta_{R}
\end{array}\right\}=\frac{2 \xi}{\omega_{(m)}+\omega_{(n)}}\left\{\begin{array}{c}
\omega_{(m)} \omega_{(n)} \\
1
\end{array}\right\}
$$

## Extended Rayleigh viscous damping

Clough [28] specifies that the mass and stiffness matrices used to formulate Rayleigh damping are not the only matrices to which the free-vibration mode-shape orthogonality conditions apply; in fact, it can be shown that an infinite number of matrices have this property. This leads to a general relation for the estimation of the proportional damping matrix build up out of any combination of these matrix's (see Equation 4.7).

$$
\begin{equation*}
c=m \sum_{b} a_{b}\left[m^{-1} a\right]^{b} \equiv \sum_{b} c_{b} \tag{4.7}
\end{equation*}
$$

For stiffness proportional damping a similar system of equations can be obtained for four natural frequency modes ( $m, n, o$ and $p$ ) as drawn out by Clough [28] (see Equation 4.8) corresponding to four arbitrary chosen Rayleigh damping parameters ( $\alpha_{R}, \beta_{R}, \gamma_{R}$ and $\lambda_{R}$ ). The resulting variation of viscous damping corresponding to the frequency applying the Rayleigh damping formulation is drawn out in Figure 4.9.

$$
\frac{1}{2}\left[\begin{array}{cccc}
\frac{1}{\omega_{(m)}^{2}} & \frac{1}{\omega_{(m)}^{2}} & \omega_{(m)} & \omega_{(m)}^{3}  \tag{4.8}\\
\frac{1}{\omega_{(n)}^{2}} & \frac{1}{\omega_{(n)}^{2}} & \omega_{(n)} & \omega_{(n)}^{3} \\
\frac{1}{\omega_{(o)}^{2}} & \frac{1}{\omega_{(o)}^{2}} & \omega_{(o)} & \omega_{(o)}^{3} \\
\frac{1}{\omega_{(p)}^{2}} & \frac{1}{\omega_{(p)}^{2}} & \omega_{(p)} & \omega_{(p)}^{3}
\end{array}\right]\left\{\begin{array}{c}
\alpha_{R} \\
\beta_{R} \\
\gamma_{R} \\
\lambda_{R}
\end{array}\right\}=\left[\begin{array}{c}
\xi_{(m)} \\
\xi_{(n)} \\
\xi_{(o)} \\
\xi_{(p)}
\end{array}\right]
$$



Figure 4.9: Extended Rayleigh damping after Clough [28]
[Note that: "To simplify the figure it has been assumed here that the same damping ratio, $\xi_{x}$, was specified for all four frequencies; however, each of the damping ratios could have been specified arbitrarily" Clough [28]. This also outlines the down-point of the application of this method, namely the complexity in the use (due to the difficulty in parameter determination) and the model implementation.]

### 4.2.2. Hysteretic Damping



Figure 4.10: Figurative representation of a hysteretic damping element modified after Jia [60]
In a linear hysteretic damping consideration no accumulation of strains is considered. The accumulation of strains are related to non-linear non-elastic small-strain stiffness principle as proposed by Benz [19]. Elastic behavior is adopted to model the stiffness response of the soil, to the dynamic waves induced by pile driving and the non-linear small-strain stiffness is therefore not further considered. Tatsuoka [110] outlined the relatron of the shear-strain modulus $G(\mathrm{kPa})$ to hysteretic damping and the determination of the damping ratio $\eta$ $(-)$ (see Equation 4.9).

$$
\begin{equation*}
\eta=\frac{1}{2 \pi} \frac{\Delta W}{W}=\frac{2 K_{1}}{\pi}\left(1-\frac{G}{G_{\max }}\right) \tag{4.9}
\end{equation*}
$$

Tatsuoka [110] relates the damping ratio to the dissipation of energy by the shear modulus $G$ and the maximum shear modulus $G_{\max }$ (related to elastic strains). The constant $K_{1}$ influences the size of the area $\Delta W$, as shown in Figure 4.11, which represents the energy dissipated per cycle. The stored energy (or also called elastic energy) in the soil per cycle is defined as $W$. Jia [60] defined a figurative representation of a hestertic damping element used for modeling hysteretic damping of soils. Cornejo [29] studies the phenomena hysteretic damping extensively in the reaction of a moving sinusoidal load by the model representation of an elastic half-space.

### 4.3. Models of continuous systems (CS)

For systems with similar properties throughout the 1D-domain Continuous system (CS) representation is beneficial (as explained earlier in this chapter). In this section various Continuous System models are discussed including their broad potentials as well as argument outlines are accomplished.


Figure 4.11: Stress-strain hysteresis loop for reverse loading according to Tatsuoka [110], Benz [19]

### 4.3.1. Euler-Bernoulli Beam model

Basis for most of the beam models is the Euler-Bernoulli beam approach, or also named the bending-beam approach. This theory assumes that deformations from the beam are generated by a bending motion as stated by van Dijk [117]. Boundary-conditions are essential for the beam's behavior and are therefore extensively considered in the model assumptions and initial conditions. According to Spijkers [109] these boundaryconditions will be introduced in the solution manner of the differential equations applied.

## Model assumptions

Derivation of the Euler-Bernoulli beam model is carried out by van Dijk [117], Spijkers [109] and de Brabander [33] including the following model assumptions:

1. Perpendicular plane cross-sections, with respect to the neutral axis of the beam, in initial state will remain a plane cross-section and perpendicular to the neutral axis of the beam.
2. Only small deformations can occur.
3. A linear elastic isotropic material is adopted ignoring the effect of the Poisson's ratio of the material.

## Derivation of the equations



Figure 4.12: Sign convention of the prismatic beam modified after Spijkers [109]
Derivation of the equations start with understanding the basics of the system. Therefore the normal force $N(\mathrm{~N})$, moment $M(\mathrm{Nmm})$ and the shear force $V\left(\mathrm{~N} / \mathrm{mm}^{2}\right)$ are displayed in Equation 4.10, Equation 4.11 and Equation 4.12 respectively. The stress in the direction of the positive normal force is described by $\sigma_{11}$
$\left(\mathrm{N} / \mathrm{mm}^{2}\right)$ and the stress in the upward direction is the positive shear stress $\sigma_{12}\left(\mathrm{~N} / \mathrm{mm}^{2}\right)$. All the forces and stresses in the outward direction of the paper are assumed to be zero. The situation is displayed in Figure 4.12, were the negative direction of $X_{2}$ and the positive direction of $X_{1}$ are shown.

$$
\begin{gather*}
N=\int \sigma_{11} d A  \tag{4.10}\\
M=-\int X_{2} \sigma_{11} d A  \tag{4.11}\\
V=-\int \sigma_{12} d A \tag{4.12}
\end{gather*}
$$

From the equations above can be concluded that the normal force $N$ is the stress in the normal direction integrated over the entire cross-sectional area. The moment is the stress in the normal direction multiplied by the distance $X_{2}$ (the eccentric distance the stress has with respect to the central line of the beam) and then integrated over the entire cross-sectional area. And the shear force is calculated in a similar way as the normal force, but then with the use of the shear stress (see Figure 4.13). Under the assumption that the shear force and the moment are continuous and smooth along the entire beam the equilibrium of a two closely space cross-sectional areas can be made. The equilibrium equations are stated for the normal force Equation 4.13,


Figure 4.13: Sign convention of the prismatic cross-section modified after Spijkers [109]
the moment Equation 4.14 and the shear force Equation 4.15 were the left side of the equation represents the left cross-section and the right side of the equation the right cross-section of the beam. Since the crosssections are very closely space by a distance $d X_{1}$ the left and right side are nearly equal to each other.

$$
\begin{align*}
& N \approx N+\frac{\partial N}{\partial X_{1}} d X_{1}  \tag{4.13}\\
& M \approx M+\frac{\partial M}{\partial X_{1}} d X_{1}  \tag{4.14}\\
& V \approx V+\frac{\partial V}{\partial X_{1}} d X_{1} \tag{4.15}
\end{align*}
$$

With the use of the assumptions made in 1) and 2), stating that only small deformations will occur and the planes stay perpendicular to the neutral axis, the following relations can be obtained:

$$
\begin{gather*}
\theta=\tan (\theta)=\sin (\theta)=\frac{d y}{d X_{1}}  \tag{4.16}\\
\cos (\theta)=1  \tag{4.17}\\
x_{1}=X_{1}-X_{2} \sin (\theta)=X_{1}-X_{2} \frac{d y}{d X_{1}}  \tag{4.18}\\
x_{2}=y+X_{2} \cos (\theta)=y+X_{2} \tag{4.19}
\end{gather*}
$$

The small deformation of the beam, with respect to the neutral axis, is represented as $y ; x_{1}$ and $x_{2}$ are the new positions of the beam and $X_{1}$ and $X_{2}$ the old positions respectively; the rotation of the beam element is described by the angle $\theta$. Due to the assumption made in 2), related to small deformations Equation 4.20
states the use of the small deformation matrix $\epsilon_{\text {small }}{ }^{-}$. Further more the deformation matrix $\bar{w}$ and the derivative of the deformation matrix $\nabla \bar{w}$, that are used to express the small deformation matrix, are carried out in Equation 4.21 and Equation 4.22 respectively.

$$
\begin{gather*}
\bar{\epsilon}_{\text {small }}=\frac{1}{2}\left[\nabla w+\nabla w^{T}\right]=\left[\begin{array}{ccc}
-X_{2} \frac{d^{2} y}{d X_{1}^{2}} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right]  \tag{4.20}\\
\bar{w}=\left[\begin{array}{l}
x_{1}-X_{1} \\
x_{2}-X_{2} \\
x_{3}-X_{3}
\end{array}\right]=\left[\begin{array}{cc}
-X_{2} \frac{d y}{d X_{1}} \\
y \\
0
\end{array}\right]  \tag{4.21}\\
\nabla \bar{w}=\left[\begin{array}{lll}
\frac{\partial w_{1}}{\partial X_{1}} & \frac{\partial w_{1}}{\partial X_{2}} & \frac{\partial w_{1}}{\partial X_{3}} \\
\frac{\partial w_{2}}{\partial X_{1}} & \frac{\partial w_{2}}{\partial X_{2}} & \frac{\partial w_{2}}{\partial X_{3}} \\
\frac{\partial w_{3}}{\partial X_{1}} & \frac{\partial w_{3}}{\partial X_{2}} & \frac{\partial w_{3}}{\partial X_{3}}
\end{array}\right]=\left[\begin{array}{ccc}
-X_{2} \frac{d^{2} y}{d X_{1}^{2}} & -\frac{d y}{d X_{1}} & 0 \\
\frac{d y}{d X_{1}} & 0 & 0 \\
0 & 0 & 0
\end{array}\right] \tag{4.22}
\end{gather*}
$$

The constitutive law stated in point 3) of the assumptions states that the entire beam consists of a linear elastic isotropic material. Therefore Hooke's law, as stated by Ainola [7], of isotropic behavior can be applied relating to a linear stress and strain distribution along the cross-section of the beam (see Equation 4.23).

$$
\begin{equation*}
\sigma_{11}=E \epsilon_{11}=-E X_{2} \frac{\partial \theta}{\partial X_{1}}=-E X_{2} \frac{d^{2} y}{d X_{1}^{2}} \tag{4.23}
\end{equation*}
$$

Equation 4.23 uses the fact that in Equation 4.20 only the first entry of the matix, namely $\epsilon_{11}$, consists of a non-zero term. From this relation the moment $M$ can be expressed by means of Equation 4.24 with the use of Equation 4.25 describing the moment of inertia $I\left(\mathrm{~mm}^{4}\right)$.

$$
\begin{gather*}
M=-\int X_{2} \sigma_{11} d A=E \frac{d^{2} y}{d X_{1}^{2}} \int X_{2}^{2} d A=E I \frac{d^{2} y}{d X_{1}^{2}}  \tag{4.24}\\
I=\int X_{2}^{2} d A \tag{4.25}
\end{gather*}
$$

The derivation of the shear force can be made with the use of the equilibrium of moments around a point ( $\sum M=0$ ):

$$
\begin{gather*}
M-\left(M+\frac{\partial M}{\partial X_{1}} d X_{1}\right)+\left(V+\frac{\partial V}{\partial X_{1}} d X_{1}\right)-q d X_{1} \frac{d X_{1}}{2}=0  \tag{4.26}\\
V=\frac{\partial M}{\partial X_{1}} \tag{4.27}
\end{gather*}
$$

Using Newton's Second law stating the equilibrium of forces $\left(\sum F=m a\right)$ and $\rho\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ as the material density of the beam this leads to the final relation describing the Euler-Bernoulli-beam equation of motion(see Equation 4.32). In this representation the internal self-weight of the structure is neglected (in the final representation of the equation) and the external force $q(x, t)$ is stated with a relation depending on position $x$ and time $t$.

$$
\begin{gather*}
\sum F_{y}=m a  \tag{4.28}\\
V+q(x, t) d X_{1}-\left(V+\frac{\partial V}{\partial X_{1}} d X_{1}\right)+F_{z}=\rho A d X_{1} \frac{\partial^{2} w}{\partial t^{2}}  \tag{4.29}\\
q(x, t)+\rho A\left(X_{1}\right) g-\frac{d V}{d X_{1}}=\rho A \frac{\partial^{2} w}{\partial t^{2}}  \tag{4.30}\\
q(x, t)+\rho A\left(X_{1}\right) g=\rho A \frac{\partial^{2} w}{\partial t^{2}}+\frac{\partial^{2}}{\partial X_{1}^{2}}\left(E I \frac{d^{2} y}{d X_{1}^{2}}\right) \tag{4.31}
\end{gather*}
$$

When making $X_{1}$ equal to $x$ and working out the equation the final equation of motion for an Euler-Bernoulli beam is obtained.

$$
\begin{equation*}
\rho A \frac{\partial^{2} w}{\partial t^{2}}+E I \frac{d^{4} y}{d x^{4}}=q(x, t) \tag{4.32}
\end{equation*}
$$

The generalized Euler-Bernoulli beam model considered no material damping, assuming an idealized linear elastic spring as material behavior.

### 4.3.2. Kelvin-Voigt model

When steel is considered as the construction material of the pipe, assuming a visco-elastic solid material, the Kelvin-Voigt model is a good model choice. The model represents a linear elastic perfectly plastic behavior that coincides well with the material behavior of steel. Taking into consideration that a concrete pipe is modeled, this model approach would only be valid dealing with no cracks within the material.
The model builds up on the knowledge declared by the Euler-Bernoulli beam model and extends it with a viscosity term. According to Meyers [87] and van Dijk [117] the stress-strain relation for this model is expressed the relation in Equation 4.33. Hooke's law is extended with an additional viscosity term, the Dynamic Modulus of Elasticity $E^{*}$, that is dependent on a time-related strain term. The model approach can be characterized by a parallel spring-dashpot system, representing the material damping.

$$
\begin{equation*}
\sigma(t)=\sigma_{\text {elastic }}+\sigma_{v i s c o u s}=E \epsilon(t)+E^{*} \frac{d \epsilon(t)}{d t} \tag{4.33}
\end{equation*}
$$

The equation of motion can be derived in a similar manner as performed for the Euler-Bernoulli model and is stated in Equation 4.34.

$$
\begin{equation*}
\rho A \frac{\partial^{2} w}{\partial t^{2}}+E I \frac{d^{4} y}{d x^{4}}+E^{*} I \frac{\partial}{\partial t} \frac{d^{4} y}{d x^{4}}=q(x, t) \tag{4.34}
\end{equation*}
$$

Specification of the Dynamic Modulus of Elasticity is given by the relation in Equation 4.35. This relation consists of a real part, the Elasticity modulus $E$ and an imaginary part directly proportional to the frequency $\omega$ and the viscosity index $\eta$.

$$
\begin{equation*}
E^{*}=E+i \eta \omega \tag{4.35}
\end{equation*}
$$

To emphasize the inner workings of the model recall of the basic principles for damping mechanisms is required. In addition to elastic behavior, this model applies viscous damping. The viscous damping mechanism is specified by the Dynamic Modulus of Elasticity $E^{*}$. This parameters contains a real part emphasized by the elasticity modulus $E$ and a frequency dependent imaginary part. The principle used in this model is similar to viscous Rayleigh damping (conducting the same frequency dependent imaginary part) described by Lanzo [70], Youssef [129], Jia [60] and also carried out earlier this chapter.

### 4.3.3. Timoshenko beam model

In addition to the disadvantages of the model assumptions made for the development of the Euler-Bernoulli beam model, Timoshenko [113] established a model named Timoshenko beam model (see also Chen [27], Ross [105] and Kocaturk [67]). The concept of perpendicular plane cross-sections, with respect to the neutral axis of the beam, do not necessarily stay in plane after deformation has occurred (see Figure 4.14). Therefore shear and rotational deformation phenomena are included within this model. Accumulation of shear and


Figure 4.14: Deformation of a Timoshenko beam (blue) compared with that of an Euler-Bernoulli beam (red) after Banerje [16]
rotational deformation spectra results in an additional term to the existing Euler-Bernoulli beam equation of motion.

## Quasi-static application

The supplementary discrete description of the equation of motion for a Timoshenko beam model is presented by Equation 4.36 and Equation 4.37, finally leading to the general equation described by Equation 4.38.

$$
\begin{equation*}
\frac{d^{2}}{d x^{2}}\left(E I \frac{d \phi}{d x}\right)=q(x, t) \tag{4.36}
\end{equation*}
$$

$$
\begin{equation*}
\frac{d w}{d x}=\phi-\frac{1}{\kappa A G} \frac{d}{d x}\left(E I \frac{d \phi}{d x}\right) \tag{4.37}
\end{equation*}
$$

Leading to the general equation of motion for quasi-static Timoshenko beam model:

$$
\begin{equation*}
E I \frac{d^{4} w}{d x^{4}}+\frac{E I}{\kappa A G} \frac{d^{2} q}{d x^{2}}=q(x, t) \tag{4.38}
\end{equation*}
$$

The displacement $w(\mathrm{~mm})$ is here proportional to the rotation $\phi$ (degrees); $A\left(\mathrm{~mm}^{2}\right)$ is the cross-sectional area of the beam; $E\left(\mathrm{~N} / \mathrm{mm}^{2}\right)$ is the Young's or elasticity modulus of the material (of the beam); $G\left(\mathrm{~N} / \mathrm{mm}^{2}\right)$ is the shear modulus of the material (of the beam); $I\left(\mathrm{~mm}^{4}\right.$ ) the moment of inertia; $\kappa$ is named Timoshenko's shear coefficient and can be determined, according to Timoshenko [114], for round solid pipes by Equation 4.39 with the use of the void ratio of the material $v$.

$$
\begin{equation*}
\kappa=\frac{6(1+v)}{7+6 v} \tag{4.39}
\end{equation*}
$$

## Dynamic application

Thomson [112] derived with use of the quasi-static equation, the dynamic relationship describing the Timoshenko beam model. Similar to the quasi-static case the model can be represented as a time depended system of equations determined by Equation 4.40 and Equation 4.41. The final relationship, for the dynamic application of the Timoshenko beam model, is therefore captured by Equation 4.42. Numerical application of the sophisticated model requires skilled computational knowledge and a large simulation time for the computer to obtain the solution of the system.

$$
\begin{gather*}
\rho A \frac{\partial^{2} w}{\partial t^{2}}-q(x, t)=\frac{\partial}{\partial x}\left[\kappa A G\left(\frac{\partial w}{\partial x}-\phi\right)\right]  \tag{4.40}\\
\rho I \frac{\partial^{2} \phi}{\partial t^{2}}=\frac{\partial}{\partial x}\left(E I \frac{\partial \phi}{\partial x}\right)+\kappa A G\left(\frac{\partial w}{\partial x}-\phi\right)  \tag{4.41}\\
E I \frac{\partial^{4} w}{\partial x^{4}}+\rho A \frac{\partial^{2} w}{\partial t^{2}}-\left(\rho I+\frac{E I P A}{\kappa A G}\right) \frac{\partial^{4} w}{\partial x^{2} \partial t^{2}}+\frac{\rho^{2} A I}{\kappa A G} \frac{\partial^{4} w}{\partial t^{4}}-\frac{\rho I}{\kappa A G} \frac{\partial^{2} q}{\partial t^{2}}+-\frac{E I}{\kappa A G} \frac{\partial^{2} q}{\partial x^{2}}=q(x, t) \tag{4.42}
\end{gather*}
$$

### 4.4. Methods for linear systems

Main characteristics of linear systems are symbolized by linear independent systems of equations. In strait conjunction with the linear independent systems are the symmetric matrices of the model, which are straightforward for a computer to solve. Methods to get solutions for these systems are drawn out in this section. Eigen-frequencies and mode shapes, depending on the material properties, geometry of the structure and the boundary conditions, are used in most methods to achieve the solution spectrum. Each mode-shape depends on the number of degrees of freedom in a system and corresponds to its own natural frequency. A very important principle holds only for linear systems: the superposition principle. The net response at a given place and time of a system to individual induced forces is equal to the response of the system to all forces combined as stated in Equation 4.43, Illingworth [55].

$$
\begin{equation*}
F\left(x_{1}, x_{2}, \ldots, x_{n}\right)=F\left(x_{1}\right)+F\left(x_{2}\right)+\ldots+F\left(x_{n}\right) \tag{4.43}
\end{equation*}
$$

### 4.4.1. Modal superposition approach

The Modal analysis corresponds to a solving algorithm that uses natural vibration modes to determine the solution spectrum of a vibrating system. "Rayleigh showed that if the damping matrix is a linear combination of the stiffness and inertia matrices, the damped system will have classical normal modes" Caughey [26]. He derived the relation of the normal modes to a linear system of equations including damping. "The essence of Modal Analysis is the assumption that the response can be represented as a summation of eigenvectors or mode shapes multiplied by an unknown time function" Metrikine [86]. The determination method, for a system without damping, reveals the eigenvector $\underline{\hat{x}}$ corresponding to the eigenvalues of the matrices. The system of equations solved will be:

$$
\begin{equation*}
\mathbf{M} \ddot{u}+\mathbf{K} \dot{u}=0 \tag{4.44}
\end{equation*}
$$

In order to find the natural frequencies of the system the determinant of the following system should be solved:

$$
\begin{equation*}
\operatorname{det}\left[-\omega^{2} \mathbf{M}+\mathbf{K}\right]=0, \quad\left(-\omega^{2} \mathbf{M}+\mathbf{K}\right) \underline{\hat{x}}=0 \tag{4.45}
\end{equation*}
$$

The solution of the solved system of equations is the matrix $\mathbf{E}$ (see Equation 4.46), consisting of all the eigenvectors of the system.

$$
\begin{equation*}
\mathbf{E}=\left[\underline{\hat{x}_{1}} \underline{\hat{x}_{2}} \ldots \ldots . . \hat{\hat{x}_{n}}\right] \tag{4.46}
\end{equation*}
$$

According to van Dijk [117] the solution to the homogeneous, forced or damped system is found in the form of a summation of the eigen-frequencies and the modes. The solution of the system is determined for each mode, corresponding to a natural frequency and thus to an eigen-vector, by means of uncoupled equations with use of the orthogonality principle applied to the modal mass and stiffness matrices of the system.
Down points of the method are related to the linearity properties of the system linked to the solution obtained by summation of natural modes of the system. Non-linear matrices, comprehensive with frequency dependent damping and leading to coupled natural modes, will not be solvable using Modal Analysis. "This deviates from reality where increasing or decreasing damping ratios for higher natural frequencies are possible" Metrikine [86]. Rayleigh damping uses a proportional modal damping matrix related to the modal stiffness and modal mass matrix to describe the damping in all modes Spijkers [109].

### 4.4.2. Fourier transform

The Fourier transform, also known as the discrete Fourier transform, emphasizes a determination method of a system by means of solving the scheme within the frequency domain, Loof [73]. The general Fourier transform is defined by Equation 4.47 according to Ratzkin [102].

$$
\begin{equation*}
\mathscr{F}(f)(\omega)=\hat{f}(\omega)=\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{\infty} e^{-i x \omega} f(x) d x \tag{4.47}
\end{equation*}
$$

Fourier transformation is a method, using a finite sum of small intervals, to describe a function (in the solution spectrum) by means of harmonic functions (with different amplitudes), corresponding to these small intervals. With other words an infinite sum of harmonic functions to approximate an existing function in the frequency domain (see Equation 4.48, Loof [73]). "Here $u$ is an integer for the Fourier coefficients and $j$ is an integer for the spatial domain of the input" Loof [73]. "The Fourier series of a vector with length $n$ results in a sum of sinusoidal terms, because terms of the form $e^{i a x}$ for real valued a can be decomposed into $\cos (a x)+i \sin (a x)$, so a sine and cosine part" Loof [73].

$$
\begin{equation*}
F(x)=\sum_{u=0}^{n-1}\left[\sum_{j=0}^{n-1} v[j] e^{-i \frac{2 \pi}{n} u j}\right] e^{i \frac{2 \pi}{n} u x} \quad \text { for } n=0,1, \ldots, n-1 \tag{4.48}
\end{equation*}
$$

When the response signal contains various frequencies and therefore time dependent, obtaining a solution of the system in the frequency domain with help of the Fourier transform is mandatory. The method is also very powerful, in combination with numerical approximation methods, when dealing with non-linear systems of equations as used in Itoh [57]. The solution can then be approximated within the frequency domain and thereafter be transformed back to get the result.

### 4.5. Methods for non-linear systems

In contradiction with linear systems, non-linear systems of differential equations include coupled processes that makes Modal Analysis (using the general solution approach) inaccessible. Modal Analysis defines the general solution of the system to obtain the answer. Non-linear systems provoke an undetermined number of outcomes, where no general solution exists.
Approximation methods for non-linearity use a linear viscous dashpot approach to model the combined damping of the system to avoid complex mathematical analysis, van Dijk [117], Jia [60]. There are various factors influencing the non-linearity of a system and the response. According to van Keulen [118] and van Dijk [117] there are four main factors:

1. Non-linearity in the boundary conditions: Independent of the nature of the system itself, the boundary conditions can result in a non-linear response of the system.
2. Geometrical non-linearity's: The buckling of rods is an example that leads to a non-linear relationships between deformation, displacement and forces and thus a non-linear response of the medium.
3. Interaction with the environment: When construction material properties, like stiffness and strength, change due to interaction with the surrounding environment this leads to non-linear behavior of the system (related to time non-linearity characteristics of the material). An example of such a coupled material behavior is investigated by Zhou [131], where the investigation of degradation of sheet pile walls due to chemical processes in the subsurface was investigated.
4. Physical non-linearity: A system relates to linear behavior when the material behaves elastic. So in case of small deformations due to induced vibrations, linear system behavior can be expected. But when larger deformations occur, in for instance soil material, plastic or elasto-plastic non-linear behavior of the system takes place. Elasto-plastic soil behavior leads to hardening or softening of the material and therefore influences the material stiffness that leads to non-linear stress-strain relationship of the system. Benz [19] delineates an extensive study to non-linear soil behavior associated with small strain stiffness.

### 4.5.1. Galerkin Method

The Galerkin method is categorized in the so called heuristic techniques, that approximate the solution with periodic spectra. According to van Dijk [117] the accuracy of these methods is unknown and stability of the solution spectrum obtained should be carefully considered.
Rao [101] states that the Galerkin Method is also classified as a weighted residual method, that works directly with the governing differential equation and boundary conditions of a problem. The approximation method needs a trial solution $\bar{\phi}$, that in general does not have to satisfy the general solution, to obtain a result. This results in a measure of error defined by Equation 4.49 (for a one-dimensional problem). $A$ and $B$ are linear operators of the general differential equation $A W=\lambda B W$; $\lambda$ is the corresponding eigenvalue.

$$
\begin{equation*}
R(\bar{\phi}, x)=A \phi-\lambda B \phi \tag{4.49}
\end{equation*}
$$

Equation 4.49 shows that if the trial solution $p \bar{h} i(x)$ is exactly the eigenfunction $W_{i}(x)$ and $\lambda$ the eigenvalue $\lambda_{i}$, the residual equals zero.
The Galerkin Method can be described, according to Rao [101], as a method where the solution of the eigenvalue problem is assumed in the form of a series of $n$ comparison functions which satisfy all the boundary conditions of the problem and is described by Equation 4.50.

$$
\begin{equation*}
\bar{\phi}^{(n)}(x)=\sum_{i=1}^{n} c_{i} \phi_{i}(x) \tag{4.50}
\end{equation*}
$$

The individual coefficients of the $\mathrm{z}^{\text {th }}$ eigenvector, $c_{i}^{(z)}$, that need to be determined to relate the known comparison functions, $\phi_{i}(x)$. The Galerkin Weighted Residual is obtained by substitution of Equation 4.50 into the general differential equation and results in Equation 4.51. In this residual $\lambda^{(n)}$ associates with eigenvalue of the $\mathrm{n}^{n t}$ trial solution. "The Galerkin Method is applicable to both conservative and non-conservative systems" Rao [101].

$$
\begin{equation*}
R=A \bar{\phi}^{(n)}-\lambda^{(n)} B \bar{\phi}^{(n)} \tag{4.51}
\end{equation*}
$$

Pesheck et al. [96], Devulder et al. [38] and Rao [101] define a procedure where assuming the error of the system goes to zero is allowed (as described by Equation 4.52). The procedure exists of: multiplication of the Galerkin Weighted Residual with the comparison functions $\phi_{1}(x), \phi_{2}(x), \ldots, \phi_{n}(x)$; integrating over the entire domain; setting the obtained answer to zero.

$$
\begin{equation*}
\int_{0}^{l} R\left(\bar{\phi}^{(n)}\right) \phi_{i}(x) d x=0, \quad i=1,2, \ldots, n \tag{4.52}
\end{equation*}
$$

### 4.5.2. Numerical approach

Powerful numerical tools are nowadays available to approximate the solution of complicated systems of equations and are therefore the most frequently used approach. Van Dijk [117] states that due to the complexity of the non-linear systems, the quality of the obtained solution spectrum is hard to determine and therefore validation by means of analytic or physical approaches is mandatory to verify the validity. "Exact/analytical solutions to equations of motions are usually not possible if the excitations vary arbitrarily with time or if the system is nonlinear" Kramer [69]. A more simplified non-linear approach of the system, serving as validation model, is therefore required.

The Direct Time Integration Method (DTIM) is a step-by-step numerical procedure to solve convoluted systems of equations. "In DTIM the general equations of motion are integrated using a numerical step-by-step procedure, the term direct meaning that prior to the numerical integration, no transformation of the equations into a different form is carried out" Bathe [18]. Obligatory for the system is that all individual components, including the excitation function, are deterministic (time-dependent), since the excitations varying with time must be defined at every time step, Jia [60]. According to Jia [60] there are generally three types of time-stepping procedures available for obtaining the responses:

1. Interpolation (normally linearly) of the excitation input between two adjacent time instants $t_{i}$ and $t_{i+1}$. This method is only applicable for linear systems.
2. Finite difference expression of acceleration and velocity. Non-linear systems can be solved with this method.
3. Variation of accelerations. Applicable for non-linear systems.
"The most commonly used methods are the central difference method, the Houbolt method, the Wilson- $\theta$ method, and the Newmark method" Bathe [18]. Numerical methods exist in two categories, namely explicit and implicit numerical approach methods. These two categories are explained with the use of the general equation of motion, taking place at time $t_{i}$ carried out in Equation 4.53 and with the use of $\Delta t_{i}=t_{i+1}-t_{i}$. [Note: The size of the time step $\Delta t_{i}$ chosen can NOT exceed the critical time step of the system $\Delta t_{c r}$ ]

$$
\begin{equation*}
M \ddot{W}^{t_{i}}+C \dot{W}^{t_{i}}+K W^{t_{i}}=F^{t_{i}} \tag{4.53}
\end{equation*}
$$

## Explicit Numerical Method

Central difference method:
The central difference method adopts a linear approximation, for the value of time $t_{i}$, in the middle of the two points $t_{i-1}$ and $t_{i+1}$ (between one calculation point earlier and one calculation point further in time complementary with the time step $\Delta t_{i}$ ). The first-and second order derivatives are estimated, according to Bathe [18], with use of Equation 4.54 and Equation 4.55, respectively. Substitution of both approximations and solving thereafter the system leads to the final solution of the general equation. The error made by the estimation method is defined of the second-order $\left(\mathrm{O}\left(\mathrm{h}^{2}\right)\right.$.

$$
\begin{gather*}
\dot{W}^{t_{i}}=\frac{1}{2 \Delta t_{i}}\left(W^{t_{i}+\Delta t_{i}}-W^{t_{i}-\Delta t_{i}}\right)  \tag{4.54}\\
\ddot{W}^{t_{i}}=\frac{1}{\Delta t_{i}^{2}}\left(W^{t_{i}-\Delta t_{i}}-2 W^{t_{i}}+W^{t_{i}+\Delta t_{i}}\right) \tag{4.55}
\end{gather*}
$$

## Implicit Numerical Method

## The Houbolt method:

Similar to the Central difference method, this method uses displacement components to determine the velocity and acceleration terms. The difference here is that the method is implicit, meaning that it does not determine the result at time $t_{i}$ but at $t_{i}+\Delta t_{i}$. Equation 4.56 and Equation 4.57, according to Bathe [18], comprise approximations for the velocity and acceleration terms at time $t_{i}+\Delta t_{i}$ with use of the Houbolt method.

$$
\begin{gather*}
\dot{W}^{t_{i}+\Delta t_{i}}=\frac{1}{6 \Delta t_{i}}\left(11 W^{t_{i}+\Delta t_{i}}-18 W^{t_{i}}+9 W^{t_{i}-\Delta t_{i}}-2 W^{t_{i}-2 \Delta t_{i}}\right)  \tag{4.56}\\
\ddot{W}^{t_{i}+\Delta t_{i}}=\frac{1}{\Delta t_{i}}\left(2 W^{t_{i}+\Delta t_{i}}-5 W^{t_{i}}+4 W^{t_{i}-\Delta t_{i}}-W^{t_{i}-2 \Delta t_{i}}\right) \tag{4.57}
\end{gather*}
$$

## Newmark Method:

Depending on the choice of the value $\alpha$ and $\delta$ the Newmark method is either an extension of the linear acceleration method ( $\alpha=\frac{1}{6}$ and $\delta=\frac{1}{2}$ ) or the trapezoidal rule ( $\alpha=\frac{1}{4}$ and $\delta=\frac{1}{2}$ ). The method is visually represented in Figure 4.15, were the terminology is outlined in detail. The approximation method consists of an error of order two $\left(\mathrm{O}\left(\mathrm{h}^{2}\right)\right)$, relating to an average accurate estimation method.

$$
\begin{equation*}
W^{t_{i}+\Delta t_{i}}=W^{t_{i}}+\dot{W}^{t_{i}} \Delta t_{i}+\left[\left(\frac{1}{2}-\alpha\right) \ddot{W}^{t_{i}}+\alpha \ddot{W}^{t_{i}+\Delta t_{i}}\right] \Delta t_{i}^{2} \tag{4.58}
\end{equation*}
$$



Figure 4.15: Newmark method/trapezoidal rule, modified after Bathe [18]

$$
\begin{equation*}
\dot{W}^{t_{i}+\Delta t_{i}}=\dot{W}^{t_{i}}\left[(1-\delta) \ddot{W}^{t_{i}}+\delta \ddot{W}^{t_{i}+\Delta t_{i}}\right] \Delta t_{i} \tag{4.59}
\end{equation*}
$$

Implicit Euler Method:
Closely related to the Newmark method and the Central difference method, the implicit Euler method estimates the value of the next time step $W^{t_{i}+\Delta t_{i}}$ by means of a linear extrapolation of the present value (see Equation 4.60) and consists of an error of order one $(\mathrm{O}(\mathrm{h})$ ), relating to a low accurate estimation method.

$$
\begin{equation*}
W^{t_{i}+\Delta t_{i}}=W^{t_{i}}+\Delta t_{1} f\left(t_{i}, W^{t_{i}}\right) \tag{4.60}
\end{equation*}
$$

Runge-Kutta Method:
The general approximation approach of the Runge-Kutta method is described by Equation 4.61 and consists of an error of order four $\left(\mathrm{O}\left(\mathrm{h}^{4}\right)\right)$, relating to a highly accurate estimation method.

$$
\begin{equation*}
W^{t_{i}+\Delta t_{i}}=W^{t_{i}}+\frac{\Delta t_{i}}{6}\left(k_{1}+2 \cot \left(k_{2}\right)+2 k_{3}+k_{4}\right) \tag{4.61}
\end{equation*}
$$

Four different components are used to determine the displacement value of the next time step $W^{t_{i}+\Delta t_{i}}$. The four components of the approximation equation are carried out by Equation 4.62, Equation 4.63, Equation 4.64 and Equation 4.65.

$$
\begin{gather*}
k_{1}=f\left(t_{i}, W^{t_{i}}\right)  \tag{4.62}\\
k_{2}=f\left(t_{i}+\frac{\Delta t_{i}}{2}, W^{t_{i}}+\frac{k_{1} \Delta t_{i}}{2}\right)  \tag{4.63}\\
k_{3}=f\left(t_{i}+\frac{\Delta t_{i}}{2}, W^{t_{i}}+\frac{k_{2} \Delta t_{i}}{2}\right)  \tag{4.64}\\
k_{4}=f\left(t_{i}+\Delta t_{i}, W^{t_{i}}+k_{3} \Delta t_{i}\right) \tag{4.65}
\end{gather*}
$$

### 4.6. Soil structure interaction

Comprehensive proficiency about the soil-structure interaction (soil-pipe interaction) is requisite to evolve model boundaries that symbolize the wave transmission from the soil to the structure. Simplicity is aimed for without reduction in model performance. The following section exhibit two possible model boundaries that symbolize the soil-structure interaction for wave transmission related problems.

### 4.6.1. Direct force or displacement

The most simplified approach is the addition of a displacement or force, induced by ground vibrations in the soil, directly on the pipe structure by means of a force or displacement vector, $\bar{F}(x)$ or $\bar{w}(x)$, respectively, depending on $\underline{x}$ (as represented in Figure 4.16). Assumption of direct transmission of forces/displacements, from the soil to the pipe, is made to obtain this simplified approach. No soil-structure interaction effects are hereby taken into account.

### 4.6.2. Indirect force or displacement

The indirect force or displacement method, to account for soil-structure interaction effects, is the extension of the direct force/displacement model. The extension accounts for the delayed and damped effect caused by the soil during transmission of the force/displacement. A higher accuracy can be reached when precise soil conditions around the pipe structure are known. This fact directly relates to the down-side of the model, were uncertainties of soil variation, soil contact area between the pipe and the soil and uncertainties in soil properties lead to lower model accuracy compared with the direct force/displacement model.


Figure 4.16: Direct force or displacement on pipe structure using Klevin-Voigt Foundation, modified after Metrikine [86]


Figure 4.17: Indirect force or displacement on pipe structure using Klevin-Voigt Foundation, modified after Metrikine [86]

### 4.7. Conclusion

Focus on the investigation of dynamic motion accompanying slender structures has been completed with the aim for a simplified model approach. Modeling the pipe structure with the help of a structural dynamics approach and coupling this with the wave properties from the wave propagation model will satisfy the research aim and the intention for simplification without a decrease in prediction performance.
System representations are revealed by three different descriptions: The Single Degree Of Freedom system (SDOF); Multiple Degree Of Freedom system (MDOF); Continuous system (CS). As the pipe structure is an embodiment for a CS likewise a MDOF system, the SDOF system is not further studied. The MDOF system as well as CS can both be considered as model build up system representations.
To accomplish pipe structure modeling various approach methods for continuous system representations are carried out. The Euler-Bernoulli method discussed is based on three model assumptions to retrieve a continuous derived equation, that directly relates to the configurations down points. Shear forces within the beam model can not be taken into account due to the fact that perpendicular plane cross-sections stay in plane related to the neutral axis. On the other hand, the pipe will not only experience small oscillating deformations induced by the dynamic wave, but also exist of a homogeneous material (steel). Both point 2) and point 3) of the model assumptions, mentioned in this chapter, are therefore not down points related to this problem case. The Kelvin-Voigt model has the Euler-Bernoulli theory integrate with an additional frequency dependent material damping as also used in the Rayleigh damping methodology. As complexity of the theory is higher than Euler-Bernoulli's theory and in this case complexity does not necessarily improve model quality, this model is not recommended in further research.
Timoshenko [113] developed an improved model for the Euler-Bernoulli beam model. Shear forces in the beam are incorporated by means of rotational capability of the planar cross-sections. Situations where shear forces play a role this model fits the choice. As for the circumstances examined in this thesis (the pipe structure in the subsurface exposed to vibration waves) does not necessarily involve shear forces within the pipe structure, Timosheko's beam model is too evolved and sophisticated as a model choice.
Pipe structures susceptible to oscillations induced by vibration waves undergo a damped motion as the soil retains the structure in place. The soil reaction to the oscillating pipe structure can be described by numerous damping representations. Rayleigh damping and hysteretic damping both deliberate predilection. Rayleigh damping is frequency depended and can lead to model stability with a variety of forced frequencies. Hysteretic damping on the other hand comprise of mainstream "easy to use" geo-technical parameters, what could be a great benefit.
To obtain a solution spectrum, the differential system of equations need to be solved. As frequency dependent damping or hysteretic damping will be applied, the system of equations will not be linear and numerical solution methods are mandatory. To gain calculation speed, the Fourier analysis can be applied prior to numerical utilization. A simplified model approach should be used as a numerical investigation to obtain the most efficient numerical method practiced.
To finalize the chapter, a direct force/displacement soil-structure interaction scheme should be applied to
the Euler-Bernoulli beam representation of the pipe structure. Since the aim for simplification is a required matter combined with the fact that limited knowledge on the subsurface conditions adjacent to the pipe is present, this will gain the best model outcome.

