Entanglement entropy in shock wave collisions

Based on work with Daniel Grumiller, Stefan Stricker 1506.02658 (JHEP);
& Wilke van der Schee, Philipp Stanzer 1609.03676

Christian Ecker
(TU-Wien)

Tampere
September 29, 2016

FWF

TU WIEN TECHNISCHE UNIVERSITÄT WIEN Vienna University of Technology

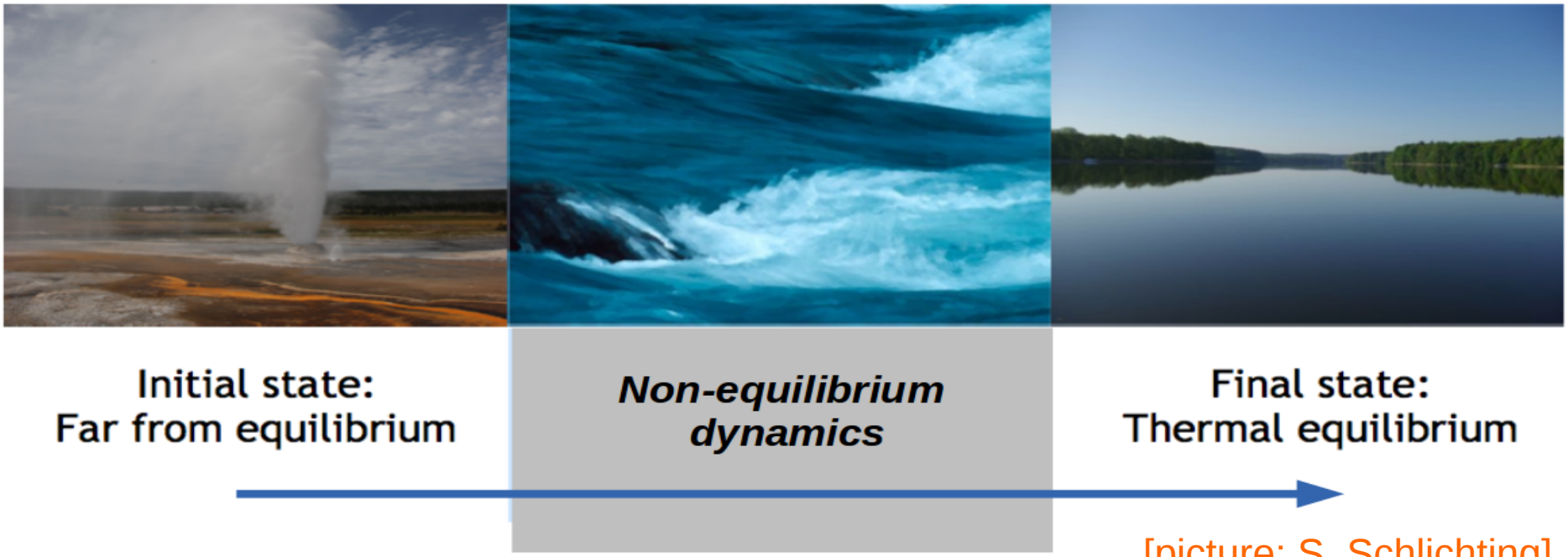
DOKTORATSKOLLEG PI
$\int dk\, \Pi$
Particles and Interactions

