STEADY TRANSONIC DENSE GAS FLOW PAST A TWO - DIMENSIONAL COMPRESSION/EXPANSION RAMP

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Summary The behaviour of steady transonic dense gas flow is essentially governed by two nondimensional parameters characterising the magnitude and sign of the fundamental derivative of gas dynamics (Γ) and its derivative with respect to the density at constant entropy (Λ). The resulting response to external forcing is surprisingly rich and studied in detail for the canonical problem of 2D flow past compression/expansion ramps.

MOTIVATION

The possibility that compression as well as rarefaction shocks may form in single phase vapours was envisaged first by Bethe [1]. However his calculations based on the Van der Waals equation of state indicated that the latter type of shock is possible only if the ratio c_v/R (where c_v and R denote the specific heat at constant volume and the universal gas constant respectively) is larger than about 17.5 which he considered too large to be satisfied by real fluids. This conclusion was contested by Thompson (see Thompson [6]) and coworkers who showed that this required condition for the existence of rarefaction shocks is indeed satisfied for a large number of Fluorocarbon vapours. For these vapours the ratio c_v/R far exceeds the critical value so that the required condition for the existence of rarefaction shocks is satisfied in the general neighbourhood of the thermodynamic critical point. For a detailed review see Kluwick [5]. This finding spawned a burst of theoretical studies elaborating on the unusual and often counterintuitive behaviour of flows with rarefaction shocks present. These produced both results of a fundamentally theoretical character (addressing, among other issues, questions of admissibility and existence) but also results suggesting the practical importance of rarefaction shocks including the observation that the entropy increase and thus losses resulting from weak rarefaction shocks may be much smaller than those associated with compression shocks in perfect gases.

Expectations were high that the first experimental observation of a rarefaction shock was just around the corner providing verification of the accumulating theoretical predictions. Unfortunately, however, attempts to reach this goal by means of classical shock tube experiments have so far failed. Studies analysing the reasons for this failure have been carried out by a number of research groups resulting in a much better understanding of the thermodynamic properties of possible fluids and identifying a new family of siloxanes as testing fluids (see Colonna, Guardone & Nannan [3]). Finally, and most importantly, the properties of vapour mixtures have recently been investigated by Guardone, Colonna, Casati & Rinaldi [4] allowing for the optimisation of experimental work as well as leading to new practical applications. Therefore we think that the time is ripe for a second round of experiment efforts to settle the long standing question "do rarefaction shocks exist?" but more than this to further explore the unusual behaviour of dense gas flows theoretically. The results presented here aim to contribute to our understanding of dense gas flows for a practically realisable flow setup.

OUTLINE

Analytical studies of dense gas flows have in the past mainly concentrated on 1D unsteady flows. In contrast to such flows which are of strictly hyperbolic type 2D steady flows are of mixed type, i.e. hyperbolic or elliptic depending on whether the Mach number M is larger or smaller than one. In 1D unsteady flows it is not necessary to distinguish between the properties of shock polars and characteristics as they agree automatically as long as changes of the entropy are negligible small. In contrast, for the case of 2D steady flows, however, the construction of shock polars has to be considered separately and their connection with characteristics deserves special attention. To provide insight into the more complex flow behaviour is the aim of the present investigation. In this connection we note that 2D steady flows have been treated already in Cramer and Tarkenton [2] but as the authors approach was predominantly a numerical investigation the present analysis is considered to close a "hole" in our understanding of such flows. To make things as clear as possible it is necessary to keep the geometrical complexity as simple as possible. A generic problem which satisfies this requirement is given by supersonic flow past a compression/expansion ramp. Specifically we assume that the corner of the ramp is located at the origin of a Cartesian coordinate system (x, y), that the upstream flow $\mathbf{u}_\infty = (u_\infty, 0)$ is aligned with the x - axis and the corner geometry is: $y = 0, x < 0; y = \delta a x, x > 0$. Here the parameter $a = O(1)$ while the positive parameter δ with $\delta \ll 1$ ensures that the subsequent