## Conclusion literature review

The Master Thesis project Literature Review will be completed in this chapter. Reexamination of the plan of work [103] is mandatory and hence contained in the conclusion.
Essential wave behavior in elasto-dynamics is delineated by means of consequence and origin assessment. As main topic "pile installation effects on adjacent pipe structures in the elasto-dynamics", distinctive literature investigation is worked out. Finally advise from the collected literature review can be contrived.
All wave movements can find their base in the D'Alambert solution. P-waves contain the most input energy (of P, S-and R-waves), according to Wolf [126], and exist of the highest propagation velocity. As both factors have great impact on the nearby pipe construction, great care needs to be taken when P-waves are the dominant wave types. Situations where body waves progress near the surface, their wave energy is quickly paved out making them irrelevant to consider in this thesis.
Total damping of waves arises from both a materialistic role as well as a geometric damping process. Restrictions of buried constructions, by the Eurocode 3 [2] with a maximum allowable PPV of $25 \mathrm{~mm} / \mathrm{s}$, are strongly correlated to the quantity of the damping process. Supplementary components that control this process are: the method of installation; soil properties; layering; whether or not resonance occurs.

Range of vibration evaluation and estimation methods are accomplished within the literature study. Extensive review and specification of the approach schemes led to conclusions concerning the research goals as stated in the Plan of Work [103] and Chapter 1.
High performance analytic approach as accomplished by Jongmans [62] shows a high model performance, but complexity is a draw-back of the model. As the model structure is too complex it is irrelevant for this Mater thesis.
By the use of physical laws Massarsch and Fellenius [80] achieved a method capable of handling spherical waves produced by the pile tip; cylindrical waves arising from the pile shaft friction with the soil and surface waves originating when body waves come in contact with the surface. Integration of vibrator-pile and pilesoil interaction schemes in combination with the formulation of the wave propagation within the soil led to realistic model results. The use of simple physical laws including known model parameters make it possible to obtain model results without the use of empirical factors (except the use of the remoldiation factor, which is a limitation of the model). The implementation of the critical distance, which describes the distance from the pile where Rayleigh/surface waves are developed translates into pragmatic model outcomes. "Another uncertainty is the superposition of ground vibrations during pile penetration, as the wave propagation process from different depths and sources (at different frequencies) can lead to superposition or canceling of vibration amplitudes" Massarsch and Fellenius [80]. Furthermore the effect of reflection and refraction within the soil layers is not incorporated into the model, what affects the outcome. Although this model represents a simplified approach, it gains it strength in usability and reliability due to the possibility to recognize the effect of different input parameters and their response on ground vibrations. Impact pile driving is the only application on which this model is applicable, making it not possible to use it for vibratory pile driving without modifications. By simple model modifications, vibratory pile driven methods can be implemented in this methods as well. Due to its strength and simplified approach, this method is advised in the application of the research to predict vibration propagation waves through the subsurface.

The pipe structure is not build at the surface level. As the model proposed by Massarsch and Fellenius predicts vibrations at surface level, model modifications need to be applied. Circumstances where pipe constructions are placed further than the critical distance, where emergence of Rayleigh waves takes place, application of the graphical relation proposed by Richard et al. [104] should be applied (over the depth).
The engineering methods as achieved in the literature review are over-simplified methods regarding the complexity of the problem spectrum. As comparison methods they can be applied to parts of the model build for the project investigation. Another method that can be practiced in a similar manner is the use of the program ABAQUS. Verification of the vibration propagation through the soil specimen can be modeled by elastic as well as elasto-plastic soil behavior. The program performance is entailed by Pichler [97] and reveals its strong and weak points. Care with boundary implementation is required to obtain compatible results.

Finite Element Method (FEM) approach for the pipe structure makes it possible to couple the boundaries of the model with the wave properties obtained with the wave propagation model. Masoumi [79] used a coupled Boundary Element Method - Finite Element Method (BEM-FEM) approach to model a similar problem. As stated in the Plan of Work [103], the approach can be applied to the investigation of pipe structures. As the aim of the investigation is to simplify the approach, the complicated BEM-FEM concept does not satisfy the aim for simplification and will therefore not by applied. Furthermore the FEM approach can be applied to SDOF systems and couple them to a MDOF system to justify the pipe structure behavior. In the further research this approach should be considered as a model application method.
The Euler-Bernoulli method discussed is based on three model assumptions to retrieve a continuous derived equation, that directly relates to the configuration's down points. Shear forces within the beam model can not be taken into account due to the fact that perpendicular plane cross-sections stay in plane related to the neutral axis. On the other hand, the pipe will not only experience small oscillating deformations induced by the dynamic waves, but also exist of a homogeneous material (steel). Both point 2 ) and point 3 ) of the model assumptions, mentioned in this chapter, are therefore not down points related to this problem case. As simplification is mandatory in the master thesis investigation project, the Euler-Bernoulli approach should be considered as model choice for the pipe structure.
Pipe structures susceptible to oscillations induced by vibration waves undergo a damped motion as the soil retains the structure in place. The soil reaction to the oscillating pipe structure can be described by numerous damping representations. Rayleigh damping and hysteretic damping both deliberate predilection. Rayleigh damping is frequency dependent and can lead to model stability with a variety of forced frequencies. Hysteretic damping on the other hand comprise of mainstream "easy to use" geo-technical parameters, what could be a great benefit.
To obtain a solution spectrum, the system of differential equations need to be solved. As frequency dependent or hysteretic damping will be applied, the system of equations will not be linear and numerical solution methods are mandatory. To gain calculation speed, the Fourier analysis can be applied prior to numerical utilization. A simplified model approach should be used as a numerical investigation to obtain the most efficient numerical method practiced.
To finalize the conclusion, a direct force/displacement soil-structure interaction scheme should be applied to the Euler-Bernoulli beam representation of the pipe structure. Since the aim for simplification is a required matter combined with the fact that limited knowledge on the subsurface conditions adjacent to the pipe is present, this will gain the best model outcome.

## Final Model Choice

Prediction of dynamic problems by means of any approach method is a big engineering challenge. The correctness and uncertainties of the approximation method used, plus the unpredictability and spread of the model and material parameters (representing the material and therefore model behavior), control the development a proper simulation method.
Massarsch and Fellenius [80] developed an analytic approach method that forms a base for the wave attenuation model used in this Master thesis research project. Application of this method enhance the possibility to obtain the magnitude of vibration, on surface level, as a result of impact pile driving can be estimated. As this model is developed for impact pile driving, modification of the enforcement method into vibratory pile driving is mandatory. The approach proposed by Massarsch and Fellenius [80] retrieves a prognoses values for the amplitude and frequency of waves at any distance from the pile on surface level. These prognoses values will serve as forced values for the pipe structure model. Modification of the predicted amplitude and frequency rates on surface level (which results from the calculation method) is required since the pipe structure is located at depth and not at surface level.
Verification of the new approach methodology for wave propagation modeling (after modifications have been applied) is required to prove the effectiveness of the methodology applied. Verification on the other hand can only be accomplished by field measurements or laboratory executed simulations (also known as physical modeling). Due to the lack of research time within this project, comparison with numerical (by means of ABAQUS) as well as empirical methods (codes of conduct as well as other empirical methods) will be made. All methods are used to replicate the wave propagation during vibratory pile driving. Important to note is that the implementation of S-waves, related to shaft friction of the pile, is not taken into account in any model throughout this Master Thesis research project. The reason refers to the magnitude of the S-waves as a result of the pile shaft friction, which is very small with respect to the P-waves and R-waves resulting from the toe impact with the soil. This makes consideration of this process for wave development irrelevant. Wolf [126] also stated a similar conclusion for the application of "high frequency rates: as used for pile installation by a vibrating hammer". In the literature study of this Master Thesis the development of P-waves at the pile toe was related to the volume change under the pile toe due to downward pile toe movement. The downward pile toe movement by itself is induced by the compression wave (reflected back as a dilative wave at the pile toe) caused by the kinetic energy of the vibrating hammer. This process enhances that the development of P-waves indicates big energy transfer as the vibrating hammer excites its energy in the same direction.
The simulation of the pipe structure in the subsurface, induced by vibrations of the pile installation, is the second part that will be converted into a model. Resulting from the conclusion of the literature study: EulerBernoulli's beam method is used as governing equation for the model. The pipe structure is supported by the surrounding soil and hence included in the model behavior. Gunakala et al. [48], Desai [37], Thomas and Abbas [111] and the University of Colerado [116] implemented beam theory into a Finite Element approach by means of Galerkin's weighted residual method with the application of Green's theorem. A similar approach is used in this Thesis to implement the governing equation's into a Finite Element approach with use of Hermite cubic interpolation (or cubic spline) functions. According to Fereira [42] a cubic interpolation polynomial satisfies the requirement to be thrice differentiable, which is required for the fourth-order Euler-Bernoulli differential equation.
The constitutive behavior is implemented by means of the general equation of motion (see Equation 6.68).

The mass matrix, $[M]$, and stiffness matrix, $[K]$, are obtained by applications of Galerkin's weighted residual method. Damping behavior, represented by the damping matrix, [ $C$ ], is modeled by Rayleigh-damping: The damping matrix is calculated according to the method described in Chapter 4. Hysteretic damping on the other hand could be a choice of implementation as well: The dynamic soil behavior could be represented by spring-and-dashpot supports of the beam model. The damping behavior of the dashpot is then calculated according to Equation 4.9 and Equation 4.35. The damping behavior is strain and frequency dependent (meaning it relates to strain and frequency dependent stiffness of the soil), which is typical for real soil behavior. Due to the lack of time within this Master Thesis Project, implementation of the hysteretic damping approach is not applied. The force acting on the structure, $F$, is linked (coupled) to the output of the modified wave propagation model of Massarsch and Fellenius [80]. The Galerkin's weighted residual method with use of Greens theorem is also used to convert the force vector into Finite Element form.
A numerical approach is used to resolve the solution of the system of Ordinary Differential Equations (ODE). The Fourth-order Runge-Kutta (RK4) numerical approach has an order 4 truncation error and is therefore chosen technique to gain the solution spectrum. This technique is implemented in the Ode-Solver of Matlab and therefore chosen as numerical solving method. The strains within the pipe structure are obtained as a function of time and are used to calculated the corresponding stresses and momentum within the pipe structure as a function of time.

### 6.1. Total program

The model description of a coupled approach for the prediction of vibratory pile induced vibrations and the effects on buried pipe structures, used for this Master Thesis research project, is carried out in this chapter. In addition the model properties regarding the small-strain soil behavior are accomplished. Furthermore, different verification methods used in the model are entailed and explained in detail.

## Model definition

This section is to clarify and underline the total coupled program and the verification with the different approach methods. All of the individual components of the approach include verification as shown in Figure 6.1. The wave propagation program is verified by the codes of conduct and by a model build with the commercial software ABAQUS [3]. Furthermore the pipe structure Finite Element Model is studied by a sensitivity analysis in the form of a parameter study.


Figure 6.1: Model flow for the total program

## Material properties

The wave propagation model is defined according to linear elastic and elasto-plastic material laws and therefore linear elastic wave propagation behavior as well as non-elastic wave propagation behavior will be taken into account. To verify the calculations carried out by the Wave propagation model (WPM), linear elastic material behavior is used for the ABAQUS model. In addition to the purely elastic model an elasto-plastic (linear elastic - perfect plastic) material model in ABAQUS is used to compare its results with the linear elastic
material model. This comparison is made with a smaller excitation pressure amplitude (pressure as a result of the impact force of the pile toe) to reduce calculation time and to reduce the large plastic strains occurring under the pile toe, which can lead to local failure of the material.


Figure 6.2: Mohr-Coulomb's material model for soils, after Midas Information Technology Co., Ltd. [88]
The linear elastic - perfect plastic model includes Mohr-Coulomb's criterion of failure as this is one of the most suited criterion of failure for soils (the entire plasticity model adopted is also know as the Mohr-Coulomb model, see Figure 6.2). The material properties for the ABAQUS Finite Element Model are stated in Table 6.1 and are chosen according to Geotechdata [45] and Benz [19]. The dilatation angle $\psi$ is calculated with Equation 6.1 according to Bartlett [17]. The cohesion $c$ for the medium dense sand is 0 , but is held 1.0 for numerical purposes.

$$
\begin{equation*}
\psi=\phi-30, \quad \text { for } \phi>30 \tag{6.1}
\end{equation*}
$$

The correlation for the small-strain stiffness for all soils, $E_{0}(\mathrm{kPa})$, can be determined according to Benz [19], Biarez and Hicher [21] by Equation 6.2. In this formulation the void ratio, $e(-)$, relates to the predefined reference pressure, $p^{r e f}=100 \mathrm{kPa}$ (equal to the atmospheric pressure), and the mean effective pressure, $p^{\prime}(\mathrm{kPa})$. In the case of this Master Thesis Project, the small-strain stiffness is chosen equal to the reference stiffness (according to Benz [19]) and is therefore not depth dependent. A well graded sand is assumed leading to a Poisson's ratio of $v=0.23(-)$, according to Biarez and Hicher [21]. According to Benz [19] the range of Poisson's ratio for small-strain applications is $0.1 \leq v \leq 0.3$. For all wave propagation models considered in this Master Thesis project $5 \%$ material damping, $\xi$ (\%), is considered. For the pipe structure model, a sensitivity analysis related to material damping is carried out.

$$
\begin{equation*}
E_{0}=\frac{140}{e} \sqrt{\frac{p^{\prime}}{p^{r e f}}} \tag{6.2}
\end{equation*}
$$

Heinoord Tunnel small-strain parameters

| Material properties |  | Sand, medium dense | Clay |
| :--- | :--- | :--- | :--- |
| $\gamma$ | $\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ | 2000 | 1600 |
| $E_{0}$ | $[\mathrm{kPa}]$ | 240000 | 210000 |
| $\nu$ | $[-]$ | 0.23 | 0.40 |
| $\phi$ | $\left[{ }^{\circ}\right]$ | 35.0 | - |
| $\psi$ | $\left[{ }^{\circ}\right]$ | 5.0 | - |
| $c$ | $[\mathrm{kPa}]$ | 1.0 | - |
| $\alpha$ | $[-]$ | 0.7020 | 0.8815 |
| $\beta$ | $[-]$ | $2.9963 \quad 10^{-3}$ | $1.8762 \quad 10^{-3}$ |
| $\xi$ | $[\%]$ | 5.0 | 5.0 |

Table 6.1: Material properties for the ABAQUS Finite Element Model and the wave propagation model, Geotechdata [45] and Benz [19]

### 6.2. Wave propagation model (WPM)

Massarsch and Fellenius [80] developed a method for the prediction of wave speed at surface level, at a predefined distance from the pile. Chapter 3 defines a detailed model description and its inner workings according to the paper of Massarsch and Fellenius [80].

### 6.2.1. Model definition

This section entails an adopted strategy with respect to the paper carried out by Massarsch and Fellenius [80]. Model modifications related to the enforcement method and Rayleigh-wave correction (with respect to
the depth of the pipe) are required to attain comprehensive model results. Swapping from an impact pile driving method, as used by Massarsch and Fellenius [80], to a vibratory pile driving method, as adopted in this Master Thesis research project, modification related to this changed enforcement method are mandatory. Conversion from retrieving wave speeds at the surface level (at a predefined distance from the pile) to an at depth situation requisite Rayleigh-wave magnitude adjustment with respect to the embedded depth of the pipe.
Pile depth progression in the subsurface, associated with the induced force of the vibrator machine, is simulated by a semi-static calculation. The arithmetic scheme uses a one-dimensional grid equally distributed over the depth profile associated with the pile trajectory. For every individual grid point, relating to the depth of the pile toe, the dynamic load of the vibrating hammer is applied and the corresponding wave velocity at the pipe is calculated.
The program is developed with the commercial software Matlab [82] and undertakes all the calculation steps for wave predictions at depth. Since the Finite Element Model for the pipe structure is developed using the same software package, the predicted wave velocities can easily be coupled to the Finite Element Model to form an interactive coupled program to achieve good understanding of the system as a whole.

### 6.2.2. Model structure



Figure 6.3: Model flow for the wave propagation model
Figure 6.3 indicates the model flow for the Wave Propagation Model (WPM). The end result, elaborated in a vertical wave velocity vector at the surface level, $\left\{v_{2}\right\}(\mathrm{mm} / \mathrm{s})$, calculated by Equation 6.4 and a horizontal wave velocity vector at the surface level, $\left\{v_{1}\right\}(\mathrm{mm} / \mathrm{s})$, calculated by Equation 6.3 (here still indicated without geometric and material damping), is linked to the input of the Finite Element Model of the pipe structure. As indicated in the introduction of this chapter, only the waves excited by the toe are taken into consideration. More information on the one-sided coupled approach, with the Finite Element Model of the pipe structure, follows later on in this chapter.

$$
\begin{gather*}
v_{1}=k_{s} F_{h} E_{T} \frac{\sqrt{F^{H} W_{0}}}{r_{r}} \cos \left(90-\theta_{P}\right)  \tag{6.3}\\
v_{2}=k_{s} F_{\nu} E_{T} \frac{\sqrt{F^{H} W_{0}}}{r_{r}} \cos \left(\theta_{P}\right) \tag{6.4}
\end{gather*}
$$

## Initialization

The initialization process is modeled to obtain the model parameters and dimensions that are required to undertake further calculation steps. Parameters related to the material behavior as well as spacial parameters are prescribed by the user or calculated according to the user specified input.

Sheet pile parameters
The pile impedance, $Z^{P}(\mathrm{kNs} / \mathrm{m})$, and specific pile impedance, $z^{P}\left(\mathrm{kNs} / \mathrm{m}^{3}\right)$, are determined with Equation 6.5 and Equation 6.6, respectively. These parameters are related to the resistance of the pile in a complex valued form. In these equations the following parameters are used: $A^{P}\left(\mathrm{~m}^{2}\right)$ regarding the pile cross-sectional area; $\rho^{P}\left(\mathrm{~kg} / \mathrm{m}^{3}\right)$ indicating the density of the pile material; $c^{P}(\mathrm{~m} / \mathrm{s})$ referring to the compression wave ( P wave) speed in the pile. The properties of the sheet pile are outlined in Table 6.2.

$$
\begin{gather*}
Z^{P}=A^{P} \rho^{P} c^{P}  \tag{6.5}\\
z^{P}=\rho^{P} c^{P} \tag{6.6}
\end{gather*}
$$

The sheet pile wall is modeled as a strait rectangular plate (instead of the bended shape sheet pile walls have), as a simplistic approach, with thickness $T^{P}(\mathrm{~m})$ and the corresponding theoretical length (of one pile/element) $b^{P}$ calculated by Equation 6.7..

$$
\begin{equation*}
B^{P}=\frac{A^{P}}{T^{P}} \tag{6.7}
\end{equation*}
$$

| Wall properties |  | AZ32-750 |  |
| :--- | :--- | :--- | :--- |
| Cross-sectional area | $A^{P}$ | $\left[\mathrm{~m}^{2}\right]$ | $14.87 \mathrm{e}^{-3}$ |
| Wall thickness | $T^{P}$ | $[\mathrm{~m}]$ | $13.5 \mathrm{e}^{-3}$ |
| Density | $\rho^{P}$ | $\left[\mathrm{~kg} / \mathrm{m}^{3}\right]$ | 7850 |
| P-wave speed | $c^{P}$ | $[\mathrm{~m} / \mathrm{s}]$ | 5100 |

Table 6.2: Properties of an AZ32-750 sheet pile wall used for the model simulations, according to Arcelor Mittal [90]

## Vibrator parameters

The hammer efficiency factor $F^{H}$ relates to the amount of energy the vibrating hammer can transfer to the pile. This factor is held equal to 1.0 , since the vibrator is mounded to the sheet pile wall. In reality this relates to the movement of the vibrator relative to the movement of the top of the pile, but is chosen this value for simplicity. The inverse of the frequency of excitation, $f(\mathrm{~Hz})$, is called the impact time of one vibration cycle by the hammer, $t_{\text {vibro }}$, and is obtained by Equation 6.8. The impact time of the pile toe with the soil, $t^{P}(\mathrm{~s})$, is equalized to the impact time of one vibration cycle by the hammer, $t_{\text {vibro }}$ as in this case dynamic shaft friction of the pile is not taken into account (and the pile therefore serves as ridged body).

$$
\begin{equation*}
t_{\text {vibro }}=t^{P}=\frac{1}{f}=f^{-1} \tag{6.8}
\end{equation*}
$$

Pressure under the sheet pile wall caused by the gravitational force related to the own weight of the vibrator, $F_{0}$ $(\mathrm{N})$, and the transition vibrating force, $F_{\nu i b r o}(\mathrm{~N})$, is calculated with the use of Equation 6.9 and Equation 6.10 and are directly related to $P_{0}(\mathrm{~Pa})$ and $P_{\nu i b r o}(\mathrm{~Pa})$, respectively.

$$
\begin{gather*}
P_{0}=\frac{F_{0}}{A^{P}}  \tag{6.9}\\
P_{\nu i b r o}=\frac{F_{\nu i b r o}}{A^{P}} \tag{6.10}
\end{gather*}
$$

The two corresponding forces $F_{0}(\mathrm{~N})$ and $F_{\nu i b r o}(\mathrm{~N})$ are calculated by Equation 6.11 and Equation 6.12, respectively. These equations are dependent on the mass of the vibrator, $M_{s}(\mathrm{~kg})$; eccentric moment, $M_{e}(\mathrm{kgm})$ and the corresponding driving frequency, $\omega(\mathrm{rad} / \mathrm{s})$. The loading on the pile and the corresponding pressure at the toe of the pile are visualized in Figure 6.4. From this visualization is clear that the shaft resistance for the model is taken zero resulting in a model representing just the pile toe resistance. In reality there will be shaft resistance, lowering the applied pressure on the toe, resulting in a smaller wave magnitude (which is


Figure 6.4: Representation of the force applied to the pile with the corresponding toe resistance force, modified after Van Den Berghe [20]
not a conservative approach for this analytic approach method). Another factor lowering the pressure at the toe is the interlock resistance of the sheet pile walls as visualized in Figure 6.5. As this phenomena is not present when the first sheet pile wall is installed, the interlocking effect will NOT be considered as the normative situation. The simplified model, as considered in this Master Thesis project, is therefore considered as a conservative approach.

(a) Description of the test setup with the installation (left) and the pull-out test (right)

(b) Results of the comparison of quasi-static, extraction, interlock resistance in saturated and dry sands

Figure 6.5: Interlocking effect of sheet pile walls during installation, after Van Den Berghe [20] and Viking [120]

$$
\begin{gather*}
F_{0}=M_{s} g  \tag{6.11}\\
F_{\nu i b r o}=m_{e} \omega^{2} \sin (\omega t) \tag{6.12}
\end{gather*}
$$

The vibrator parameters are summarized in Table 6.3. In the calculation the own weight of the sheet pile wall is not taken into account as assumed that the internal friction with the surrounding soil at the shaft will reduce the point pressure to nearly zero.

| Vibrating hammer properties |  | PVE 55M |  |
| :--- | :--- | :--- | :--- |
| Eccentric moment | $m_{e}$ | $[\mathrm{kgm}]$ | 54 |
| Own weight | $M_{z}$ | $[\mathrm{~kg}]$ | 7000 |
| Driving frequency | $f$ | $[\mathrm{~Hz}]$ | 25 |

Table 6.3: The properties of the PVE 55M vibrating hammer used for the model simulations, according to Diesko Groep [46]

## Soil parameters

The depth profile is build up out of different soil types and the corresponding soil layers have a particular thickness. As the pile (toe) progresses in depth it experiences resistance of the soil resulting in compression and shear waves. The magnitude of resistance is correlated to the soil properties. These properties are depth dependent and therefore included in the simulation as a depth associated property distribution with respect to the grid points. All parameters are resolved according to the Massarsch and Fellenius model [80] explained in detail in Chapter 3.
The compression wave velocity, $c_{P}(\mathrm{~m} / \mathrm{s})$, and shear wave velocity, $c_{S}(\mathrm{~m} / \mathrm{s})$, in a soil layer, calculated with use of Equation 6.13 and Equation 6.14 respectively, are subsidiary to the Poisson's ratio, $v(-)$, the soil density, $\rho\left(\mathrm{kg} / \mathrm{m}^{3}\right)$, and the elasticity modulus of the soil, $E(\mathrm{~Pa})$. The wave speed for Rayleigh-waves, $c_{R}(\mathrm{~m} / \mathrm{s})$, is calculated with the use of the shear wave velocity (see Equation 6.15). These formulations are complementary with the expressions outlined in the literature study, Chapter 2, of this Master Thesis project.

$$
\begin{gather*}
c_{P}=\sqrt{\frac{E(1-v)}{\rho(1-2 v)(1+v)}}  \tag{6.13}\\
c_{S}=\sqrt{\frac{E}{2 \rho(1+v)}}  \tag{6.14}\\
c_{R}=\frac{0.87+1.12 v}{1+v} c_{S} \tag{6.15}
\end{gather*}
$$

According to Massarsch and Fellenius [80] Rayleigh-waves develop at a so called 'critical radial distance', $r_{c r i t}$ (m), from the pile when P-waves and or S-waves interfere with the ground surface. This span in the horizontal surface plane is obtained with Equation 6.16 and depends on the critical direct distance, $d_{\text {crit }}$ (m) (see Equation 3.23), the current depth of the pile toe, $h_{z}(\mathrm{~m})$, and the critical angle, $\theta_{\text {crit }}\left({ }^{\circ}\right)$ (see Equation 3.22). When the pile progresses in depth, the critical distance increases. The critical distance (which relates to the summation of the critical direct distances relates to the previous exceeded soil layers and the current depth. The direct radial distance, $r_{r}(\mathrm{~m})$, used to determine the magnitude of P -waves traveling directly to the pipe structure, is calculated with use of Equation 6.17. In this equation the depth of the center of the pipe, $h_{\text {pipe }}$ $(\mathrm{m})$, and the center-to-center distance of the pipe structure and the pile, $d_{c t c}(\mathrm{~m})$, correlates to the obtained direct radial distance.

$$
\begin{gather*}
r_{c r i t}=\sqrt{h_{z}^{2}+d_{c r i t}^{2}}  \tag{6.16}\\
r_{r}=\sqrt{\left(h_{z}-h_{p i p e}\right)^{2}+d_{c t c}^{2}} \tag{6.17}
\end{gather*}
$$

Related to material damping, the material absorption parameters, $\alpha\left(\mathrm{m}^{-} 1\right)$, is required. This soil parameters is calculated according to Equation 2.21 and also indicated by Equation 6.18.

$$
\begin{equation*}
\alpha=\frac{2 \pi D f}{c_{P}} \tag{6.18}
\end{equation*}
$$

In the model the average material absorption parameter for the soil profile, with respect to the current pile toe depth, is used for the damping of compression waves along the distances $r_{c r i t}$ and $r_{r}$. For the damping of the Rayleigh-wave, the absorption parameter related to the upper soil layer is used (as these waves travel along the surface).
The total energy of one cycle, $W_{\text {tot }}(\mathrm{J})$, induced by the vibrating hammer, is calculated by Equation 6.20. This formulation is according to Newton's second law of motion for the definition 'Work' and relates to: Equation 6.19 (where $W$ is the amount of energy in Joule; $F$ is the force in Newton needed to move the mass over the distance $u(\mathrm{~m})$ ).

$$
\begin{gather*}
W=F u  \tag{6.19}\\
W_{\text {tot }}=W_{0}+W_{v i b r o}=W_{0}+\int_{0}^{\frac{t_{v i b r o}}{2}} F_{\nu i b r o} u d t \tag{6.20}
\end{gather*}
$$

The vibratory energy is related to half of a sinus period, $\frac{t_{\text {vibro }}}{2}$, representing the input energy of the vibrator (the first half of the sine). A cumulative distribution function of the energy over time for one cycle is shown in Figure 6.6. The energy related to the own-weight of the vibrator, $W_{0}$ (J), and the energy induced by the moving center mass of the vibrator, $W_{v i b r o}(\mathrm{~J})$, can be calculated with Equation 6.19.


Figure 6.6: Cumulative distribution function of the input energy over time for one cycle

The distance, $u(\mathrm{~m})$, related to the induced force is determined according to Boussinesq's exact solution, Pistrol [98] and Boussinesq [23]. This method, visualized in Figure 6.7, relates the pressure of a plate-like geometry, as calculated by Equation 6.9 and Equation 6.10, to the displacement, $w(\mathrm{~m})$, resulting from this pressure (and is determined by Equation 6.21). The maximum displacement, in the middle of the sheet pile geometry (see deformation shape in Figure 6.7), given by Equation 6.21 is the exact solution for the soil represented as an linear elastic medium.

## Grundriss:



Figure 6.7: Calculation method for the static displacement due to a surface plate load according to Boussinesq's methodology, according to Pistrol [98]

$$
\begin{equation*}
w=\frac{2 P}{\pi C}\left[\ln \left(\sqrt{a^{2}+b^{2}}+a\right) b+\ln \left(\sqrt{a^{2}+b^{2}}+b\right) a-\ln \left(\sqrt{a^{2}+b^{2}}-a\right) b-\ln \left(\sqrt{a^{2}+b^{2}}-b\right) a\right] \tag{6.21}
\end{equation*}
$$

The arbitrary calculation factor, $C\left(\mathrm{~N} / \mathrm{m}^{2}\right)$, used in Equation 6.21 is calculated by Equation 6.22 with the use of the Poisson's number, $m(-)$ (see Equation 6.23). The parameters $a$ and $b$ are half of the theoretical length of the sheet pile wall and half of the theoretical thickness of the wall, respectively.

$$
\begin{align*}
C & =\frac{m^{2} E}{m^{2}-1}  \tag{6.22}\\
m & =\frac{1}{v}=v^{-1} \tag{6.23}
\end{align*}
$$

The factor $s$, used for the calculation of the vibration amplification factor, is calculated by Equation 3.29 and relates the angles of incidence for P-waves, $\theta_{P}\left({ }^{\circ}\right)$, and S-waves, $\theta_{S}\left({ }^{\circ}\right)$, to the Poisson's ratio, $v(-)$. The
incidence angle for P-waves, determined by Equation 6.24 is known. Related to these two parameters, the incidence angle for S-waves, $\theta_{S}\left({ }^{\circ}\right)$, is calculated with use of Equation 6.25.
[Note: The incidence angle for both P-and S-waves is different for the direct radial distance as for the critical radial distance]

$$
\begin{gather*}
\theta_{P}=\tan ^{-1}\left(\frac{d_{c t c}}{h_{z}}\right)  \tag{6.24}\\
\theta_{S}=\sin ^{-1}\left(\sin \left(\theta_{P}\right) s\right) \tag{6.25}
\end{gather*}
$$

The vibration amplification factor for horizontal, $F_{h}(-)$, and vertical, $F_{\nu}(-)$, propagating waves are calculated by Equation 3.28 and Equation 3.27, respectively. The amplification factors are NOT used in this case. The reason for this is that the factors should only be applied when a P-wave hit the ground surface, which is not the case for the buried pipe structure.
The vibration transmission efficacy of the pile toe, $E_{T}(-)$, in accordance to the literature study (Equation 3.18) should be determined by Equation 6.26.

$$
\begin{equation*}
E_{T}=\frac{R_{T}}{F_{d}} \tag{6.26}
\end{equation*}
$$

This factor depends on the dynamic resistance force, $R_{T}(\mathrm{kN})$, of the pile toe-soil interaction. The dynamic resistance force is in accordance with Equation 6.27 and relates to the damping factor of the pile toe, $J_{c}(-)$ (see Equation 6.28); the soil and pile specific impedance, $z_{P}$ and $z^{P}\left(\mathrm{kNs} / \mathrm{m}^{3}\right)$, respectively; the pile impedance, $Z^{P}(\mathrm{kNs} / \mathrm{m})$; the pile velocity, $v^{P}(\mathrm{~m} / \mathrm{s})$ calculated in accordance with the literature study and given by Equation 6.29; and the total force of the vibrating hammer, $F_{d}(\mathrm{kN})$. All of the individual terms are extensively explained in Chapter 2.

$$
\begin{gather*}
R_{T}=R_{R} J_{c} Z^{P} v^{P}  \tag{6.27}\\
J_{c}=2 \frac{z_{P}}{z^{P}}  \tag{6.28}\\
v^{P}=\frac{F_{d}}{Z^{P}} \tag{6.29}
\end{gather*}
$$

The particle velocity influence parameter for spherical waves (P-waves), $k_{S}\left(\sqrt{\frac{m^{2}}{k g}}\right.$ ), as drawn out in Equation 6.30, depends on the wave length (for P-waves), $\boldsymbol{\lambda}_{P}(\mathrm{~m})$ (see Equation 6.31), and the material density of the soil, $\rho\left(\mathrm{kg} / \mathrm{m}^{3}\right)$.

$$
\begin{gather*}
k_{S}=\frac{1}{\sqrt{2 \pi \rho \lambda_{P}}}  \tag{6.30}\\
\lambda_{P}=t^{P} c_{P} \tag{6.31}
\end{gather*}
$$

The wave length for Rayleigh-waves is dependent on the frequency of excitation, $f(\mathrm{~Hz})$, and the wave speed for Rayleigh-waves, $c_{R}(\mathrm{~m} / \mathrm{s})$.

$$
\begin{equation*}
\lambda_{R}=\frac{c_{R}}{f} \tag{6.32}
\end{equation*}
$$

Rayleigh-wave correction at depth
Richart et al. [104] and Das and Ramana [31] represented a method for the correlation of Rayleigh-wave amplitude with depth. In the methodology used for this Master Thesis project, only the vertical component of the Rayleigh-waves are taken into consideration (as they enhance the largest magnitude over depth).
Since the pipe structure is located at depth, depth $_{\text {pipe }}(\mathrm{m})$, the calculation method will incorporate average values for all parameters required namely:

- $c_{S}=\operatorname{mean}\left(c_{S}\left(\right.\right.$ surface $\left.\left.: \operatorname{depth}_{\text {pipe }}\right)\right)$
- $c_{R}=$ mean $\left(c_{R}\left(\right.\right.$ surface $:$ depth $\left.\left._{\text {pipe }}\right)\right)$
- $v=$ mean $\left(v\left(\right.\right.$ surface $:$ depth $\left.\left._{\text {pipe }}\right)\right)$
- $\lambda_{R}=$ mean $\left(\lambda_{R}\left(\right.\right.$ surface $:$ depth $\left.\left._{\text {pipe }}\right)\right)$

The Rayleigh-wave correction factor, $W_{\text {correction }}(-)$, is a normative factor with respect to the surface level and is calculated by Equation 6.33. The general calculation methodology for the Rayleigh-wave correction factor, $W_{\text {correction }}(-)$, is determined by Equation 6.34.

$$
\begin{gather*}
W_{\text {correction }}=\frac{W_{\text {correction }, z=\text { pipe }}}{W_{\text {correction }, z=0}}  \tag{6.33}\\
W_{\text {correction }}(z)=-\exp \left(-\frac{q}{f} f_{\text {corr }} z\right)+\frac{2}{\frac{s}{f}+1} \exp \left(-\frac{s}{f} f_{\text {corr }} z\right) \tag{6.34}
\end{gather*}
$$

The individual components for the calculation of the Rayleigh-wave correction factor in Equation 6.34 are given by Equation 6.35, Equation 6.36, Equation 6.37, Equation 6.38 and Equation 6.39. The substitution of $z=0$ and $z=$ dept $_{\text {pipe }}$ will give the two unknown values used in Equation 6.33.

$$
\begin{gather*}
V_{c o r r}^{2}=\frac{c_{R}^{2}}{c_{S}^{2}}  \tag{6.35}\\
\alpha_{c o r r}^{2}=\frac{1-2 v}{2-2 v}  \tag{6.36}\\
f_{c o r r}^{2}=\frac{2 \pi}{\lambda_{R}}  \tag{6.37}\\
\frac{q^{2}}{f^{2}}=1-\alpha_{c o r r}^{2} V_{c o r r}^{2}  \tag{6.38}\\
\frac{s^{2}}{f^{2}}=1-V_{c o r r}^{2} \tag{6.39}
\end{gather*}
$$

## Wave speed determination

Primary compression waves (P-waves) are enforced as a result of the pile toe impact with the soil. As they travel through the soil medium the waves experience damping, resulting in a decrease of wave amplitude. Massarsch and Fellenius [80] developed a model that defines the wave velocity at the surface level. This value is then adopted as input for Equation 2.22, which defines the wave velocity at surface level after geometric damping and material damping enforce the amplitude decrements. The damping in itself relates to the energy loss due to plastic soil deformations.
For the calculation of the horizontal and vertical wave velocity at the pipe, located at an embedded depth, the proposed formulations in Equation 6.3 and Equation 6.4 need to be modified to Equation 6.40 and Equation 6.41. In these equations, the amplification factor, $F_{\nu}$ and $F_{h}$, are erased and the equations are generalized by making them dependent on the radius of the wave propagation path, $r(\mathrm{~m})$. In this way the formulations can both be used for the propagation of P-wave along the critical radius, $r_{\text {crit }}(\mathrm{m})$, and the direct radius, $r_{r}$ (m), by the method of substitution.

$$
\begin{gather*}
v_{1}=k_{s} E_{T} \frac{\sqrt{F^{H} W_{0}}}{r} \cos \left(90-\theta_{P}\right)  \tag{6.40}\\
v_{2}=k_{s} E_{T} \frac{\sqrt{F^{H} W_{0}}}{r} \cos \left(\theta_{P}\right) \tag{6.41}
\end{gather*}
$$

Prediction of wave speed at the pipe's surface, emerged by Rayleigh-waves, will be accomplished in the following manner:

1. The wave speed at surface level induced by primary compression waves (also named spherical waves or P-waves) traveling along the critical radius, $r_{\text {crit }}$. The damping behavior will be accomplished by Equation 2.22 with substitution of $n=1$.
2. The P-wave will be reflected at the surface and continue as a Rayleigh-wave. [Note: the calculation method assumes that $100 \%$ of the P-wave energy will be converted into Rayleigh-wave behavior]
3. The propagating Rayleigh-wave will experience damping behavior as it propagates along the surface to the pre-described position of the pipe. Calculation is accomplished by Equation 2.22 with substitution of $n=0.5$.
4. The simulated wave speed from the Rayleigh-wave is determined at surface level. The value needs to be corrected to the pre-defined embedded depth of the pipe structure according to Figure 2.7.
5. The end result is a vertical Rayleigh-wave oscillation at surface level and should be corrected by: $v_{R, c o r r}=$ $W_{\text {correction }} \nu_{R}$, with $W_{\text {correction }}$ determined with Equation 6.33.

The wave propagation (Ray-)path for Rayleigh-waves is visualized in Figure 6.8 and indicated with the redcolored (Ray-)path.

In a similar manner the wave speed at the pipe excited by primary compression waves is calculated. These waves on the other hand are traveling directly to the pipe structure. The following steps are undertaken to accomplish the indication value of the direct P -waves:

1. The wave speed at surface level induced by primary compression waves (also named spherical waves or P-waves) traveling along the direct radius, $r_{r}$. The damping behavior will be accomplished by Equation 2.22 with substitution of $n=1$.
2. The end result can be presented by horizontal and vertical oscillation (or a combination of the two).

The wave propagation (Ray-)path for P-waves is visualized in Figure 6.8 and indicated with the blue colored (Ray-)path.


Figure 6.8: Wave propagation behavior of Rayleigh-waves and P-waves in an elastic half-space, modified after Massarsch and Fellenius [80]

### 6.3. Verification with Finite Element Model

Conceptual models, like the analytic wave propagation model used in this Master Thesis project, require validation by means of measured data from field tests and/or laboratory tests. Validation of a model is performed by comparison of its calculated conceptual data with corresponding measured data of field or laboratory tests.
The constitutive behavior of the material applied in the ABAQUS model relates to the general equation of motion given by Equation 6.42, which is solved. In this equation the displacement $w$ is related to: the mass matrix $M$; damping matrix [C]; stiffness matrix [K]; force vector $F$. Since the mass matrix, stiffness matrix and the damping matrix are symmetric (the damping matrix is a linear function of the stiffness and mass matrix as can be seen later on in this chapter), the system can be solved relatively easy.

$$
\begin{equation*}
[M] \ddot{w}(t)+[C] \dot{w}(t)+[K] w(t)=F(w, t) \tag{6.42}
\end{equation*}
$$

Due to the lack of time in this Master Thesis project validation of the wave propagation model (WPM) is not possible. Instead Finite Element Model (FEM) (as a commercial software ABAQUS application) calculations are carried out and compared with the obtained solutions of the WPM. The data comparison between the WPM calculations and the FEM calculations will give good insight in suitability of the wave propagation model.
With the FEM a parametric study will be performed to investigate: the influence of a change in pile driving force using different operating amplitudes of the vibrating hammer; changing soil stiffness and soil types; and varying the magnitude of material damping for the soil domain.