INSTITUTE FOR THEORETICAL PHYSICS

Motivation

<u>Central question:</u>

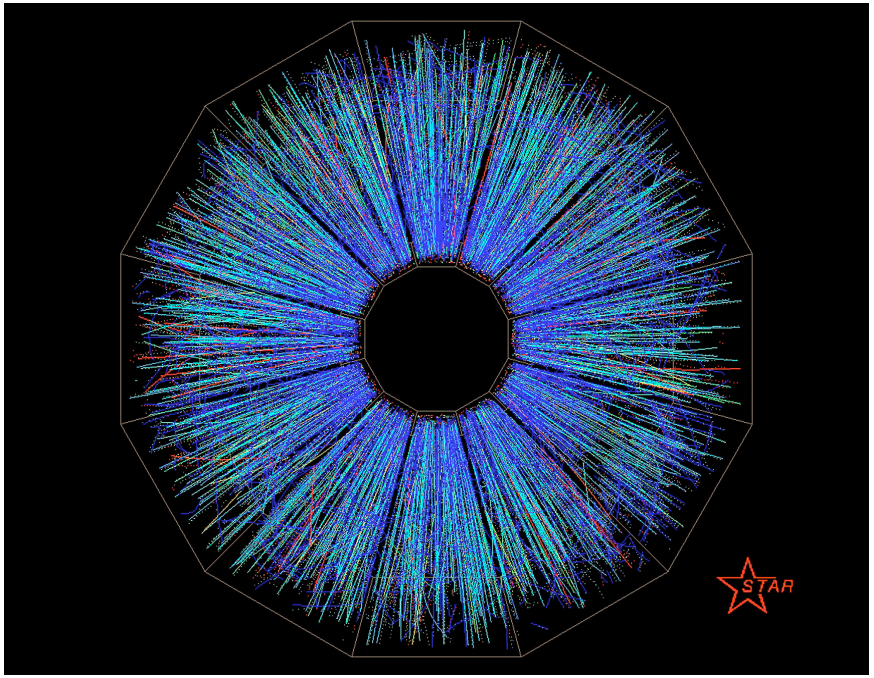
How does a **strongly coupled** quantum system which is initially **far-from equilibrium** evolve to its **equilibrium state?**

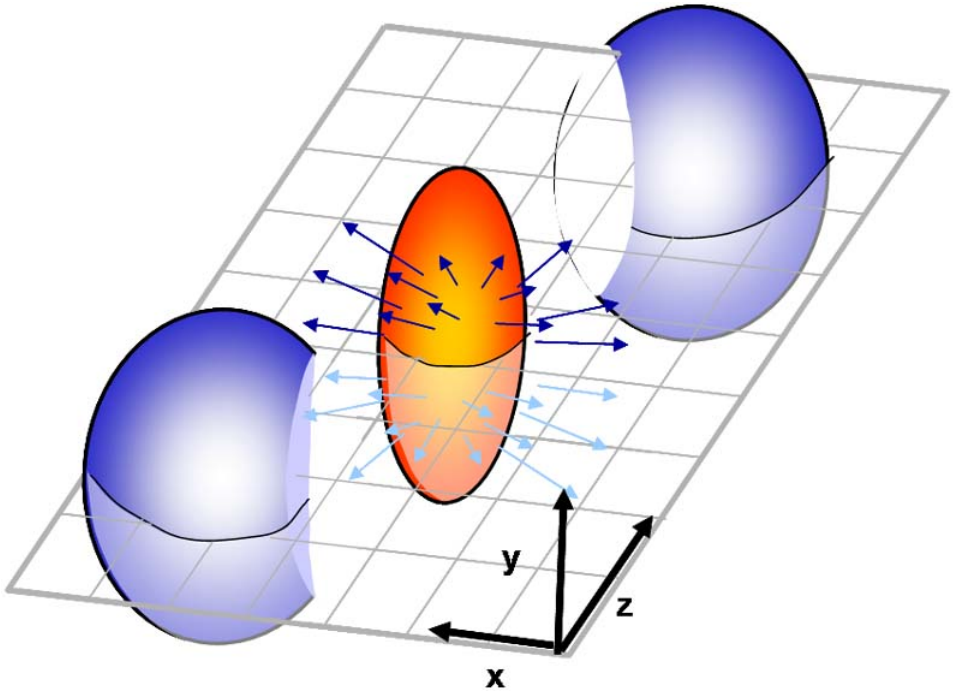


[picture: S. Schlichting]

