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Hopf-Takens-Bogdanov interaction for a fluid-conveying tube

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We investigate the dynamics after loss of stability of the downhanging configuration of a fluid conveying tube with a small end mass and an elastic support.

By varying the fluid flow rate and the stiffness and location of the elastic support, different degenerate bifurcation scenarios can be observed. In this article we investigate the bifurcating solution branches of the codimension 3 interaction between a Hopf bifurcation and a Bogdanov-Takens bifurcation.

A complete discussion of the primary and secondary solution branches was already given by W. F. Langford and K. Zhan. After reducing the system to the three-dimensional Normal Form equations we apply a numerical continuation procedure to locate the expected higher order bifurcation branches and detect more complicated dynamics, like Shilnikov orbits.

1 Bifurcation Equations for the Hopf-Bogdanov-Takens Mode Interaction

We investigate the dynamics close to the so-called 0:1-resonance: At a certain parameter constellation simultaneously a Bogdanov-Takens bifurcation with a non-semisimple pair of zero eigenvalues and a pair of purely imaginary eigenvalues occurs. This scenario can be observed at the loss of stability of a down-hanging fluid-conveying tube, which is clamped at the upper end and elastically supported at the relative position $\xi\ell$. A corresponding stability chart in the (c, ϱ) parameter plane,

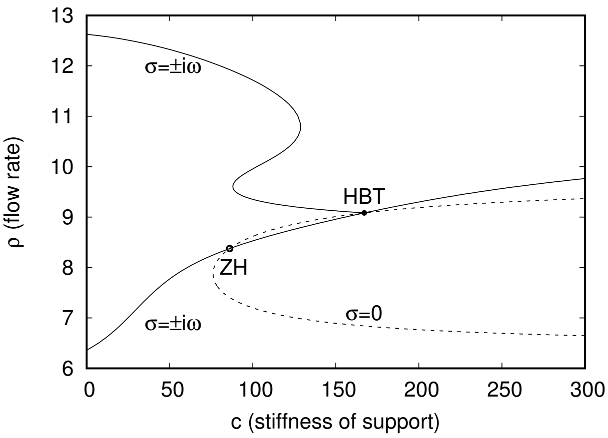

Fig. 1: Stability boundary in the (c, ϱ) parameter plane for a fluid-conveying tube for $\xi \approx 0.8763$. The stability boundaries with a zero eigenvalue ($\sigma = 0$) and with purely imaginary eigenvalue ($\sigma = \pm i\omega$) intersect at the Hopf-Bogdanov-Takens ("HBT") interaction, where the Jordan normal form in the critical eigenspace is given by

$$\mathbf{J}_c = \begin{pmatrix} i\omega & 0 & 0 & 0 \\ 0 & -i\omega & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

where c denotes the (non-dimensional) stiffness of the support and ϱ represents the flow velocity, is shown in Fig. 1: For a stiff support the tube becomes unstable by a zero-eigenvalue, leading to steady buckling of the initially straight configuration, while for a soft support a Hopf bifurcation with a pair of purely imaginary eigenvalues occurs, leading to an oscillating behaviour. These stability boundaries usually intersect either in a Zero-Hopf interaction ("ZH"), as it can be found in Fig. 1 at the left intersection of the lower $\sigma = \pm i\omega$ and the $\sigma = 0$ boundaries, or at a Bogdanov-Takens bifurcation, where a non-semisimple pair of zero eigenvalues occur. In Fig. 1 that interaction takes place at the intersection of the upper Hopf-bifurcation boundary and the static bifurcation. Along the Hopf-bifurcation boundary the frequency ω decays to zero upon approaching the Bogdanov-Takens point. By varying another parameter in the system, e.g. the location ξ of the support, these intersection points can coalesce, as shown in Fig. 1.

While for the considered configuration the trivial state is linearly unstable in the whole vicinity of the "HBT"-point, a small stable triangular shaped stable area occurs, if ξ is increased slightly; in that case the "HBT" point breaks up into a Zero-Hopf point, a Hopf-Hopf interaction, and a Bogdanov-Takens point, respectively, which form the vertices of the small stability region.