### 6.3.1. Model definition

The dynamic Finite Element analysis uses ABAQUS/Explicit Finite Element software [3] to analyze the vibratory pile problem investigated in this thesis. Numerical techniques are used by the program to estimate the wave propagation behavior as a result of pile installation pressure caused by the pile tip. ABAQUS/Explicit uses an explicit central difference time integration to solve the differential equations linked to the integration points of the elements. "The integration scheme satisfies the dynamic equilibrium equations at the beginning of the time step, $t^{\prime \prime}$ Ekanayake et al. [41]. The displacement, velocity and acceleration are determined at time $t+\Delta t$ and $t+(\Delta t / 2)$ by Equation 6.43, Equation 6.44 and Equation 6.45. In these equations $P_{t}$ is the external force vector, $I_{t}$ the internal force vector and $M$ the mass matrix.

$$
\begin{align*}
w_{t+\Delta t}^{N} & =w_{t}^{N}+\dot{w}_{t+(\Delta t / 2)}^{N} \Delta t  \tag{6.43}\\
\dot{w}_{t+(\Delta t / 2)}^{N} & =\dot{w}_{t-(\Delta t / 2)}^{N}+\ddot{w}_{t}^{N} \Delta t  \tag{6.44}\\
\ddot{w}_{t}^{N} & =M^{-1}\left(P_{t}-I_{t}\right) \tag{6.45}
\end{align*}
$$

## Model and grid Geometry

The ABAQUS Finite Element Model is divided into two (linked) domains (as can be seen in Figure 6.9):

- The soil domain: Existing of linear quadratic 4-node axis-symmetric elements (CAX4), see Figure 6.10
- The infinite soil domain: Existing of linear quadratic 4-node axis-symmetric infinite elements (CINAX4), see Figure 6.10


Figure 6.9: Radial symmetric domain of the ABAQUS model: Soil domain existing of CAX4 elements; infinite domain existing CINAX4 elements

The dimensions of the model are not based on the more often used 30D or 40D rules, according to Ekanayake et al. [41], were $D$ is the diameter of the pile driven. In this case one so called 'base-model' is developed for all the different calculations performed, where the grid and dimensions of the model stay constant (varying the pile depth and different soil types). Nodal values can be obtained within the vertical direction in the domain
of 0 meter to 30 meters of surface level (nodal values outside this domain are influenced by the boundaries and are therefore not used). In horizontal direction values within 1 meters and 30 meters of the pile can be obtained. Due to the required domain, the horizontal as well as vertical dimensions of the soil domain are chosen 50 meters. As rule of thumb, according to Katzenbach [64], the dimensions of the infinite elements are chosen equal to the soil domain and are therefore chosen a length of 50 meters.

(a) 4-node axis-symmetric elements (CAX4), ABAQUS Inc. [4]

(b) 4-node axis-symmetric infinite elements (CINAX4), ABAQUS Inc. [4]

Figure 6.10: ABAQUS elements for the Finite Element calculations
The elements size is regulated by the length of the propagating waves within the domain. According to ABAQUS Inc. [3] the element size can be determined by Equation 6.46, where $l_{\max }(\mathrm{m})$ is the maximum element size and $\lambda_{\min }(\mathrm{m})$ the minimum wave length. The factor 10 relates to a minimum requirement of 10 elements per wave length as stated in ABAQUS Inc. [3].

$$
\begin{gather*}
l_{\max }=\frac{\lambda_{\min }}{10}  \tag{6.46}\\
\lambda_{\min }=\frac{c}{f_{e x}} \tag{6.47}
\end{gather*}
$$

The minimum wave length (see Equation 6.47) is dependent on:

- The wave type and soil material. This affects the wave speed in the material, $c(\mathrm{~m} / \mathrm{s})$, which can be resolved with Equation 2.15 (and leads to $c_{S}=88 \mathrm{~m} / \mathrm{s}$ for $S$-waves in the medium dense sand material and $c_{S}=46 \mathrm{~m} / \mathrm{s}$ for $S$-waves in the clay material)
- Frequency of excitation, $f_{e x}(\mathrm{~Hz})$

With the use of Table 2.1 and the excitation frequency equal to 25 Hz the element size is chosen a safe value of 0.25 m (whereas the maximum element length is 0.18 m according to the calculated value of $c_{S}=46 \mathrm{~m} / \mathrm{s}$, which might seem low compared to the value $c_{S}=150 \mathrm{~m} / \mathrm{s}$ for clay given in Table 2.1). Figure 6.11 shows the finite element mesh-distribution along the domain, where the finer elements of $0.01 \times 0.25$ (horizontal,vertical) are applied near the pile and the coarser elements of $0.25 \times 0.25$ are chosen further away from the pile. The choice for the finer grid near the pile is not only for numerical purposes, but also to be able to replicate the applied force, with respect to its surface area (more detailed information follows further on in this chapter). Grid optimization could have been performed to minimize calculation time, but is not taken into consideration due to the focus of this Master Thesis research project and the limited time frame of its application.

## Material properties

The material properties for the ABAQUS model are defined earlier in this chapter (see Table 6.1 and Table 6.2).

## Damping in the system

Damping relates to energy loss generated by plastic deformations within a material. In the calculation of a purely elastic medium, plastic deformations can not occur as there is no plastic limit defined. When considering damping for a purely elastic material, the material will behave purely elastic and no energy loss will occur. In the model energy loss as a result of damping will be modeled by means of Rayleigh viscous damping (related in reality to plastic material behavior) according to Equation 4.4 and Equation 6.48, ABAQUS Inc. [3]


Figure 6.11: Grid distribution for the axisymmetric ABAQUS model
(see also Chapter 4). This formulation is used in the general equation of motion, Equation 6.42, (that is used as constitutive behavior and solved for all grid components) and will serve as damping matrix [C].
[Note: Application of an additional damping factor for an elasto-plastic material calculation will result in too much damping as damping due to plastic material behavior will already occur. Applying the linear elastic - perfect plastic material behavior without Rayleigh-damping will result in too less damping of the higher frequencies, resulting in a more oscillatory behavior of the Finite Element Model. With this last application, the plastic strains will only occur at the location of the pile toe, as here the excitation pressure is very high.]

$$
\begin{equation*}
[C]=\alpha[M]+\beta[K] \tag{6.48}
\end{equation*}
$$

The two factors $\alpha$ and $\beta$ (see also Table 6.1) are determined by Equation 4.5 and relate to mass proportional damping and stiffness proportional damping, respectively. In this formulation $\omega_{m}(\mathrm{rad} / \mathrm{s})$ relates to the eigenfrequency of the first mode and $\omega_{n}(\mathrm{rad} / \mathrm{s})$ to the eigenfrequency where $95 \%$ of the model mass is activated. The Rayleigh damping formulation for sand and clay are shown in Figure 6.12 and Figure 6.13 (the red lines indicate the eigenfrequency of the first mode and the frequency where $95 \%$ of the model mass is activated). From both the figures it can be concluded that an increase in frequency leads to an increase in damping (only for frequencies bigger then the first mode). This statement relates to Colerado University [115] saying: "Damping generally increases with frequency, the reason being that more hysteresis cycles take place within a fixed time interval". The Rayleigh-damping procedure ensures that all frequencies outside the domain (indicated with the two red lines in the figures) will be damped out (due to high damping factors). This only applies to frequencies higher than the upper limit as the lower limit is equal to the eigenfrequency of the first mode. Another important feature to notice is that the higher the frequency, the more sensitive to damping the mechanism becomes. With other words: for high frequencies and increase in damping coefficient has a bigger impact on the damping within the system than for lower frequency rates. For all the purely elastic models considered in this Master Thesis project, $5 \%$ material damping is taken into account (as defined earlier in this chapter). The frequencies used in the calculation of the $\alpha$ and $\beta$-parameters are drawn out in Table 6.4 and Table 6.5.

Rayleigh- Damping sand


Figure 6.12: Variation of viscous damping, for medium dense sand, as a function of period and frequency for different values of damping using Rayleigh's damping formulation

Rayleigh- Damping clay


Figure 6.13: Variation of viscous damping, for medium packed clay, as a function of period and frequency for different values of damping using Rayleigh's damping formulation

| Mode | Freq. $[\mathbf{r a d} / \mathbf{s}]$ | Freq. $[\mathrm{Hz}]$ |
| :--- | :--- | :--- |
| 1 | 10.040 | 1.598 |
| 6 | 23.334 | 3.714 |

Table 6.4: Eigenmodes with corresponding eigenfrequencies used for the Rayleigh-damping determination of the system with Sand as material

| Mode | Freq. $[\mathbf{r a d} / \mathbf{s}]$ | Freq. [Hz] |
| :--- | :--- | :--- |
| 1 | 11.145 | 1.774 |
| 13 | 42.155 | 6.709 |

Table 6.5: Eigenmodes with corresponding eigenfrequencies used for the Rayleigh-damping determination of the system with Clay as material

The boundaries, modeled as infinite elements, include the geometric damping behavior of the model. The algorithm for the infinite elements works on basis of stress increases. The damping effect is ensured by minimization of the stress increase at the boundaries. This process is carried out by keeping the stress increase at the boundary as close to zero as possible by geometric damping within its element length. Therefore the length of the elements has a big impact on the geometric damping ensured by the infinite elements. A rule of thumb, as explained earlier, is therefore to equalize the length of the elements to the model geometry
as stated by Katzenbach [64]. To assure that the reflection of waves at the boundary is minimized, circular shaped boundaries are normally chosen. The simulation time applied in the calculations $0.4-0.8$ seconds, is relatively short. Therefore this more complex geometry is not considered as reflection of waves at the boundaries will not occur in this small time frame.

## Loads on the system

The method of implementation of the forces on a system is an essential factor for the reliability of the outcome prediction of a model. In the case of the earlier described ABAQUS model the impact of the vibrator on the system will result in nodal displacements. Furthermore the initial conditions, implemented as a geostatic stress distribution along the entire domain, will positively impact the reliability of the outcome as well (as this is the case in reality).
The soil weight is implemented as geostatic stress, $\sigma_{\text {geo }}(\mathrm{kPa})$, over depth (see Figure 6.14) and is calculated by Equation 6.49. In this relation: $\gamma_{\text {soil }}\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ is the relative density of the soil; $g\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ the gravitational acceleration; and $d(\mathrm{~m})$ the depth with respect to the surface level.

$$
\begin{equation*}
\sigma_{\text {geo }}=\gamma_{\text {soil }} g d \tag{6.49}
\end{equation*}
$$

Forces resulting from the vibrating pile point are applied as a pressure on the grid cells with respect to the pile toe (depth) position. This Master Thesis research project governs sheet pile walls. On the other hand pressure resulting from a circular pile tip is implemented in the ABAQUS model, since an axisymmetric model build up is applied. Therefore conversion of the pile tip from a sheet pile geometry to a round pile geometry needs to be undertaken (to prevent the need for 2-dimensional or 3-dimensional plane-strain calculations when applying a sheet pile wall geometry). The conversion is a normal-stress-based conversion method, where the activated sliding wedge in the soil is not taken into consideration (as this is smaller for the sheet pile wall than for the round pile geometry). This means that the conversion is based on pressure generated by the pile geometry. This application method could result in the application of to high local stresses, which is important in the case of the plastic material model.
The conversion of the sheet pile wall geometry to the round pile geometry is given by Equation 6.50, where the cross-sectional area of the sheet pile equals the cross-sectional area of the round pile (see also Figure 6.14). This relation relates to the required radius of the pile $r=0.069(\mathrm{~m})$, for a type AZ32-750, Arcelor Mittal [90], sheet pile wall applied in the ABAQUS model and is given in Equation 6.51.

$$
\begin{equation*}
A_{\text {sheet }}=A_{\text {round }} \tag{6.50}
\end{equation*}
$$



Figure 6.14: Conversion of surface area from sheet pile wall to round pile for the ABAQUS model

$$
\begin{equation*}
r_{\text {pile }}=\frac{\sqrt{\frac{4 A_{\text {sheet }}}{\pi}}}{2} \tag{6.51}
\end{equation*}
$$

The pressure, $P_{\text {cell }}\left(\mathrm{N} / \mathrm{m}^{2}\right)$, applied on the grid cells depends on the chosen horizontal size of the cells, $l_{\text {cell, hor }}(\mathrm{m})$; the number of horizontal cells chosen, $N_{\text {cells }}(-)$; the applied force of the vibrator, $F_{d}(\mathrm{~N})$, and is calculated with Equation 6.54. The applied force of the vibrator is calculated with use of Equation 6.52 and Equation 2.43. A PVE55M vibrating machine is used for the calculation leading to $F_{d}=1.410^{6} \mathrm{~N}$, Diesco Group B.V. [46]. This force relates to pressure by multiplication with the cross-sectional area of the sheet pile wall, $A_{\text {sheet }}\left(\mathrm{mm}^{2}\right)$.

$$
\begin{gather*}
F_{d}=F_{v i b r o}+F_{z}  \tag{6.52}\\
P_{\text {cell }}=\frac{F_{d}}{A_{\text {sheet }}} \frac{r_{\text {pile }}}{N_{\text {cells }} l_{\text {cell,hor }}} \tag{6.53}
\end{gather*}
$$

$$
\begin{equation*}
P_{\text {cell }, \sin }=P_{\text {cell }} \sin (\omega t) \tag{6.54}
\end{equation*}
$$

The stress, $P_{\text {cell }, \text { sin }}\left(\mathrm{N} / \mathrm{m}^{2}\right)$, is equally distributed over the number over cells, $N_{\text {cell }}(-)$, according to a sinusoidal amplitude function given by Equation 6.54. In this function the frequency, $\omega$ (Hz), is equal to the excitation frequency of the vibrator. The result of one cycle of the vibrator hammer is modeled, as more cycli would result in wave reflection at the boundaries (and therefore a disturbance of the outcome). The stress distribution on ABAQUS model grid is schematically represented in Figure 6.15.


Figure 6.15: Pressure distribution of the pile toe on the grid cells in the ABAQUS model with a sinusoidal pressure distribution

### 6.3.2. Solution method for non-linear problems

Non-linear soil behavior is related to non-linear differential equations (or so called non-linear problems). When not approached in a correct way the behavior of non-linear problems in the program ABAQUS can relate to an unstable solution spectrum. To obtain convergence when solving non-linear problems ABAQUS uses the so called Newton-Raphson method, as carried out by Katzenbach [64]. The following section declares the Newton-Raphson method applied in the ABAQUS program to obtain convergence.

## Newton-Raphson method

To obtain convergence in the calculation for every iteration made by the ABAQUS model, the Newton-Raphson approach calculates a tangential stiffness, $k$. This stiffness is then used to build up the new stiffness matrix, $K$ for the next calculation step.


Figure 6.16: Calculation of the first increment with the Newton-Raphson approach, according to Katzenbach [64]

In the first calculation step the start-stiffness matrix, $K_{0}$, is calculated. With this matrix the displacement, $u_{0}$, can be determined. The criterion for residual error, as later on in this section is carried out, can be used. If the error is to big the Newton-Raphson approach will be applied.

The first step is the determination of the displacement correction, $c_{a}$, by means of Equation 6.58. In this formulation the force vector, $F$, relates to the increment in force, $\Delta P$. The new displacement, $u_{a}$, is then obtained (see Figure 6.16).

$$
\begin{equation*}
c_{a}=K_{0}^{-1} F \tag{6.55}
\end{equation*}
$$

The new stiffness matrix, $K_{a}$, is then resolved by the tangential method as used to obtain the original stiffness matrix, $K_{0}$, but is then corrected to the new position of the graph as shown in Figure 6.16. With the use of the newly calculated stiffness matrix, $K_{a}$, the internal force matrix, $I_{a}$, can be calculated with use of Equation 6.56. With the use of the internal force vector, the error can be calculated according to Equation 6.57 and used for the check.

$$
\begin{align*}
I_{a} & =K_{a} u_{a}  \tag{6.56}\\
R_{a} & =P-I_{a} \tag{6.57}
\end{align*}
$$



Figure 6.17: Calculation of the first increment with the Newton-Raphson approach, according to Katzenbach [64]

This process can be repeated until convergence is found as shown in Figure 6.17. The only difference in the following calculation steps is the determination of the correction for the displacement. In Equation 6.58 this is disclosed for the second calculation step, $b$.

$$
\begin{equation*}
c_{b}=K_{a}^{-1} R_{a} \tag{6.58}
\end{equation*}
$$

## Convergence criterion

To check if the calculated value satisfies the prerequisite convergence criterion, the following two principles, within the model at every calculation step, are checked:

1. Check the calculated residual error
2. Check the magnitude of displacement

The benchmark for the check of the calculated residual error, $R_{\text {max }}^{\alpha}$, is given by Equation 6.59. In this equation $R_{n}^{\alpha}$ relates to the predefined tolerance of the error and $\tilde{q}^{\alpha}$ to the average force per time increment (calculated with Equation 6.60. Furthermore time step, $i$, and average force, $\bar{q}^{\alpha}$ ) are applied.

$$
\begin{gather*}
\frac{R_{\max }^{\alpha}}{\tilde{q}^{\alpha}} \leq R_{n}^{\alpha}=5 e^{-3}  \tag{6.59}\\
\tilde{q}^{\alpha}=\frac{\sum_{i} \bar{q}^{\alpha}}{i} \tag{6.60}
\end{gather*}
$$

The check for the magnitude of displacement is implemented by use of Equation 6.61 with the substitution of the increment of the displacement, $\Delta u_{\text {max }}^{\alpha}$, and the magnitude of the displacement correction, $c_{\text {max }}^{\alpha}$.

$$
\begin{equation*}
c_{\max }^{\alpha} \leq 0.01 \Delta u_{\max }^{\alpha} \tag{6.61}
\end{equation*}
$$

## Quasi-Newton

According to Katzenbach [64] ABAQUS uses a Quasi-Newton equation solver that remembers the stiffness matrix, $K$, of the previous step. This stiffness matrix is then used in the next attempt so that not for every time step the stiffness matrix needs to be calculated. With implementation of this efficient technique the calculation time is shorted.

### 6.4. Comparison with codes of conduct

Codes of conduct are used as guidelines in the design process of all engineering applications. Therefore they will be utilized in the comparison with the obtained particle velocities of the Abaqus model as well as the Wave Propagation Model.
Different guidelines for the maximum allowable particle velocity are presented throughout the world. On the other hand the Eurocode 3 [2] is the only code of conduct that outlines Peak Particle Velocities (PPVs) for buried structures. And as this code is the main code of conduct adopted in Europe's civil engineering projects, the comparison will be made with use of its advised Peak Particle Velocities. Ekanayake et al. [41] outlined the different levels of the Eurocode 3 with respect to Peak Particle Velocities. These quantities are summarized in Table 6.6.

| Building type | PPV $[\mathbf{m m} / \mathbf{s}]$ |
| :--- | :--- |
| Architectural merit | 2 |
| Residential area | 5 |
| Light commercial | 10 |
| Heavy industrial | 20 |
| Buried structures | 25 |

Table 6.6: Eurocode 3: Maximum acceptable vibration levels to prevent structural damage, according to Ekanayake et al. [41] and Eurocode 3 [2]

### 6.5. Coupling function

Interaction between the impeding wave and the pipe structure is handled through the one-sided coupling approach. Particle velocities calculated with the Wave Propagation Model (WPM) relate to pressure waves on the pipe structure as impellent.

### 6.5.1. Model structure

The coupling function is build up out of two important stages:

1. Calculation of wave velocities at elements
2. Determination of force at elements

These stages are outlined in detail in the following section.

## Calculation of wave velocities at elements

The maximum allowed particle velocity at the boundary, $v_{\text {bound }}(\mathrm{mm} / \mathrm{s})$, calculated with Equation 6.62, relates directly to the size of the Finite Element Model for the pipe structure.

$$
\begin{equation*}
v_{\text {bound }}=\gamma v_{\max } \tag{6.62}
\end{equation*}
$$

The maximum particle velocity, $v_{\max }(\mathrm{mm} / \mathrm{s})$, relates to the direct distance, $L_{\text {direct }}(\mathrm{m})$, as shown in Equation 6.62. The factor, $\gamma(-)$, is a predefined reduction factor for the particle velocity level at the boundaries. Using an iterative approach the wave velocities in the domain from the direct distance to the boundaries are calculated for all the elements. The calculation runs until the Peak Particle velocity level at the element equals the predefined level at the boundary. The number of elements is therefore dependent on the Peak Particle velocity level defined at the boundaries.
As the problem is symmetric, only half of the values are calculated and mirrored to the remaining grids cells to shorten calculation time. With the same method the travel time of the wave, $t_{\text {wave }}(\mathrm{s})$, calculated with use of Equation 6.63, to the corresponding element is calculated. The travel time depends on the horizontal distance
the wave needs to travel from the pile toe the pipe element, Distance $_{\text {element }}(\mathrm{m})$, and the progression speed of impeding P-waves in the soil. For this calculation the direct distance is taken, since the maximum level of particle velocity at the pipe structure is obtained when the pile toe is at the same depth as the pipe structure in the subsurface. This travel time directly relates to the time shift of the forcing functions applied on the pipe structure elements.

$$
\begin{equation*}
t_{\text {wave }}=\frac{\text { Distance }}{\text { element }} \text { } \tag{6.63}
\end{equation*}
$$

[Note: As the distance from the pile to the pipe Finite Element increase (when moving more to the outer boundaries), the wave velocity, $v$, will decrease due to damping properties of the soil]


Figure 6.18: Grid distribution of the pipe Finite Element Model dependent on the percentage of maximum wave velocity chosen at the outer boundary

## Determination of Force at elements

For every element the maximum particle velocity is distributed over time with the application of Equation 6.64. The sinusoidal particle velocity behavior is replicated by the sinusoidal distribution with the application of the maximum particle velocity, calculated with the Wave Propagation Model, as amplitude. In the relation described by Equation 6.64, $\omega(\mathrm{rad} / \mathrm{s})$ relates to the angular excitation frequency of the system (the frequency corresponding with the vibratory frequency) and $t_{\text {simulation }}(\mathrm{s})$ to the simulation time.

$$
\begin{equation*}
v_{\text {Sin }}=v_{\max } \sin \left(\omega t_{\text {simulation }}\right) \tag{6.64}
\end{equation*}
$$

To convert the particle velocity, $v(\mathrm{~m} / \mathrm{s})$, to a force distribution along the elements, $F(\mathrm{~N})$, the following algorithm enforced:

1. Displacement: Calculation of the corresponding particle displacement, $w(\mathrm{~m})$, in the soil.
2. Pressure: The corresponding pressure, $P(\mathrm{~Pa})$, is calculated with use of the pressure-displacement method according to Boussinesq's methodology as entailed by Pistrol [98] (explained in detail earlier in this chapter).
3. Force: The force working on the pipe structure, $F(\mathrm{~N})$, is obtained by translation of the pressure onto the pipe structure area, $A_{\text {element }}\left(\mathrm{m}^{2}\right)$, for all the elements.

The conversion methodology from peak particle velocity (PPV) to particle displacement, $w(\mathrm{~m})$, requisites integration of Equation 6.64 over its entire time domain (for every element independently). The integration is performed with use of the cumulative Trapezoidal Rule, as described in Equation 6.65, and implemented by Yeh [128] and Jones [61]. This results in a cumulative distribution function.

$$
\begin{equation*}
\text { Cum. Area }=\int_{0}^{t(i)} f(t) d t=\frac{(t(i)-0)}{i} \frac{f(0)+f(t(i))}{2} \tag{6.65}
\end{equation*}
$$

Correction of the cumulative function, described in Equation 6.65, is performed by fitting a linear curve to the function and disregarding it from the obtained function outcome (see Figure 6.19). With this procedure a sinusoidal function is regained, describing the particle displacements resulting from the pressure wave. Due to the error, resulting from the application of the Trapezoidal Rule, the sinusoidal function achieved for the
particle displacements will not be a $100 \%$ horizontal, which could lead to an increasing displacement of the pipe structure.
[Note: Expected function values for a velocity distribution is zero for time is equal to zero and is increasing/decreasing according to a sinusoidal function over a time period modeled. Similar function behavior is expected for the displacement and is with the application of this technique obtained as model behavior (see Figure 6.19)]


Figure 6.19: Displacement calculation method with the help of the cumulative Trapezoidal Rule and disregarding linear fit curve

With use of Boussinesq's methodology, as entailed by Pistrol [98], the required pressure, $P(\mathrm{~Pa})$, can be calculated. This formulation is described in Equation 6.66.

$$
\begin{equation*}
P=\frac{w \pi C}{2}\left[\ln \left(\sqrt{a^{2}+b^{2}}+a\right) b+\ln \left(\sqrt{a^{2}+b^{2}}+b\right) a-\ln \left(\sqrt{a^{2}+b^{2}}-a\right) b-\ln \left(\sqrt{a^{2}+b^{2}}-b\right) a\right] \tag{6.66}
\end{equation*}
$$

The final step is to convert the pressure to force per unit length of the pipe, $F$ ( $\mathrm{N} / \mathrm{m}$ ), (for every element) by application of Equation 6.67. In this equation the pressure, $P(\mathrm{~Pa})$, is multiplied by the soil-contact area of the pipe, $A_{\text {element }}\left(\mathrm{m}^{2}\right)$. It is uncertain what the exact contact area between the pipe and the soil includes (due to all kinds of circumstances like for instance animal live. The animal live could effect the contact area and therefore the contact area could be lower as simulated with the method presented in Equation 6.67.) The conversion from the equally distributed pressure on the pipe to the force is shown in Figure 6.20.

$$
\begin{equation*}
F=P A_{\text {element }}=P \frac{1}{2} \pi D_{\text {Pipe }} \tag{6.67}
\end{equation*}
$$

More research is required to achieve the required knowledge about the pressure distribution on the pipes surface. No further comments can be made to what extent this method is conservative or not.


Figure 6.20: Conversion from pressure to force on pipe structure

Very important on the other hand is to keep in mind is that for every pipe element the force vector entails a time shift with respect to the travel time. This travel of the wave depends on the distance from the pile to the pipe element. These time shifts will result in a different pipe response behavior to the force than when this time shift would not be applied (which is not a realistic).
To shorten calculation time and to ensure the force distribution relates to a realistic situation the calculated force is multiplied with a so called "Block function" (see Figure 6.21). The application of this methodology ensures that the force is applied on the structure with a gradual increase in sine amplitude as can be seen in Figure 6.22.


Figure 6.21: Block function applied on the force


Figure 6.22: Modified force on pipe structure by means of the application of the block function

### 6.6. Pipe structure model

The embedded pipe structure is excited by a dynamic load resulting from nearby pile installations. Predicting the high complex behavior of this structure requisites good understanding of the system as a whole. Prognosis of the time dependent movement of the pipe, with resulting stress increases, is of high importance to prevent damage to the structure. Therefore a sophisticated Finite Element Model (FEM) approach is developed with the software Matlab [82]. With this FEM the time dependent oscillatory behavior of the pipe structure will lead to a good understanding of the mechanism.

### 6.6.1. General model description

A Finite Element (FE) approach with use of the Euler-Bernoulli beam theorem as governing equation offers a possibility to describe the behavior of the pipe structure as a finite length beam (see Figure 6.23). The


Figure 6.23: Finite Element Model schematic representation, modified after Metrikine [86]
choice for the Euler-Bernoulli beam theorem is made, since the Euler-Bernoulli method deals with 4 x 4 local matrices whereas the Timoshenko theory deals with $8 \times 8$ matrices. The Timoshenko theory involves a longer calculation time and not only includes a sophisticated model approach that includes the displacement and internal rotations of the beam (see Thomas and Abbas [111]), but also describes the shear behavior of the pipe, which makes it very complicated.
The Euler-Bernoulli Finite Element scheme will be represented with use of the general equation of motion (see Equation 6.68) as constitutive behavior, which includes the dynamic manner into the FE approach. The elements are modeled with Hermite cubic interpolation ( or cubic spline) functions as they fit the requirement to be thrice differentiable, Fereira [42]. Galerkin's weighted residual method with use of Greens theorem, as described by Gunakala et al. [48], Desai [37], Thomas and Abbas [111], Pesheck et al. [96] and the University of Colerado [116], offers the possibility to implement the Euler-Bernoulli beam theorem into a FE approach. By means of conversion of mass and stiffness properties (of the pipe) into corresponding coupled global mass matrix, $[M]$, and stiffness matrix, $[K]$ the FE approach is achieved. More information on its application and implementation follows in this chapter. The force acting on the structure is coupled to the output of the modified wave propagation model of Massarsch and Fellenius [80], with the coupling function as described earlier in this chapter. The Galerkin's weighted residual method with use of Greens theorem is also used to convert the force vector into Finite Element form.

$$
\begin{equation*}
[M] \ddot{w}(t)+[C] \dot{w}(t)+[K] w(t)=F(w, t) \tag{6.68}
\end{equation*}
$$

### 6.6.2. Model structure FEM with Rayleigh-damping

Figure 6.24 declares the Finite Element program flow (also known as pseudo-code) for the pipe structure modeled with Rayleigh-damping. A Finite Element Model (FEM) is in general build up out of eight steps:

1. Governing equation
2. Constitutive behavior
3. Element discritization
4. Weighted residual method
5. Domain discritization
6. Element assembly
7. Boundary and initial conditions
8. Solve the system

These steps can be related to the model flow given in Figure 6.24 and will be declared in detail in this section.

## Initialization

The initialization process is modeled to obtain the model parameters that required to undertake further calculation steps. Parameters related to the material behavior as well as spacial and time dimensions are prescribed by the user and used for the model calculation.


Figure 6.24: Model flow for the Finite Element approach of the pipe structure with Rayleigh-damping

The model dimensions are determined in the coupling function. The rod-element length is the same for every element and is set at 0.1 m . The quantity of elements (number of elements) is dependent on the distance between the pile and the pipe, plus the user defined limit value of peak particle velocity, $v(\mathrm{~mm} / \mathrm{s})$, at the boundaries of the model. The rod elements consist of 4 integration points, that are declared in detail in the next sections. To clarify the system for the numbering of the nodes and elements, three coupled rod-elements including their numbering are shown in Figure 6.25.

|  | E1 |  | E2 |  |  | E3 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\oint$ | + | + | $\dagger$ | + | + | $\phi$ | + | + | $\bigcirc$ |
| N1 |  |  | N2 |  |  | N3 |  |  | N4 |

Figure 6.25: Element configuration pipe structure Finite Element Model

## Construct matrices

Construction of the matrices requires the governing equation. Euler-Bernoulli's beam conceptual model, described in Equation 6.69, is used to describe the conservation of mass and energy of the system by means of an equilibrium. The derivation of this method is given in Chapter 4.

$$
\begin{equation*}
\frac{d^{2}}{d x^{2}}\left[r(x) \frac{d^{2} w}{d x^{2}}\right]=q(w), \quad 0 \leqslant x \leqslant L_{\text {Beam }} \tag{6.69}
\end{equation*}
$$

The rigidity function $r(x)=E I\left(\mathrm{~N} / \mathrm{m}^{2}\right)$ is build up out of the Young's modulus, $E(\mathrm{kPa})$, and the moment of inertia $I\left(\mathrm{~m}^{4}\right)$. In this case the rigidity function is equal throughout the domain and therefore the equation
can be written as Equation 6.70.

$$
\begin{equation*}
E I \frac{d^{4} w}{d x^{4}}=q(w), \quad 0 \leqslant x \leqslant L_{\text {Beam }} \tag{6.70}
\end{equation*}
$$

The element discritization, the weighted residual method and the domain discritization are all carried out to form the Galerkin FEM. The domain, characterized by the length of the beam, $L_{\text {Beam }}(\mathrm{m})$, is divided into a finite number of rod elements coupled with each other by their nodes to form the entire beam (pipe) model. To be able to apply the Hermite cubic interpolation functions in combination with Equation 6.70, this equation needs to be represented in the so called "weak form" of the differential equation, as stated by Gunakala et al. [48]. A supplemental weight function, $s(x)$, is introduced and the entire differential equation will be integrated by parts resulting in Equation 6.71 (for convinces $L_{\text {Elem }}=$ element length is replaced by $L$ for the derivation of the Galerkin FEM).

$$
\begin{equation*}
\int_{0}^{L}\left[E I \frac{d^{4} w}{d x^{4}}-q(w)\right] s d x=\left[E I \frac{d^{3} w}{d x^{3}} s\right]_{0}^{L}-\left[E I \frac{d^{2} w}{d x^{2}} \frac{d s}{d x}\right]_{0}^{L}+\int_{0}^{L}\left[E I \frac{d^{2} s}{d x^{2}} \frac{d^{2} w}{d x^{2}}-q s\right] d x=0 \tag{6.71}
\end{equation*}
$$

The Hermite cubic interpolation functions are given by Equation 6.72, Equation 6.73, Equation 6.74 and Equation 6.75. Each of these functions is graphically represented by Figure 6.26.

$$
\begin{gather*}
N_{1}=\frac{1}{L^{3}}\left(L^{3}-3 L x^{2}+2 x^{3}\right)  \tag{6.72}\\
N_{2}=\frac{1}{L^{2}}\left(L^{2} x-2 L x^{2}+x^{3}\right)  \tag{6.73}\\
N_{3}=\frac{1}{L^{3}}\left(3 L x^{2}-2 x^{3}\right)  \tag{6.74}\\
N_{4}=\frac{1}{L^{2}}\left(x^{3}-L x^{2}\right) \tag{6.75}
\end{gather*}
$$

Each shape function corresponds with an integration point on the rod element, referring to 4 integration points equally distributed on each element. The graphical representation given in Figure 6.26 delineates the inner workings of the shape functions by means of an element with an arbitrary length 1 . Every function has its maximum value with respect to its position on the element. Furthermore the plots show that there is a correlation between the integration points and that they therefore influence each other.
When substituting the shape functions into Equation 6.71, with the assumption that $w=\sum_{j=1}^{4} w_{j} N_{j}$, this results in Equation 6.76.

$$
\begin{equation*}
\int_{0}^{L}\left[E I \frac{d^{4} w}{d x^{4}}-q(w)\right] s d x=\left[E I w_{x x x} N_{i}\right]_{0}^{L}-\left[E I w_{x x} N_{i, x}\right]_{0}^{L}+\int_{0}^{L} E I N_{i, x x} w_{x x} d x-\int_{0}^{L} q N_{i} d x=0 \tag{6.76}
\end{equation*}
$$

The stiffness matrix, $\left[K^{P i p e}\right]$, is given by Equation 6.84, the force vector, $\{F\}$, by Equation 6.78 and in a similar way the mass matrix, $[M]$, is calculated and represented by Equation 6.79 (here $\rho$ represents the density of the material and $A$ the area of the cross-sectional area of the beam).

$$
\begin{gather*}
K_{i j}^{\text {Pipe }}=E I \int_{0}^{L} \frac{d^{2} N_{i}}{d x^{2}} \frac{d^{2} N_{j}}{d x^{2}} d x  \tag{6.77}\\
F_{i}=\int_{0}^{L} q N_{i} d x  \tag{6.78}\\
M_{i j}=\rho A \int_{0}^{L} N_{i} N_{j} d x \tag{6.79}
\end{gather*}
$$

The final result of the matrices $\left[K_{\text {elem }}^{\text {Pipe }}\right]$ and $\left[M_{\text {elem }}\right]$ and the force vector, $\left\{F_{\text {elem }}\right\}$, are given by Equation 6.80, Equation 6.82 and Equation 6.81, respectively. The derivations can be found in Appendix A.

$$
\left[K_{\text {elem }}^{\text {Pipe }}\right]=\frac{E I}{L^{3}}\left[\begin{array}{cccc}
12 & 6 L & -12 & 6 L  \tag{6.80}\\
6 L & 4 L^{2} & -6 L & 2 L^{2} \\
-12 & -6 L & 12 & -6 L \\
6 L & 2 L^{2} & -6 L & 4 L^{2}
\end{array}\right]
$$



Figure 6.26: Hermite cubic interpolation functions (or also known as cubic shape functions) used for the Galerkin Finite Element Model

$$
\begin{gather*}
\left\{F_{\text {elem }}\right\}=\frac{L q}{12}\left\{\begin{array}{c}
6 \\
L \\
6 \\
-L
\end{array}\right\}  \tag{6.81}\\
{\left[M_{\text {elem }}\right]=\frac{\rho A L}{420}\left[\begin{array}{cccc}
156 & 22 L & 54 & -13 L \\
22 L & 4 L^{2} & 13 L & -3 L^{2} \\
54 & 13 L & 156 & -22 L \\
-13 L & -3 L^{2} & -22 L & 4 L^{2}
\end{array}\right]} \tag{6.82}
\end{gather*}
$$

Hereafter the global matrix assembly can be accomplished. To outline the process of assembly an example is given for the assembly of two stiffness matrices relating to two elements, as outlined in Equation 6.83. The second element, in the global ( $6 \times 6$ ) stiffness matrix, is highlighted in red. The process is repeated in a similar manner for the case of $N$ elements, for the force vector, stiffness matrix and mass matrix.

$$
\left[K^{\text {Pipe }}\right]=\frac{E I}{L^{3}}\left[\begin{array}{cccccc}
12 & 6 L & -12 & 6 L & 0 & 0  \tag{6.83}\\
6 L & 4 L^{2} & -6 L & 2 L^{2} & 0 & 0 \\
-12 & -6 L & 12+12 & -6 L+6 L & -12 & 6 L \\
6 L & 2 L^{2} & -6 L+6 L & 4 L^{2}+4 L^{2} & -6 L & 2 L^{2} \\
0 & 0 & -12 & -6 L & 12 & -6 L \\
0 & 0 & 6 L & 2 L^{2} & -6 L & 4 L^{2}
\end{array}\right]
$$

The next step is to implement the boundary conditions and initial conditions into the global matrices, $[K]$ and $[M]$ and displacement vector, $\{w\}$. But first the determination of the soil stiffness matrix is carried out.

## Soil stiffness

To incorporate the soil stiffness reaction on the beam the decoupled Winkeler model using a unit subgrade spring coefficient, according to Adhikary et al [6], is applied. Assumed is that the soil reaction is only activated in the direction of the force applied on the pipe structure. That means that the pipe structure model is simulated as a horizontal beam structure supported by decoupled Winkler springs and with a vertical excitation force (see Figure 6.23).

The stiffness matrix for the soil is defined in a similar way as the stiffness matrix for the pipe structure (defined in Equation 6.80). The soil stiffness per meter length of the pipe structure, $k_{\text {soil }}(\mathrm{Pa} / \mathrm{m})$, is obtained with the use of Equation 6.84 as defined by Adhikary et al [6]. When the quadratic shape functions are applied this leads to the element stiffness matrix, $\left[K_{\text {elem }}^{\text {Soil }}\right]$, as indicated in Equation 6.85.

$$
\begin{gather*}
k_{\text {soil }}=\frac{1.3 G}{D_{\text {Pipe }}(1-v)}  \tag{6.84}\\
{\left[K_{\text {elem }}^{\text {Soil }}\right]=\frac{k_{\text {soil }}}{L^{3}}\left[\begin{array}{cccc}
12 & 6 L & -12 & 6 L \\
6 L & 4 L^{2} & -6 L & 2 L^{2} \\
-12 & -6 L & 12 & -6 L \\
6 L & 2 L^{2} & -6 L & 4 L^{2}
\end{array}\right]} \tag{6.85}
\end{gather*}
$$

The element matrix is then transformed to the global matrix in a similar manner as performed in Equation 6.83. The total stiffness matrix, $[K]$, is then calculated by Equation 6.86 and contains the stiffness response of both the pipe structure and soil.

$$
\begin{equation*}
[K]=\left[K^{\text {Pipe }}\right]+\left[K^{S o i l}\right] \tag{6.86}
\end{equation*}
$$

## Boundary implementation

Application of boundary conditions in a model is very important as it can have a big influence on the outcome of the model. Correctness of the application is in most engineering cases not possible as it implies complicated physical behavior.
The boundaries of the pipe structure model should be able to represent an infinite structure where no reflection of waves at the boundaries is present. Furthermore an infinite boundary is in conjunction to the adjacent grid cell and its movement/behavior.
Two different boundaries are applied for this Master Thesis:

1. Experimental boundary: These so called "experimental boundaries" exist of normal pipe elements, translational as well as rotational stiffness applications, but with the extension of an additional parameter, $\alpha_{\text {trans }}(-)$, and, $\alpha_{\text {rot }}(-)$, respectively (see Equation 6.87 and Equation 6.88). These parameters will be implemented in a parameter study, where the sensitivity of the boundary conditions can be investigated.
2. Extension boundary: This boundary exists of an additional 200 elements on both sides of the pipe structure NOT excited by external forces. These boundary elements have the same properties as the pipe, but are retrained with respect to rotational and transnational movements at the ends of the boundary.