Quark-gluon plasma in heavy ion collisions

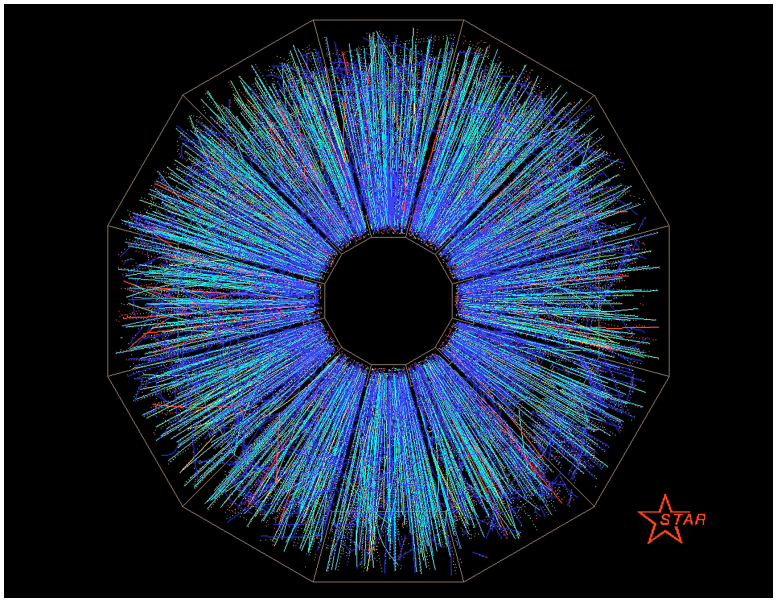
Quark-gluon plasma (QGP) is a **deconfined phase of quarks and gluons** produced in **heavy ion collision** (HIC) experiments at **RHIC** and **LHC**.

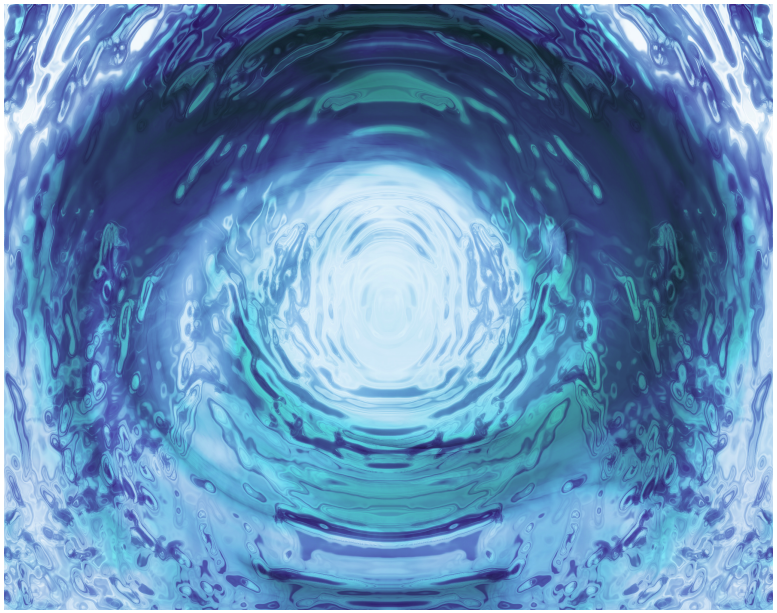




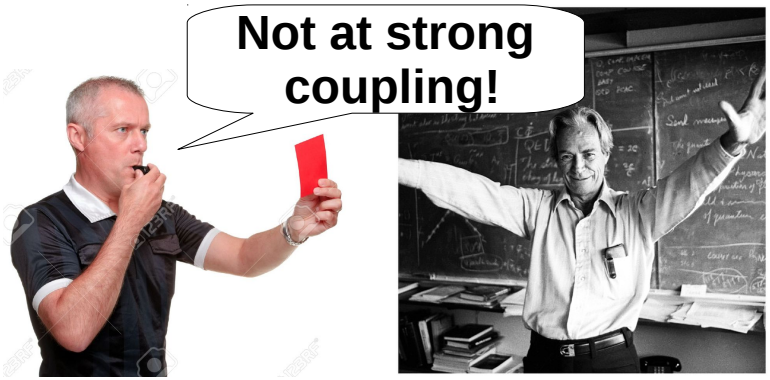
Why AdS/CFT?