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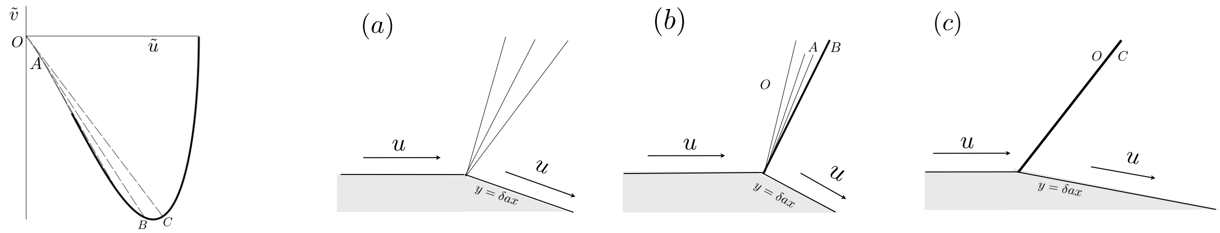

Figure 1: Fan, compound shock-fan and single shock structures for $K_1 > 0$ and $K_2 > 0$ in hodograph plane (— characteristic, ━ shock polar, - - - Rayleigh lines) and with the resulting flow geometry (— wavefronts, ━ shocks). Results are presented for $K_2 < \frac{3}{2K_1}$ and $\Gamma_\infty > 0$.

flow generated around the corner is a small perturbation of the uniform flow. Introducing suitably nondimensionalised and scaled quantities $\xi, \eta, \tilde{u}, \tilde{v}$ in place of x, y, u, v the problem under consideration can be cast into the form

$$J(\tilde{u})_\xi + \tilde{v}_\eta = 0, \quad \tilde{u}_\eta - \tilde{v}_\xi = 0. \tag{1}$$

The boundary conditions on $\eta = 0$ become

$$\tilde{v} = 0, \quad \xi \leq 0, \qquad \tilde{v} = a \operatorname{sgn}(\Gamma_\infty), \quad \xi > 0. \tag{2}$$

The function $J(\tilde{u})$ in (1) is given by

$$J(\tilde{u}) = -K_1\tilde{u} - \tilde{u}^2 + \frac{K_2}{3}\tilde{u}^3 \tag{3}$$

and measures the perturbed mass flux where K_1, K_2 are the similarity parameters

$$K_1 = \frac{M_\infty^2 - 1}{(\delta\Gamma_\infty)^{\frac{2}{3}}}, \quad K_2 = \frac{\delta^{\frac{2}{3}}\Lambda_\infty}{\Gamma_\infty^{\frac{4}{3}}}. \tag{4}$$

Here $M_\infty, \Gamma_\infty, \Lambda_\infty$ denotes the freestream Mach number and the values of the thermodynamic quantities Γ, Λ evaluated in the unperturbed flow. Also we require $K_1 > 0$ (supersonic flow) while $K_2 \gtrless 0$ depending on the signs of $\Gamma_\infty, \Lambda_\infty$. Characteristics of (1) are given by the integral curves of

$$\frac{d\tilde{v}}{d\tilde{u}} = \pm\sqrt{K_1 + 2\tilde{u} - K_2\tilde{u}^2} = \pm\sqrt{-\frac{dJ}{d\tilde{u}}} \tag{5}$$

and shock polars in the hodograph $(\tilde{u}, \tilde{v})$ plane are given by

$$\frac{[\tilde{v}]}{[\tilde{u}]} = \pm\sqrt{-\frac{[J]}{[\tilde{u}]}}. \tag{6}$$

Equations (5), (6) provide the necessary information to construct solutions of the problem posed. Equation (3) suggests that five parameter ranges of K_2: $K_2 < -\frac{1}{K_1}$, $-\frac{1}{K_1} < K_2 < -\frac{3}{4K_1}$, $-\frac{3}{4K_1} < K_2 < 0$, $K_2 = 0$, $K_2 > 0$ have to be distinguished. This reflects the fact that, in contrast to classical gasdynamics, the perturbation mass flux $J(\tilde{u})$ is not a monotonic function of the perturbed velocity if K_2 is nonzero. As an example, Figure 1 displays results for $K_2 > 0$. In this parameter range shock solutions exist which lead to a reduction of the velocity and give qualitatively similar results to those observed in perfect gases. These shocks are not included here. In contrast to the classical case however an additional branch of the shock polar is seen to exist for $\tilde{u} > 0$ and to connect with the characteristic describing accelerated flows. Increasing values of the downstream velocity (and associated with a non-monotonic variation of the parameter a then causes a transition from a centered wave fan (a) to a wave fan terminated by a sonic rarefaction shock (b) and finally to a single rarefaction shock (c).

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