A comparison between the two boundary conditions is made.

$$
\begin{gather*}
k_{\text {bound,trans }}(x=0 V x=\text { end })=\alpha_{\text {trans }} k_{\text {elem }}  \tag{6.87}\\
k_{\text {bound,rot }}(x=0 V x=e n d)=\alpha_{\text {rot }} k_{\text {elem }} \tag{6.88}
\end{gather*}
$$

## Damping matrix

Rayleigh-damping is chosen as the damping method for the system. As outlined in Chapter 4 this relates to the determination of the damping matrix, $[C]$, that is described by Equation 6.89.

$$
\begin{equation*}
[C]=\alpha[M]+\beta[K] \tag{6.89}
\end{equation*}
$$

The parameters $\alpha$ and $\beta$, are in the ABAQUS model calculated by Equation 6.90, relate to the eigenfrequency of the first-mode, $\omega_{m}$; the eigenfrequency were $95 \%$ of the model mass is activated, $\omega_{n}$; and the damping ratio, $\xi$, as variable factor defined by the program user. This case differs from the ABAQUS model. $\omega_{n}$ is a chosen value by the user. A sensitivity study will be carried out related to the choice of upper limit eigenfrequency, $\omega_{n}$.

$$
\left\{\begin{array}{l}
\alpha_{R}  \tag{6.90}\\
\beta_{R}
\end{array}\right\}=\frac{2 \xi}{\omega_{(m)}+\omega_{(n)}}\left\{\begin{array}{c}
\omega_{(m)} \omega_{(n)} \\
1
\end{array}\right\}
$$

Complementary to the calculation of the parameters $\alpha$ and $\beta$, the vector with eigenvalues need to be computed. Metrikine [86] declares that the eigenvalues of a Multiple Degree of Freedom System (MDOF) can be determined by solving Equation 6.91. The real eigenvalues of the system are used in the parameter determination for the Rayleigh-damping matrix, [ $C$ ]. Related to the choice of the sensitivity analysis, the first modal frequency and the Nth modal frequency (chosen by the user) are taken to resolve the parameters $\alpha$ and $\beta$.

$$
\begin{equation*}
\operatorname{det}\left(-\omega^{2}[M]+[K]\right)=\{0\} \tag{6.91}
\end{equation*}
$$

## Solve the system for all time steps

The general equation of motion, given by Equation 6.68, comprises a second-order differential equation (DE). This second-order DE can be re-written as a system of first-order DEs, leading to Equation 6.94. In this equation the substitution of Equation 6.93 is undertaken. In this way the system can be solved numerically by the fourth/fifth-order Runga-Kutta method applied as the build in ODE15s solver from Matlab. The choice for this ode-solver type is made according to Mathworks [83]. The Finite Element system enhances a stiff problem with a change in the so called mass matrix of the system (this mass matrix is not the same mass matrix as the mass matrix of the pipe structure, $[M]$, but indicates the mass matrix of the entire system). This mass matrix includes the properties of: the stiffness matrix, $[K]$; the mass matrix, $[M]$; the force vector, $\{F\}$; and the damping matrix, $[C]$. The stiffness of the problem depends on all the properties of the system that are included by the user like: soil stiffness, pipe material, pipe geometry, boundary conditions, etc..
The ode-solver uses a similar numerical solution resolving technique as the solver used in the ABAQUS software. Here the step-size is chosen according to the absolute tolerance, AbsTol $=1 e-6(-)$, and the relative tolerance, RelTol $=1 e-4(-)$. According to Mathworks [84] the local error, $e$, can not exceed a certain value as shown in Equation 6.92.

$$
\begin{equation*}
|e|<\max ([\operatorname{RelTol} * \operatorname{abs}(y(i))], \operatorname{AbsTol}(i)) \tag{6.92}
\end{equation*}
$$

[Note: Be aware that the force vector is time dependent and therefore for every independent time step needs to be pre-multiplied as global force vector, $\{F\}$, by the inverse mass-matrix $\left.\left[M^{-1}\right]\right]$

$$
\begin{align*}
& w=y_{1} \\
& \dot{w}=\frac{\partial w}{\partial t}=\frac{\partial y_{1}}{\partial t}=y_{2}  \tag{6.93}\\
& \ddot{w}=\frac{\partial^{2} w}{\partial t^{2}}=\frac{\partial^{2} y_{1}}{\partial t^{2}}=\frac{\partial y_{2}}{\partial t}=\dot{y}_{2}
\end{align*}
$$

The solutions of the system will be obtained using the fourth/fifth-order Runga-Kutta numerical method as explained in Chapter 4 and solve the system of equations to obtain the displacement vector, $\{w\}$. This numerical method is implemented by the build-in Matlab ODE15s-solver (as explained earlier).

$$
\left\{\begin{array}{l}
\left\{\dot{y_{1}}\right\}=\left\{y_{2}\right\}  \tag{6.94}\\
\left.\left\{\dot{y_{2}}\right\}=\left[M^{-1}\right]\right]\{F\}-\left[M^{-1}\right][C]\left\{y_{2}\right\}-\left[M^{-1}\right][K]\left\{y_{1}\right\}
\end{array}\right.
$$

## Calculation of stresses and momentum

Additional stresses in the pipe as a result of the induced vibrations is one of the topics of interest in this Master Thesis research project. Momentum in the pipe structure is caused by rotational movement in the pipe structure. The momentum is directly related to the stresses within the pipe structure. Massachusetts Institute of Technology [92] outlined a method to calculate the increase in stress within a beam element as a result of angular rotation at the boundaries of an element (leading to bending of the element). "Plane crosssections remain perpendicular to the longitudinal axis of the beam" Massachusetts Institute of Technology [92].
[Note:This restriction, which does not take shearing of the individual elements into account, is in agreement with the restrictions used to build up the Euler-Bernoulli beam model (see Chapter 4)]

Figure 6.27 shows that the change in rotational movement, $d \theta\left({ }^{\circ}\right)$, of an element is directly related to the rotation angle, $\theta\left({ }^{\circ}\right)$. The rotation angle is an outcome of the Matlab Ode-Solver and is calculated according to Equation 6.95.

$$
\begin{equation*}
d \theta(i)=\theta(i+1)-\theta(i) \tag{6.95}
\end{equation*}
$$



Figure 6.27: Deformed element after rotation $\theta_{1}$ and $\theta_{2}$, modified after Massachusetts Institute of Technology [92]

The stresses strains and momentum in the pipe are all related to the center of gravity for a cross-section of half of the pipe structure. The center of gravity of the pipe, $y_{z}(\mathrm{~m})$, relates to the inner radius of the pipe, $R_{I n}$ $(\mathrm{m})$ and the outer radius, $R_{\text {Out }}(\mathrm{m})$. The center of gravity is determined by Equation 6.96 and its derivation is given in Appendix A.

$$
\begin{equation*}
y_{z}=\frac{4}{3 \pi} \frac{\left(R_{O u t}^{3}-R_{I n}^{3}\right)}{\left(R_{O u t}^{2}-R_{I n}^{2}\right)} \tag{6.96}
\end{equation*}
$$

The change in length, $\Delta s(\mathrm{~m})$, at the center of gravity, due to the change in rotational angle, $d \theta\left({ }^{\circ}\right)$, can be obtained with use of Equation 6.97. The strain at the center of gravity, $\epsilon_{z}(-)$, is resulting from the change in length at the center of gravity, $\Delta s(\mathrm{~m})$, according to Equation 6.98.

$$
\begin{gather*}
\Delta s=y_{z} d \theta  \tag{6.97}\\
\epsilon_{z}=\frac{\Delta s}{\text { Leng thElem }} \tag{6.98}
\end{gather*}
$$

The stress in the pipe structure, $\sigma_{z}\left(\mathrm{~N} / \mathrm{mm}^{2}\right)$, is calculated with the application of Hooke's law and is given in Equation 6.99. The momentum in the pipe is related to the stress level in the pipe and the position of the center of gravity. The momentum, $M_{z}(\mathrm{kNm})$, can be obtained by Equation 6.100.

$$
\begin{gather*}
\sigma_{z}=E_{\text {Pipe }} \epsilon_{z}  \tag{6.99}\\
M_{z}=\sigma_{z} \frac{A_{\text {Pipe }}}{2} y_{z} \tag{6.100}
\end{gather*}
$$

## Results

Development of the Matlab coupled Wave Propagation - Pipe Structure Model (WPM-PSM) is carried out in this Master Thesis research project. Furthermore an ABAQUS [3] model is created to compare the outcome of the Wave Propagation Model (WPM). Calculations with the use of the Matlab coupled Wave Propagation Pipe Structure Model (WPM-PSM) and comparison with ABAQUS are accomplished. Results and correlations corresponding with the outcome are implemented in this chapter. A parameter sensitivity study is practiced to enhance the reliability of its model results and the sensitivity to parameter changes. By means of comparison in calculation time with the model outcome, the efficiency of the WPM-PSM is investigated to reduce calculation time and maintain model performance. As no field measurements are available for vibration excited pipes in the subsurface, validation of the entire model is inconceivable. On the other hand, the Wave Propagation Model (WPM) results are correlated with subsurface ground vibration measurements of Peak Particle Velocities (PPV) of vibratory sheet pile installation found in the literature to validate the outcome. The measurements are converted to a formulation, describing the PPV for vibratory sheet pile installation, by means of parameter fitting by Athanasopoulos and Pelekis [11] and is used as comparison method. The Eurocode 3 [2] is used to relate the results of the simulations with maximum values used in engineering practice.

### 7.1. WPM and ABAQUS

The propagation behavior of vibratory pile excited waves is investigated with the use of two models namely:

1. Wave Propagation model: The basis for this model is the model developed by Massarsch and Fellenius [80]. Modifications are implemented and explained in detail in Chapter 6.
2. ABAQUS: The commercial software ABAQUS [3] is used to build up an axi-symmetric model to simulate the wave propagation behavior through soils.

In this section the results of the Master Thesis investigation are carried out. Various model parameter correlations and dependencies are accomplished and visualized in graphical format. A comparison of the two different models is made to underline the model behavior and parameter sensitivities of both models.

### 7.1.1. Explanation of the result representation

To understand the result presentation, this section reveals the parameters applied as well as the domain location of the graphs drawn out.

## Model parameter

Chapter 6 outlined the model parameters applied for the models. Clarification of the parameters used for the parameter sensitivity study is required to acquire a good understanding of the model behavior. Table 7.1 gives the parameters used for the WPM and ABAQUS calculations and Table 7.2 the for the PSM simulations. Throughout this chapter, different parameter ranges are used for the sensitivity analysis whereas other parameters are remained constant. The parameter spectrum applied can be obtained from the context of the result representation.

Heinoord Tunnel small-strain parameters

| Material properties |  | Sand, medium dense | Clay |
| :--- | :--- | :--- | :--- |
| $\gamma$ | $\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ | 2000 | 1600 |
| $E_{0}$ | $[\mathrm{kPa}]$ | 240000 | 210000 |
| $v$ | $[-]$ | 0.23 | 0.40 |
| $\phi$ | $\left[{ }^{\circ}\right]$ | 35.0 | - |
| $\psi$ | $\left[{ }^{\circ}\right]$ | 5.0 | - |
| $c$ | $[\mathrm{kPa}]$ | 1.0 | - |
| $\alpha$ | $[-]$ | 0.7020 | 0.8815 |
| $\beta$ | $[-]$ | $2.996310^{-3}$ | $1.8762 \quad 10^{-3}$ |
| $\zeta$ | $[\%]$ | 5.0 | 5.0 |

Table 7.1: Material properties for the ABAQUS Finite Element Model and the wave propagation model, Geotechdata [45] and Benz [19]

PSM parameters

| Material properties |  | Soil | Pipe |
| :--- | :--- | :--- | :--- |
| $d_{\text {hth }}$ | $[\mathrm{m}]$ | 5.0 | - |
| $d_{\text {pipe }}$ | $[\mathrm{m}]$ | - | 3.0 |
| $D_{\text {pipe }}$ | $[\mathrm{mm}]$ | - | 500.0 |
| $\rho_{\text {pipe }}$ | $\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ | - | 7850 |
| $E_{\text {pipe }}$ | $[\mathrm{MPa}]$ | - | 210 |
| $t_{\text {pipe }}$ | $[\mathrm{mm}]$ | - | 1.0 |
| $A_{\text {pipe }}$ | $\left[\mathrm{mm}^{2}\right]$ | - | 6271 |
| $I_{\text {pipe }}$ | $\left[\mathrm{mm}^{4}\right]$ | - | 48793646 |
| $f$ | $[\mathrm{~Hz}]$ | 28.0 | - |
| $A_{\text {sheet }}$ | $\left[\mathrm{mm}^{2}\right]$ | 14870 | - |
| $t_{f}$ | $[\mathrm{~mm}$ | 13.5 | - |
| $\xi_{\text {wave,soil }}$ | $[\%]$ | 5.0 | - |
| $\xi_{\text {pipe,soil }}$ | $[\%]$ | - | 5.0 |
| $\xi_{\text {bound }}$ | $[\%]$ | - | 10.0 |
| $E_{\text {soil }}$ | $[\mathrm{MPa}]$ | 500 | - |
| $L_{\text {bound }}$ | $[\mathrm{m}]$ | - | 40.0 |

Table 7.2: Material and model properties for the PSM

## Domain representation of WPM

Every graphical representation includes an $x$-and $z$-coordinate regarding its location. To understand the system of the location representation, Figure 7.1 schematically outlines the point of interest.


Figure 7.1: The location representation of the graphical results regarding the WPM and ABAQUS calculations

## Domain representation of PSM

Figure 7.2 displays the pipe model over its length with respect to the displacement, momentum and stress. The two vertical (blue) lines indicate the boundaries of the model (the extended boundary method). As can be seen, the boundaries include most of the model's domain. Further on in this chapter is explained why the choice for this boundary length is made.

Figure 7.2: WPM-PSM calculation, displacement, momentum, stress shown over the length of the pipe at $t=0.07 \mathrm{~s}$

### 7.1.2. Different distances from pile, for R-waves WPM

Figure 7.3, Figure 7.4 and Figure 7.5 show the Rayleigh-wave PPV as a result of the model described in Chapter 6. The maximum PPV rate of the WPM should be taken as normative value for further calculations. The maximum value is in good agreement with the ABAQUS calculations.
The application of the theory discussed in the literature study makes it possible to define the PPV rate for different soil types and their individual behavior with respect to their properties. Richart et al. [104] declared that the Rayleigh-waves decrease in depth according to an exponential function, as outlined in Figure 2.7. Das and Ramana [31] described the relation by a mathematical, depth dependent, correlation as outlined in Chapter 6. The methodology is applied to the calculated Rayleigh-wave speed, to correct for the predefined depth of the data point (in this case 1 meter below the surface level).
Due to the application of a constant critical angle (as in this case one soil type is applied), the critical distance, with a pile toe near the surface, grows to an infinite small value. Dividing by this infinite small value results in an infinite big PPV rate. Therefore the maximum value at the depth of the pipe (in this case 1 meter below surface), is applied for further calculations. The utilization of the constant critical angle also controls the shape of the WPM outcome. The shape of the graphical representation does not entirely coincide with the ABAQUS model results, but since the maximum value is taken as normative PPV value, the deviating shape of the WPM is not further considered.


Figure 7.3: WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths for Rayleigh-waves. Data positioning point is 5.0 meters from center pile at 1 meter below surface level.


Figure 7.4: WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths for Rayleigh-waves. Data positioning point is 10.0 meters from center pile at 1 meter below surface level.


Figure 7.5: WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths for Rayleigh-waves. Data positioning point is 20.0 meters from center pile at 1 meter below surface level.

### 7.1.3. Different distances from pile, for P-waves WPM

Correlation between the horizontal distance from the pile and the PPV is important to analyze. This result representation, which is a function of the pile toe depth, underlines the behavior of both the WPM and ABAQUS model with respect to different materials. A comparison between the individual models and their contrast with the Eurocode 3 [2] will emphasize the importance of the inner workings of the WPM and its limitations.
For four horizontal distances from the pile ( 0.5 meter; 5 meter; 10 meter; 20 meter) there are three different plots made namely: for the surface level; -1 meter depth; -2 meters depth; -3 meters depth. Because pipe structures are present in most cases near the surface deeper depths relative to the surface level are not considered. Not all of the plots are outlined in this chapter, but the rest of the plots can be found in Appendix B.

## Location: 0.5 meters from the center of the pile

Figure 7.6 shows a similar behavior for both models. On the other hand the ABAQUS indicates clearly a distinct behavior for both materials (their material properties are outlined in Chapter 6), whereas the WPM does not. In addition the ABAQUS model exhibits a gradual increase in PPV while the WPM increase very slowly until a certain pile toe depth and then displays a rapid increase (an almost horizontal line). The gradual increase in PPV of the ABAQUS model tends more to reality than the rapid increase of the WPM.
Considering the location is close to the pile ( 0.5 meters horizontal distance), the PPV will grow to very high levels near the surface when the pile toe is near the surface. As the pile progresses in depth, the PPV decreases to low levels. This type of behavior is in accordance with reality and can be confirmed. An increase in depth will result in an increase in distance with respect to the data receiving point. This by itself means a longer traveling distance for the wave resulting in a smaller wave amplitude due to soil damping behavior.
The limit value for buried structure, in accordance with Eurocode 3 [2], outlines, in case of the ABAQUS model, that for pile toe depths deeper than 8 meters the limit value is not reached (for the WPM this is 3.5 meters). In accordance with the Eurocode 3 [2] this means that vibratory pile installation at 0.5 meters distance would lead to an unsafe situation. This also will be addressed later on in this chapter.


Figure 7.6: ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 0.50 meters from center pile at 1 meter below surface level.

## Location: 5, 10 and 20 meters from the center of the pile

The distinct behavior for both materials is conspicuous for both models in Figure 7.7, Figure 7.8 and Figure 7.9. The complementary shape of the lines indicates that the two models act in a analogous manner. The WPM implies lower values of PPV for the deeper pile toe depths, whereas the ABAQUS model shows lower values for the sand material in the upper region of the pile toe penetration depth. The models demonstrate that near the pile for deeper pile toe depth, the difference in material properties becomes of less importance as the PPV values lie close to one another. Further from the pile this is not the case.
For the ABAQUS model the higher PPV values for the clayey soil can be related to the almost similar stiffness property ( 240 MPa for the sandy material and 210 MPa for the clayey material) and the higher Poisson's
ratio ( 0.40 for the clayey material and 0.23 for the sandy material). According to Figure 2.7 soils with high Poisson's ratio have a bigger Rayleigh-wave PPV than soils with low Poisson's ratio. This is represented by the big difference between the two materials when the pile toe is near the surface and the decreasing difference with an increasing pile toe depth. Furthermore the difference in PPV values between the two soil types is related to the critical angle in which the Rayleigh-waves are developed (as explained in Chapter 6). The clayey material properties result in a smaller critical angle than the sandy material. A smaller critical angle is associated with Rayleigh-wave development nearer to the pile than with a bigger critical angle. Rayleigh-waves undergo less damping than body waves and therefore the difference between PPV values of the two soil types increases with an increasing horizontal distance from the center of the pile. The ABAQUS model reveal this in its outcome PPV values by an increasing difference between the lines of the two soil types with an increasing horizontal distance from the center of the pile.
With an increase in horizontal distance from the pile there is an increase in distinct material behavior. With other words: as the distance from the pile increases the relative difference in PPVs between the materials increases too, for both models. This can be related to both the wave propagation speed for both materials as the stiffness of the materials relating to the response of the soil to the pile (modeled in the case of the WPM by means of Bousinesq's theory according to Pistrol [98]). The difference between the models could be related to the soil reaction with the impact of the pile (does not have to be the only influence factor). In case of the ABAQUS model, the sheet pile wall is converted to a round pile, as explained in Chapter 6, whereas the WPM converts the sheet pile geometry to a long thin wall (which could be taken as a more realistic approach). In the ABAQUS model this could mean that the wave excitation manner of this model relates to too high PPV values.


Figure 7.7: ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 5.0 meters from center pile at 1 meter below surface level.


Figure 7.8: ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 10.0 meters from center pile at 1 meter below surface level.


Figure 7.9: ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 20.0 meters from center pile at 1 meter below surface level.

### 7.1.4. Influence Rayleigh-waves ABAQUS

At 10 meters distance (horizontally) from the center of the pile a depth profile is made for four different pile toe depths. With this procedure an insight in the various waves (R-waves, P-waves and S-waves) propagating through the soil is perceived for different pile toe depths. These plots are shown in Figure 7.10 and a comprehensive examination of the graphs are accomplished in this section.


Figure 7.10: ABAQUS calculation, Peak Particle Velocity (PPV) as a result of a pile toe depth of $-3,-9,-15$ and -24 meters. Data positioning point is 10.0 meters from center pile at depths between surface and -24 meters.

## 24 meters

The graphical format for the 24 meters depth pile toe displays a relationship where the influence of both the compression waves (P-waves) and Rayleigh-waves are distinguishable. The highest PPV is found at 24 meters depth. This indicates that the compression wave, which in this case traveled along the direct horizontal distance (shortest ray-path), generated the largest PPV. This is the case for all depths except the upper 3 meters of the modeled soil. Here the relation demonstrates a curve change (a bending of the curve to the right), indicating the influence of Rayleigh-waves near the surface.

## 15 meters

Complementary behavior to a pile toe of 24 meters depth is apparent for 15 meters depth. The maximum PPV value is associated with the pile toe depth and between the surface and 3 meters depth the domination of Rayleigh-waves is pointed out by the curvature change (bending to the right). In this case the pile toe is located nearer to the surface level, meaning less damping of the wave traveling to the surface, leading to a larger wave amplitude reflected at the surface, resulting in a Rayleigh-wave with larger particle velocity.
[Note: Although the Rayleigh-waves have, in this case a bigger magnitude, the maximum PPV is located at 15 meters depth, corresponding with compression waves traveling horizontally from the pile toe.]

## 9 meters

Unlike the 15 meters depth plot (previously discussed), the maximum value of the PPV is indicated at the surface level. The domination of Rayleigh-waves can be declared by its position of the pile toe. The pile toe depth of 9 meters, relates in this case to a position where the compression wave propagating towards to surface undergo less damping than in the case of a pile toe depth of 15 meters. Therefore the PPV resulting from Rayleigh-waves also increases. Under the conditions used for this simulation, 9 meters pile toe depths is the turning point for dominance of the two individual wave types ( P -and R-waves).

## 3 meters

A combination of P-wave and R-wave energy results in the plot found, for a pile toe of 3 meters depth, in Figure 7.10. In Figure 7.10 the lack of a peak value for the PPV at the pile toe depth indicates the dominance of R-waves over direct P -waves near the surface. The influence of the direct compression waves ( P -waves), increases with depth. This phenomena can be assigned to the diminishing influence of Rayleigh-waves with
depth. The circulating motion of the particles, caused by the Rayleigh-waves, lowers the strength of the direct compression waves. This proportion of the circular motion decreases with depth, leading to a higher influence on PPV for the direct waves, for the deeper pile toe depths. This is in accordance with the literature study Figure 2.7, which declares that a big decrease in Rayleigh-wave energy with depth can be expected.
Also the clear decrease of PPV with depth is visible. This decrease is related to the damping of P-waves in the soil as they propagate in depth. As there is a gradual increase in wave propagation distance, with an increase in depth, the P-waves experience more damping resulting in a decrease in PPV. The combination of the decrease in R-wave and P-wave energy is evident to the graph of the outcome of the ABAQUS model.

### 7.1.5. Variation with distance ABAQUS

Investigating the impact of material properties on wave propagation behavior is accomplished in this section by increasing the horizontal distance of the data point with respect to the center of the pile for the ABAQUS model calculations. The results are compared with the Eurocode 3 [2] prescribed limit value of PPV for buried structures.

## Different depths

The graphical representation, in Figure 7.11, of the ABAQUS model results underline the correlation between the PPV and the distance to the center of the pile (for a pile toe depth of -1 meters). Increasing the horizontal distance from the center of the pile leads to a decrease in PPV. Declaration to the decreasing relation can be linked to the damping phenomena that takes place within the continuum as the stress waves propagate.
According to the literature study of this Master Thesis research project, at shallow depths, near the pile the direct compression waves dominate. In the far-field the Rayleigh-waves will dominate at shallow depths (since they undergo less damping than body-waves). In agreement with the statement of the literature study the ABAQUS model results show a similar behavior: The pile toe is at -1 meters depth, the line for 1 meter depth exhibits a fast increase in PPV, near the pile, due to the direct compression waves. With an increasing distance, the Rayleigh-waves dominate over the compression waves resulting in a similar relation for all depths. The decrease in PPV over depth (thus the difference of the lines) relates to the reduction of R-wave strength, according to Figure 2.7 and to the increasing distance with the pile toe with respect to the depth (since the pile toe is located at -1 meter depth). Near the pile toe, the lines show different behavior with increasing depth associated to the previous mentioned case.


Figure 7.11: ABAQUS calculation, Peak Particle Velocity (PPV) as a result of a pile toe depth of -1 meter. Data positioning point is between 0.5 and 20.0 meters from center pile at depths of $0,-1,-2,-3,-4$ and -5 meters.

## Frequency dependency

Applying different frequencies to the system will reflect the driving frequency influence on wave propagation behavior through the soil body. As indicated in the literature study, driving frequencies will impact the damping behavior of the soil (the soil will react different when applying a load with a short pulse than applying it gradually). With an increase in driving frequency, the period of the applied force will decrease, resulting in a
shorter load pulse. The soil will react as a viscous material and therefore stiffer to a shorter pulse load. On the other hand, this only relates to nonlinear soil behavior. In this case linear elastic soil behavior is modeled including the plastic component by means of Rayleigh-damping (as indicated in Figure 6.12). Rayleighdamping includes a certain frequency domain in which the frequencies undergo lesser damping than relative low or high frequencies. For Figure 7.12 the indicated boundaries for the $5 \%$ material damping are given by Table 6.4. As a result, an increase in frequency will relate to more damping within the system arising in lower PPV values.
In Figure 7.12 the comparison is based on driving frequencies with a constant amplitude. The decrease in PPV is also related to a decrease in input energy with an increase in frequency. Applying a higher frequency (with the application of the same force amplitude) is a technique to diminish the PPV and meet the requirement of the Eurocode 3 limit value nearer to the pile. According to Ekanayake et al. [41] resonant frequency vibrator machines, as given in [56], can offer this diminishing effect. The behavior related to the different soil stiffness's are explained in the next subsection.

Different frequencies, 3 meter deep pile toe


Figure 7.12: ABAQUS calculation, Peak Particle Velocity (PPV) as a result of a pile toe depth of -3 meter for different driving frequencies. Data positioning point is between 0.5 and 20.0 meters from center pile at -3 meter depth.

## Soil stiffness dependency

Variation in soil stiffness delineates wave propagation behavior with respect to energy dissipation in the soil as a result of damping. Figure 7.13 outlines the ABAQUS model results for soil stiffness varying from 25 MPa to 200 MPa .
Figure 7.13 clearly indicates that the PPV near the pile (so in the range of 0.5 meters to 5 meters from the center of the pile) increases with a decrease in stiffness. Contrary behavior can be assigned to the far field (distances bigger than 5 meters from the center of the pile). Soft soils, with low stiffness, undergo more deformation, according to Hooke's law, as a result of the pile toe pressure applied. The deformation under the pile toe is directly related to the PPV. Therefore the PPV for low soil stiffness arises in high PPV values, as shown in Figure 7.13 for the near field. According to the literature study, Chapter 2, low soil stiffness is associated with high damping behavior, resulting in lower PPV values in the far-field (distances bigger than 5 meters from the center of the pile) for small soil stiffness's.

### 7.1.6. Variation with distance WPM

Investigating the impact of material properties on wave propagation behavior is accomplished in this section by increasing the horizontal distance of the data point with respect to the center of the pile for the WPM model calculations. The results are compared with the Eurocode 3 [2] prescribed limit value of PPV for buried structures.

## Soil stiffness dependency

In contradiction with the ABAQUS calculation, the WPM calculation results show a less sensitive behavior related to soil stiffness, for the application of just P-waves. In Figure 7.14 the graphical lines follow a small


Figure 7.13: ABAQUS calculation, Peak Particle Velocity (PPV) as a result of a pile toe depth of -3 meter for different soil stiffness and constant Poisson's ratio and soil density. Data positioning point is between 0.5 and 20.0 meters from center pile at -3 meter depth.
band-width, indicating a low sensitivity to a change in soil stiffness. Furthermore the influence in the near field, where the PPV values are higher for a low soil stiffness, is extended to a distance of approximately 15 meters (differing from the ABAQUS calculations where this value is approximately 5 meters, see Figure 7.13). The difference between the model results in the far field can be associated with damping of pressure waves. Higher frequencies are are less sensible to damping than lower frequencies. R-waves experience less damping than P-waves and therefore the results in the far field for the ABAQUS calculations (where both P-waves and Rwaves are taken into account) show a wider solution spectrum than the WPM, which only takes P-waves into consideration (see Figure 7.14). Bare in mind that the ABAQUS results are presented at a depth of -3 meters below the surface, whereas the WPM results at a depth of -1 meters below surface level. This influences the results as well.


Figure 7.14: WPM calculation for P-waves, Peak Particle Velocity (PPV) as a result of a pile toe depth of - 1 meter, for different soil stiffness and constant Poisson's ratio and soil density. Data positioning point is between 0.5 and 20.0 meters from center pile at -1 meter depth.

The application of a combined model for R-waves and P-waves, as suggested in Chapter 6 indicates a correlation that satisfies the ABAQUS calculation (see Figure 7.15). Although the calculation is taken at -1 meters below the surface, in contrast with the -3 meters below surface for the ABAQUS calculation, the results do show a similar model behavior for the far field. The results for the near field indicate a less sensible model with respect to changes in soil stiffness. The difference can be related to the different damping models used. The WPM damping model, according to the Bornitz equation as stated by Athanasopoulos et al. [12], shows to be less sensitive to soil parameter changes, in the near field of the pile, as the Rayleigh-damping model


Figure 7.15: WPM calculation for R+P-waves, Peak Particle Velocity (PPV) as a result of a pile toe depth of -1 meter, for different soil stiffness and constant Poisson's ratio and soil density. Data positioning point is between 0.5 and 20.0 meters from center pile at -1 meter depth.
used in the ABAQUS simulations.
A comparison between the WPM model with just P-waves and the extended model with $\mathrm{R}+\mathrm{P}$-waves it can be observed that for the near field (from the pile to a distance of 5 meters from the pile) the results show analogous behavior. From this statement can be concluded that in the near field of the pile, the P-waves have a similar magnitude as the R-waves. This is in agreement with reality, where due to the lack of damping of both P-waves and R-waves in the near field of the pile, the direct P-waves have proportional magnitude to the R-waves. The extended model with R+P-waves is therefore more realistic than the simple WPM with just P-waves (in the far-field).

## Frequency dependency

The increase in frequency leads to a higher amplitude of the applied force in the WPM. On the other hand in the ABAQUS calculations the amplitude of the force is taken as a constant (so no increase with an increase in frequency) and thus only the frequency is varied. Therefore complementary methodology is applied in the results shown in Figure 7.16 for the WPM. As explained earlier, input energy will therefore decrease with an increase in frequency.
Stiff soils show less sensitive behavior to driving frequency changes than soft soils. The sensitivity to driving frequency grows for all cases with an increase in horizontal distance from the center of the pile. This means that the WPM displays analogous behavior to the ABAQUS model. On the other hand, the WPM aims to be less sensitive in all cases in the near-field of the pile, leading to difference with the ABAQUS model. Earlier in this chapter is stated that the damping models/methods applied to the ABAQUS model and the WPM differ from each other. This declares also the difference in behavior in the near field of the pile for both models. Due to the lack in measurement data, a comparison with reality, to outline to correctness of both models, can not be made.

### 7.1.7. Linear elastic versus linear elastic-perfect plastic ABAQUS model

The linear elastic-perfect plastic model, also named 'Mohr-Coulomb material model', is very sensitive to big localized loads, since this may lead to local failure of the material (very big plastic strains). The ABAQUS calculation will than be aborted due to the instability in the calculation. Too small loads on the other hand may lead to no plastic strains and the material will then react purely elastic.
In the attempt to show the difference between the linear elastic (LE) and the Mohr-Coulomb (MC) model, the application of a very high local load led to abortion of the ABAQUS calculation for the MC model. By lowering the load, stability in the model (but including plastic strains) was aimed for, but did not succeeded. Therefore a comparison between the LE model and the MC model is not taken into further consideration. Suggestions to improve the stability of the calculations are given in Chapter 10.


Figure 7.16: WPM calculation, Peak Particle Velocity (PPV) as a result of a pile toe depth of -1 meter for different driving frequencies. Data positioning point is between 0.5 and 20.0 meters from center pile at -1 meter depth.

### 7.2. Coupled WPM - PSM

The coupled WPM-PSM approach enhances possibilities to investigate pressure wave impact on pipe structures in the subsurface. In the following section, the results of the coupled approach are carried out. The focus on the other hand, is on the behavior and sensitivity analysis of the PSM. Therefore parameters related to the WPM are held constant unless mentioned.

### 7.2.1. Boundary condition

Implementation of the correct boundary condition is of high importance as it has a big influence on the model outcome. To what extend the boundary has an impact on the model outcome and the methodology to lower its effect are carried out in this section by means of a sensitivity analysis.

## Boundary condition comparison

Two different boundary methodologies are proposed in Chapter 6 . The so called "alpha-boundary" and the "extended-boundary". Figure 7.17 presents the two different boundaries by means of stress in the pipe as a relation of a varying soil stiffness around the pipe. The alpha-boundary is implemented with an alpha-factor of 2.0 and 3.0 and the extended-boundary is achieved with a 20.0 meters extended length.
The fluctuating model response to the alpha-boundary can be related to the difference in stiffness between the last elements and the other elements of the model. This stiffness contrast causes a big increase in momentum at the transition location leading to a large angular rotation and thus a large stress in the pipe. The sensitivity to an increase in soil stiffness is large, since the over all stiffness increases and accordingly the relative difference becomes larger at the transition point. On the other hand, the outcome does not show a smooth decrease in stress with an increase in stiffness (it shows even an increase in stress at some point with an increase in stiffness). This proves that the methodology does with its fluctuating model results is not a reliable boundary technique.
A more smooth correlation is concluded from Figure 7.17 for the extended-boundary method. This indicates that the boundary does not cause model instability problems and is therefore applied for further system calculations.

## Boundary condition size

Inquisition to the impact of the extended-boundary method through a variation in boundary length substantiates the selected confined calculation domain. Figure 7.18 displays a solution spread associated with the boundary length. As could be expected, this distribution reveals that an increasing boundary length relates to a decreasing band-width of the solution spread with respect to its average. The chosen 40.0 meter boundary length as standard application is based on its calculation time (explained later on in this chapter) and on the accuracy (no higher accuracy is required) and is indicated in Figure 7.2.


Figure 7.17: WPM-PSM calculation, relation of the stress in the pipe structure - soil stiffness around the pipe structure. Comparison of different boundary conditions


Figure 7.18: WPM-PSM calculation, relation of the stress in the pipe structure - extended boundary length

### 7.2.2. Frequency

The response of the system to different excitation frequencies is already investigated earlier in this chapter. Not only will the excitation frequency influence the wave propagation behavior in the WPM, but different frequencies will also lead to a distinctive behavior of the PSM. Hence, investigation is carried out by minimizing the influence of the WPM on the PSM to trace the excitation frequency impact on the PSM only. Reduction of the WPM impact on the outcome is performed by the following steps:

- Keeping the excitation force identical for all simulations and only vary the frequency. Therefore the difference in response of the soil under the pile will only be related to the frequency and not to the force. A down-side of this method is that the input energy will decrease with an increase in frequency (as explained earlier in this chapter).
- An increase in frequency will lead to more damping of the propagating wave. On the other hand, in the far field of the pile the Rayleigh-wave will dominate over the direct P-wave. Figure 7.16 indicates that for an increase in driving frequency there will by an increase in PPV, in case of the far field. Associated to the increasing pressure on the pipe at the boundaries (related to an increasing PPV) by means of an increasing driving frequency the domain size will also differ (as it is dependent on the ratio of maximum PPV at the boundary, see Chapter 6). The maximum allowed percentage at the boundaries is changed in such a way, that the domain size, and its corresponding mode shapes and Rayleigh-damping frequencies, are remained constant.
- The number of excitation cycles is kept as a constant therefore the simulation time is different for every calculation performed. In this way the response of the system is analyzed based on the number of excitation cycles, which is assumed as a reliable approach.


Figure 7.19: WPM-PSM calculation, stress in the pipe structure - frequency of excitation of the vibrating hammer

The system does not seem to be sensitive to an excitement in its eigenfrequency. The excitation pressure on the pipe follows a time and position relate distribution. Every element is excited with the same frequency, but with a different initial time position. For that reason the eigenfrequency of the system does not have to correspond with an increase in stress within the pipe.
The research to the behavior of the pipe, related to frequency, is in the model almost impossible. This is due to the fact that it is one-to-one related to the WPM model. The research undertaken is performed with a force that stays constant, the amount of elements of the model is held constant and only the frequency is changed. The increase in frequency leads to a decrease in stress within the pipe structure. Higher frequencies are damped more than lower frequencies. Therefore, the deformation of the pipe will be lower for higher excitation frequencies leading to a decreasing stress. Moreover, a decrease in excitation frequency, with the application of a constant force amplitude, will also lead to a decrease in input energy (as explained earlier) leading to a lower pressure on the pipe (and associated to that a lower stress in the pipe).
The application of Rayleigh-damping of the system is based on the frequency of the first eigenmode and the eigenfrequency near the excitation frequency. When the excitation frequency proceeds the eigenfrequency used in the previous calculation as upper-boundary value, another upper-boundary value will be applied in to the calculation (leading to a different Rayleigh-damping behavior). Therefore the system, as can be seen in Figure 7.19, reacts different after proceeding the eigenvalue of the system (used as upper boundary for the determination of the Rayleigh-damping parameters).
When considering the entire coupled WPM-PSM system, an increase in excitation frequency would lead to an increase in stress within the pipe structure. Declaration can be made by: an increase in driving frequency will lead to and increase in driving force; the bigger amplitude but shorter excitation frequency will still lead to an increase in input energy; the PPV at all the elements will increase; the pipe displacement will increase; leading to an increase in stress within the pipe structure.

### 7.2.3. Soil properties

System response is strongly correlated to soil configuration in circumference of the pipe structure likewise the trajectory of the propagating pressure wave. Accordingly, interrelationship between soil composition and the models (WPM and PSM) is investigated by means of a sensitivity study. The following subsections entail this extensive soil material sensitivity examination.

## Soil type WPM model

Study to the system response by means of variation in soil properties will give a good insight in parameter sensitivity of the model. In case of this investigation, the material density, $\rho\left(\mathrm{kg} / \mathrm{m}^{3}\right)$, is kept constant to
delineate system reaction to the soil stiffness, $E(\mathrm{MPa})$, and Poisson's ratio, $v(-)$. Both material parameters are affecting more than just the wave propagation speed parameter, $c(\mathrm{~m} / \mathrm{s}$ ) (which is not the case for the material density parameter). On the other hand, it should be noticed that the mass of a material does effect its damping behavior, like incorporated in the Rayleigh-damping model (mass-proportional damping) for instance, but is not investigated in this Master Thesis research project.
Figure 7.20 indicates that for different Poisson's ratios there is a maximum PPV in the soil stiffness range of $100-200 \mathrm{MPa}$. The PPV rate of the WPM is dependent on:

- Input energy: The energy input of the system depends on the stiffness of the soil and the vibrator installation machine.
- Deformation under the pile toe: The deformation under the pile toe directly correlates to the pressure wave propagating through the soil. The degree of deformation depends on the input energy and the the pile cross-sectional area.
- Damping of the pressure wave: Material damping is associated with: the excitation frequency; soil stiffness; Poisson's ratio; soil density (in this case a constant value and therefore not of influence on the outcome); degree of damping (taken as a constant value of $5 \%$ ). The wave velocity, $c(\mathrm{~m} / \mathrm{s})$, that is directly connected with the soil stiffness, Poisson's ratio and soil density (which is taken as a constant), is integrated in the material damping parameter, $\alpha$, carried out in the literature study by Equation 2.21. By an increase in soil stiffness the wave velocity will also increase (see Figure 7.21). In reality as stiffer soil relates to a more packed grain configuration than a less stiff soil. This packed grain configuration transports energy from one grain to the other more easily than less packed soil, which is in agreement with Figure 7.21.

The model outcome is therefore a combination of material damping and input energy in the system (directly correlated with the deformation under the pile toe). The soil stiffness range of $100-200 \mathrm{MPa}$ shows to be the an optimal combination of material damping and input energy for this model setup configuration.
[Note that: Although a stiffer soil is related to less material damping, this does not necessarily relate to a higher PPV at the receiver point (as explained earlier in this section)].


Figure 7.20: WPM calculation, Peak Particle Velocity (PPV) as a result of a pile toe depth between 0.0 and - 24 meters: for different soil stiffness and Poisson's ratio and a constant soil density.

## Soil stiffness around pipe

Figure 7.17 displays not only a comparison between different boundaries, but also the relation between stress and soil stiffness in the circumference of the pipe structure. The extended boundary follows a fourth-order polynomial stress - soil stiffness relation. An increase in soil stiffness lead to a decrease in stress, which is realistic according to Hooke's law of elasticity. Hooke's law of elasticity declares that the displacement vector decreases when the stiffness increases (under the assumption that the stress remains constant in time).

(a) General P-wave velocity used for the WPM model

(b) General S-wave velocity used for the WPM model

Figure 7.21: General wave velocity used for the WPM model

## Rayleigh-damping parameter, $\xi$

The excitation frequency ( 28 Hz ) is near the eigenfreqeuncy $(28.1 \mathrm{~Hz}$ ) of the system. Earlier in this chapter is concluded that this situation does not necessarily means that this leads to a large deformation within the pipe structure. In case of no damping, Figure 7.22 shows that this will lead to a very large stress within the pipe structure (the line temps to infinity) and therefore also a large deformation. This implies that without damping (which in reality is never the case) an excitation frequency near the eigenfrequency of the system leads to very large stresses in the pipe structure. The stress within the pipe structure temps to zero with the application of $100 \%$ (Rayleigh-)damping (as indicated by Figure 7.22), which can be assumed realistic. On the other hand, due to numerical inaccuracies, this value is not equal to zero, which is the case in reality.


Figure 7.22: WPM-PSM calculation, stress in the pipe structure - Rayleigh damping parameter

### 7.2.4. Pipe dimensions

Soil stiffness behavior is investigated earlier in this chapter. The research results showed that the impact on the model stiffness proportionality was noticeable. A variation in pipe dimensions is directly related to a change in stiffness properties of the pipe and therefore its reaction to the pressure applied on the pipe. Momentum and stress in the pipe are examined by varying pipe properties.

## Diameter pipe

The diameter of the pipe structure is not only in direct relation to its stiffness proportions, but also to the magnitude of pressure applied on its surface. With an increase in pipe diameter its surface area increases and the associated pressure per unit length of the pipe also increases. Figure 7.23 indicates an exponential relation for both the stress and momentum in the pipe to an increase in pipe diameter. Increasing the pipe diameter will lead to a higher pressure per unit length of the pipe, but also to a stiffer structure and a larger cross-sectional area. All these non-linear relationships lead to a coherence displayed in Figure 7.23.
[Note: It seems that the momentum increases more rapidly than the stress in the pipe. Since the stress and momentum are both displayed by different $y$-axis, this conclusion can not be made.]


Figure 7.23: WPM-PSM calculation, stress and momentum in the pipe structure in relation to the diameter of the pipe structure with constant pipe thickness of 1 mm .