The QGP produced in HIC's behaves like a **strongly coupled liquid** rather than a **weakly coupled gas.**

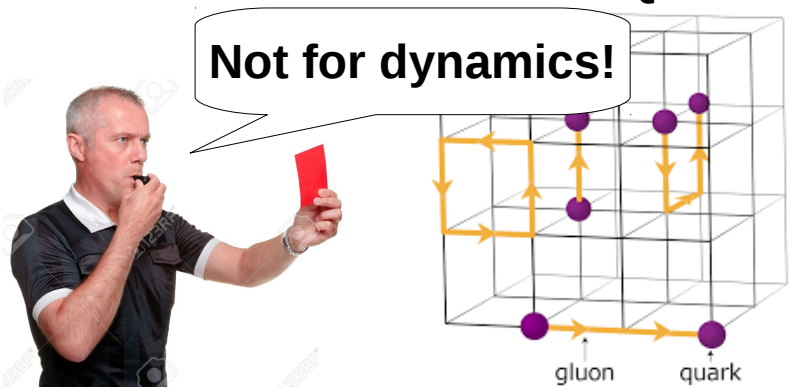




Perturbative QCD?



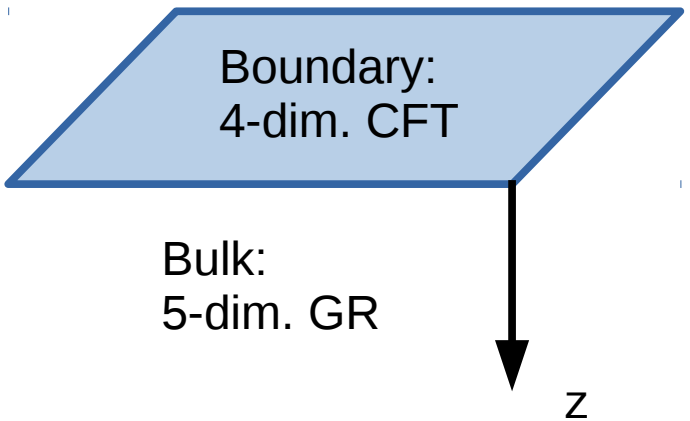
Lattice QCD?



AdS/CFT correspondence

AdS/CFT correspondence: [Maldacena 97]

Type IIB string theory on $AdS_5 \times S^5$ is equivalent to $\mathcal{N}$=4 super symmetric $SU(N_C)$ **Yang-Mills theory** in 4D.



Supergravity limit:

Strongly coupled large N_c $\mathcal{N}$=4 $SU(N_C)$ SYM theory is equivalent to **classical supergravity** on AdS_5

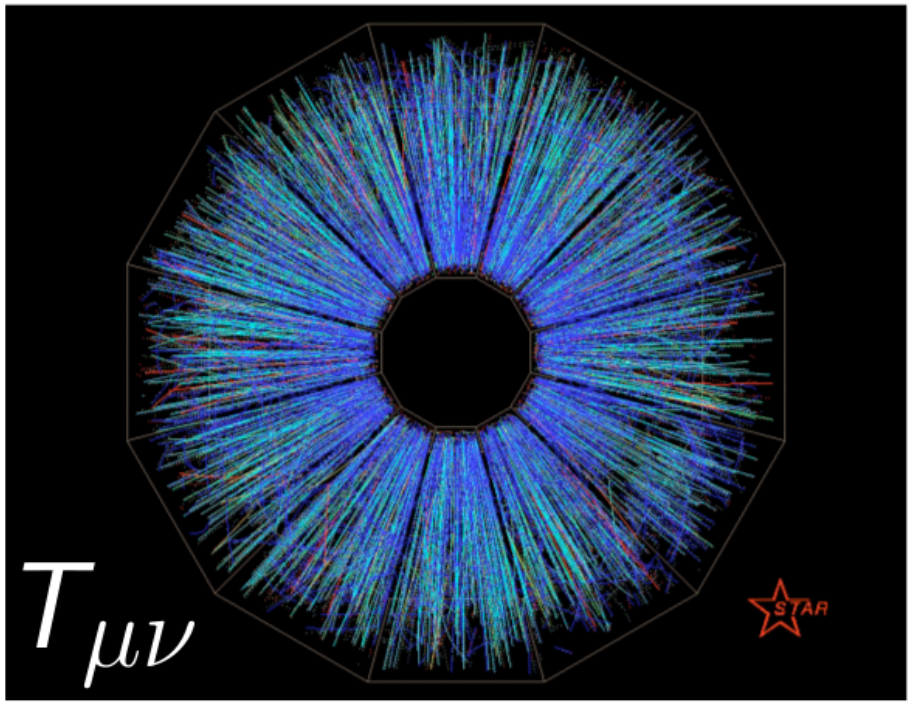
$$e^{-S_{\mathrm{sugra}}[\phi]}\Big|_{\lim_{z\to 0}\phi=\phi_0} = \left\langle \exp\left(\int \mathrm{d}^d x \phi(x) \mathcal{O}(x) \right) \right\rangle_{CFT} .$$