Reducing the system to the critical subspace (due to the mirror reflection symmetry of the model the nonlinearities start at third order, so the Center Manifold contributions can be safely neglected), applying normal form theory for the cubic tems and the linear perturbations, introducing polar coordinates for the Hopf modes and eliminating the angular variable, the bifurcation

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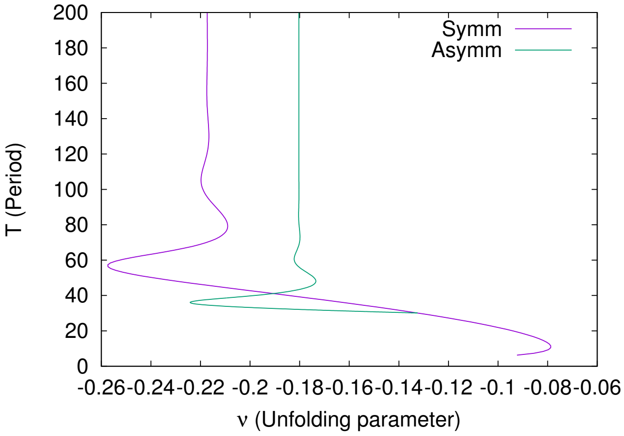

Fig. 2: Bifurcation diagram for fast-slow oscillations in the (ν, T)-parameter plane.

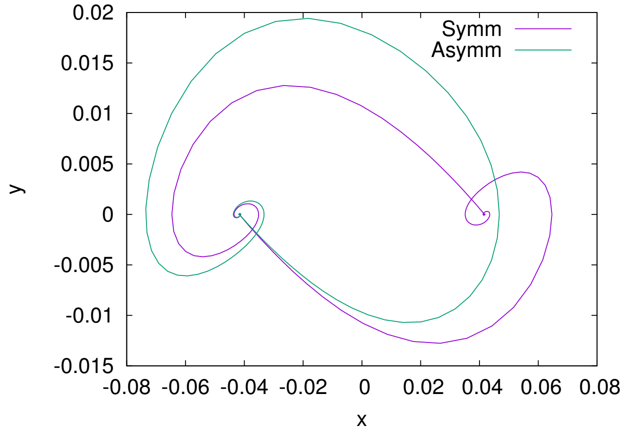

Fig. 3: Symmetric and asymmetric homoclinic orbits.

equations become

$$\dot{r} = (\lambda + c_1 r^2 + c_2 x^2) r, \tag{1a}$$

$$\dot{x} = y, \qquad \dot{y} = (\mu + c_3 r^2) x + (\nu + c_4 r^2) y + c_5 x^3 + c_6 x^2 y, \tag{1b}$$

where λ, μ, and ν denote the linear unfolding parameters, and the c_i are the coefficients of the cubic terms in the normal form. For the considered model these coefficients are given by the rescaled values

$$c_1 = 1, \quad c_2 = 2.16093, \quad c_3 = -266.391, \quad c_4 = 9.23651, \quad c_5 = -1, \quad c_6 = 3.40442$$

2 Investigation of the Bifurcation Equations

The system (1) was already studied in [1]. Its steady state solutions correspond either to static solutions for $r = 0$ or to periodic solutions for $r \neq 0$. System (1) admits the following primary solution branches:

1. Fast oscillation: $r \neq 0$, $x = y = 0$. Bifurcation equation: $\lambda + c_1 r^2 = 0$.

   It is supercritical ($\lambda > 0$) for $c_1 < 0$. Secondary bifurcations occur along the rays $\mu + c_3 r^2 = 0$ (oscillation about a buckled configuration) and $\nu + c_4 r^2 = 0$ (secondary Hopf bifurcation).

   Results for the numerical path-following ([3]) from the secondary Hopf bifurcation point are shown in Figs. 2 and 3: First a slow symmetric periodic solution branches off from the fast oscillation; this periodic solution becomes asymmetric and both branches converge to homoclinic orbits, giving rise to the Shilnikov scenario.

2. Static buckling: $r = y = 0$, $x \neq 0$. Bifurcation equation: $\mu + c_5 x^2 = 0$.

   The solution is supercritical ($\mu > 0$) for $c_5 < 0$. Secondary bifurcations occur along the rays $\lambda + c_2 x^2 = 0$ (fast oscillation about the buckled state) and $\nu + c_6 x^2 = 0$ (slow oscillation about the buckled state.

3. Slow oscillation: $r = 0$, $x \neq 0$, $y \neq 0$. The bifurcation equations are governed by the Bogdanov-Takens bifurcation, which are investigated in detail in [2]. The slow oscillations bifurcate supercritically ($\nu > 0$) from the ray $\nu = 0, \mu < 0$, if $c_6 < 0$. Besides the secondary branches of the Bogdanov-Takens sub-system, also the fast oscillation can become unstable, if

   $$\lambda + \frac{c_2}{T} \int_0^T x^2(t) dt > 0.$$

References

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