## Thickness pipe

A change in pipe structure stiffness can also be reached by a variation in pipe material thickness. The pressure per unit length on the pipe structure remains constant and as a result the methodology decreases the number of influence factors. Figure 7.25 indicates the growth in cross-sectional area and moment of innersia (which is in direct relation to the stiffness of the pipe, E I) by a change in pipe material thickness. Both relations are not linear and reveal non-linear WPM-PSM behavior in Figure 7.24. Non-linearity is even more likely as a model outcome, since the stiffness of the pipe structure is sheared corporation with the soil stiffness (which is taken as a constant value in these calculations).
The model behavior shown in Figure 7.24 can be declared by its correlation with the relations displayed in Figure 7.25. An increase in pipe thickness leads to a higher pipe stiffness. A stiffer pipe structure leads to less deformation, but therefore also a higher momentum in the structure. A higher momentum would indicate higher stresses in the pipe structure. However, the relationship between cross-sectional area and internal forces decreased, leading to a lower internal stress. Both the stress and momentum relations converge, because their proportionality to the pressure on the structure grows to a very large value.


Figure 7.24: WPM-PSM calculation, stress and momentum in the pipe structure - thickness of the pipe structure with constant pipe diameter of 500 mm .


Figure 7.25: Cross-sectional area and moment of innersia of a pipe - thickness of a pipe structure with constant pipe diameter of 500 mm .

### 7.2.5. Calculation time

The calculation time does not only depend on the domain size and the simulation time (although these two factors have a big influence on the calculation time). The parameters used for the calculation determine the stiffness of the system. The stiffness of the system influences the time step size taken by the ODE-solver to solve the system within the pre-defined tolerances. The step size taken by the ODE-solver is the main factor influencing the calculation time. If the system is very stiff the calculation steps will be very small. The calculation time is also dependent on the amount of failed attempts by the ODE-solver. The failed attempts are in direct relation to the stiffness of the problem. The stiffer the problem, the more failed attempts the ODE-solver needs to sustain numerical stability by application of the correct calculation step.

Aside from the parameters that regulate the stiffness-matrix and mass-matrix of the problem, the excitation frequency drives the force-matrix. A direct relation between the frequency and the stiffness of the problem influences the calculation time of the ODE-solver. An increase in frequency lead to a more rapid increase in pressure applied on the pipe. To sustain numerical stability, smaller calculation steps are required and thus the calculation time increases to.

Figure 7.26 indicates the exponential increase in calculation time with the increase in boundary length (of the extended boundary). The choice to maintain a standard model with a boundary length of 40.0 meters is also based on the ODE-calculation time shown in Figure 7.26.

As the application of 11 frequency periods are used as a standard simulation length, the simulation time of the model is related to the frequency. With an increase in frequency, there is an decrease in simulation time. No conclusions related to the calculation time can be made as its length depends on the increasing factor connected with numerical stability problems of the ODE-solver and on the other hand its decreasing factor related to the 11 frequency periods applied.

### 7.2.6. Inaccuracy of calculation

Instabilities in model calculations can lead to inaccurate results. The coupled WPM-PSM suffers from small model instabilities as can be seen in Figure 7.27 and Figure 7.28. The figures show that the displacements grows in time and does not find a converge (which should be expected). On the other hand, this problem is mastered maximization of the simulation time by 11 frequency cycles, so that the inaccuracy remains small. Furthermore, the maximum stress and momentum is found to be at $t=0.06 \mathrm{~s}$, which confirms this statement. Also should be noticed that the small growth of the pressure, as shown in Figure 7.28, is due to the small inaccuracy of the Trapezoidal rule as stated in Chapter 6. Appendix C shows a suggestion to solve the stability problem by a Jacobian based numerical-solver method. Further inaccuracies of the calculation relate to the model limitations carried out in Chapter 8.

## Influence boundary length on calculation time



Figure 7.26: WPM-PSM calculation, calculation time - boundary length


Figure 7.27: WPM-PSM calculation, displacement as a relation of the simulation time in the middle of the pipe

### 7.3. Conclusion

Correlations between the different model results are made to describe the inner workings of the coupled WPM-PSM and the ABAQUS model. Conclusion about the inter-relations and parameter sensitivity study are given in Chapter 9. From the results carried out in this chapter, clear model limitations came forward. These limitations are described in detail in Chapter 8. Model limitations aim for improvement and therefore recommendations for further research are suggested in Chapter 10.


Figure 7.28: WPM-PSM calculation, results obtained from the midpoint of the pipe

## Model limitations

Simulations performed by computer models are a perception of reality. Each model is based on assumptions and simplifications based on engineering judgment and grounded statements. These simplifications can lead to model uncertainties that let the results differ from reality, which can bring risk in design statements. Additional safety factors are used in engineering designs to cover the risk, but lead to a more conservative approach. Model validation by means of measurements can lower the risk and lead to less conservative judgments.
In this Master Thesis research project, risk is not a factor taken into account. Reasoning leads to the experimental basis of the research. The preliminary coupled Wave Propagation Model-Pipe Structure Model (WPM-PSM), developed for this Master Thesis research project, does imply a good practical application. On the other hand, due the lack of validation by means of measurements its correctness is not yet proven. The comparison with the ABAQUS model does lead to a more reliable knowledge of the Wave Propagation Model (WPM), but without the comparison with real field data the model will be experimental. Furthermore, no comparison is made for the Pipe Structure Model (PSM), leaving this part of the model entirely unproved. However, the parameter sensitivity study applied to both the WPM and the coupled WPM-PSM can outline the model behavior. Therefore a good understanding of the responsiveness of the models are outlined.

### 8.1. ABAQUS model

The axi-symmetric ABAQUS model, applied for this Master Thesis research project, represents a 2D axisymmetric situation of a round pile installed in the subsurface. Round piles have complementary stiffness behavior with respect to the axi-symmetric axis. In the case of this Master Thesis research project a situation for a sheet pile is being investigated. Due to its complex stiffness behavior, investigation by means of a 3-dimensional approach would be optimal. Limited time frame, for the research project, led to a simplified approach by means of axi-symmetry. 3D-effects are therefore neglected and stiffness-behavior in all directions is assumed similar. As a matter of fact, the stiffness behavior is not at all considered, since assumptions that wave development related to the shaft of the pile is neglected. The following points of attention outline the limitations of the situation declared above:

- The cross-sectional area of the pile is taken equal. But due to the different shape, the soil reaction to the applied pressure of the round pile geometry will be different from the sheet pile geometry (a different deformation/failure wedge according to Prandl).
- Horizontal bending of the sheet pile can cause big horizontal pressure waves (depending on the stiffness behavior of the sheet pile wall), as declared by Liden [71]. Neglecting this 3D effect, as done in this Master Thesis research project, could lead to the risk of under-estimated PPV levels.
- Dynamic shaft resistance lead to the development of pressure waves traveling through the soil. The PPV could, in case of very stiff clay's, could have higher rates than the point resistance. Therefore the application of very stiff clay's, as performed in the case of this research project, is not prerequisite.

During pile installation, energy dissipation, due to plastic soil deformations, will occur near the pile. Soil models, complementary to the soil-type applied, that take plastic deformations and hardening/softening
behavior into account, can predict realistic PPV rates near the pile. As stated by Masoumi [79], further from the pile the soil behavior, to the progressing pressure wave, will more or less linear. The ABAQUS model, used for this Master Thesis research project, uses a linear elastic soil model with the application of Rayleighdamping. Predicted PPV rates near the pile are therefore too high and can not by taken as realistic values.
Minimizing the refection at the boundaries of the model can be performed by addition of "rounded shape" of the boundary. In this way the shape of the boundary is in agreement with the shape of the approaching pressure waves and therefore minimize reflection of waves (no reflection of waves will take place when they approach under a 90 degree angle at the boundary absorption elements). In the case of the ABAQUS model, used for this research, rounded boundaries are not applied due to the short model calculation time (where reflection is not expected to take place). A long simulation time will result in an unreliable model outcome, which is a limitation of this simplified approach.

### 8.2. Wave Propagation Model

In accordance to the ABAQUS model, the Wave Propagation Model (WPM) does not take wave development by means of shaft resistance into consideration. Horizontal bending of the sheet pile wall (horizontal deformation) will result in pressure wave development, as explained earlier in this chapter. Limitations corresponding to the two previously discussed points has similar effects on the model outcome as to ABAQUS model (discussed in the previous section). The lack of measurement data makes it difficult to predict the effect of these two model limitations, since no comparison with reality can be made.
The prediction method for Rayleigh-wave development is based on a critical distance terminology. Application will result in an exponential PPV response that relates to unrealistic high rates near the surface. On the other hand, application at depths more than 1 meter below surface result in complementary maximum values with respect to the ABAQUS model. This proves reliability at near surface application. However, no corresponding Rayleigh-wave distribution with respect to pile toe depth is found. The shape of the output does not entirely fit the ABAQUS model outcome. When this relation is requested as model output, the limitations of its shape should be taken into consideration.
The soil disturbance factor for the remoldiation of the soil under the pile toe is taken as a constant value (= 2.0). This factor is directly linked to the energy transfer between the pile toe and the soil. Different soil types result in different behavior. Therefore this averaged value can relate to too high or too low model PPV output rates. Further investigation is required.

### 8.3. Pipe Structure Model

The Pipe Structure Model is developed as a 1D-Finite Element Model that only accounts for bending stresses. 2D-and 3D-effects, like torsional stresses, are therefore not taken into consideration. These torsional stresses could be caused by waves approaching the surface of the pipe in different angles. The combination of bending and torsional stresses could lead to a higher total stress in the pipe material. The simplified approach for this Master Thesis project therefore can not be considered a valid simulation method, unless a comparison/validation with field measurements, 2D-and or 3D-calculation is performed.
Application of numerical methods always lead, in some extend, to model errors. Utilization of higher-order numerical methods, as used for the PSM, lowers the truncation error, but on the other hand still includes an error. Using a build-in Matlab ode-solver, with variable oder method and variable calculation step method, composes a more stable solution spectrum. However, this dynamic Finite Element problem implies a very stiff differential equation and leads to a very sensitive solution spectrum towards the calculation step chosen. Due to this sensitivity the calculation will become unstable after 10-15 forcing periods making it not possible to run longer simulations without obtaining unreliable model results.
The Euler-Bernoulli method has 3 basic assumptions, as mentioned in Chapter 6. These assumptions make it possible to develop a relative simple dynamic Finite Element Model (in comparison with the Timoshenko Beam model as stated by Banerje [16], Chen et al. [27], Kocaturk and Simsek [67] and Thomas and Abbas [111]), but its response does not take shearing forces between the elements into account. This makes the model outcome less reliable, but decreases model calculation time.
Chapter 6 proposes two distinct boundary conditions to minimize their impact on the model result spectrum. Chapter 7 makes clear that the extended boundary enhances the most stable model behavior. Further investigation to what extend the boundary length affects the maximum stress within the pipe structure proves that with an increasing boundary length, the impact on stress development decreases. However, the decreasing influence develops according to an average trend-line distribution that covers a wide spread of model results.

A wide distributed solution spectrum can be associated with a sensitive model response to, in this case, its boundary. In addition, the application of no rotation and no vertical displacement at the outer limits of the boundary could be questioned. In reality rotation stiffness from the pipe as well as translation stiffness of the pipe and the soil will be present. Clarification towards the sensitive boundary behavior could be associated with the utilization of the fixed outer limit of the boundary elements. The conditions applied for the outer limit of the model can be addressed as a responsive methodology and requires further investigation.
Concrete pipes for the application of water transport require to be water tight. Judgment of limit material stress, with respect to the enforced circumstances, is of importance to authorize the quality of the pipe structure. The PSM is developed with application of average stress distribution along the cross-section of the pipe. Therefore limit stresses on its surface are not considered. Model adjustments should be applied to trace limit stresses.
Lack of knowledge on the soil-pipe contact surface require the assumption of homogeneous conditions. Peak stresses caused by locally stiffer or weaker zones are therefore neglected. Further more the contrast between 3D soil stiffness's (horizontal, upper and lower soil stiffness differ from each other) is also not taken into account. Both model simplifications could lead to higher stresses in the pipe, which makes the PSM used for this Master Thesis research project less conservative. The application of an empirical formulation to emphasize the soil stiffness, as suggested by Adhikary et al. [6], includes (as it includes empirical parameters) an uncertainty with respect to pipe deformations and directly linked pipe stress-distributions. Real soil behavior includes hardening and softening effects that effect the pipes oscillatory deflection. Linear elastic soil behavior, as included in the model, could lead to lower calculated stresses than in reality will be produced by the pressure waves. However, the Rayleigh-damping as applied to the model influences the pipes behavior, lowering the deflection of the pipe (representing the non-linear soil behavior as in reality would take place). The program suggest that the pipe is not filled with a fluid. In case of a filled pipe, the total mass will be higher, lowering the pipes oscillatory acceleration and therefore damping the deflection. Calculations for these situation should therefore be considered conservative with the application of the coupled WPM-PSM. The lack of implementation of the geostatic stress around the pipe does imply the pipe can move without the additional pre-stressed situation. This can lead to higher or lower pipe deflections, depending on the direction of the load caused by the pressure wave.

## Final Conclusion

This Master Thesis research project aims for a simplified method to investigate the affect of vibratory pile installations on pipes and ducts in the subsurface. The extensive literature study accomplished, resulted in a adequate approach methodology for the development of a coupled analytic model. The Wave Propagation Model is based on the model developed by Massarsch [80]. Implementation of modifications to this model are made with respect to vibratory pile driving arising pressure waves in a soil body. Therefore it is possible to describe the wave origination of vibratory sheet pile installations and the progression of the pressure waves through the soil body. Two distinguished pressure wave types are incorporated in the model. The P-waves and Rayleigh-waves as indicated in Chapter 2 and Chapter 6.
A 1D Finite Element approach, by utilization of the Euler-Bernoulli method, describes the pipe structure excited by the progressing pressure wave. One-directional coupling of the two individual models, by means of the forcing approach as addressed in Chapter 6, gains optimal simulation of the entire problem statement. The methodology, developed for this Master Thesis research project, not only permits excellent insight in the problem spectrum, but also adequate system results in a relative short time can be obtained. Due to its beneficial techniques a good estimation of the influence of the pressure waves on the pipe can be accomplished. No time expensive and complicated 3D commercial Finite Element (FE) calculations need to be made. Although in case of a critical situation additional 3D FE calculations should be made, since the model satisfies a simplified approach method and does not take 3D effects into account.
The development of the system approach is seen as conceptual research, since the lack of measurement data make validation of the model not possible. However, results related to the parameter sensitivity study do address the inner workings of the model and the interconnections between the mechanisms modeled. Judgment of the approach methodologies used to investigate the research problem are defined in detail throughout the content of this chapter, model limitations of the other hand are carried out in Chapter 8.

### 9.1. Wave propagation approaches

Chapter 7 delineates the system response to certain parameter input values. A comparison between the ABAQUS model and the WPM declares that their model behavior is complementary. This means that the inner workings of the WPM, resulting in its system response, are concluded to behave correctly (under the assumption that the ABAQUS model is complementary to reality). On the other hand, small differences lead to a certain degree of uncertainty for both the model approaches. Only by a comparison with real field measurement data these uncertainties can be validated. From the research carried out in this Master Thesis project can be concluded that the proposed Wave Propagation Model proves to be of similar high quality prediction method for the prediction of PPV values induced by vibratory pile installation.

### 9.2. Pipe Structure Model

The simplified 1D Finite Element approach of the pipe structure represents its response to the excitation pressure of the vibration wave in the subsurface. The reaction is investigated by means of a sensitivity study. From this study can be concluded that the methodology can lead to a good estimation of the vibration behavior in reality. On the other hand, modification and further research is mandatory to reach excellent system quality. For instance the boundaries influence the model outcome to a certain degree. Utilization of another
approach could lead to better model performance. Another very important model limitation that need to be solved, as mentioned in Chapter 8, is the model instability related to larger simulation times (larger than 11 frequency cycles). This part of the coupled WPM-PSM also requires field measurements to prove its validity. Lack of field measurements and unknown soil-pipe contact circumstances made it not possible to carry out this very important research step and confirm the correctness of the model simulations.

### 9.3. Research objectives and questions

Although the coupled WPM-PSM consists of limitations regarding its performance, focus should be laid on the research goals and research questions regarding the model results.

### 9.3.1. Research questions

The following section reveals the answers to the research questions regarding the investigation results of the Master project.

- What are the additional stresses in the pipe caused by the vibration wave?: The additional stresses are dependent on numerous factors. The coupled WPM-PSM can predict the additional stresses on the basis of a set of input parameters. The conclusion that can be made regarding this research question are the graphical relations presented in Chapter 7. These graphs describe the relation between the input parameters and the resulting stresses. The conceptional model developed for this thesis needs to be validated by means of displacement data. Back calculation of the stresses is than possible. Therefore no conclusion can be made about the degree of reliability of the calculated stresses within the pipe structure.
- What are the factors influencing the wave propagation through the soil and how do they influence this process?: By means of the parameter sensitivity study, carried out in Chapter 7, all of the factors that influence wave propagation behavior are investigated. The damping process is one of the main factors manipulating the PPV of the propagating pressure wave. Different wave types relate to distinctive wave development and propagation behavior. P-waves and Rayleigh-waves are integrated in the WPM and their apparent mobilization through the soil body is taken into account. Similar to the previous question, Chapter 7 displays the answer by means of graphical representation and the description of inter-connection between the properties.
- To what extend will damping influence the behavior of the pipe structure?: Figure 7.22 delineates the relation between the degree of (Raleigh-)damping and the stress in the pipe structure. From this relation can be concluded that the degree of damping effects the behavior of the pipe dramatically. Low degree of damping relates to very high stresses in the pipe, whereas a high degree of damping to low stresses. The relation is described by an semi-asymptotic function that grows to a very high value when no damping is applied and almost to zero when $100 \%$ damping is enforced. Due to its sensitivity, care needs to be taken choosing the degree of damping.
Soil damping in the WPM also effects the pipe structure. Application of low damping results in higher PPV rates and therefore a higher pressure on the pipe structure. Also will the degree of soil damping result in a higher contrast of pressure, enforced on the pipe structure, regarding its geometry. The contrast of induced pressure, on the pipes surface along its length, will arise in different stress distributions in the pipe.
- What will be the preferred method to solve the problem statement and to what extend do model assumptions influence the reliability of the end result?: In the literature study an in depth investigation is accomplished to retrieve the "ideal" methodology to approach the problem statement. From the literature study is concluded, in Chapter 5, that the combination of the wave propagation Massarsch and Fellenius [80] model (including the modification with respect to vibratory sheet pile driving) and the 1D Euler-Bernoulli Finite Elements pipe structure model would lead to optimal efficiency and reliability. Further investigation of the model limitations, regarding the approach assumptions is delineated in Chapter 8.


### 9.3.2. Research objectives

The research objectives are drawn out in Chapter 1. This section gives feedback on the research by examining the requested objectives regarding the research outcomes.

- Aim for simplification to obtain a fast result: Throughout the entire Master project the aim to develop a simplified model approach was mandatory. Over-simplification of the other hand leads to less reliability. Therefore ensuring the quality and reliability of the model was considered of high importance. The coupled WPM-PSM model is a specified model made to investigate vibratory pile induced vibrations and their effects on pipes and duct in the subsurface. The model retrieves a relative fast approximation of the problem situations, since the entire pipe geometry is linked to different pressure wave values (regarding their decrease in magnitude and time shift). Different problem situations can be accomplished by quick changes in the parameter input. Related to time expensive and complicated 3D commercial software models that need to be developed the coupled WPM-PSM is considered a beneficial time saving approach methodology.
- Development of a model that describes pressure waves excited by vibratory sheet pile installation and their progression through a soil body: Both the ABAQUS model and the WPM have the ability to for-fill this requirement. On the other hand, the ABAQUS-model is radial symmetric, making it impossible to apply a sheet pile geometry. A risk of to much input energy regarding the round pile geometry application can lead to unreliable system results. The progression of the waves is better implemented in the ABAQUS model, since 2D effects of wave interactions are taken into account, as by the WPM only 1D wave progression is modeled.
- Predict and describe the behavior of an embedded pipe structure excited pressure waves: The requirement is implemented in the coupled WPM-PSM. The behavior of the embedded pipe can be analyzed both graphically as well as visually making the prediction and description of its behavior possible.
- Incorporation of a layer soil body: The WPM is capable of handling layered soil bodies. However, layered soil bodies are not considered in this Master Thesis research project. Therefore this research objective is considered as not for-filled.


### 9.3.3. Hypothesis

## The ground vibrations induced by vibratory sheet pile driving can lead to failure of the pipe structure.

So many factors influence the stresses in the pipe, induced by vibratory sheet pile installation, that not a strait answer to the hypothesis can be given. In case of this research, only steel pipes are taken into consideration, since steel material behaves more or less purely elastic. Situations of a horizontal center-to-center distance of 5 meters between the pipe and the pile leads to a maximum stress increase of approximately $0.7 \mathrm{~N} / \mathrm{mm}^{2}$. This stress increase is considered very low regarding the material yielding stress of $235 \mathrm{~N} / \mathrm{mm}^{2}$. Therefore can be concluded that for steel pipes, located 5 meters and further from the sheet pile, will not fail due to the vibratory sheet pile installations.
In case of concrete pipes, the situation still needs to be investigated. For this particular material can be expected that stress increases of $0.7 \mathrm{~N} / \mathrm{mm}^{2}$ can lead to material failure, since its yielding stress is a factor 50 lower than that of steel. Also in case of no failure, small cracks in the material can lead to leakage of water disposal systems. Extended research suggestions, regarding concrete pipes, are made in Chapter 10.
The Eurocode 3 [2] states a limit Peak Particle Velocity (PPV) for buried structures at $25.0 \mathrm{~mm} / \mathrm{s}$. From Chapter 7 can be concluded that in most cases this value is reached in the range of 0 to 5 meters from the center of the pile. Since the increase in stress of the investigated situations is assumed low regarding the yielding stress of the steel material, this limit value is not considered useful in case of steel pipe structures. The limit value could be of importance for concrete structures, since cracks in the structure can relate to leakage (as explained earlier in this section). Further investigation of the limit value suggested by the Eurocode 3 [2] in contrast with the stress increase in the pipe is therefore advised.

### 9.4. Final comment

In accordance with Chapter 8 a variety of model modifications and further research need to be carried out to higher system quality and lower risk. These suggestions are drawn out in Chapter 10.

## Recommendations

Optimal research outcome is the aim of every investigation project. On the other hand, due to time and money issues, research simplification are requisite to fore fill research goals and requirements. The proposed and applied model for this Master Thesis research project does have limitations, as outlined in Chapter 8. To minimize its limitations further research proposals are suggested in this chapter.

- Investigating a multiple-layered soil in ABAQUS will allow a good insight in the wave propagation behavior of a layer soil. Also a comparison with a multiple-layered situation in the WPM and reality will than be possible. Validation of the WPM model without the use of the ABAQUS model, for a multiplelayered soil, is possible just by means of field measurement PPV values and laboratory data to obtain the system parameters.
- An more extensive parameter and boundary impact study, for the ABAQUS model, will further outline its sensitivity to system parameter changes. Comparison of its study outcome to the WPM will outline an even more detailed solution spectrum.
- The hammer efficiency factor for the WPM is taken as 1.0. This factor indicates the amount of energy transferred from the hammer to the pile toe. A more extensive study to the transfer of energy from the hammer to the pile and the energy loss in the pile (towards the pile toe) should result in a more reliable model outcome. A suggestion is to calculate the hammer efficiency factor, $F^{H}(-)$, according to $F_{H}=\frac{F_{d} z}{F_{d} \text { pilepointmovement }}$. The efficiency factor is dependent on the pile point movement - vibrator movement ratio.
- Utilization of the Modified Drucker-Prager + cap model (see Han et al. [50]) will lead to better material behavior for the ABAQUS model. Hypo-plasticity models could be even better. These material models on the other hand could lead to too less damping without the application of Rayleigh-damping.
- In addition to the Drucker-Prager model, saturated soil behavior for sand and clay could be implemented, resulting in undrained behavior under the pile tip as a result of the rapid increase in local pressure (see Katzenbach [64])
- Implementation of entire pile in ABAQUS, similarly like Ekanayake et al. [41] implemented in their ABAQUS model. This model includes a friction surface at the shaft. In this way the shear waves resulting from the shaft friction can be investigated better leading to an even better understanding of the wave propagation behavior. An extension of this model can be performed by extending this model into a 3D-model where bending of the sheet pile wall will be taken into account.
- Implementation of hysteretic damping as material behavior for the Pipe Structure Model (PSM). This could be applied in a similar manner to the approaches presented by Orologopoulos and Loukidis [95].
- Use the differential quadrature method to investigate cracked beams under dynamic loads. This application could be of big importance with old concrete waterdisposal systems. Matbuly et al. [81] implemented a method to examine cracked beams under dynamic loads.
- Further investigation to the boundaries of the PSM is required to minimize boundary influence on the model output.
- The PSM suffers still from stability problems related to the numerical solving method for the system of differential equations. In Appendix C a methodology with the use of a numerical generated Jacobian matrix is outlined. This could be an approach to generate numerical stability.
- Verification of the entire model with use of field and laboratory measurements. Use of a Marcov-Chain Monte-Carlo analysis would offer the possibility to fit all the model parameters based on a probabilistic approach and measurement data.
- Inelastic soil behavior under structures and its corresponding energy dissipation could be applied to the PSM. Mergos and Kawashima [85]; Anastasopoulos et al. [9]; Kourkoulis et al. [68]; Zafeirakos and Gerolymos [130] give methods for the application of this theory. The inelastic soil behavior can lead to more realistic soil behavior and therefore a more reliable model outcome.
- Further investigation remoldiation factor for pile toe - soil interaction should give a better insight in the energy loss due to soil disturbance (as outlined in Chapter 8).
- Investigation of the limit value, suggested by the Eurocode 3 [2], in contrast with the stress increase in concrete pipe structures (as explained in Chapter 9.


## Bibliography

[1] Cur166, civieltechnisch centrum uitvoering research regelgeving, 2005.
[2] En 1993, eurocode 3 - design of steel structures, part 5: Piling, 2010.
[3] Inc. ABAQUS. ABAQUS version 6.11 user's manual. Providence, Rhode Island, USA, 2011.
[4] Inc. ABAQUS. Abaqus manual version 6.7. http://www.egr.msu.edu/software/abaqus/ Documentation/docs/v6.7/books/gsk/default.htm?startat=ch03s01.html, viewed 9th February, 2016.
[5] ABI. Mrzv s. http://www.abi-gmbh.com/attachments-for-mobilram-system.html, viewed 27th October, 2011.
[6] S. Adhikary, J. Singh, and D.K. Paul. Modeling of soil-foundation structure system. Technical report, Department of Earthquake Eingineering, IIT Roorkew, 2008.
[7] L.Ia. Ainola. A reciprocal theorem for dynamic problems of the theory of elasticity. Journal of Applied Mathematic and Mechanics, 31(1):176-177, 1967.
[8] H. Amick and M. Gendreau. Construction vibrations and their impact on vibration-sensitive facilities. Proceedings of the 6th ASCE Construction Congress, February 22 2000. Orlando, Florida.
[9] I. Anastasopoulos, G. Gazetas, M. Loli, M. Apostolou, and N. Gerolymos. Soil failure can be used for seismic protection of structures. Bulletin of Earthquake Engineering, 8(2):309-326, 2010.
[10] J. Andrews and P.E.D. Buehler. Transportation and construction induced vibration guidance manual, 2004.
[11] G.A. Athanasopoulos and P.C Pelekis. Ground vibrations from sheetpile driving in urban environment: Measurements, analysis and effects on building and occupants. Soil Dynamics and Earthquake Engineering, 19(5):371-387, July 2000.
[12] G.A. Athanasopoulos, P.C. Pelekis, and G.A. Anagnostopoulos. Effect of soil stiffness in the attenuation of rayleigh-wave motions from field measurements. Soil Dynamics and Earthquake Engineering, 19(4): 277-288, June 2000.
[13] P.B. Attewell and I.W. Farmer. Attenuation of ground vibrations from pile driving. Ground Engineering, (6):26-29, 1973. Department of Geological Science, University of Durham.
[14] P.B. Attewell, A.R. Selby, and L. O' Donnell. Estimation of ground vibration from driven piling based on statistical analyses of recorded data. Geotechnical and geological engineering, (10):41-59, 1992. School of Engineering and Computer Science, University of Durham.
[15] D. Aubry and D. Clouteau. A subdomain approach to dynamic soil-structure interaction. Recent Advances in Earthquake Engineering and Structural Dynamics, pages 251-272, 1992. Ouest Editions: AFPS.
[16] B. Banerje. Deformation of a timoshenko beam and coordinate system. https://en.wikipedia. org/wiki/Timoshenko_beam_theory/media/File:TimoshenkoBeam.svg, December 2011. Online accessed November 25th 2015.
[17] S.F. Bartlett. Mohr-coulomb model. Technical report, 2010.
[18] K.J. Bathe. Finite Element Procedures. Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1996. Professor of Mechanical Engineering Massachuetts Institute of Technology.
[19] T. Benz. Small-strain stiffness of soils and its numerical consequences. PhD thesis, Universität Stuttgart, Fakultät für Bau- und Umweltingenieurswissenschaften, September 2006.
[20] J.F. Van Den Berghe. Interlock friction in a sheet-pile: laboratory testing. International Conference on Soil Mechanics and Geotechnical Engineering, August 2001.
[21] J. Biarez and P.Y. Hicher. Elementary Mechnics of Soil Behavior. Balkema, 1994.
[22] A. Bodare. Kompendium jordoch bergdynamik. Technical Report 1B1435, Division for Soil and Rock Mechanics, Royal Institute of Technology, Stockholm, Sweden, 1996.
[23] J. Boussinesq. Application des potentiels a l etude de l equilibre et du mouvement des solides elastiques. Gauthier-Villars, Paris, 1885.
[24] L.M. Brekhovskikh. Waves in a layered media, volume 16. Academic Press Inc., 2 edition, 1980.
[25] R.B.J. Brinkgreve, S. Kumarswamy, and W.M. Swolfs. Plaxis 2015 users manual. Plaxis BV., 2015.
[26] T.K. Caughey. Classical normal modes in damped linear dynamic systems. Journal of Applied Mechanics, 27(2):269-271, 1960. Division of Engineering, California Institute of Technology, Pasadena, Calif.
[27] G. Chen, L. Qian, and Q. Yin. Dynamic analysis of a timoshenko beam subjected to an accelerating mass using spectral element method. Hindawi publishing corporation, shock and vibration, 2014, February 2014.
[28] R.W. Clough and J. Penzien. Dynamics of Structures. Computer and Structures Inc., 3 edition, 2003.
[29] C.J. Cornejo Cordova. Elastodynamics with Hysteretic Damping. PhD thesis, Delft University of Technology, 2002.
[30] M. Costabel. Principles of Boundary Element Methods. Technische Hochschule Darmstadt, 1986.
[31] B.M. Das and G.V. Ramana. Principles of soil dynamics. Cengage Learning, 3 edition, 2010.
[32] D. Davis. A review of prediction methods for ground-borne noise due to construction activities. Proceedings of the 20th International Congress on Acoustics, Sydney, Australia, pages 23-27, August 2010.
[33] W.H. de Brabander. Dynamic behaviour of long jetty structures under seismic conditions. Master's thesis, Delft University of Technology, October 2006.
[34] F. Deckner. Ground vibrations due to pile and sheet pile driving, influencing factors, predictions and measurements. PhD thesis, Division of Soil and Rock Mechanics Department of Civil and Architectural Engineering School of Architecture and the Built Environment KTH, Royal Institute of Technology, 2013.
[35] F. Deckner, K. Viking, and S. Hintze. Ground vibrations due to pile and sheet pile driving, prediction models of today. Proceedings of the 22nd European Young Geotechnical Engineers Conference, Gothenburg, Sweden, pages 26-29, August 2012.
[36] A.J. Deeks and M.F. Randolf. Axisymmetric time-domain transmitting boundaries. Journal of Engineering Mechanics, 120(1):25-40, January 1994.
[37] J.N. Desai. Implementation of a beam element in fea using matlab. Technical Report EML 5226.
[38] C. Devulder, M. Marion, and E.S. Titi. On the rate of convergence of the nonlinear galerkin method. Mathematics of Computation, 60(202):495-514, April 1993.
[39] C.H. Dowding. Construction Vibrations. Prentice Hall Inc., Upper Saddle River, USA, 1996.
[40] X. Du and M. Zhao. A local time domain transmitting boundary for simulating cylindrical elastic wave propagation in infinite media. Soil Dynamics and Earthquake Engineering, 30(10):937-946, October 2010.
[41] S.D. Ekanayake, D.S. Liyanapathirana, and C.J. Leo. Influence zone around a closed ended pile during vibratory driving. Soil Dynamics and Earthquake Engineering, 53:26-36, June 2013.
[42] A.J.M. Fereira. Matlab codes for finite element analysis. Solids and Structures, 2009.
[43] S. Francois, H.R. Masoumi, and G. Degrande. An iterative coupled boundary-finite element method for dynamic response of structures. III European Conference on Computational Mechanics, June 2006.
[44] L. Gaul. The influence of damping on waves and vibrations. Mechanical Systems and Signal Processing, 13(1):1-30, January 1999.
[45] Geotechdata. Geotechnical parameters. www.geotechdata.info/parameter/parameter.html, viewed 11th of February, 2016.
[46] Dieseko Groep. Vibrator pve55m. http://www.pve-holland.com/producten/34/36506/ PVE-55M/Trilblokken/, viewed 11th of February, 2016.
[47] C. Guillemet. Pile-soil interaction during vibratory sheet pile driving. Master's thesis, Royal Institude of Technology Stockholm, May 2013.
[48] S.R. Gunakala, D.M.G. Comissiong, K. Jordan, and A. Sankar. A finite element solution of the beam equation via matlab. International Journal of Applied Science and Technology, 2(8):80-88, October 2012.
[49] T.G. Gutowski and C.L. Dym. Propagation of ground vibration: A review. Journal of Sound and Vibration, 49(2):179-193, July 1976.
[50] L.H. Han, J.A. Elliott, A.C. Bentham, A. Mills, G.E. Amidon, and B.C. Hancock. A modified druckerprager cap model for die compaction simulation of pharmaceutical powders. International journal of solids and structures, 45(10):3088-3106, May 2008.
[51] J.M. Head and F.M. Gardine. Ground-borne vibrations arising from piling. CIRIA Technical Note 142, 1992.
[52] D.M. Hiller and V.S. Hope. Groundborne vibration generated by mechanized construction activities. Proceedings of the ICE: Geotechnical Engineering, 131(4):223-232, 1998.
[53] D.A.J. Hilster. Viscoelastic damping in high rise structures. Literatuur review, Delft University of Technology, October 2013.
[54] R. Holmberg, P.W. Arnberg, O. Bennerhult, L. Forssblad, L. Gereben, L. Hellman, K. Olsson, G. Rundqvist, C. Sjoberg, K. Sjokvist, and G. Wallmark. Vibrations generated by traffic and building construction activities. Swedish Council for Building Research, 1984.
[55] V. Illingworth. The Penguin Dictionary of Physics. Penguin Books, London, 2 edition, 1991. First published 1977.
[56] Resonance Technology International. Resonant pile driver. http://www.resonancetechnology.ca/ rd260\%20rd140\%20brochure\%2004. pdf, viewed 23rd March 2012.
[57] T. Itoh. Damped vibation mode superposition method for dynamic response analysis. Earthquake Engineering and Structural Dynamics, 2(1):47-57, 1973.
[58] IVA. Jordoch bergdynamik. ivas kommitte for vibrationsfragor, arbetsgrupp 4: Jordoch bergdynamik, volym 225 av ingenjorsvetenskapsakademien. Technical report, IVA, Stockholm, Sweden, 1979.
[59] N.M. Jackson. Use of nondestructive techniques to estimate the allowable vibratory compaction level during construction. State Materials Office, 2007.
[60] J. Jia. Essentials of Applied Dynamic Analysis. Springer Verlag Berlin Heidelberg, Vienne, Austria, 2014.
[61] N.H. Jones. Finding the area under a curve using jmp and a trapezoidal rule. http://www. jmpdiscovery.com/news/jmpercable/fall97/trapezoidal_rule.html, 1997. SAS Institute.
[62] D. Jongmans. Prediction of ground vibrations caused by pile driving: A new methodology. Engineering Geology, 42(1):25-36, March 1996.
[63] A. Kareem, T. Kijewski, and Y. Tamura. Mitigations of motions of tall buildings with specific examples of recent applications. Wind and structures, 2(3):201-239, 1999.
[64] R. Katzenbach. Anwendung der FEM in der Geotechnik. Technische Universitat Darmstadt, 2004.
[65] D.S. Kim and J.S. Lee. Propagation and attenuation characteristics of various ground vibrations. Soil Dynamics and Earthquake Engineering, 19(2):115-126, February 1999.
[66] S. Kirkup and J. Yazdani. A gentle introduction to the boundary element method in matlab or freemat. East Lancashire Institute of Higher Education Blackburn College Blackburn, 2008.
[67] T. Kocaturk and M. Simsek. Dynamic analysis of eccentrically prestressed viscoelastic timoshenko beams under a moving harmonic load. Computers and Structures, 84(31-32):2113-2127, December 2006.
[68] R. Kourkoulis, F. Gelagoti, and I. Anastasopoulos. Rocking isolation of frames on isolated footings: design insights and limitations. Journal of Earthquake Engineering, 16(3):374-400, 2012.
[69] S.L. Kramer. Geotechnical Earthquake Engineering. Prentice-Hall International Series in Civil Engineering and Engineering Mechanics, 1996.
[70] G. Lanzo, A. Pagliaroli, and D Elia B. Influenza della modellazione di rayleigh dello smorzamento viscoso nelle analisi di risposta sismica locale. Dipartimento di Ingegneria Strutturale e Geotecnica, Universita di Roma, pages 25-29, January 2004.
[71] M. Liden. Ground vibrations due to vibratory sheet pile driving. Master's thesis, Royal Institude of Technology Stockholm, June 2012.
[72] G. Liu and S.S. Quek Jerry. A non-reflecting boundary for analyzing wave propagation using the finite element method. Finite Elements in Analysis and Design, 39(5-6):403-417, March 2003.
[73] P. Loof. Het versnellen van contact programmatuur met behulp van Fouriertransformaties. Delft University of Technology, 2011.
[74] J. Lysmer and R. L. Kuhlemeyer. Finite dynamic model for infinite media. Journal of the Engineering Mechanics Division, 95(4):859-878, July, Augustus 1969.
[75] J. Mackerele. Fem and bem in geomechanics - foundations and soil structure interaction - a bibliography. Finite Elements in Analysis and Design, 22(3):249-263, July 1996.
[76] D.J. Martin. Ground vibrations from impact pile driving during road construction. Technical Report Report 544, Transport and Road Research Laboratory Supplementary, England, 1980.
[77] H.R. Masoumi and G. Degrande. Numerical modeling of free field vibrations due to pile driving using a dynamic soil-structure interaction formulation. Journal of Computational and Applied Mathematics, 215(2):503-511, June 2005.
[78] H.R. Masoumi, G. Degrande, and G. Lombaert. Prediction of free field vibrations due to pile driving using a dynamic soil-structure interaction formulation. Soil dynamics and earthquake engineering, 27 (2):126-143, February 2007.
[79] H.R. Masoumi, S. Francois, and G. Degrande. A non-linear coupled finite element-boundary element model for the prediction of vibrations due to vibratory and impact pile driving. International journal for numerical and analytical method in geomechanics, 33(2):245-274, February 2009.
[80] K.R. Massarsch and B.H. Fellenius. Ground vibrations induced by impact pile driving. The 6th International Conference on Case Histories in Geotechnical Engineering, August 2008.
[81] M.S. Matbuly, O. Ragb, and M. Nassar. Natural frequencies of a functionally graded cracked beam using the differential quadrature method. Applied Mathematic and Computation, 215(6):2307-2316, November 2009.
[82] Mathworks. Matlab user's manual. https://www.ma.utexas.edu/users/haack/getstart.pdf, vieuwed 26th of February, 2016.
[83] Mathworks. Choose an ode solver. http://nl.mathworks.com/help/matlab/math/ choose-an-ode-solver.htmlzmw57dd0e25753, viewed 31th of March, 2016.
[84] Mathworks. Create or modify options structure for ode solvers. http://nl.mathworks.com/help/ matlab/ref/odeset.html, viewed 31th of March, 2016.
[85] P.E. Mergos and K. Kawashima. Rocking isolation of a typical bridge pier on spread foundation. Journal of Earthquake Engineering, 9(2):395-414, 2005.
[86] A.V. Metrikine. Dynamics, Slender Structures and an Introduction to Continuum Mechanics CT4145, volume Lecture Notes. Delft University of Technology, 2014.
[87] M.A. Meyers and K.K. Chawla. Mechanical behavior of Materials. Prentice Hall Inc., 1999.
[88] Ltd. Midas Information Technology Co. Mohr-coulomb model. http://manual.midasuser.com/ EN_common/SoilWorks/250/Start/02-Model/01-Property/01-Ground_Material_Property/ Model_Types/09-Mohr-Coulomb.htm, viewed 10th February, 2016.
[89] G.F. Miller and H. Pursey. On the partition of energy between elastic waves in a semiinfinite solid. Proceedings of the Royal Society of London, 233:55-69, 1955.
[90] Arcelor Mittal. Az32-750 sheet pile wall. http://www.arcelorprojects.com/projects/europe/ foundationsolutions/EN/sheet_piling/AZ_sections/AZ32-750.htm, viewed, 17th of February, 2016.
[91] S. Nordal. Lecture notes: PhD course BA8305 Geodynamics. Norwegian University of Science and Technology, Trondheim, Norway, 2009.
[92] Massachusetts Institute of Technology MIT. Chapter 9, stresses: Beams in bending. http://ocw.mit.edu/courses/civil-and-environmental-engineering/ 1-050-solid-mechanics-fall-2004/readings/emech9_04.pdf, 2004.
[93] S. Oller. Nonlinear Dynamics of Structures, First edition. Springer Science+Business Media B.V, 2014.
[94] E. Olsson. A dynamic response study on optimal piling depth with respect to ground vibations. Master's thesis, Chalmers University of Thechnology, May 2014.
[95] N. Orologopoulos and D. Loukidis. Soil non-linear behavior and hysteretic damping in the spring dashpot analog. Second European conference on earthquake engineering and seismology, 2014.
[96] E. Pesheck, C. Pierre, and S.W. Shaw. A new galerkin-based approach for accurate non-linear normal modes through invariant manifolds. Journal of Sound and Vibration, 249(5):971 - 993, 2002.
[97] T. Pichler, T. Pucker, T. Hamann, S. Henke, and G. Qiu. High performance abaqus simulations in soil mechanics. Simulia Community Conference, 2008. Institute for Geotechnics and Construction Management, Hamburg University of Technology, Germany.
[98] J. Pistrol. Gegenuberstellung von verschiedenen berechnungsverfahren zur erfassung der wechselwirkung bauwerk-untergrund. Master's thesis, Technischen Universitat Wien, June 2011.
[99] L. Pyl, D. Clouteau, and G. Degrande. A weakly singular boundary integral equation in elastodynamics for heterogeneous domains mitigating fictitious eigenfrequencies. Engineering Analysis with Boundary Elements, 28(2004):1493-1513, 2004.
[100] A.S. Ramkisoen. Predictie en analyse van trillingen veroorzaakt door het intrillen van damwandplanken. Master's thesis, Technical University Delft, 2012.
[101] S. Rao. Vibration of continuous systems. John Wiley \& sons Inc., Coral Gables Florida, department of Mechanical and Aearospace Engineering, University of Miami, 2007.
[102] J. Ratzkin. The fourier transform and related topics. November 172003.
[103] G. Reuver. Plan of work, master thesis evaluation of pile installation effects on adjacent structures: A numerical approach. September 2015.
[104] F.E. Richart, R.D., and J.R. Hall. Vibrations of Soils and Foundations. Prentice-Hall Inc., Engelwood Cliffs, USA, 1970.
[105] T. J. Ross. Dynamic rate effects on timoshenko beam response. Journal of Applied Mechanics, 52(2): 439-445, June 1985.
[106] M. Schevenels, S. Francois, and G. Degrande. User's guide elastodynamic toolbox for matlab version 2.0 build 13. Kasteelpark Arenberg 40, B3001 Leuven, Belgium, July 2010.
[107] M.S. Serdaroglu. Nonlinear analysis of pile driving and ground vibrations in saturated cohesive soils using the finite element method. PhD thesis, University of Iowa, 2010.
[108] I.M. Smith, D.V. Griffiths, and L. Margetts. Programming the Finite element method. John Wiley and sons Ltd., October 2013.
[109] J.M.J. Spijkers. Dynamics of Structures, volume Part 1 and Part 2. Delft University of Technology, 2014.
[110] F. Tatsuoka, T. Iwasaki, and Y. Takagi. Hysteretic damping of sands under cyclic loading and its relation to shear modulus. Soils and foundations, 18(2):25, June 1978.
[111] J. Thomas and B.A.H. Abbas. Finite element model for dynamic analysis of timoshenko beam. Journal of Sound and Vibration, 41(3):291-299, January 1975.
[112] W. T. Thomson. Theory of Vibration with Applications. Nelson Thornes Ltd., 1981.
[113] S. Timoshenko. Vibration Problem in Engineering. D. Van Nostrant Company, Inc., 1937.
[114] S. Timoshenko. Mechanics of Materials. Van Nostrand Reinhold Co., 1977.
[115] Colerado University. Modal analysis of mdof forced damped systems. http://www. colorado.edu/ engineering/CAS/courses.d/Structures.d/IAST.Lect22.d/IAST.Lect22.pdf, uploaded, 24th of February, 2016.
[116] Colerado University. One-dimensional finite element methods. http://www.cs.rpi.edu/ ~flaherje/pdf/fea2.pdf, uploaded, 24th of February, 2016.
[117] S.A. van Dijk. The dynamic behavior of floors in high rise buildings and their contribution to damping: An analytical model. Master's thesis, Delft University of Technology, July 2015.
[118] F. van Keulen. Collegedictaat Stijfheid en Sterkte III. Delft University of Technology, Faculteit Ontwerp, Constructie en Productie Werktuigbouwkunde, 2000.
[119] A. Verruit. An introduction to soil dynamics. Springer Science+Business Media B.V, 2010.
[120] K. Viking. Vibro driveability A field study of vibratory driven sheet piles in non cohesive soils. PhD thesis, Royal Institude of Technology Stockhom, May 2002.
[121] M. Vucetic and R. Dobry. Effect of soil plasticity on cyclic response. Geotechnical Engineering, 117(1): 89-107, 1991.
[122] P.H. Waarts and M.S. De Wit. Does more sophisticated modeling reduce model uncertainty? a case study on vibration predictions. TNO Building and Construction Research, Delft, The Netherlands, 49(2), 2004.
[123] C. Wersall, K.R Massarsch, and A. Bodare. Planning and execution of rock blasting adjecent to tunnels. Proceedings, 4th International Symposium on Enviremental Vibrations, 1:749-756, 2009.
[124] V. Whenham. Power transfer and vibrator-pile-soil interactions within the framework of vibratory pile driving. PhD thesis, Universite catholique de Louvain, May 2011.
[125] J.F. Wiss. Construction vibrations: State-of-the-art. discussion and closure. American Society of Civil Engineers, 108(GT3):510-512, March 1982.
[126] J.P. Wolf. Foundation Vibration Analysis Using Simple Physical Models. Prentice Hall Inc., Upper Saddle River, USA, 1994.
[127] R.D. Woods. Dynamic effect of pile installations on adjacent structures. National Academy Press, Washington, DC, 1997.
[128] S.T. Yeh. Using trapezoidal rule for the area under a curve calculation. http://www2.sas.com/ proceedings/sugi27/p229-27.pdf. Paper 229-27, GlaxoSmithKline, Collegeville, PA.
[129] M.A. Youssef and D. Park. Viscous damping formulation and high frequency motion propagation in non linear site response analysis. Soil Dynamics and Earthquake Engineering, 22(7):611-624, September 2002.
[130] A. Zafeirakos and N. Gerolymos. On the seismic response of under-designed caisson foundations. Bulletin of Earthquake Engineering, 11(5):1337-1372, 2013.
[131] J. Zhou. Modeling mic and metal precipitation with a ld reactive transport model. Master's thesis, Delft University of Technology, 2015.
[132] O.C. Zienkiewicz, C. Emson, and P. Bettess. A novel boundary infinite element. International Journal for Numerical Methods in Engineering, 19(3):393-404, March 1983.

## Derivations

## A.1. Center of gravity pipe

The center of gravity of half of the pipe is used in the calculation of the momentum in the pipe structure. The derivation is based on Figure A. 1 where:

- $R_{1}(\mathrm{~m})$ : The outer radius of the pipe
- $R_{2}(\mathrm{~m})$ : The inner radius of the pipe
- $\theta$ (rad): Angle with the horizontal
- $y_{z}(\mathrm{~m})$ : Center of gravity for half a pipe


Figure A.1: Center of gravity of half of a pipe cross-section

$$
\begin{gather*}
\iint y d A=\iint r \sin (\theta) d A=\iint r \sin (\theta) r d \theta d r  \tag{A.1}\\
\frac{1}{2} \pi\left(R_{1}^{2}-R_{2}^{2}\right) y_{z}=\int_{\theta=0}^{\theta=\pi} \int_{r=R_{1}}^{r=R_{2}} \sin (\theta) d \theta r^{2} d r  \tag{A.2}\\
\frac{1}{2} \pi\left(R_{1}^{2}-R_{2}^{2}\right) y_{z}=\int_{\theta=0}^{\theta=\pi} \sin (\theta) d \theta \int_{r=R_{1}}^{r=R_{2}} r^{2} d r  \tag{A.3}\\
\frac{1}{2} \pi\left(R_{1}^{2}-R_{2}^{2}\right) y_{z}=\frac{1}{3}[-\cos (\theta)]_{0}^{\pi}\left[\frac{1}{3} r^{3}\right]_{R_{1}}^{R_{2}}  \tag{A.4}\\
\frac{1}{2} \pi\left(R_{1}^{2}-R_{2}^{2}\right) y_{z}=\frac{1}{3}\left(R_{1}^{3}-R_{2}^{3}\right) 2  \tag{A.5}\\
y_{z}=\frac{4}{3 \pi} \frac{\left(R_{1}^{3}-R_{2}^{3}\right)}{\left(R_{1}^{2}-R_{2}^{2}\right)} \tag{A.6}
\end{gather*}
$$

## A.2. Element matrices used in the FEM

The individual components of the stiffness matrix, $[K]$, mass matrix, $[M]$ and the force vector, $\{F\}$, are obtained in this section. By means of Galerkin's Weighted Residual method with the use of Greens Theorem and the application of the Hermite cubic interpolation functions two-noded rod-elements with four integration points are created. The cubic shape functions and their first-and second derivative are shown in Equation A. 7 to Equation A. 18.

$$
\begin{gather*}
N_{1}=\frac{1}{L^{3}}\left(L^{3}-3 L x^{2}+2 x^{3}\right)  \tag{A.7}\\
N_{2}=\frac{1}{L^{2}}\left(L^{2} x-2 L x^{2}+x^{3}\right)  \tag{A.8}\\
N_{3}=\frac{1}{L^{3}}\left(3 L x^{2}-2 x^{3}\right)  \tag{A.9}\\
N_{4}=\frac{1}{L^{2}}\left(x^{3}-L x^{2}\right)  \tag{A.10}\\
\frac{\partial N_{1}}{\partial x}=\frac{1}{L^{3}}\left(-6 L x+6 x^{2}\right)  \tag{A.11}\\
\frac{\partial N_{2}}{\partial x}=\frac{1}{L^{2}}\left(L^{2}-4 L x+3 x^{2}\right)  \tag{A.12}\\
\frac{\partial N_{3}}{\partial x}=\frac{1}{L^{3}}\left(6 L x-6 x^{2}\right)  \tag{A.13}\\
\frac{\partial N_{4}}{\partial x}=\frac{1}{L^{2}}\left(3 x^{2}-2 Ł x\right)  \tag{A.14}\\
\frac{\partial^{2} N_{1}}{\partial x^{2}}=\frac{1}{L^{3}}(-6 L+12 x)  \tag{A.15}\\
\frac{\partial^{2} N_{2}}{\partial x^{2}}=\frac{1}{L^{2}}(-4 L+6 x)  \tag{A.16}\\
\frac{\partial^{2} N_{3}}{\partial x^{2}}=\frac{1}{L^{3}}(6 L-12 x)  \tag{A.17}\\
\frac{\partial^{2} N_{4}}{\partial x^{2}}=\frac{1}{L^{2}}(6 x-2 Ł) \tag{A.18}
\end{gather*}
$$

## A.2.1. Components element stiffness matrix [ $\mathbf{K}^{\text {Pipe }}$ ]

The stiffness matrix can be obtained by solving the integral as given in Equation A. 19 (for the pipe structure). This will result in a $4 \times 4$ symmetric stiffness matrix.

$$
\begin{equation*}
K_{i j}^{P i p e}=E I \int_{0}^{L} \frac{d^{2} N_{i}}{d x^{2}} \frac{d^{2} N_{j}}{d x^{2}} d x \tag{A.19}
\end{equation*}
$$

The individual components of the equation are given by Equation A. 20 and Equation A.21. Their partial derivatives are obtained in a similar manner.

$$
\begin{gather*}
\left\{N_{i}\right\}=\left\{\begin{array}{c}
\frac{1}{L^{3}}\left(L^{3}-3 L x^{2}+2 x^{3}\right) \\
\frac{1}{L^{2}}\left(L^{2} x-2 L x^{2}+x^{3}\right) \\
\frac{1}{L^{3}}\left(3 L x^{2}-2 x^{3}\right) \\
\frac{1}{L^{2}}\left(x^{3}-L x^{2}\right)
\end{array}\right\}  \tag{A.20}\\
\left\{N_{j}\right\}=\left[\frac{1}{L^{3}}\left(L^{3}-3 L x^{2}+2 x^{3}\right) \quad \frac{1}{L^{2}}\left(L^{2} x-2 L x^{2}+x^{3}\right) \quad \frac{1}{L^{3}}\left(3 L x^{2}-2 x^{3}\right) \quad \frac{1}{L^{2}}\left(x^{3}-L x^{2}\right)\right] \tag{A.21}
\end{gather*}
$$

Individual component $k_{11}$

$$
\begin{array}{r}
k_{11}=E I \int_{0}^{L}\left[\frac{1}{L^{3}}(-6 L+12 x)\right]\left[\frac{1}{L^{3}}(-6 L+12 x)\right] d x \\
=E I \int_{0}^{L} \frac{1}{L^{6}}\left(36 L^{2}-72 L x+144 x^{2}\right) d x  \tag{A.22}\\
=E I \frac{1}{L^{6}}\left[36 L^{2} x-72 L x^{2}+144 x^{3}\right]_{0}^{L}=E I\left(\frac{12}{L^{3}}\right)
\end{array}
$$

Individual component $k_{12}$ and $k_{21}$

$$
\begin{array}{r}
k_{12}=k_{21}=E I \int_{0}^{L}\left[\frac{1}{L^{3}}(-6 L+12 x)\right]\left[\frac{1}{L^{2}}(-4 L+6 x)\right] d x \\
=E I \int_{0}^{L} \frac{1}{L^{5}}\left(24 L^{2}-84 L x+72 x^{2}\right) d x  \tag{A.23}\\
=E I \frac{1}{L^{5}}\left[24 L^{2} x-42 L x^{2}+24 x^{3}\right]_{0}^{L}=E I\left(\frac{6 L}{L^{3}}\right)
\end{array}
$$

## Individual component $k_{22}$

$$
\begin{array}{r}
k_{22}=E I \int_{0}^{L}\left[\frac{1}{L^{2}}(-4 L+6 x)\right]\left[\frac{1}{L^{3}}(-4 L+6 x)\right] d x \\
=E I \int_{0}^{L} \frac{1}{L^{4}}\left(16 L^{2}-48 L x+36 x^{2}\right) d x  \tag{A.24}\\
=E I \frac{1}{L^{4}}\left[16 L^{2} x-24 L x^{2}+12 x^{3}\right]_{0}^{L}=E I\left(\frac{4 L^{2}}{L^{3}}\right)
\end{array}
$$

Individual component $k_{13}$ and $k_{31}$

$$
\begin{array}{r}
k_{13}=k_{31}=E I \int_{0}^{L}\left[\frac{1}{L^{3}}(-6 L+12 x)\right]\left[\frac{1}{L^{2}}(6 L-12 x)\right] d x \\
=E I \int_{0}^{L} \frac{1}{L^{6}}\left(-36 L^{2}+144 L x-144 x^{2}\right) d x  \tag{A.25}\\
=E I \frac{1}{L^{6}}\left[-36 L^{2} x+72 L x^{2}-48 x^{3}\right]_{0}^{L}=E I\left(\frac{-12}{L^{3}}\right)
\end{array}
$$

## Individual component $k_{23}$ and $k_{32}$

$$
\begin{array}{r}
k_{23}=k_{32}=E I \int_{0}^{L}\left[\frac{1}{L^{2}}(-4 L+6 x)\right]\left[\frac{1}{L^{2}}(6 L-12 x)\right] d x \\
=E I \int_{0}^{L} \frac{1}{L^{5}}\left(-24 L^{2}+84 L x-72 x^{2}\right) d x  \tag{A.26}\\
=E I \frac{1}{L^{5}}\left[-24 L^{2} x+42 L x^{2}-24 x^{3}\right]_{0}^{L}=E I\left(\frac{-6 L}{L^{3}}\right)
\end{array}
$$

## Individual component $k_{33}$

$$
\begin{align*}
k_{33}= & E I \int_{0}^{L}\left[\frac{1}{L^{2}}(6 L-12 x)\right]\left[\frac{1}{L^{2}}(6 L-12 x)\right] d x \\
& =E I \int_{0}^{L} \frac{1}{L^{6}}\left(36 L^{2}-144 L x+144 x^{2}\right) d x  \tag{A.27}\\
& =E I \frac{1}{L^{6}}\left[36 L^{2} x-72 L x^{2}+48 x^{3}\right]_{0}^{L}=E I\left(\frac{12}{L^{3}}\right)
\end{align*}
$$

Individual component $k_{14}$ and $k_{41}$

$$
\begin{array}{r}
k_{14}=k_{41}=E I \int_{0}^{L}\left[\frac{1}{L^{3}}(-6 L+12 x)\right]\left[\frac{1}{L^{2}}(-2 L+6 x)\right] d x \\
=E I \int_{0}^{L} \frac{1}{L^{5}}\left(12 L^{2}-60 L x+72 x^{2}\right) d x  \tag{A.28}\\
=E I \frac{1}{L^{5}}\left[12 L^{2} x-30 L x^{2}+24 x^{3}\right]_{0}^{L}=E I\left(\frac{6 L}{L^{3}}\right)
\end{array}
$$

## Individual component $k_{24}$ and $k_{42}$

$$
\begin{array}{r}
k_{24}=k_{42}=E I \int_{0}^{L}\left[\frac{1}{L^{2}}(-4 L+6 x)\right]\left[\frac{1}{L^{2}}(-2 L+6 x)\right] d x \\
 \tag{A.29}\\
=E I \int_{0}^{L} \frac{1}{L^{4}}\left(8 L^{2}-36 L x+36 x^{2}\right) d x \\
=E I \frac{1}{L^{4}}\left[8 L^{2} x-18 L x^{2}+12 x^{3}\right]_{0}^{L}=E I\left(\frac{2 L^{2}}{L^{3}}\right)
\end{array}
$$

## Individual component $k_{34}$ and $k_{43}$

$$
\begin{array}{r}
k_{24}=k_{42}=E I \int_{0}^{L}\left[\frac{1}{L^{3}}(6 L-12 x)\right]\left[\frac{1}{L^{2}}(-2 L+6 x)\right] d x \\
=E I \int_{0}^{L} \frac{1}{L^{5}}\left(-12 L^{2}+60 L x-72 x^{2}\right) d x  \tag{A.30}\\
=E I \frac{1}{L^{5}}\left[-12 L^{2} x+30 L x^{2}-24 x^{3}\right]_{0}^{L}=E I\left(\frac{-6 L}{L^{3}}\right)
\end{array}
$$

## Individual component $k_{44}$

$$
\begin{array}{r}
k_{44}=E I \int_{0}^{L}\left[\frac{1}{L^{2}}(-2 L+6 x)\right]\left[\frac{1}{L^{2}}(-2 L+6 x)\right] d x \\
=E I \int_{0}^{L} \frac{1}{L^{4}}\left(4 L^{2}-24 L x+36 x^{2}\right) d x  \tag{A.31}\\
=E I \frac{1}{L^{4}}\left[4 L^{2} x-12 L x^{2}+12 x^{3}\right]_{0}^{L}=E I\left(\frac{4 L^{2}}{L^{3}}\right)
\end{array}
$$

## Final stiffness matrix

The stiffness matrix obtained is a combination of all the individual matrix components derived and is given by Equation A. 32 .

$$
\left[K_{\text {elem }}^{\text {Pipe }}\right]=\frac{E I}{L^{3}}\left[\begin{array}{cccc}
12 & 6 L & -12 & 6 L  \tag{A.32}\\
6 L & 4 L^{2} & -6 L & 2 L^{2} \\
-12 & -6 L & 12 & -6 L \\
6 L & 2 L^{2} & -6 L & 4 L^{2}
\end{array}\right]
$$

## A.2.2. Components element mass matrix [ M ]

The mass matrix can be obtained by solving the integral as given in Equation A.33. This will result in a $4 \times 4$ symmetric mass matrix.

$$
\begin{equation*}
M_{i j}=\rho A \int_{0}^{L} N_{i} N_{j} d x \tag{A.33}
\end{equation*}
$$

The individual components of the equation are given by Equation A. 20 and Equation A. 21 .

## Individual component $m_{11}$

$$
\begin{array}{r}
m_{11}=\rho A \int_{0}^{L}\left[\frac{1}{L^{3}}\left(L^{3}-3 L x^{2}+2 x^{3}\right)\right]\left[\frac{1}{L^{3}}\left(L^{3}-3 L x^{2}+2 x^{3}\right)\right] d x \\
=\rho A \int_{0}^{L} \frac{1}{L^{6}}\left(L^{6}-6 L^{4} x^{2}+4 L^{3} x^{3}+9 L^{2} x^{4}-12 L x^{5}+4 x^{6}\right) d x  \tag{A.34}\\
=\rho A \frac{1}{L^{6}}\left[L^{6} x-2 L^{4} x^{3}+L^{3} x^{4}+\frac{9}{5} L^{2} x^{5}-2 L x^{6}+\frac{4}{7} x^{7}\right]_{0}^{L}=\rho A\left(\frac{156 L}{420}\right)
\end{array}
$$

Individual component $m_{12}$ and $m_{21}$

$$
\begin{array}{r}
m_{12}=m_{21}=\rho A \int_{0}^{L}\left[\frac{1}{L^{3}}\left(L^{3}-3 L x^{2}+2 x^{3}\right)\right]\left[\frac{1}{L^{2}}\left(L^{2} x-2 L x^{2}+x^{3}\right)\right] d x \\
=\rho A \int_{0}^{L} \frac{1}{L^{5}}\left(L^{5} x-2 L^{4} x^{2}-2 L^{3} x^{3}+8 L^{2} x^{4}-12 L x^{5}+4 x^{6}\right) d x  \tag{A.35}\\
=\rho A \frac{1}{L^{5}}\left[\frac{1}{2} L^{5} x^{2}-\frac{2}{3} L^{4} x^{3}-\frac{1}{2} L^{3} x^{4}+\frac{8}{5} L^{2} x^{5}-\frac{7}{6} L x^{6}+\frac{2}{7} x^{7}\right]_{0}^{L}=\rho A\left(\frac{22 L^{2}}{420}\right)
\end{array}
$$

## Individual component $m_{22}$

$$
\begin{array}{r}
m_{22}=\rho A \int_{0}^{L}\left[\frac{1}{L^{2}}\left(L^{2} x-2 L x^{2}+x^{3}\right)\right]\left[\frac{1}{L^{2}}\left(L^{2} x-2 L x^{2}+x^{3}\right)\right] d x \\
=\rho A \int_{0}^{L} \frac{1}{L^{4}}\left(L^{4} x^{2}-4 L^{3} x^{3}+5 L^{2} x^{4}-4 L x^{5}+L^{2} x^{4}+x^{6}\right) d x  \tag{A.36}\\
=\rho A \frac{1}{L^{4}}\left[\frac{1}{3} L^{4} x^{3}-L^{3} x^{4}+L^{2} x^{5}-\frac{2}{3} L x^{6}+\frac{1}{5} L^{2} x^{5}+\frac{1}{7} x^{7}\right]_{0}^{L}=\rho A\left(\frac{4 L^{3}}{420}\right)
\end{array}
$$

## Individual component $m_{13}$ and $m_{31}$

$$
\begin{array}{r}
m_{13}=m_{31}=\rho A \int_{0}^{L}\left[\frac{1}{L^{3}}\left(L^{3}-3 L x^{2}+2 x^{3}\right)\right]\left[\frac{1}{L^{3}}\left(3 L x^{2}-2 x^{3}\right)\right] d x \\
=\rho A \int_{0}^{L} \frac{1}{L^{6}}\left(3 L^{4} x^{2}-9 L^{2} x^{4}+12 L x^{5}-2 L^{3} x^{3}-4 x^{6}\right) d x  \tag{A.37}\\
=\rho A \frac{1}{L^{6}}\left[L^{4} x^{3}-\frac{9}{5} L^{2} x^{5}+2 L x^{6}-\frac{1}{2} L^{3} x^{4}-\frac{4}{7} x^{7}\right]_{0}^{L}=\rho A\left(\frac{54 L}{420}\right)
\end{array}
$$

## Individual component $m_{23}$ and $m_{32}$

$$
\begin{array}{r}
m_{23}=m_{32}=\rho A \int_{0}^{L}\left[\frac{1}{L^{2}}\left(L^{2} x-2 L x^{2}+x^{3}\right)\right]\left[\frac{1}{L^{3}}\left(3 L x^{2}-2 x^{3}\right)\right] d x \\
=\rho A \int_{0}^{L} \frac{1}{L^{5}}\left(3 L^{3} x^{3}-8 L^{2} x^{4}+7 L x^{5}-2 x^{6}\right) d x  \tag{A.38}\\
=\rho A \frac{1}{L^{5}}\left[\frac{3}{4} L^{3} x^{4}-\frac{8}{5} L^{2} x^{5}+\frac{7}{6} L x^{6}-\frac{2}{7} x^{7}\right]_{0}^{L}=\rho A\left(\frac{13 L^{2}}{420}\right)
\end{array}
$$

## Individual component $m_{33}$

$$
\begin{array}{r}
m_{33}=\rho A \int_{0}^{L}\left[\frac{1}{L^{3}}\left(3 L x^{2}-2 x^{3}\right)\right]\left[\frac{1}{L^{3}}\left(3 L x^{2}-2 x^{3}\right)\right] d x \\
=\rho A \int_{0}^{L} \frac{1}{L^{6}}\left(9 L^{2} x^{4}-12 L x^{5}+4 x^{6}\right) d x  \tag{A.39}\\
=\rho A \frac{1}{L^{6}}\left[\frac{9}{5} L^{2} x^{5}-2 L x^{6}+\frac{4}{7} x^{7}\right]_{0}^{L}=\rho A\left(\frac{156 L}{420}\right)
\end{array}
$$

Individual component $m_{14}$ and $m_{41}$

$$
\begin{align*}
& m_{14}=m_{41}=\rho A \int_{0}^{L}\left[\frac{1}{L^{3}}\left(L^{3}-3 L x^{2}+2 x^{3}\right)\right]\left[\frac{1}{L^{2}}\left(-L x^{2}+x^{3}\right)\right] d x \\
&=\rho A \int_{0}^{L} \frac{1}{L^{5}}\left(-L^{4} x^{2}+3 L^{2} x^{4}-5 L x^{5}+L^{3} x^{3}+2 x^{6}\right) d x  \tag{A.40}\\
&=\rho A \frac{1}{L^{5}}\left[-\frac{1}{3} L^{4} x^{3}+\frac{3}{5} L^{2} x^{5}-\frac{5}{6} L x^{6}+\frac{1}{4} L^{3} x^{4}+\frac{2}{7} x^{7}\right]_{0}^{L}=\rho A\left(\frac{-13 L^{2}}{420}\right)
\end{align*}
$$

Geert Reuver 4226178

Individual component $m_{24}$ and $m_{42}$

$$
\begin{array}{r}
m_{24}=m_{42}=\rho A \int_{0}^{L}\left[\frac{1}{L^{2}}\left(L^{2} x-2 L x^{2}+x^{3}\right)\right]\left[\frac{1}{L^{2}}\left(-L x^{2}+x^{3}\right)\right] d x \\
=\rho A \int_{0}^{L} \frac{1}{L^{4}}\left(-L^{3} x^{3}+3 L^{2} x^{4}-3 L x^{5}+x^{6}\right) d x  \tag{A.41}\\
=\rho A \frac{1}{L^{4}}\left[-\frac{1}{4} L^{3} x^{4}+\frac{3}{5} L^{2} x^{5}-\frac{1}{2} L x^{6}+\frac{1}{7} x^{7}\right]_{0}^{L}=\rho A\left(\frac{-3 L^{3}}{420}\right)
\end{array}
$$

Individual component $m_{34}$ and $m_{43}$

$$
\begin{align*}
m_{34}=m_{43}= & \rho A \int_{0}^{L}
\end{aligned} \begin{aligned}
& {\left[\frac{1}{L^{3}}\left(3 L x^{2}-2 x^{3}\right)\right]\left[\frac{1}{L^{2}}\left(-L x^{2}+x^{3}\right)\right] d x } \\
& =\rho A \int_{0}^{L} \frac{1}{L^{5}}\left(-3 L^{2} x^{4}+5 L x^{5}-2 x^{6}\right) d x  \tag{A.42}\\
= & \rho A \frac{1}{L^{5}}\left[-\frac{3}{5} L^{2} x^{5}+\frac{5}{6} L x^{6}-\frac{2}{7} x^{7}\right]_{0}^{L}=\rho A\left(\frac{-22 L^{2}}{420}\right)
\end{align*}
$$

## Individual component $m_{44}$ and $m_{44}$

$$
\begin{array}{r}
m_{44}=m_{44}=\rho A \int_{0}^{L}\left[\frac{1}{L^{2}}\left(-L x^{2}+x^{3}\right)\right]\left[\frac{1}{L^{2}}\left(-L x^{2}+x^{3}\right)\right] d x \\
\quad=\rho A \int_{0}^{L} \frac{1}{L^{4}}\left(L^{2} x^{4}-2 L x^{5}+x^{6}\right) d x  \tag{A.43}\\
=\rho A \frac{1}{L^{4}}\left[\frac{1}{5} L^{2} x^{5}-\frac{1}{3} L x^{6}+\frac{1}{7} x^{7}\right]_{0}^{L}=\rho A\left(\frac{4 L^{3}}{420}\right)
\end{array}
$$

## Final mass matrix

The mass matrix obtained is a combination of all the individual matrix components derived and is given by Equation A. 44 .

$$
\left[M_{\text {elem }}\right]=\frac{\rho A L}{420}\left[\begin{array}{cccc}
156 & 22 L & 54 & -13 L  \tag{A.44}\\
22 L & 4 L^{2} & 13 L & -3 L^{2} \\
54 & 13 L & 156 & -22 L \\
-13 L & -3 L^{2} & -22 L & 4 L^{2}
\end{array}\right]
$$

## A.2.3. Components element force vector $F$

The force vector can be obtained by solving the integral as given in Equation A.45. This will result in a vector of length 4.

$$
\begin{equation*}
F_{i}=\int_{0}^{L} q N_{i} d x \tag{A.45}
\end{equation*}
$$

The individual components of the equation are given by Equation A. 20 .

## Individual component $F_{1}$

$$
\begin{align*}
& F_{1}=q \int_{0}^{L} \frac{1}{L^{3}}\left(L^{3}-3 L x^{2}+2 x^{3}\right) d x \\
&=q\left[\frac{1}{L^{3}}\left(L^{3} x-L x^{3}+\frac{1}{2} x^{4}\right)\right]_{0}^{L}=q L\left(\frac{6}{12}\right) \tag{A.46}
\end{align*}
$$

Individual component $F_{2}$

$$
\begin{array}{r}
F_{1}=q \int_{0}^{L} \frac{1}{L^{2}}\left(L^{2} x-2 L x^{2}+x^{3}\right) d x \\
=q\left[\frac{1}{L^{2}}\left(\frac{1}{2} L^{2} x^{2}-\frac{2}{3} L x^{3}+\frac{1}{4} x^{4}\right)\right]_{0}^{L}=q L\left(\frac{L}{12}\right) \tag{A.47}
\end{array}
$$

## Individual component $F_{3}$

$$
\begin{align*}
& F_{1}=q \int_{0}^{L} \frac{1}{L^{3}}\left(3 L x^{2}-2 x^{3}\right) d x \\
= & q\left[\frac{1}{L^{3}}\left(L x^{3}-\frac{1}{2} x^{4}\right)\right]_{0}^{L}=q L\left(\frac{6}{12}\right) \tag{A.48}
\end{align*}
$$

## Individual component $F_{4}$

$$
\begin{array}{r}
F_{1}=q \int_{0}^{L} \frac{1}{L^{2}}\left(-L x^{2}+x^{3}\right) d x \\
=q\left[\frac{1}{L^{2}}\left(-\frac{1}{3} L x^{3}+\frac{1}{4} x^{4}\right)\right]_{0}^{L}=q L\left(\frac{-L}{12}\right) \tag{A.49}
\end{array}
$$

## Final force vector

The force vector obtained is a combination of all the individual matrix components derived and is given by Equation A. 50 .

$$
\left\{F_{\text {elem }}\right\}=\frac{L q}{12}\left\{\begin{array}{c}
6  \tag{A.50}\\
L \\
6 \\
-L
\end{array}\right\}
$$



## Plots

For the plots show in this appendix the results of the Wave Propagation model only include the direct $P$ wave integration (so Rayleigh-waves are in these plots not taken into account for the WPM).


Figure B.1: ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 0.50 meters from center pile at 2 meter below surface level.


Figure B.2: ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 5.0 meters from center pile at 2 meter below surface level.


Figure B.3: ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 10.0 meters from center pile at 2 meter below surface level.


Figure B.4: ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 20.0 meters from center pile at 2 meter below surface level.


Figure B.5: ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 0.50 meters from center pile at 3 meter below surface level.


Figure B.6: ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 5.0 meters from center pile at 3 meter below surface level.


Figure B.7: ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 10.0 meters from center pile at 3 meter below surface level.


Figure B.8: ABAQUS and WPM calculation, Peak Particle Velocity (PPV) as a result of different pile toe depths. Data positioning point is 20.0 meters from center pile at 3 meter below surface level.

## Matlab code

## C.1. Coupled WPM - PSM

## C.1.1. Main code

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Influence of Pile driving on pipes in the subsurface %
% Graduation Thesis project %
% Finite Element Model Beam coupled with WPM %
% BTGeoconstult BV %
% Date: 26-04-2016 %
% Programmer: Geert Reuver %
% %
% Version: 4.6.6 %
%
%close all;
clear; clc;
%% Initialize the folders of the functions %%
addpath('./ Functions');
addpath('./ Functions/VariableFunctions') ;
addpath ('./ Functions / MatrixAssambly') ;
addpath ('. / Functions / Solver') ;
addpath('./ MainProgramFunctions');
%% Wave propagation function for the maximum wave speed %%
[v_S,v_R,Dim,Par] = WavePropagationModel();
%% Assambly and calculation of all the matrices for the FEM %%
[Par ,Dim]= GlobalMatrixAssambly (Dim, Par ) ;
%% Calculation of the stresses and strains of the pipe structure %%
[Out] = StressesAndStrains(Dim,Par) ;
%%% Plot of the Momentum, stresses and strains of the pipe over time %%
% [Plot] = PlotMomentumStressStrain(Out,Dim);
%
% %% Plot the displacements of the pipe in a 3D plot %%
% % [r] = ThreeDPlotPipe(w,t);
```


## C.1.2. Wave propagation model function

```
function [v_S,v_R,Dim,Par] = WavePropagationModel()
[Dim] = ModelDimensions();
[Par] = ModelParameters(Dim);
%% Velocity at receiving point, P-Waves %%
if Dim.Wave.depth_pipe ==0
    % Spherical wave velocity in vertical direction (P-wave) with F_v_P [mm/s]
    v_S_v_without = Par.Wave.k_s.*Par.Wave.F_v_P.*Par.Wave.E_T.*...
            ((sqrt (Par.Wave.F_P.*Par.Wave.W_tot)) .* cosd(Dim.Wave. theta_P) .*...
            1e3./(Dim.Wave.r_r));
        % Spherical wave velocity in horizontal direction (P-wave) with F_v_P [mm/s]
        v_S_h_without = Par.Wave.k_s.*Par.Wave.F_v_P.*Par.Wave.E_T.*...
            (( sqrt (Par.Wave.F_P.*Par.Wave.W_tot)) .* cosd(90-Dim.Wave.theta_P) .*...
            1e3./(Dim.Wave.r_r));
        % Spherical wave velocity in direct direction (P-wave) with F_v_P [mm/s]
        v_S_without = Par.Wave.k_s.*Par.Wave.F_v_P.*Par.Wave.E_T.*...
            ((sqrt(Par.Wave.F_P.*Par.Wave.W_tot)).*1 e3./ (Dim.Wave.r_r));
else
    % Spherical wave velocity in vertical direction (P-wave) [mm/s]
        v_S_v_without = Par.Wave.k_s.*Par.Wave.E_T.*((sqrt(Par.Wave.F_P.*...
            Par.Wave.W_tot)).* cosd(Dim.Wave.theta_P).*1e3./(Dim.Wave.r_r));
        % Spherical wave velocity in horzontal direction (P-wave) [mm/s]
        v_S_h_without = Par.Wave.k_s.*Par.Wave.E_T.*((sqrt(Par.Wave.F_P.*...
            Par.Wave.W_tot)).* cosd(90-Dim.Wave.theta_P).*1e3./(Dim.Wave.r_r));
        % Spherical wave velocity in direct direction (P-wave) [mm/s]
        v_S_without = Par.Wave.k_s.*Par.Wave.E_T.*((sqrt(Par.Wave.F_P.*...
            Par.Wave.W_tot)).*\operatorname{cosd(90-Dim.Wave.theta_P).*1e3./ (Dim.Wave.r_r));}
end
% Geometric and material damping respectively damping
v_S_v = v_S_v_without.*(1./Dim.Wave.r_r).*exp(-Par.Wave.alpha_P.*Dim.Wave.r_r);
v_S_h = v_S_h_without.*(1./Dim.Wave.r_r).*exp(-Par.Wave.alpha_P.*Dim.Wave.r_r);
v_S = v_S_without.*(1./Dim.Wave.r_r).*exp(-Par.Wave.alpha_P.*Dim.Wave.r_r);
%% Rayleigh-waves %%
% R-wave correction factor
[W_correction] = R_WaveDepthCorrection(Dim, Par);
Par.Wave.W_correction = W_correction;
% Spherical wave (P-wave) velocity in vertical direction without damping
% traveling along the critical radius distance [mm/s]
v_R_P_without = Par.Wave.k_s.*Par.Wave.F_v_R.*Par.Wave.E_T.*...
        (sqrt(Par.Wave.F_P.*Par.Wave.W_tot)./ Par.Wave.r_crit) .* cosd(Dim.Wave.theta_P) .*1
            e3;
% Damped spherical wave along the critical radius distance [mm/s]
v_R_P = v_R_P_without.*(0.5./ Par.Wave.r_crit).*exp(-Par.Wave.alpha_P.*...
        Par.Wave.r_crit);
% Geometric and material damping respectively damping. As the R-wave arises
% at the surface, the material damping will be according to the soil layer
% at surface level.
v_R = W_correction.*v_R_P.*(Par.Wave.d_crit./Dim.Wave.d_hth).^0.5.*...
    exp(-Par.Wave.alpha_P(1).*(Dim.Wave.d_hth-Par.Wave.d_crit));
%% Creating plot %%
% figure(1)
```

```
% plot(v_R(10:end),Dim.Wave.zN(10:end))
% hold on
% plot(v_S_v,Dim.Wave.zN)
% y = -[lllllllllllll}
% x (1,:) = [\begin{array}{lll}{5.75468 5.307665 4.84064 3.186245 1.777718 0.9870115 0.6805705 0.529602}\end{array}]
    0.4237465 0.338137];
% % x (1,:) = [4.783315 3.899155 3.07085 1.998315 1.194665 0.5793095 0.370474
        0.2614615 0.21215 0.1732105];
% plot(x,y,'-o')
% xlabel('Vertical vibration velocity [mm/s]')
% ylabel('Pile toe depth [m]')
% title(['Location: ', num2str(Dim.Wave.d_hth),' meters distance from the pile at , ,
    num2str(Dim.Wave.depth_pipe), ' meters depth, Sand medium dense'])
% legend('Analytical R-waves','Analytical P-waves', 'ABAQUS', 'Location ','southeast ')
% grid on
%
% figure(2)
% plot(v_S_h,Dim.Wave.zN)
% hold on
% y = -[lllllllllllll}
```



```
    0.4237465 0.338137];
% % x (1,:) = [lll.783315 3.899155 3.07085 1.998315 1.194665 0.5793095 0.370474
        0.2614615 0.21215 0.1732105];
% plot(x,y,'-o')
% xlabel('Horzontal vibration velocity [mm/s]')
% ylabel('Pile toe depth [m]')
% title(['Location: ', num2str(Dim.Wave.d_hth),' meters distance from the pile at , ,
        num2str(Dim.Wave.depth_pipe), ' meters depth, Sand medium dense'])
% legend('Analytical P-waves','ABAQUS','Location','southeast')
% grid on
% figure(3)
% plot(v_S,Dim.Wave.zN)
% hold on
% y = -[lllllllllllll}
%x(1,:)=[[\begin{array}{llllllllllll}{5.75468}&{5.307665}&{4.84064}&{3.186245}&{1.777718}&{0.9870115}&{0.6805705}&{0.529602}\end{array}]
        0.4237465 0.338137];
%%x(1,:) = [lll.783315 3.899155 3.07085 1.998315
        0.2614615 0.21215 0.1732105];
% plot(x,y,'-o')
% xlabel('Vibration velocity [mm/s]')
% ylabel('Pile toe depth [m]')
% title(['Location: ', num2str(Dim.Wave.d_hth),' meters distance from the pile at , ,
        num2str(Dim.Wave.depth_pipe), , meters depth, Sand medium dense'])
% legend('Analytical P-waves','ABAQUS','Location', 'southeast')
% grid on
end
```


## C.1.3. Stresses and strains function

```
function [Out] = StressesAndStrains(Dim,Par)
```

$\% \%$ Calculation of the stresses and strains of the pipe \%\%
\% The stresses are calculated with use of the rotation angles calculated by
\% the matlab ODE-solver. The strains are also folowing from the matlab
\% ODE-solver.
\%\% Localizing the matrices required for the ODE-Solver \%\%
F = Par. Matrix.F;
M_INV = Par.Matrix.M_INV;
K = Par. Matrix.K;
C = Par. Matrix.C;
\%\% Initial conditions of the FEM \%\%
x0 $=$ zeros(length (K), 1);
$\mathrm{v} 0=$ zeros(length (K) , 1);
$y_{-} 0=[x 0 ; v 0] ;$
\%\% Localized parameters required \%\%
NumElem = Dim. Pipe .NumElem;
NumNode = Dim. Pipe .NumNode;
\%\% Simulation time vector \%\%
TimeOde $=0$ :Par. Pipe.t_Step $:$ Par.Pipe.tEnd;
\%\% Solving the equations of motion with the ODE Solver \%\%
opts $=$ odeset('Stats ', 'on','RelTol' , 1e-4,' $A b s T o l ', 1 e-6)$;
tic
$[\mathrm{t}, \mathrm{y}]=$ odel5s $\left(\mathrm{O}(\mathrm{t}, \mathrm{y})\right.$ FunctionTotalOde ( $\mathrm{t}, \mathrm{y}, \mathrm{F}, \mathrm{M}$ _INV, $\mathrm{K}, \mathrm{C}$, TimeOde, Dim) ,TimeOde, $\mathrm{y} \_0$, opts
);
toc
$\% \%$ Determining the rotations and displacements of the Pipe \%\%
$\mathrm{L}=$ length $(\mathrm{y}(1,:)) / 2$;
\% Displacements Pipe Elements [m]
$\mathrm{w}=\mathrm{y}(:, 1: 2: \mathrm{L}-1)$;
\% Rotation angles Pipe Elements [rad]
theta $=\mathrm{y}(:, 2: 2: \mathrm{L})$;
\% Velocity Pipe Elements [m/s]
$\mathrm{v}=\mathrm{y}(:, \mathrm{L}+1: 2: 2 * \mathrm{~L}-1)$;
\% Angular speed Pipe Elements [rad/s]
omega $=y(:, L+2: 2: 2 * L) ;$
$\% \%$ Determination of the differential rotation angle \%\%
for ii = 1: length $(\mathrm{w}(1,:))-1$,
Deltatheta $(:, i i)=\operatorname{theta}(:, i i+1)-\operatorname{theta}(:, i i) ;$
end
\%\% Determination of the Momentum and stress on every element
\% Outer diameter pipe [m]
D_out = Par.Pipe.D_out;
\% Outer radius pipe [m]
R_Out = D_out/2;
\% Innter radius pipe [m]
R_In = R_Out-(Par.Pipe.tf_Pipe/1000) ;
\% Center of gravity half Pipe [m]

```
yz_Pipe = (4/(3*pi()))*((R_Out^3-R_In^3)/(R_Out^2-R_In^2));
% Extension of the Pipe element at the distance yz_Pipe (this extension is
% circlar chaped) [m]
ds_z = yz_Pipe.*Deltatheta;
% Strain of the Pipe element at the distance yz_Pipe [-]
epsilon_z = ds_z./Dim.Pipe.LengthElem;
% Stress of the Pipe element at the distance yz_Pipe [N/mm^2]
sigma_Pipe = Par.Pipe.E_Pipe.*epsilon_z.*1e-6;
% Momentum of the Pipe element at the distance yz_Pipe [kNm]
Momentum_Pipe = sigma_Pipe.*(Par.Pipe.A_Pipe/2).*yz_Pipe.*1 e3;
% Maximum momentum [kNm]
Max_Momentum = max(abs(Momentum_Pipe(:)));
%% Maximum stress plus position and moment in time %%
% Maximum stress in pipe [N/mm^2]
Max_sigma = max(abs(sigma_Pipe(:)));
% Position of the maximum stress in the pipe:
[num idx] = max(sigma_Pipe(:));
[time_max_stress Element_max_stress] = ind2sub(size(sigma_Pipe),idx);
% Position in time [s]
time_max_stress = time_max_stress*Par.Pipe.t_Step;
% Position on the beam from the left boundary [m]
Position_max_stress = (Element_max_stress - 1)*Dim.Pipe.LengthElem +...
    (Dim. Pipe.LengthElem/2);
%% Print the imporatant values to the screen %%
% Print the postion of the mamimum values on the screen
fprintf('\n\n');
fprintf('Position of the maximum stress and Moment: \n');
fprintf('x = %.2fm (Position from the left boundary) \n', Position_max_stress);
fprintf('t = %.2f s (Position in time) \n\n', time_max_stress);
fprintf('Maximum values at the position \n');
% Print maximum value momentum to screen
fprintf('M = %.2f kNm \n', round(Max_Momentum,2));
% Print maximum value stress to screen
fprintf('sigma = %.2f N/mm^2 \n\n', round(Max_sigma,2));
%% Define outcome workspace
Out.w = w; Out.theta = theta; Out.v = v; Out.omega = omega;
Out.sigma_Pipe = sigma_Pipe; Out.Momentum_Pipe = Momentum_Pipe;
Out.Max_sigma = Max_sigma; Out.Max_Momentum = Max_Momentum; Out.t = t;
end
```


## C.1.4. Animation plot function

```
function [Plot] = PlotMomentumStressStrain(Out,Dim)
%% Plots of the Momentum, displacements and stresses over time %%
sigma_Pipe = Out.sigma_Pipe;
Momentum_Pipe = Out.Momentum_Pipe;
w = Out.w;
t = Out.t;
PosX = Dim.Pipe.PosX_new;
PosX(1) = [];
PosX(end) = [];
% Calculate the positions of the moments stresses
PosX_new = PosX-(Dim.Pipe.LengthElem/2);
PosX_new(1) = [];
% writerObj = VideoWriter('Pipe_Ingeklemt');
% open(writerObj);
figure('units','normalized','position',[\begin{array}{llll}{0}&{0}&{1}&{1}\end{array}])
for ind=1:10:length(t)
    % Displacement
    subplot(3,1,1)
    plot(PosX,w(ind,:))
    axis([0 max(PosX) min(w(:)) max(w(:))])
    xlabel('Length Pipe [meter]')
    ylabel('Displacement [m]')
    title('Displacement Pipe structure')
    % Momentum
    subplot(3,1,2)
    plot(PosX_new,Momentum_Pipe(ind,:))
    axis([0 max(PosX_new) min(Momentum_Pipe(:)) max(Momentum_Pipe(:))])
    xlabel('Length Pipe [meter]')
    ylabel('Moment [kNm]')
    title('Momentum in Pipe structure')
    % Stresses
    subplot(3,1,3)
    plot(PosX_new,sigma_Pipe(ind,:))
    axis([0 max(PosX_new) min(sigma_Pipe(:)) max(sigma_Pipe(:))])
    xlabel('Length Pipe [meter]')
    ylabel('Stress [N/mm^2]')
    title('Stresses in Pipe structure')
    pause(0.01)
% frame = getframe;
% writeVideo(writerObj, frame);
end
Plot = [];
end
```


## C.1.5. 3D plot animation function (Not used)

```
function Plot = ThreeDPlotPipe(w,t)
%% 3D plot function %%
% The pipe will be plotted in 3D, but the view angle is so specified that
% only a 2D plot is visible for the pipe length.
figure('units','normalized','position',[[0}00.5 1 0.3]
Plot = 50;
for ind=1:1:length(t)
    [X,Y,Z] = cylinder (max(w(:)),Plot);
    Z = (0:0.1:(length(w(1,:))*0.1)-0.1)';
    wTemp = w';
    wTemp = wTemp (: , ind);
    wTemp2 = wTemp;
    for i = 1:Plot+1,
    Z(:,i) = (0:0.1:(length (w(1,:))*0.1)-0.1)';
    wTemp2(:,i) = wTemp;
    end
    for i = 1:length(Z(:,1)),
    X(i,:) = X(1,:);
    Y(i,:) = Y(1,:);
    end
    Y = Y + wTemp2;
    surf(Z,X,Y)
    view (0,0);
    axis([0 ((length (w(1,:))*0.1)-0.1) 4*min(w(:)) 4*max(w(:)) 4*min(w(:)) 4*max(w
        (:))])
    title(['Time = ' num2str(t(ind)) ' seconds'])
    zlabel('Displacement [meter]')
    xlabel('Length Pipe [meter]')
    %drawnow
    %z(ind) = getframe(1);
    pause(0.01)
end
end
```


## C.1.6. Model dimensions function

```
function [Dim] = ModelDimensions()
%% Reading excel document information %%
data_sheet = xlsread('Effects_pile_pipe.xlsx');
Dim.data_sheet = data_sheet;
```



```
%
```



```
%% Discretisation %%
% The discritisation is defined from the top to the bottom; as a negative
% defintion
% Thickness of the subsurface layers
d(1:9,1) = data_sheet(1:9,19);
% Layer count
nd = size(d);
% Domain of the grid in vertical direction (z-direction)
hz_tot = sum(d); % [m] total height grid
hz_grid = 0.10; % [m] grid size
% Intermediate point of grid
zIN = (0:-hz_grid:-hz_tot)'; % positive defenition intermediate grid points
nzIN = length(zIN); % count intermediate grid points
% Main grid points
zN(1:nzIN-1,1) = (zIN(1:nzIN-1,1)+zIN(2:nzIN,1))./2;
nnz = length (zN); % count of main grid points
% Determine stap size (delta z)
dzN = zN(2:end,1)-zN(1:end-1,1);
dzIN = zIN(2:end,1)-zIN (1:end-1,1);
%% Coordinates of pipe relative to pile %%
% Horizontal hart-to-hart distance pile-pipe [m]
d_hth = data_sheet(1,30); %Dim.Wave.Distance;
% Depth of the pipe, relative to the surface level [m]
depth_pipe = data_sheet (2,30);
% Radial distance from pile toe to pipe [m]
r_r = sqrt(d_hth^2+(abs(zN)-depth_pipe).^2);
% Incidental angle with the P-wave [degrees]
theta_P = atand(d_hth./ abs(zN));
%% Create Dimension model space for wave propagation model part %%
Dim.Wave.d = d; Dim.Wave.nd = nd; Dim.Wave.hz_tot = hz_tot;
Dim.Wave.hz_grid = hz_grid; Dim.Wave.zIN = zIN; Dim.Wave.nzIN = nzIN;
Dim.Wave.zN = zN; Dim.Wave.nnz = nnz; Dim.Wave.dzN = dzN;
Dim.Wave.dzIN = dzIN; Dim.Wave.depth_pipe = depth_pipe;
Dim.Wave.r_r = r_r; Dim.Wave.theta_P = theta_P; Dim.Wave.d_hth = d_hth;
```



```
% BEAM FINITE ELEMENT MODEL %
% ________________________________________________________________________
%% Defining the element length for the Beam model %%
% Length of element [m]
LengthElem = data_sheet (5,40);
```

56
${ }_{57} \% \%$ Percentage of the velocity $\% \%$
${ }_{58} \%$ The percentage of the maximum velocity (at the direct distance), use to
59 \% determine the size of the pipe domain
${ }_{60}$ PercentageVelocity $=$ data_sheet $(4,40)$;
${ }^{61}$
${ }_{62} \% \%$ Create Dimension model space for the pipe model $\% \%$
${ }_{63}$ Dim. Pipe.LengthElem $=$ LengthElem;
${ }_{64}$ Dim. Pipe. PercentageVelocity $=$ PercentageVelocity;
65 end

## C.1.7. Model parameters function

```
function [Par] = ModelParameters(Dim)
```



```
% WAVE PROPAGATION MODEL %
```



```
%% Recall model dimensions from model dimension space %%
nnz = Dim.Wave.nnz;
data_sheet = Dim.data_sheet;
hz_grid = Dim.Wave.hz_grid;
depth_pipe = Dim.Wave.depth_pipe;
zN = Dim.Wave.zN;
d = Dim.Wave.d;
hz_tot = Dim.Wave.hz_tot;
%% Sheetpile parameters
% Surface area sheetpile [mm2]
A_P_pile = data_sheet(1,40);
% Thickness sheet pile [mm]
t_P_flens = data_sheet (2,40);
% Length of one sheet pile element (converted to rectangular shape) [mm
b_P_pile = A_P_pile/t_P_flens;
% Half of the length of one sheet pile element [m]
a = (b_P_pile/1000)/2;
% Half of the thickness of sheet pile [m]
b = (t_P_flens/1000)/2;
% Unit weight pile/steel [kg/m3]
rho_pile = 7850;
% Wave speed sheet pile [m/s]
c_P_pile = 5100;
% Pile impedance [kNs/m3]
Z_P_pile = (A_P_pile/1000000)*rho_pile*c_P_pile *0.00981;
% Specific pile impedance [kNs/m]
z_P_pile = rho_pile*c_P_pile*0.00981;
%% Vibrator parameters %%
% Hammer efficiency factor [-]
F_P = 1.0;
% Driving frequency vibrator [Hz]
f = data_sheet (2,35);
% Impact time of the hammer [s]
t_vibro = l/f;
% Excentrical moment vibrator [kg m]
m_ex = data_sheet (1,35);
% Maximal driving force vibrator [N]
F_vibro = data_sheet (4,35)*1000;
% Force dead weight vibrator [N]
F_0 = data_sheet (3,35)*9.81;
% Pressure at the sheet pile tip as a result of the force F_0 [Pa]
Pressure_F_0 = (F_0/A_P_pile) *1000000;
% Pressure at the sheet pile tip as a result of the force F_vibro [Pa]
Pressure_F_vibro = (F_vibro/A_P_pile)*1000000;
%% Creating model parameter vectors %%
% Fill in the parameter value of the first ground layer:
```

```
% Wave speed P-Waves [m/s]
c_P(1:nnz,1) = data_sheet(1,20);
% Wave speed S-Waves [m/s]
c_S(1:nnz,1) = data_sheet(1,21);
% Poisson's ratio [-]
nu(1:nnz,1) = data_sheet(1,22);
% Reduction factor for disturbance or remolding [-]
R_R(1:nnz,1) = data_sheet(1,23);
% Unit weight soil [kg/m3]
rho_soil(1:nnz,1) = data_sheet(1,24);
% Elasticity modulus [N/m2]
E(1:nnz,1) = data_sheet (1,25)*1000;
% Fill in the other values (over depth) of the vector by over writing the
% values of the previous ground layer:
UpdateVector = data_sheet (1,19);
Update = data_sheet (1,19);
for i = 1:1:8,
    if hz_tot > Update
        c_P((((abs(data_sheet(i,19))+hz_grid)/hz_grid)):end,1) = data_sheet(i+1,20);
        c_S((((abs(data_sheet(i,19))+hz_grid)/hz_grid)):end,1) = data_sheet(i+1,21);
        nu((((abs(data_sheet(i,19))+hz_grid)/hz_grid)):end,1) = data_sheet(i+1,22);
        R_R((((abs(data_sheet (i,19))+hz_grid)/hz_grid)):end,1) = data_sheet(i+1,23);
        rho_soil((((abs(data_sheet(i,19))+hz_grid)/hz_grid)):end,1) = data_sheet(i
                +1,24);
        E((((abs(data_sheet(i,19))+hz_grid)/hz_grid)):end,1) = data_sheet(i+1,25)
            *1000;
        else
        end
        % Make condition for if statement (the current depth of the layer)
        UpdateVector = [UpdateVector; data_sheet(i+1,19)];
        Update = sum(UpdateVector);
end
%% Shear modulus soil [N/m2]
G = E./(1+nu);
%% Determination of the critical distance %%
% Critical angle [rad] %%
theta_crit = asin(c_S(1:d(1)/hz_grid)./c_P(1:d(1)/hz_grid));
% Tangent theta critical used for calculation only [rad]
tan_theta_crit = tan(theta_crit);
% Critical horizontal distance from the pile [m]
d_crit = tan_theta_crit.*abs(zN(1:d(1)/hz_grid));
for i = 2:1:length(d),
    if d(i) == 0,
    else
        % Create temporary (empty) vectors
        theta_crit_temp = zeros(d(i)/hz_grid,1);
        tan_theta_crit_temp = zeros(d(i)/hz_grid,1);
        d_crit_temp = zeros(d(i)/hz_grid,l);
        % Determine for a certain ground layer (number i) the properties
        for ii = 1:1:d(i)/hz_grid,
            theta_crit_temp(ii,1) = asin(c_S(ii+(sum(d(1:i-1))/hz_grid))/c_P(ii+(sum
                (d(1:i-1))/hz_grid)));
            tan_theta_crit_temp(ii,l) = tan(theta_crit_temp(ii,1));
```

```
            d_crit_temp(ii ,l) = tan_theta_crit_temp(ii ,l)*abs(zN(ii ,1));
        end
        % Create total properties of the entire soil by adding the new
        % vector to the total vector
        theta_crit = [theta_crit;theta_crit_temp];
        tan_theta_crit = [tan_theta_crit;tan_theta_crit_temp];
        % Add to the calculated distance vector of a particular soil layer i, the
        % last digid of the total critical distance vector
        d_crit_temp_new = d_crit_temp + d_crit(end);
        % Create total properties of the entire soil by adding the new
        % vector to the total vector
        d_crit = [d_crit;d_crit_temp_new];
    end
end
% Critical radial distance [m]
r_crit = sqrt(d_crit.^2+zN.^2);
%% Specific impedance soil %%
% P-waves [kNs/m3]
z_P = rho_soil.*c_P.*0.00981;
%% Factor "s" for P-waves %%
s = sqrt((1-2.*nu)./(2.*(1-nu)));
% Incedental angle of the S-wave [degrees]
theta_S = asind(sind (Dim.Wave.theta_P).*s);
% Incidental angle of the R-wave with critical distance [degrees]
theta_R_P = atand(d_crit./abs(zN));
theta_R_S = asind(sind(theta_R_P).*s);
%% Material absorption parameter %%
% Material damping coeffiecient [%/100]
D = data_sheet (3,40)/100;
% Calculate the material absorpotion parameter for all soil layers
% individually
alpha_P_Temporary = (2.*pi().*D.*f)./c_P;
% Pre-allocation for speed algorithm
alpha_P = zeros(length(c_P),1);
% Calculate the material absorption parameter with repsect to the average
% over the height
for i = 1:1:length(c_P),
    % P-waves and R-waves
    alpha_P(i,1) = sum(alpha_P_Temporary(1:i))/i;
end
%% Sleicher's solution (for spread surface loads) for Boussinesk's algorithm
% Poisson's number [-]
m = 1./nu;
% Arbitrary calculation number related to the Elasticity modulus [N/m2]
C = (m.^2.*E)./ (m.^2-1);
% Displacement sheet pile toe by the load F_0 [m]
w_F_0 = ((2.*Pressure_F_0)./(pi ().*C)).*(log(sqrt (a.^2+b.^2)+a) .*b+\ldots
    log(sqrt(a.^2+b.^2)+b).*a-log(sqrt(a.^2+b.^2)-a).*b-log(sqrt(a.^2+b.^2)-b).*a);
% Displacement sheet pile toe by the load F_vibro [m]
```

```
w_F_vibro = ((2.*Pressure_F_vibro)./(pi().*C)).*(log(sqrt(a.^2+b.^2)+a).*b+\ldots
    log(sqrt(a.^2+b.^2)+b).*a-log(sqrt(a.^2+b.^2)-a).*b-log(sqrt(a.^2+b.^2)-b).*a);
%% Total Energy [Joule] = [Nm] --> Nmm = Joule * 10^3
W_F_0 = w_F_0.*F_0.*1000;
W_F_vibro = w_F_vibro.*F_vibro.*1000;
W_tot = W_F_0 + W_F_vibro;
%% Calculation "Pile-soil wave transitivity" %%
% Damping factor pile toe [-]
J_c = 2.*(z_P./z_P_pile);
% Pile velocity [m/s]
v_P_pile = ((F_vibro+F_0)./1000)./Z_P_pile;
% Dynamic resistance of the pile "TOE" [kN]
R_T = R_R.*J_c.*Z_P_pile.*v_P_pile;
% Vibration transmission efficacy of the "TOE" of the pile [-]
E_T = R_T./((F_vibro+F_0)./1000);
%% Calculation "Pile wave propagation" %%
% Impact time of pile with soil [s]
t_P_pile = t_vibro;
%% Calculation "Propagation of waves through the soil" %%
% Wave length in soil (P-waves) [m]
lambda_P_soil = t_P_pile.*c_P;
% Wave speed R-waves [m/s]
c_R = ((0.87+1.12.*nu)./(1+nu)).*c_S;
% Wave length in soil (R-waves) [m]
lambda_R_soil = c_R./f;
% Determination of the material property "k"
k_s = 1./(sqrt(2.*pi().*rho_soil.*lambda_P_soil));
% Amplification factor in the vertical direction for P-waves [-]
F_v_P = 2.*((cosd (Dim.Wave.theta_P).*\operatorname{cosd}(2.* theta_S))./((s.^2.*sind (2.*Dim.Wave.
    theta_P).*...
    sind(2.*theta_S))+((\operatorname{cosd}(2.*theta_S)).^2)));
% Amplification factor in the horizontal direction for P-waves [-]
F_h_P = 2.*((cosd(Dim.Wave.theta_P).*sind (2.*theta_S))./((s.^2.*sind (2.*Dim.Wave.
    theta_P).*...
    sind(2.*theta_S)) +((\operatorname{cosd}(2.*theta_S)).^2)));
% Amplification factor in the vertical direction for R-waves [-]
F_v_R = 2.*((cosd(theta_R_P).* cosd(2.*theta_R_S))./((s.^2.*sind (2.*theta_R_P) .*...
    sind (2.*theta_R_S))+((\operatorname{cosd}(2.*theta_R_S)).^2)));
% Amplification factor in the horizontal direction for R-waves [-]
F_h_R = 2.*((cosd(theta_R_P).*sind(2.*theta_R_S))./((s.^2.*sind (2.*theta_R_P).*...
    sind (2.*theta_R_S))+((\operatorname{cosd}(2.* theta_R_S)).^2)));
%% Creating Parameter model space for wave propagation model %%
Par.Wave.c_P = c_P; Par.Wave.c_S = c_S; Par.Wave.c_R = c_R; Par.Wave.nu = nu;
Par.Wave.rho_soil = rho_soil;
Par.Wave.R_R = R_R; Par.Wave.z_P = z_P; Par.Wave.d_crit = d_crit; Par.Wave.s = s;
Par.Wave.theta_S = theta_S; Par.Wave.theta_R_P = theta_R_P; Par.Wave.theta_R_S =
        theta_R_S;
    Par.Wave.f = f; Par.Wave.t_vibro = t_vibro; Par.Wave.alpha_P = alpha_P;
Par.Wave.r_crit = r_crit; Par.Wave.F_P = F_P; Par.Wave.A_P_pile = A_P_pile;
Par.Wave.t_P_flens = t_P_flens; Par.Wave.b_P_pile = b_P_pile; Par.Wave.a = a; Par.
        Wave.b = b;
```

Par.Wave.z_P_pile = z_P_pile; Par.Wave. Z_P_pile = Z_P_pile; Par.Wave.m_ex = m_ex;
Par.Wave.F_vibro = F_vibro; Par.Wave.F_0 = F_0; Par.Wave. Pressure_F_0 = Pressure_F_0 ;

Par.Wave.Pressure_F_vibro = Pressure_F_vibro; Par.Wave.m = m; Par.Wave.E = E; Par. Wave. C = C;
Par.Wave.w_F_0 = w_F_0; Par.Wave.w_F_vibro = w_F_vibro; Par.Wave.W_F_vibro = W_F_vibro;
Par.Wave.W_F_0 = W_F_0; Par.Wave.W_tot = W_tot; Par.Wave.J_c = J_c; Par.Wave. v_P_pile = v_P_pile;
Par.Wave.R_T = R_T; Par.Wave.E_T = E_T; Par.Wave.t_P_pile = t_P_pile;
Par.Wave.lambda_P_soil = lambda_P_soil; Par.Wave.lambda_R_soil = lambda_R_soil;
Par.Wave.k_s = k_s; Par.Wave.F_v_P = F_v_P;
Par.Wave.F_v_R = F_v_R; Par.Wave.F_h_P = F_h_P; Par.Wave.F_h_R = F_h_R;
Par.Wave. $\mathrm{G}=\mathrm{G}$; Par.Wave.D = D; Par.Wave.theta_crit = theta_crit;

```
% BEAM FINITE ELEMENT MODEL %--m%--m
% __________________________________________________________________
```

\%\% Input paramters of the pipe structure \%\%
\% Young's modulus [ $\mathrm{N} / \mathrm{m}^{\wedge} 2$ ]
E_Pipe = data_sheet $(6,30) * 1000$;
\% Desity pipe material $\left[\mathrm{kg} / \mathrm{m}^{\wedge} 3\right]$
rho_Pipe $=$ data_sheet $(5,30)$;
\% Thickness wall pipe [mm]
tf_Pipe = data_sheet $(4,30)$;
\% Diameter pipe [mm]
D_Pipe = data_sheet $(3,30)$;
\% Outer diameter pipe [m]
D_out = D_Pipe/1000;
\% Inner diameter pipe [m]
D_in = D_out-(2*(tf_Pipe / 1000)) ;
\% Moment of inertia pipe $\left[\mathrm{m}^{\wedge} 4\right]$
I_Pipe $=($ pi ()$/ 64) *\left(D \_\right.$out^4 $\left.-D \_i n \wedge 4\right)$;
\% Cross-sectional area pipe $\left[\mathrm{m}^{\wedge} 2\right.$ ]
A_Pipe $=($ pi ()$/ 4) *($ D_out^2-D_in^2) ;
\% Stiffness of the soil spring, k, according to "Decoupled Winkler Spring
\% Method" [N/m/(m pipe)]
if data_sheet $(7,32)==1$
\% Elastic modulus ground next to pipe [Pa]
E_ground_pipe $=$ data_sheet $(7,30) * 1000$;
\% Poisson's ratio ground next to pipe [-]
nu_ground_pipe $=0.25$;
\% Shear modulus soil $[\mathrm{N} / \mathrm{m} 2=\mathrm{Pa}]$
G_ground_pipe = E_ground_pipe / (1+nu_ground_pipe) ;
\% [N/m/(m pipe)]
k_SoilSpring = (1.3*G_ground_pipe)/(D_out*(1-nu(depth_pipe/hz_grid)));
else
\% [ $\mathrm{N} / \mathrm{m} /(\mathrm{m}$ pipe) $]$
k_SoilSpring $=\left(1.3 * G\left(\right.\right.$ depth_pipe $/ h z \_$grid $\left.)\right) /($D_out*(1-nu(depth_pipe/hz_grid) $\left.)\right)$;
end
\%\% Time related properties pipe model \%\%
t_Step = le-4;
tEnd = data_sheet $(6,40)$;
\%\% Create Parameters model space for the pipe model \%\%
Par.Pipe.E_Pipe = E_Pipe; Par.Pipe.rho_Pipe = rho_Pipe;
272 Par.Pipe.tf_Pipe = tf_Pipe; Par.Pipe.D_pipe = D_Pipe;
${ }_{273}$ Par. Pipe.I_Pipe = I_Pipe; Par.Pipe.A_Pipe = A_Pipe;
${ }_{274}$ Par.Pipe.tEnd = tEnd; Par.Pipe.D_out = D_out;
275 Par.Pipe.k_SoilSpring = k_SoilSpring; Par.Pipe.t_Step = t_Step;
${ }_{276}$ end

## C.1.8. Rayleigh-wave correction factor

```
function [W_correction] = R_WaveDepthCorrection(Dim,Par)
%% Rayleigh-wave correction factor %%
% The fomulation is according to Das and Ramana
% Take average value of the soil parameters from surface level to pipe
% structure depth
c_R_average = mean(Par.Wave.c_R(1:Dim.Wave.depth_pipe/0.1));
c_S_average = mean(Par.Wave.c_S(1:Dim.Wave.depth_pipe/0.1));
nu_average = mean(Par.Wave.nu(1:Dim.Wave.depth_pipe/0.1));
lambda_R_soil_average = mean(Par.Wave.lambda_R_soil(1:Dim.Wave.depth_pipe/0.1));
% Calculate correction factor R-waves
V = sqrt(c_R_average^2/c_S_average^2);
alpha_corr = sqrt((1-2*nu_average)/(2-2*nu_average));
f_corr = (2*pi)/lambda_R_soil_average;
q2_f2 = 1 - alpha_corr^2*V^2;
s2_f2 = 1 - V^2;
q_f = sqrt(q2_f2);
s_f = sqrt(s2_f2);
W_correction_0 = -exp(-q_f*f_corr *0) +(2/(s2_f2+1))*...
    exp(-s_f*f_corr*0);
W_correction_temp = -exp(-q_f*f_corr *Dim.Wave.depth_pipe)+(2/(s2_f2+1)) *...
    exp(-s_f*f_corr *Dim.Wave.depth_pipe);
W_correction = W_correction_temp/W_correction_0;
% z = 0:0.001:5;
% for i = l:length(z),
% W_correction_temp2(i) = -exp(-q_f*f_corr*z(i))+(2/(s2_f2+1))*...
% exp(-s_f*f_corr*Z(i));
% end
% W_correction_temp3 = W_correction_temp./W_correction_temp2(1);
% figure(1)
% plot(W_correction_temp3,-z)
end
```


## C.1.9. Force calculation function

```
function [F_Sin,Dim] = TimeAndPositionDependentForce(Dim, Par)
% PURPOSE : This is a subprogram to obtain the forces according to the wave
% propagation model for all the elements
%% Initialize the parameters needed for the function file %%
Distance = Dim.Wave.d_hth;
PercentageVelocity = Dim.Pipe.PercentageVelocity;
d_hth = Dim.Wave.d_hth;
depth_pipe = Dim.Wave.depth_pipe;
hz_grid = Dim.Wave.hz_grid;
c_P = Par.Wave.c_P;
m = Par.Wave.m;
E = Par.Wave.E;
D_out = Par.Pipe.D_out;
% End of the simulation time
tEnd = Par.Pipe.tEnd;
% Frequency of excitation
f = Par.Wave.f;
% Shear wave distance at the depth of the pipe [m/s]
c_P_Depth = c_P(depth_pipe/hz_grid);
% Predefined length of element for pipe model [m]
LengthElem = Dim.Pipe.LengthElem;
%% The pistion of outer boundaries of the pipe + travel time wave %%
% The outer boundaries of the pipe are based on certain percentage of
% the wave velocity (determined by the user in the excel file) that is
% chosen at the boundaries and than itteratively back-calculated to the
% corresponding distance
% [!!!Keep in mind that the distance can be taken as the horizontal
% distance since the maximum value is at the same depth as the pipe!!!!]
% Variable functions are made for the wavepropagation model to be able to
% change the distance chosen (as variable input parameter for the model)
[v_max] = WavePropagationModelVariable(Distance);
% The maximum particle velocity at the corresponding distance [mm/s]
v_S_MaxInitial = max(v_max);
% The maximum particle velocity at the direct distance from the pile to
% the pipe [mm/s]
v_S_MaxElement = v_S_MaxInitial;
v_Force = v_S_MaxElement;
IncreaseDistance = 0;
% The travel time of the wave over the corresponding distance [s]
Time_WaveTravel = Distance/c_P_Depth;
h = waitbar(0,'Calculating the wave velocities at the elements');
Total = v_S_MaxElement-(v_S_MaxInitial*(PercentageVelocity/100));
while v_S_MaxElement > v_S_MaxInitial*(PercentageVelocity/100)
    TotalTemp = v_S_MaxElement-(v_S_MaxInitial*(PercentageVelocity/100));
    waitbar((Total-TotalTemp)/Total)
    IncreaseDistance = IncreaseDistance + LengthElem;
    Distance = sqrt(d_hth^2+IncreaseDistance^2);
    [v_max] = WavePropagationModelVariable(Distance);
    v_S_MaxElement = max(v_max);
```

Geert Reuver 4226178

```
    v_Force = [v_Force;v_S_MaxElement];
    t_temp = Distance/c_P_Depth;
    Time_WaveTravel = [Time_WaveTravel;t_temp];
end
close(h)
%% Defining the elements and nodes of the Beam model %%
% Predefined length of the pipe [m]
LengthPipe = 2*IncreaseDistance + LengthElem;
% Position nodes [m]
PosX = (0:LengthElem:LengthPipe+LengthElem);
% Number of nodes [-]
NumNode = length (PosX);
% Number of elements (using the 64-bit integer function) [-]
NumElem = int64 (NumNode-1);
% Size of the global matrices based on 4 x 4 local matrices
SizeGlobalMatrix = zeros(2*NumElem+2,2*NumElem+2);
%% Total velocity and total TravelTime vector with respect to elements %%
Temp = 1:1:length(v_Force);
Even = Temp(mod(Temp,2)==0);
if Even(end) == NumElem/2
    v_Force_flip = flipud(v_Force);
    v_Force = [v_Force_flip;v_Force];
    Time_WaveTravel_flip = flipud(Time_WaveTravel);
    Time_WaveTravel = [Time_WaveTravel_flip;Time_WaveTravel];
    clear v_Force_flip Time_WaveTravel_flip
else
    v_Force_flip = flipud(v_Force);
    v_Force_flip (end) = [];
    v_Force = [v_Force_flip;v_Force];
    Time_WaveTravel_flip = flipud(Time_WaveTravel);
    Time_WaveTravel_flip(end) = [];
    Time_WaveTravel = [Time_WaveTravel_flip;Time_WaveTravel];
    clear v_Force_flip Time_WaveTravel_flip
end
%% Create velocity + Force vector/Matrix with respect to shifted time %%
% The maximum time shift depends on the maximum travel time for a wave
t_Step = Par.Pipe.t_Step;
% Calculate what the roundup number of the Time_WaveTravel vector need to
% be determined
t_StepTemp = t_Step;
i = 0;
while t_StepTemp<1
    t_StepTemp = t_StepTemp*10;
    i = i + l;
end
Time_WaveTravel = round(Time_WaveTravel,i);
Time_ShiftedMax = max(Time_WaveTravel);
% Poisson's number [-]
m = m(depth_pipe/hz_grid);
% Stiffness of the present soil [N/m2 = Pa]
E = E(depth_pipe/hz_grid);
```

```
% Arbitrary calculation number related to the Elasticity modulus [N/m2]
C = (m^2*E)/(m^2-1);
% a and b for Sleichter's solution [m]
a = LengthElem;
b = 1;
% Sleicher's solution log part
log_Part = (log(sqrt(a^2+b^2)+a)*b+log(sqrt(a^2+b^2)+b)*a-\ldots
    log(sqrt(a^2+b^2)-a)*b-log(sqrt (a^2+b^2)-b) *a);
% Make empty matrices for the loop
v_Force_Sin = zeros(NumElem,round((tEnd+Time_ShiftedMax+t_Step)/t_Step));
w_Sin = zeros(NumElem,round((tEnd+Time_ShiftedMax+t_Step)/t_Step));
P_Sin = zeros(NumElem,round((tEnd+Time_ShiftedMax+t_Step)/t_Step));
F_Sin = zeros(NumElem,round((tEnd+Time_ShiftedMax+t_Step)/t_Step));
h2 = waitbar(0,'Calculating the time-related Force vector F');
NumElemTemp = round(LengthPipe/LengthElem);
i = 0;
for ii = 1:NumElem
    i = i + l;
    waitbar(i /NumElemTemp)
    TimeTemp = 0:t_Step:tEnd+Time_WaveTravel(ii);
    % Velocity [m/s]
    v_Force_Sin_Temp = v_Force(ii)*sin (2*pi () *TimeTemp*f)*le-3;
    v_Force_Sin(ii ,(round(Time_WaveTravel(ii)/t_Step)):length(v_Force_Sin_Temp) +...
                (Time_WaveTravel(ii)/t_Step)-1) = v_Force_Sin_Temp;
    % Displacement [m]
    w_Sin_Temp = cumtrapz(TimeTemp,abs(v_Force_Sin_Temp));
    for iii = 1:10,
                Factor = round(iii/f/t_Step);
                SlopeAngle(iii)=w_Sin_Temp(Factor)/(iii / f);
    end
    SlopeAngle = mean(SlopeAngle);
    LineFit = SlopeAngle *TimeTemp;
    w_Sin_Temp = w_Sin_Temp - LineFit;
    w_Sin(ii ,(round(Time_WaveTravel(ii)/t_Step)): length(w_Sin_Temp) +...
        (Time_WaveTravel(ii)/t_Step)-1) = w_Sin_Temp;
    % Pressure (Pa)
    P_Sin_Temp = (w_Sin_Temp.*pi().*C)./(2.*log_Part);
    P_Sin(ii ,(round(Time_WaveTravel(ii)/t_Step)) : length (P_Sin_Temp) +...
        (Time_WaveTravel(ii)/t_Step)-1) = P_Sin_Temp;
    % Force (N)
    F_Sin_Temp = P_Sin_Temp.*pi().*0.5.*D_out;
    % Application of the block function for a smooth distribution of the
    % first few cycles on the pipe
    if Dim.data_sheet (7,40) == 1
                Block_function = ones(1,length(TimeTemp));
                Block_function(1,1:(3/2)*int64(round((1/f)/t_Step))) = (1/((3/2)*...
                    (1/f)))*TimeTemp(1:(3/2)*int64 (round((1/f)/t_Step)));
            F_Sin_Temp = F_Sin_Temp.* Block_function;
        else
        end
        F_Sin(ii ,(round(Time_WaveTravel(ii)/t_Step)): length(F_Sin_Temp) +...
        (Time_WaveTravel(ii)/t_Step)-1) = F_Sin_Temp';
end
```

```
close(h2)
% Remove the zeros till the first value
v_Force_Sin(:,l:(min(Time_WaveTravel)/t_Step)-1) = [];
v_Force_Sin(:,tEnd/t_Step +2:end) = [];
w_Sin(:,l:(min(Time_WaveTravel)/t_Step)-1) = [];
w_Sin(:,tEnd/t_Step+2:end) = [];
P_Sin(:,1:(min(Time_WaveTravel)/t_Step)-1) = [];
P_Sin(:,tEnd/t_Step +2:end) = [];
F_Sin(:,l:(min(Time_WaveTravel)/t_Step)-1) = [];
F_Sin(:,tEnd/t_Step+2:end) = [];
%% Create Dimension model space for the pipe model %%
Dim.Pipe.LengthPipe = LengthPipe; Dim.Pipe.LengthElem = LengthElem;
Dim.Pipe.PosX = PosX; Dim.Pipe.NumNode = NumNode;
Dim.Pipe.NumElem = NumElem; Dim.Pipe.SizeGlobalMatrix = SizeGlobalMatrix;
Dim.Pipe.t_Step = t_Step; Dim.Pipe.w_Sin = w_Sin;
Dim.Pipe.v_Force_Sin = v_Force_Sin;
end
```


## C.1.10. Variable dimensions function

```
function [DimVar] = ModelDimensionsVariable(Distance)
%% Reading excel document information %%
data_sheet = xlsread('Effects_pile_pipe.xlsx');
DimVar.data_sheet = data_sheet;
%% _____________________________________________________________________________
% WAVE PROPAGATION MODEL %
% ____________________________________________________________________________
%% Discretisation %%
% The discritisation is defined from the top to the bottom; as a negative
% defintion
% Thickness of the subsurface layers
d(1:9,1) = data_sheet(1:9,19);
% Layer count
nd = size(d);
% Domain of the grid in vertical direction (z-direction)
hz_tot = sum(d); % [m] total height grid
hz_grid = 0.10; % [m] grid size
% Intermediate point of grid
zIN = (0:-hz_grid:-hz_tot)'; % positive defenition intermediate grid points
nzIN = length(zIN); % count intermediate grid points
% Main grid points
zN(1:nzIN-1,1) = (zIN(1:nzIN-1,1)+zIN(2:nzIN,1))./2;
nnz = length (zN); % count of main grid points
% Determine stap size (delta z)
dzN = zN(2:end,1) -zN(1:end-1,1);
dzIN = zIN (2:end,1)-zIN (1:end-1,1);
%% Coordinates of pipe relative to pile %%
% Horizontal hart-to-hart distance pile-pipe [m]
d_hth = Distance;
% Depth of the pipe, relative to the surface level [m]
depth_pipe = data_sheet (2,30);
% Radial distance from pile toe to pipe [m]
r_r = sqrt(d_hth^2+(abs(zN)-depth_pipe).^2);
% Incidental angle with the P-wave [degrees]
theta_P = atand(d_hth./abs(zN));
%% Create Dimension model space for wave propagation model part %%
DimVar.Wave.d = d; DimVar.Wave.nd = nd; DimVar.Wave.hz_tot = hz_tot;
DimVar.Wave.hz_grid = hz_grid; DimVar.Wave.zIN = zIN; DimVar.Wave.nzIN = nzIN;
DimVar.Wave.zN = zN; DimVar.Wave.nnz = nnz; DimVar.Wave.dzN = dzN;
DimVar.Wave.dzIN = dzIN; DimVar.Wave.depth_pipe = depth_pipe;
DimVar.Wave.r_r = r_r; DimVar.Wave.theta_P = theta_P;
DimVar.Wave.d_hth = d_hth;
end
```


## C.1.11. Variable parameters function

```
function [Par] = ModelParametersVariable(Dim)
%% _______________________________________________________________________________
% WAVE PROPAGATION MODEL %
% __________________________________________________________________________
%% Recall model dimensions from model dimension space %%
nnz = Dim.Wave.nnz;
data_sheet = Dim.data_sheet;
hz_grid = Dim.Wave.hz_grid;
zN = Dim.Wave.zN;
d = Dim.Wave.d;
hz_tot = Dim.Wave.hz_tot;
%% Sheetpile parameters
% Surface area sheetpile [mm2]
A_P_pile = data_sheet(1,40);
% Thickness sheet pile [mm]
t_P_flens = data_sheet (2,40);
% Length of one sheet pile element (converted to rectangular shape) [mm]
b_P_pile = A_P_pile/t_P_flens;
% Half of the length of one sheet pile element [m]
a = (b_P_pile/1000)/2;
% Half of the thickness of sheet pile [m]
b = (t_P_flens/1000)/2;
% Unit weight pile/steel [kg/m3]
rho_pile = 7850;
% Wave speed sheet pile [m/s]
c_P_pile = 5100;
% Pile impedance [kNs/m3]
Z_P_pile = (A_P_pile/1000000) *rho_pile*c_P_pile*0.00981;
% Specific pile impedance [kNs/m]
z_P_pile = rho_pile*c_P_pile*0.00981;
%% Vibrator parameters %%
% Hammer efficiency factor [-]
F_P = 1.0;
% Driving frequency vibrator [Hz]
f = data_sheet (2,35);
% Impact time of the hammer [s]
t_vibro = l/f;
% Excentrical moment vibrator [kg m]
m_ex = data_sheet (1,35);
% Maximal driving force vibrator [N]
F_vibro = data_sheet (4,35)*1000;
% Force dead weight vibrator [N]
F_0 = data_sheet (3,35)*9.81;
% Pressure at the sheet pile tip as a result of the force F_0 [Pa]
Pressure_F_0 = (F_0/A_P_pile)*1000000;
% Pressure at the sheet pile tip as a result of the force F_vibro [Pa]
Pressure_F_vibro = (F_vibro/A_P_pile)*1000000;
%% Creating model parameter vectors %%
% Fill in the parameter value of the first ground layer:
% Wave speed P-Waves [m/s]
```

```
c_P(1:nnz,1) = data_sheet(1,20);
% Wave speed S-Waves [m/s]
c_S(1:nnz,1) = data_sheet(1,21);
% Poisson's ratio [-]
nu(1:nnz,1) = data_sheet(1,22);
% Reduction factor for disturbance or remolding [-]
R_R(1:nnz,1) = data_sheet(1,23);
% Unit weight soil [kg/m3]
rho_soil(1:nnz,1) = data_sheet(1,24);
% Elasticity modulus [N/m2]
E(1:nnz,1) = data_sheet (1,25)*1000;
% Fill in the other values (over depth) of the vector by over writing the
% values of the previous ground layer:
UpdateVector = data_sheet (1,19);
Update = data_sheet (1,19);
for i = 1:1:8,
        if hz_tot > Update
            c_P((((abs(data_sheet(i,19))+hz_grid)/hz_grid)):end,1) = data_sheet(i+1,20);
            c_S((((abs(data_sheet(i,19))+hz_grid)/hz_grid)):end,1) = data_sheet(i+1,21);
            nu((((abs(data_sheet(i,19))+hz_grid)/hz_grid)):end,1) = data_sheet(i+1,22);
            R_R((((abs(data_sheet(i,19))+hz_grid)/hz_grid)):end,1) = data_sheet(i+1,23);
            rho_soil((((abs(data_sheet(i,19))+hz_grid)/hz_grid)):end,1) = data_sheet(i
                +1,24);
            E((((abs(data_sheet(i,19))+hz_grid)/hz_grid)):end,1) = data_sheet(i+1,25)
                *1000;
        else
        end
        % Make condition for if statement (the current depth of the layer)
        UpdateVector = [UpdateVector; data_sheet(i+1,19)];
        Update = sum(UpdateVector);
end
%% Determination of the critical distance %%
% Critical angle [rad] %%
theta_crit = asin(c_S(1:d(1)/hz_grid)./c_P(1:d(1)/hz_grid));
% Tangent theta critical used for calculation only [rad]
tan_theta_crit = tan(theta_crit);
% Critical horizontal distance from the pile [m]
d_crit = tan_theta_crit.*abs(zN(1:d(1)/hz_grid));
for i = 2:1:length(d),
    if d(i) == 0,
    else
        % Create temporary (empty) vectors
        theta_crit_temp = zeros(d(i)/hz_grid,1);
        tan_theta_crit_temp = zeros(d(i)/hz_grid,1);
        d_crit_temp = zeros(d(i)/hz_grid,1);
        % Determine for a certain ground layer (number i) the properties
        for ii = 1:1:d(i)/hz_grid,
            theta_crit_temp(ii,1) = asin(c_S(ii+(sum(d(1:i-1))/hz_grid))/c_P(ii+(sum
                (d(1:i-1))/hz_grid)));
            tan_theta_crit_temp(ii,1) = tan(theta_crit_temp(ii,1));
            d_crit_temp(ii,1) = tan_theta_crit_temp(ii , 1)*abs(zN(ii ,l));
        end
        % Create total properties of the entire soil by adding the new
        % vector to the total vector
```

```
            theta_crit = [theta_crit;theta_crit_temp];
            tan_theta_crit = [tan_theta_crit;tan_theta_crit_temp];
            % Add to the calculated distance vector of a particular soil layer i, the
            % last digid of the total critical distance vector
            d_crit_temp_new = d_crit_temp + d_crit(end);
            % Create total properties of the entire soil by adding the new
            % vector to the total vector
            d_crit = [d_crit;d_crit_temp_new];
    end
end
% Critical radial distance [m]
r_crit = sqrt(d_crit.^2+zN.^2);
%% Specific impedance soil %%
% P-waves [kNs/m3]
z_P = rho_soil.*c_P.*0.00981;
%% Factor "s" for P-waves %%
s = sqrt((1-2.*nu)./(2.*(1-nu)));
% Incedental angle of the S-wave [degrees]
theta_S = asind(sind(Dim.Wave.theta_P).*s);
% Incidental angle of the R-wave with critical distance [degrees]
theta_R_P = atand(d_crit./abs(zN));
theta_R_S = asind(sind(theta_R_P).*s);
%% Material absorption parameter %%
% Material damping coeffiecient [%/100]
D = data_sheet (3,40)/100;
% Calculate the material absorpotion parameter for all soil layers
% individually
alpha_P_Temporary = (2.*pi().*D.*f)./ c_P;
% Pre-allocation for speed algorithm
alpha_P = zeros(length(c_P),1);
% Calculate the material absorption parameter with repsect to the average
% over the height
for i = 1:1:length(c_P),
    % P-waves and R-waves
    alpha_P(i,1) = sum(alpha_P_Temporary(1:i))/i;
end
%% Sleicher's solution (for spread surface loads) for Boussinesk's algorithm
% Poisson's number [-]
m = 1./nu;
% Arbitrary calculation number related to the Elasticity modulus [N/m2]
C = (m.^2.*E)./ (m.^2-1);
% Displacement sheet pile toe by the load F_0 [m]
w_F_0 = ((2.*Pressure_F_0)./(pi ().*C)).*(log(sqrt(a.^2+b.^2)+a).*b+...
    log(sqrt(a.^2+b.^2)+b).*a-log(sqrt(a.^2+b.^2)-a).*b-log(sqrt(a.^2+b.^2)-b).*a);
% Displacement sheet pile toe by the load F_vibro [m]
w_F_vibro = ((2.*Pressure_F_vibro)./(pi ().*C)).*(log(sqrt (a.^2+b.^2)+a).*b+\ldots
    log(sqrt(a.^2+b.^2)+b).*a-log(sqrt(a.^2+b.^2)-a).*b-log(sqrt(a.^2+b.^2)-b).*a);
%% Total Energy [Joule] = [Nm] --> Nmm = Joule * 10^3
```

```
W_F_0 = w_F_0.*F_0.*1000;
```

W_F_vibro = w_F_vibro .*F_vibro .*1000;
W_tot = W_F_0 + W_F_vibro;
\%\% Calculation "Pile-soil wave transitivity" \%\%
\% Damping factor pile toe [-]
J_c = 2.*(z_P./z_P_pile);
\% Pile velocity [m/s]
v_P_pile = ((F_vibro+F_0)./1000)./Z_P_pile;
\% Dynamic resistance of the pile "TOE" [kN]
R_T = R_R.*J_c.*Z_P_pile.*v_P_pile;
\% Vibration transmission efficacy of the "TOE" of the pile [-]
E_T = R_T./((F_vibro+F_0)./1000);
\%\% Calculation "Pile wave propagation" \%\%
\% Impact time of pile with soil [s]
t_P_pile = t_vibro;
\%\% Calculation "Propagation of waves through the soil" \%\%
\% Wave length in soil (P-waves) [m]
lambda_P_soil = t_P_pile.*c_P;
\% Determination of the material property "k"
k_s = 1./(sqrt(2.*pi().*rho_soil.*lambda_P_soil));
\% Amplification factor in the vertical direction for P-waves [-]
F_v_P $=2 . *((\operatorname{cosd}(\operatorname{Dim}$. Wave. theta_P $) . * \operatorname{cosd}(2 . *$ theta_S $)) . /((s . \wedge 2 . * \operatorname{sind}(2 . * \operatorname{Dim}$. Wave.
theta_P).*...
sind (2.*theta_S)) +((cosd (2.*theta_S)).^2)));
\% Amplification factor in the horizontal direction for P-waves [-]
F_h_P $=2 . *((\operatorname{cosd}(\operatorname{Dim}$. Wave. theta_P $) . * \operatorname{sind}(2 . *$ theta_S $)) . /((s . \wedge 2 . * \operatorname{sind}(2 . * \operatorname{Dim}$. Wave.
theta_P).*...
sind (2.* theta_S)) +((cosd (2.*theta_S)).^2)));
\% Amplification factor in the vertical direction for R-waves [-]
$\mathrm{F}_{-} \mathrm{v} \_\mathrm{R}=2 . *((\cos d($ theta_R_P $) . * \operatorname{cosd}(2 . *$ theta_R_S $)) . /((\operatorname{s.\wedge } 2 . * \operatorname{sind}(2 . *$ theta_R_P) .*...
sind $(2 . *$ theta_R_S $))+((\operatorname{cosd}(2 . *$ theta_R_S $)) . \wedge 2)))$;
\% Amplification factor in the horizontal direction for $R$-waves [-]
F_h_R $=2 . *((\operatorname{cosd}($ theta_R_P).$*$ sind $(2 . *$ theta_R_S $)) . /((s . \wedge 2 . * \operatorname{sind}(2 . *$ theta_R_P) .*...
sind $(2 . *$ theta_R_S $))+((\operatorname{cosd}(2 . *$ theta_R_S $)) . \wedge 2)))$;
\%\% Creating Parameter model space for wave propagation model \%\%
Par.Wave.c_P = c_P; Par.Wave.c_S = c_S; Par.Wave.nu = nu; Par.Wave.rho_soil =
rho_soil;
Par.Wave.R_R = R_R; Par.Wave.z_P = z_P; Par.Wave.d_crit = d_crit; Par.Wave.s = s;
Par.Wave.theta_S = theta_S; Par.Wave.theta_R_P = theta_R_P; Par.Wave.theta_R_S =
theta_R_S;
Par.Wave. $\mathrm{f}=\mathrm{f}$; Par.Wave.t_vibro = t_vibro; Par.Wave. alpha_P = alpha_P;
Par.Wave. r_crit = r_crit; Par.Wave.F_P = F_P; Par.Wave.A_P_pile = A_P_pile;
Par.Wave.t_P_flens = t_P_flens; Par.Wave.b_P_pile = b_P_pile; Par.Wave.a = a; Par.
Wave. $\mathrm{b}=\mathrm{b}$;
Par.Wave. z_P_pile = z_P_pile; Par.Wave. Z_P_pile = Z_P_pile; Par.Wave.m_ex = m_ex;
Par.Wave. F_vibro = F_vibro; Par.Wave.F_0 = F_0; Par.Wave. Pressure_F_0 = Pressure_F_0
;
Par.Wave. Pressure_F_vibro = Pressure_F_vibro; Par.Wave.m = m; Par.Wave.E = E; Par.
Wave. C = C;
Par.Wave.w_F_0 = w_F_0; Par.Wave.w_F_vibro = w_F_vibro; Par.Wave.W_F_vibro =
W_F_vibro;

Par.Wave.W_F_0 = W_F_0; Par.Wave.W_tot = W_tot; Par.Wave.J_c = J_c ; Par.Wave. v_P_pile = v_P_pile;
Par.Wave.R_T = R_T; Par.Wave.E_T = E_T; Par.Wave.t_P_pile = t_P_pile;
Par.Wave.lambda_P_soil = lambda_P_soil; Par.Wave.k_s = k_s; Par.Wave.F_v_P = F_v_P; Par.Wave.F_v_R = F_v_R; Par.Wave.F_h_P = F_h_P; Par.Wave.F_h_R = F_h_R; end

## C.1.12. Variable WPM function

```
function [v_max] = WavePropagationModelVariable(Distance)
[DimVar] = ModelDimensionsVariable(Distance);
[ParVar] = ModelParametersVariable(DimVar);
%% Velocity at receiving point %%
if DimVar.Wave.depth_pipe ==0
    % Spherical wave velocity in vertical direction (P-wave) with F_v_P [mm/s]
    v_S_v_without = ParVar.Wave.k_s.*ParVar.Wave.F_v_P.*ParVar.Wave.E_T.*...
            ((sqrt(ParVar.Wave.F_P.*ParVar.Wave.W_tot)) .* cosd(DimVar.Wave.theta_P) .*...
            1e3./(DimVar.Wave.r_r));
    % Spherical wave velocity in horizontal direction (P-wave) with F_v_P [mm/s]
    v_S_h_without = ParVar.Wave.k_s .*ParVar.Wave.F_v_P.*ParVar.Wave.E_T.*...
            ((sqrt(ParVar.Wave.F_P.*ParVar.Wave.W_tot)) .* cosd(90-DimVar.Wave.theta_P)
                *...
            le3./(DimVar.Wave.r_r));
    % Spherical wave velocity in direct direction (P-wave) with F_v_P [mm/s]
    v_S_without = ParVar.Wave.k_s.*ParVar.Wave.F_v_P.*ParVar.Wave.E_T.*...
            ((sqrt(ParVar.Wave.F_P.*ParVar.Wave.W_tot)).*1e3./(DimVar.Wave.r_r));
else
    % Spherical wave velocity in vertical direction (P-wave) [mm/s]
    v_S_v_without = ParVar.Wave.k_s.*ParVar.Wave.E_T.*((sqrt(ParVar.Wave.F_P.*...
            ParVar.Wave.W_tot)).*cosd(DimVar.Wave.theta_P).*1e3./(DimVar.Wave.r_r));
        % Spherical wave velocity in horzontal direction (P-wave) [mm/s]
        v_S_h_without = ParVar.Wave.k_s.*ParVar.Wave.E_T.*((sqrt(ParVar.Wave.F_P.*...
            ParVar.Wave.W_tot)).*cosd(90-DimVar.Wave.theta_P).*1 e3./(DimVar.Wave.r_r));
    % Spherical wave velocity in direct direction (P-wave) [mm/s]
        v_S_without = ParVar.Wave.k_s.*ParVar.Wave.E_T.*((sqrt(ParVar.Wave.F_P.*...
            ParVar.Wave.W_tot)).*\operatorname{cosd}(90-DimVar.Wave.theta_P).*1 e3./(DimVar.Wave.r_r));
end
% Geometric and material damping respectively damping
v_S_v = v_S_v_without.*(1./DimVar.Wave.r_r).*exp(-ParVar.Wave.alpha_P.*DimVar.Wave.
    r_r);
v_S_h = v_S_h_without.*(1./DimVar.Wave.r_r).*exp(-ParVar.Wave.alpha_P.*DimVar.Wave.
    r_r);
v_S = v_S_without.*(1./DimVar.Wave.r_r).*exp(-ParVar.Wave.alpha_P.*DimVar.Wave.r_r);
%% Rayleigh-waves %%
% R-wave correction factor
Par.Wave.W_correction = W_correction;
% Spherical wave (P-wave) velocity in vertical direction without damping
% traveling along the critical radius distance [mm/s]
v_R_P_without = ParVar.Wave.k_s.*ParVar.Wave.F_v_R.*ParVar.Wave.E_T.*...
        (sqrt(ParVar.Wave.F_P.*ParVar.Wave.W_tot)./ ParVar.Wave.r_crit) .*...
    cosd(DimVar.Wave.theta_P) .*1 e3;
% Damped spherical wave along the critical radius distance [mm/s]
v_R_P = v_R_P_without.*(0.5./ ParVar.Wave.r_crit).*exp(-ParVar.Wave.alpha_P.*...
        ParVar.Wave.r_crit);
% Geometric and material damping respectively damping. As the R-wave arises
% at the surface, the material damping will be according to the soil layer
% at surface level.
v_R = W_correction.*v_R_P.*(ParVar.Wave.d_crit./DimVar.Wave.d_hth).^0.5.*...
    exp(-ParVar.Wave.alpha_P(1) .*(DimVar.Wave.d_hth-ParVar.Wave.d_crit));
```

${ }_{53} \% \%$ Maximum value of P-waves and R-waves for the pipe depth [ $\mathrm{mm} / \mathrm{s}$ ] \%\%
${ }_{54}$ v_max $=\max \left(\max \left(\mathrm{v} \_R(\right.\right.$ DimVar.Wave.depth_pipe /0.1:end) ) ,max(v_S (DimVar. Wave. depth_pipe /0.1:end)) ;
55 end

## C.1.13. Element damping matrix $\left[C_{E l e m}\right]$ function

```
function [C,Par] = DampingMatrix (M, K, Par,Dim)
%% Find the eigenfrequencies of the mass and stiffness matrices %%
% Use the matlab build in eigenvalue function to find the eigenvalues of
% the stiffness and mass-matrices
EigenValAll=eig (K,M) ;
% sorted natural angular frequencies, (squar-root is taken since the
% function ei gives omega^2 instead of omega) [rad/s]
EigenFreqRad=sort(real(sqrt(EigenValAll)));
% Delete the zero frequencies at the start of the vector
EigenFreqRad_Position = EigenFreqRad == 0;
EigenFreqRad(EigenFreqRad_Position) = [];
% sorted natural angular frequencies [Hz]
EigenFreq=EigenFreqRad / (2*pi);
%% Finde the frequency boundaries %%
% Position in the vector for the upper boundary frequency
omega_nPosition = find(EigenFreqRad > (Par.Wave.f*2*pi()));
omega_nPosition = omega_nPosition(1);
% Lower boundary frequency [rad/s]
omega_m = EigenFreqRad(1);
% Upper boundary frequency [rad/s]
omega_n = EigenFreqRad(omega_nPosition);
%% Determination of the alpha and beta parameters %%
% The Rayleigh-damping chosen is based on the frequency of the first mode
% and the first frequency bigger than the freqeuncy of excitation
% Damping ratio soil [%/100]
zeta = Dim.data_sheet (6,35);
% Alpha parameter
alpha = (2*zeta*omega_m*omega_n)/(omega_m + omega_n);
% Beta parameter
beta = (2*zeta)/(omega_m + omega_n);
%% Determination of the damping matrix [C] %%
C = alpha.*M + beta.*K;
%% Create Parameters model space for the pipe model %%
Par.Pipe.alpha = alpha; Par.Pipe.beta = beta; Par.Pipe.EigenFreq = EigenFreq;
Par.Pipe.EigenFreqRad = EigenFreqRad; Par.Pipe.omega_m = omega_m;
Par.Pipe.omega_n = omega_n;
end
```


## C.1.14. Element stiffness matrix $\left[K_{\text {Elem }}\right]$ function

```
function [KElem]=ElementMatrixK(prenode,postnode,Lb,E,I)
% PURPOSE : This is a subprogram as for Stiffness matrice
%
% K=[ 12 6*] -12 6*1
% 4*l^2 -6*1 2*1^2
% 12 -6*1
% symmetric 4*1^2 ];
%
% The length of the current element [m]
l=postnode-prenode;
% The length of the current element per unit length, lb:length_bar
lb=1/Lb;
KElem (1,1)=12 ;
KElem(2,1)=6*1 ; KElem(2,2)=4*1^2 ;
KElem(3,1)=-12 ; ; KElem(3,2)=-6*1 ; ; KElem(3,3)=12 ; ; ; 
KElem=KElem/lb^3*E*I /Lb^3;
for i=2:4
    for j=1:i-1
        KElem(j, i )=KElem(i,j );
    end
end
end
```


## C.1.15. Element stiffness matrix boundary condition $\left[K_{B C, E l e m}\right]$ function

```
function [KElem_BC]=ElementMatrixK_BC (prenode,postnode,Lb,k)
% PURPOSE : This is a subprogram as for Stiffness matrice
%
% K=[ lllll
% 4*lb^2 -6*lb 2*lb^2
% 12 -6*lb
% symmetric 4*lb^2 ];
%
% The length of the current element [m]
l=postnode-prenode;
% The length of the current element per unit length, lb:length_bar
lb=1/Lb;
KElem_BC(1,1)=12 ;
KElem_BC(2,1)=6*1 ; KElem_BC(2,2)=4*1^2 ;
KElem_BC(3,1)=-12 ; KElem_BC(3,2)=-6*1 ; KElem_BC(3,3)=12 ;
KElem_BC(4,1)=6*1 ; KElem_BC(4,2)=2*1^2 ; KElem_BC(4,3)=-6*1 ; KElem_BC(4,4)=4*1
    ^2;
KElem_BC=KElem_BC/lb^3*k/Lb^3;
for i=2:4
    for j=1:i-1
        KElem_BC(j , i )=KElem_BC(i,j) ;
    end
end
end
```


## C.1.16. Element matrix force vector $\left[F_{\text {Elem }}\right]$ function

```
function [FElem,qElem] = ElementMatrixF(prenode,postnode,Lb,q,q2)
% PURPOSE : This is a subprogram used to obtain the time dependent time
% vector
% F= q*L/2 [ 6
% L
% 6
% -L ]
%
% The length of the current element [m]
l=postnode-prenode;
% The length of the current element per unit length, lb:length_bar
lb=1/Lb;
FElem(1,:) = 6;
FElem(2,:) = 1;
FElem(3,:) = 6;
FElem(4,:) = -l;
FElem = FElem*lb / 12*q2*Lb;
qElem(1,:) = q;
qElem(2,:) = q;
qElem(3,:) = q;
qElem(4,:) = q;
end
```

```
C.1.17. Element mass matrix [ }\mp@subsup{M}{Elem}{}\mathrm{ ] function
function [MElem]=ElementMatrixM(prenode, postnode, Lb,pb,A)
% PURPOSE : This is a subprogram to obtain the local Mass matrix
%
% M=1/420[ 156 22*l 54 -13*1
% 4*l^2 13*1 -3*1^2
% 156 -22*1
% symmetric 4*1^2];
%
% The length of the current element [m]
l=postnode-prenode;
% The length of the current element per unit length, lb:length_bar
lb=l/Lb;
MElem(1,1)=156 ;
MElem(2,1)=22*1 ; MElem(2,2)=4*1^2 ;
MElem(3,1)=54 ; MElem(3,2)=13*1 ; MElem(3,3)=156 ;
MElem (4,1)=-13*1 ; MElem (4,2)=-3*1^2; MElem (4,3)=-22*1 ; MElem (4,4)=4*1^2 ;
MElem=MElem * lb / 420 *pb *A *Lb;
for i=2:4
    for j=1:i-1
        MElem(j , i )=MElem(i , j ) ;
    end
end
end
```


## C.1.18. Global force function

1 function [GlobalForce]=GlobalForceFunction (GlobalForce, LocalForce, ii )
2 \% PURPOSE : This is a subprogram to obtain the assembly of the global
3 \% force matrix
4
${ }_{5}$ \% (1).Verify the positions where the local matrix needs to be added to the
${ }_{6}$ \% global matrix
7 $\mathrm{a}=1+2 *(\mathrm{ii}-1): 2 *(\mathrm{ii}+1)$;
8 \% (2). assembly of global matrix
9 GlobalForce(a,:) = GlobalForce(a,:) + LocalForce;
10 end

## C.1.19. Global matrix function

function [GlobalMatrix]=GlobalMatrixFunction (GlobalMatrix , LocalMatrix , ii )
\% PURPOSE : This is a subprogram to obtain the assembly of the global
\% matrices
${ }_{6} \%(1) . V e r i f y$ the positions where the local matrix needs to be added to the , \% global matrix
8 $\mathrm{a}=1+2 *(\mathrm{ii}-1): 2 *(\mathrm{ii}+1)$;
9 \% (2). assembly of global matrix
${ }_{10}$ GlobalMatrix(a,a) = GlobalMatrix(a,a) + LocalMatrix;
1 end

## C.1.20. Global matrix assembly function

```
function [Par,Dim]=GlobalMatrixAssambly (Dim, Par)
%% Time and position dependent force behavior %%
fprintf('Constructing Force Matrix F')
fprintf('\n')
% The time and position dependent behavior is determined according to the
% wave propagation model and calculated for every element
tic
[F_Sin,Dim] = TimeAndPositionDependentForce(Dim,Par);
toc
fprintf('\n')
fprintf('Constructing Matrix M, K and C')
fprintf('\n')
tic
%% Initialize the parameters required %%
rho_Pipe = Par.Pipe.rho_Pipe;
A_Pipe = Par.Pipe.A_Pipe;
E_Pipe = Par.Pipe.E_Pipe;
I_Pipe = Par.Pipe.I_Pipe;
NumElem = Dim.Pipe.NumElem;
LengthPipe = Dim.Pipe.LengthPipe;
LengthElem = Dim.Pipe.LengthElem;
PosX = Dim.Pipe.PosX;
SizeGlobalMatrix = Dim.Pipe.SizeGlobalMatrix;
k_SoilSpring = Par.Pipe.k_SoilSpring;
%% Pre-defining the Mass and Stiffness matrices %%
% Additional boundary length of pipe on both sides [m]
length_aditional_pipe = Dim.data_sheet(8,30);
% Additional elements on both sides of the pipe [-]
N_add = length_aditional_pipe/Dim.Pipe.LengthElem;
Par.Pipe.N_add = N_add;
% Predefine matrices
M = zeros(4*N_add+length(SizeGlobalMatrix),4*N_add+length(SizeGlobalMatrix));
K=M;
K_BC=M;
F=zeros(length (M),length(F_Sin(1,:)));
q=zeros(length (M),length(F_Sin(1,:)));
% Make extended force vector, F_Sin_Temp, including the additional boundary
% elements
F_Sin_Temp = [zeros(N_add,length(F_Sin(1,:))) ; ...
    F_Sin;zeros(N_add, length(F_Sin(1,:)))];
F_Sin_Temp2 = ones(size(F_Sin_Temp));
% Temporary length of the pipe structure [m]
LengthPipeTemp = LengthPipe+2*length_aditional_pipe;
% Temporary PosX array
PosX_new = 0:Dim.Pipe.LengthElem:LengthPipe +2*...
    length_aditional_pipe+Dim. Pipe.LengthElem;
Dim.Pipe.PosX_new = PosX_new;
%% Assembly of the local to global matrices M, K and F %%
h3 = waitbar (0,'Constructing the K, M and C matrices');
NumElemTemp = round(LengthPipe/LengthElem) +2*N_add;
```

```
    i = 0;
for ii = 1:NumElem+2*N_add,
    i = i + l;
    waitbar(i /NumElemTemp)
    % Stiffness matrix K
    [KElem] = ElementMatrixK(PosX_new(ii) ,PosX_new(ii +1),LengthPipeTemp,E_Pipe,
        I_Pipe);
    [KElem_BC] = ElementMatrixK_BC(PosX_new(ii),PosX_new(ii +1),LengthPipeTemp,
        k_SoilSpring);
    [K] = GlobalMatrixFunction(K,KElem,ii);
    [K_BC] = GlobalMatrixFunction(K_BC,KElem_BC,ii);
    % Mass matrix M
    [MElem] = ElementMatrixM(PosX_new(ii),PosX_new(ii +1),LengthPipeTemp,rho_Pipe,
        A_Pipe);
    [M] = GlobalMatrixFunction(M,MElem, ii );
    % Force Matrix F
    [FElem,qElem] = ElementMatrixF(PosX_new(ii),PosX_new(ii +1) ,...
        LengthPipeTemp,F_Sin_Temp(ii ,:),F_Sin_Temp2(ii ,:));
    [F] = GlobalForceFunction(F,FElem,ii);
    [q] = GlobalForceFunction(q,qElem,ii);
end
close all
%% Create the total global force vector F %%
% The values of q need to be corrected exept for the first two time
% dependent arrays and also for the last two
q(3:end-2,:) = q(3:end-2,:)/2;
F = F.*q;
%% Include boundaries by matrix reduction%%
M_BC = M;
K_BC = K + K_BC;
F_BC = F;
% Apply fixed rotation and translation at boundaries
M_BC = M_BC(3:end-2,3:end-2);
K_BC = K_BC(3:end-2,3:end-2);
F_BC = F_BC(3:end-2,:);
% Inverse mass matrix
M_INV = inv(M_BC);
%% Obtain the damping matrix C %%
% Use the damping matrix function to obtain the damping matrix including
% the boundary conditions specified above
[C_BC,Par] = DampingMatrix(M_BC,K_BC,Par ,Dim) ;
toc
fprintf('\n')
%% Store the matrices in the parameter space %%
Par. Matrix.M = M_BC;
Par.Matrix.M_INV = M_INV;
Par.Matrix.K = K_BC;
Par.Matrix.C = C_BC;
Par.Matrix.F = F_BC;
Par.Matrix.F_Sin = F_Sin;
end
```


## C.1.21. Equation of motion function

```
function dy = FunctionTotalOde(t,y,F,M_INV,K,C,TimeOde,Dim)
%% Equation of Motion %%
% The equation of motion is re-written to a system of two first order
% differential equations
t
if t == 0
    F = F(:,1);
else
    if t == TimeOde(end)
            F = F(: , end);
    else
        i = find(TimeOde<t, 1, 'last' );
        ii = find(TimeOde>t, l, 'first' );
            if ii-i ==2
                F = F(:, i+1);
            else
                FTemp = F(:,i:ii);
                % Interpolate the data set (TimeOde,F) at time t
                F = interpl([TimeOde(i) TimeOde(ii)],FTemp',t);
                F = F';
            end
        end
end
% Predefine the length of the displacement and the velocity vector
L = length(K);
% Equation Of Motion
dy = [y(L+1:end,:);M_INV*F - M_INV*C*y(L+1:end,:) - M_INV*K*y(1:L,:)];
end
```


## C.2. Additional code suggestions

## C.2.1. Runga Kutta solver (RK4) function

```
function [t,y] = RK4Method(EquationOfMotion,t,x0,v0,Par,Dim)
%% Solver function with the Fourth-oder Runga-kutta method (RK4) %%
% Predefine the matrices required
M_INV = Par.Matrix.M_INV;
K = Par.Matrix.K;
F = Par.Matrix.F;
C = Par.Matrix.C;
% Define the time step for the calculation
dt = median(diff(t)); % time step
% Define the displacement vector and the initial conditions
y = zeros(length(Dim.Pipe.SizeGlobalMatrix)*2,length(t));
y(1:length(Dim.Pipe.SizeGlobalMatrix),1) = x0;
y(length(Dim.Pipe.SizeGlobalMatrix) +1:end,1) = v0;
% Solve with the RK4-mehtod with the application of the "Equation Of
% Motion"
for i = l:length(t),
    kl = dt*feval(EquationOfMotion,t(i),y(:,i),F(:,i),M_INV,K,C,Dim);
    k2 = dt*feval(EquationOfMotion,t(i)+dt/2,y(:,i)+kl/2,F(:,i),M_INV,K,C,Dim);
    k3 = dt*feval(EquationOfMotion,t(i)+dt/2,y(:, i)+k2/2,F(:, i),M_INV,K,C,Dim);
    k4 = dt*feval(EquationOfMotion,t(i)+dt,y(:,i)+k3,F(:,i),M_INV,K,C,Dim);
    y(:,i+1) = y(:,i) + (kl + 2*k2 + 2*k3 + k4)/6;
end
end
```


## C.2.2. Stresses and strains, including Jacobian, function

```
function [Out] = StressesAndStrains(Dim,Par)
%% Calculation of the stresses and strains of the pipe %%
% The stresses are calculated with use of the rotation angles calculated by
% the matlab ODE-solver. The strains are also folowing from the matlab
% ODE-solver.
%% Localizing the matrices required for the ODE-Solver %%
F = Par.Matrix.F;
M_INV = Par.Matrix.M_INV;
K = Par.Matrix.K;
C = Par.Matrix.C;
%% Initial conditions of the FEM %%
x0 = zeros(length (K),1);
v0 = zeros(length(K),1);
y_0 = [x0;v0];
%% Localized parameters required %%
NumElem = Dim.Pipe.NumElem;
NumNode = Dim.Pipe.NumNode;
%% Simulation time vector %%
TimeOde = 0:Par.Pipe.t_Step:Par.Pipe.tEnd;
%% Solving the equations of motion with the ODE Solver %%
opts = odeset('Jacobian',@(t,y) Jacobian(t,y,F,M_INV,K,C,TimeOde,Dim) ,...
    'Stats ', 'on' ,'RelTol',1e-2,'AbsTol',1e-4);
% opts = odeset('Stats','on','RelTol',1e-4,'AbsTol',1e-6);
tic
[t,y] = ode15s(@(t,y) FunctionTotalOde(t,y,F,M_INV,K,C,TimeOde,Dim) ,TimeOde,y_0,opts
    );
toc
%% Determining the rotations and displacements of the Pipe %%
L = length(y(1,:))/2;
% Displacements Pipe Elements [m]
w = y(:,1:2:L-1);
% Rotation angles Pipe Elements [rad]
theta = y(:,2:2:L);
% Velocity Pipe Elements [m/s]
v = y(:,L+1:2:2*L-1);
% Angular speed Pipe Elements [rad/s]
omega = y (:,L+2:2:2*L);
%% Determination of the differential rotation angle %%
for ii = 1:length(w(1,:))-1,
    Deltatheta(:,ii) = theta(:,ii+1) - theta(:,ii);
end
%% Determination of the Momentum and stress on every element
% Outer diameter pipe [m]
D_out = Par.Pipe.D_out;
% Outer radius pipe [m]
R_Out = D_out/2;
% Innter radius pipe [m]
```

```
R_In = R_Out-(Par.Pipe.tf_Pipe/1000);
% Center of gravity half Pipe [m]
yz_Pipe = (4/(3*pi()))*((R_Out^3-R_In^3)/(R_Out^2-R_In^2));
% Extension of the Pipe element at the distance yz_Pipe (this extension is
% circlar chaped) [m]
ds_z = yz_Pipe.*Deltatheta;
% Strain of the Pipe element at the distance yz_Pipe [-]
epsilon_z = ds_z./Dim.Pipe.LengthElem;
% Stress of the Pipe element at the distance yz_Pipe [N/mm^2]
sigma_Pipe = Par.Pipe.E_Pipe.*epsilon_z.*1e-6;
% Momentum of the Pipe element at the distance yz_Pipe [kNm]
Momentum_Pipe = sigma_Pipe.*(Par.Pipe.A_Pipe/2) .*yz_Pipe.*1 e3;
% Maximum momentum [kNm]
Max_Momentum = max(abs(Momentum_Pipe (:) )) ;
%% Maximum stress plus position and moment in time %%
% Maximum stress in pipe [N/mm^2]
Max_sigma = max(abs(sigma_Pipe (:)));
% Position of the maximum stress in the pipe:
[num idx] = max(sigma_Pipe (:));
[time_max_stress Element_max_stress] = ind2sub(size(sigma_Pipe),idx);
% Position in time [s]
time_max_stress = time_max_stress*Par.Pipe.t_Step;
% Position on the beam from the left boundary [m]
Position_max_stress = (Element_max_stress - 1)*Dim.Pipe.LengthElem + ...
    (Dim. Pipe.LengthElem / 2);
%% Print the imporatant values to the screen %%
% Print the postion of the mamimum values on the screen
fprintf('\n\n');
fprintf('Position of the maximum stress and Moment: \n');
fprintf('x = %.2fm (Position from the left boundary) \n', Position_max_stress);
fprintf('t = %.2f s (Position in time) \n\n', time_max_stress);
fprintf('Maximum values at the position \n');
% Print maximum value momentum to screen
fprintf('M = %.2f kNm \n', round(Max_Momentum,2));
% Print maximum value stress to screen
fprintf('sigma = %.2f N/mm^2 \n\n', round(Max_sigma,2));
%% Define outcome workspace
Out.w = w; Out.theta = theta; Out.v = v; Out.omega = omega;
Out.sigma_Pipe = sigma_Pipe; Out.Momentum_Pipe = Momentum_Pipe;
Out.Max_sigma = Max_sigma; Out.Max_Momentum = Max_Momentum; Out.t = t;
end
```


## C.2.3. Jacobian ODE-solver function

```
function [J] = Jacobian(t,y,F,M_INV,K,C,TimeOde,Dim)
% Predefine the length of the displacement and the velocity vector
L = length (K);
if t == 0
    F = F(:, 1);
else
    if t == TimeOde(end)
            F = F(: , end);
    else
            i = find(TimeOde<t, 1, 'last' );
            ii = find(TimeOde>t, 1, 'first' );
            if ii-i ==2
                F = F(:, i+1);
            else
                FTemp = F(:, i: ii);
                % Interpolate the data set (TimeOde,F) at time t
                F = interpl([TimeOde(i) TimeOde(ii)],FTemp',t);
                F = F';
            end
    end
end
JTemp1 = [];
JTemp2 = [];
for iii = 1:L
    i = iii
    yy = zeros(2*L,1);
    yy(iii) = 1;
    yy(L+iii) = 1;
    % Equation Of Motion
    f = @(c) [c(2).*yy(L+l:end,:);-c(2).*M_INV*C*yy(L+l:end,:) - c(1).*M_INV*K*yy(1:
        L,:)];
    [JTemp] = jacobianest(f,[ll 1]);
    JTemp1 = [JTemp1 JTemp(:,1)];
    JTemp2 = [JTemp2 JTemp(:,2)];
end
J = [JTemp1 JTemp2];
end
```


## C.2.4. Jacobian estimation/assembly function

```
function [jac] = jacobianest(fun,x0)
% Author: John D'Errico
% e-mail: woodchips@rochester.rr.com
% Release: 1.0
% Release date: 3/6/2007
% get the length of x0 for the size of jac
nx = numel(x0);
MaxStep = 100;
StepRatio = 2;
% was a string supplied?
if ischar(fun)
    fun = str2func(fun);
end
% get fun at the center point
f0 = fun(x0);
f0 = f0 (:);
n = length(f0);
if n==0
    % empty begets empty
    jac = zeros(0,nx);
    err = jac;
    return
end
relativedelta = MaxStep*StepRatio .^(0:-1:-25);
nsteps = length(relativedelta);
% total number of derivatives we will need to take
jac = zeros(n,nx);
err = jac;
for i = 1:nx
    x0_i = x0(i);
    if x0_i ~= 0
        delta = x0_i*relativedelta;
        else
            delta = relativedelta;
    end
    % evaluate at each step, centered around x0_i
    % difference to give a second order estimate
    fdel = zeros(n,nsteps);
    for j = l:nsteps
        fdif = fun(swapelement(x0,i,x0_i + delta(j))) - ...
            fun(swapelement(x0,i,x0_i - delta(j)));
        fdel(:,j) = fdif(:);
    end
    % these are pure second order estimates of the
    % first derivative, for each trial delta.
    derest = fdel.*repmat(0.5 ./ delta,n,1);
```

Geert Reuver 4226178

```
    % The error term on these estimates has a second order
    % component, but also some 4th and 6th order terms in it.
    % Use Romberg exrapolation to improve the estimates to
    % 6th order, as well as to provide the error estimate.
    % loop here, as rombextrap coupled with the trimming
    % will get complicated otherwise.
    for j = l:n
        [der_romb,errest] = rombextrap(StepRatio,derest(j,:),[2 4]);
    % trim off 3 estimates at each end of the scale
    nest = length(der_romb);
    trim = [1:3, nest+(-2:0)];
    [der_romb,tags] = sort(der_romb);
    der_romb(trim) = [];
    tags(trim) = [];
    errest = errest(tags);
    % now pick the estimate with the lowest predicted error
    [err(j,i),ind] = min(errest);
        jac(j,i) = der_romb(ind);
    end
end
end % mainline function end
% =========================================
% sub-functions
% ========================================
function vec = swapelement(vec,ind,val)
% swaps val as element ind, into the vector vec
vec(ind) = val;
end % sub-function end
% ==============================================
% subfunction - romberg extrapolation
% =============================================
function [der_romb,errest] = rombextrap(StepRatio,der_init,rombexpon)
% do romberg extrapolation for each estimate
%
% StepRatio - Ratio decrease in step
% der_init - initial derivative estimates
% rombexpon - higher order terms to cancel using the romberg step
%
% der_romb - derivative estimates returned
% errest - error estimates
% amp - noise amplification factor due to the romberg step
srinv = 1/StepRatio;
% do nothing if no romberg terms
nexpon = length (rombexpon);
rmat = ones(nexpon +2,nexpon+1);
```

```
% two romberg terms
rmat(2,2:3) = srinv.^rombexpon;
rmat (3,2:3) = srinv.^(2*rombexpon);
rmat(4,2:3) = srinv.^(3*rombexpon);
% qr factorization used for the extrapolation as well
% as the uncertainty estimates
[qromb,rromb] = qr(rmat,0);
% the noise amplification is further amplified by the Romberg step.
% amp = cond(rromb);
% this does the extrapolation to a zero step size.
ne = length(der_init);
rhs = vec2mat(der_init ,nexpon+2,ne - (nexpon+2));
rombcoefs = rromb \(qromb'*rhs);
der_romb = rombcoefs(1,:)';
% uncertainty estimate of derivative prediction
s = sqrt(sum((rhs - rmat*rombcoefs).^2,1));
rinv = rromb\eye(nexpon+1);
covl = sum(rinv.^2,2); % 1 spare dof
errest = s'*12.7062047361747*sqrt(covl(1));
end % rombextrap
% =============================================
% subfunction - vec2mat
% =============================================
function mat = vec2mat(vec,n,m)
% forms the matrix M, such that M(i,j) = vec(i+j-1)
[i,j] = ndgrid(1:n,0:m-1);
ind = i+j;
mat = vec(ind);
if n==1
        mat = mat';
end
end % vec2mat
```