Strategy:

- Use $\mathcal{N}$=4 SYM as **toymodel** for **QCD** in the strongly coupled regime.
- Build a **gravity model** dual to HICs, like colliding **gravitational shock waves**.
- Switch on the computer and solve the 5-dim. gravity problem **numerically**.
- Use the **holographic dictionary** to compute **observables in the 4 dim. field theory** form those gravity result.

Holographic thermalization

Thermalization = Black hole formation



Entanglement entropy

Divide the system into **two parts** A,B.
The total Hilbert space factorizes:

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

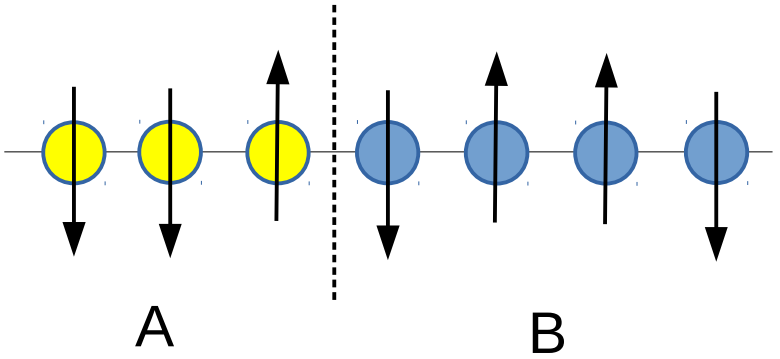
The **reduced density matrix** of A is obtained by the trace over $\mathcal{H}_\mathcal{B}$

$$\rho_A = \mathrm{Tr}_B \rho$$

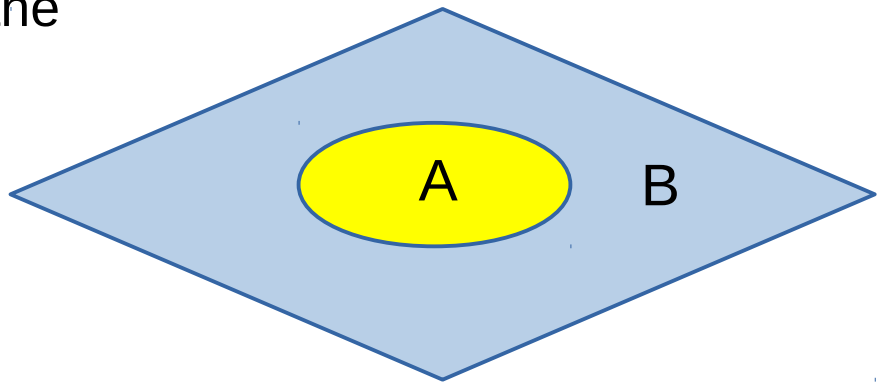
Entanglement entropy is defined as the **von Neumann entropy** of ρ_A:

$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$$

lattice model



quantum field theory

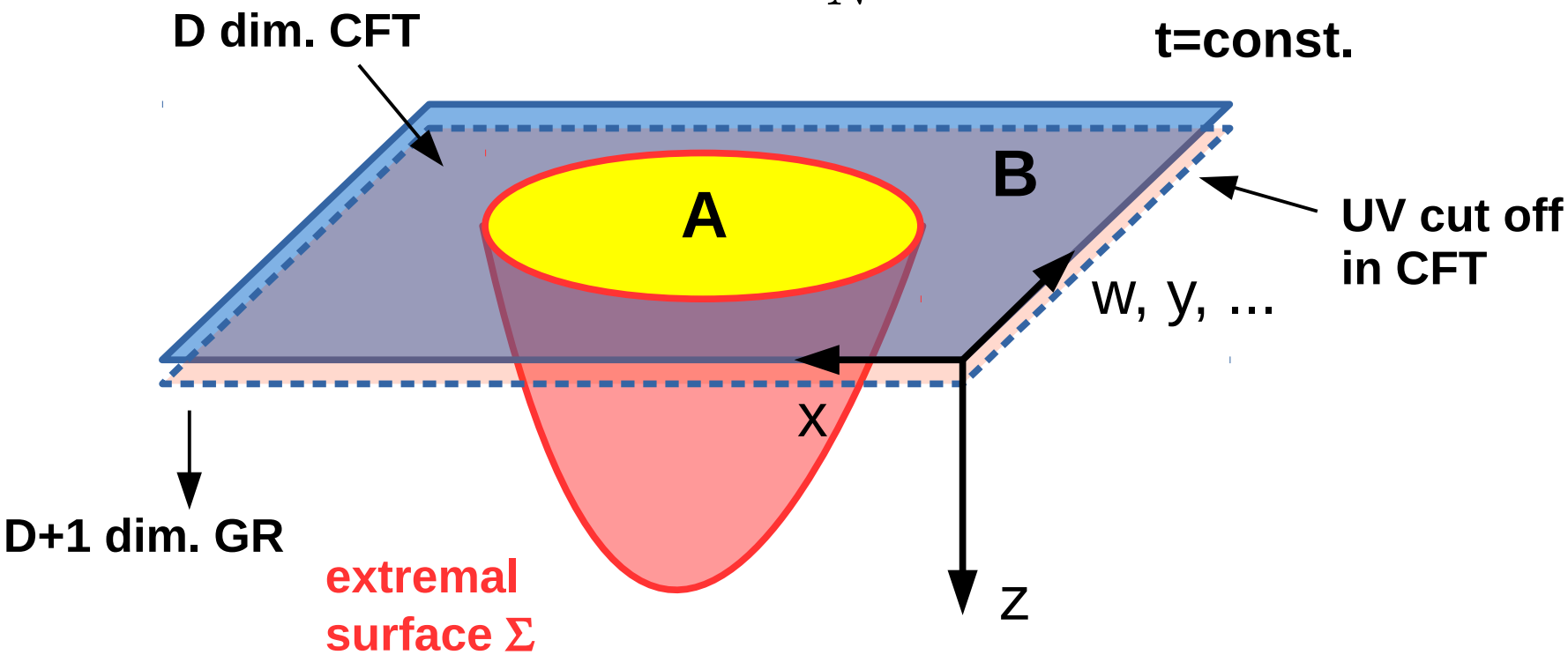


Holographic entanglement entropy

Within **AdS/CFT** entanglement entropy can be computed form the **area of minimal (extremal) surfaces** in the gravity theory.

$$S_A = \frac{\mathrm{Area}(\Sigma)}{4G_N}$$

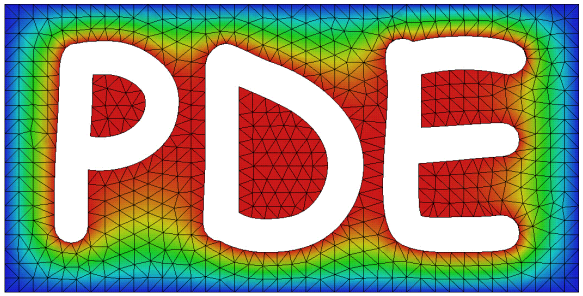
[Ryu-Takayanagi 06, Hubeny-Rangamani-Takayanagi 07]



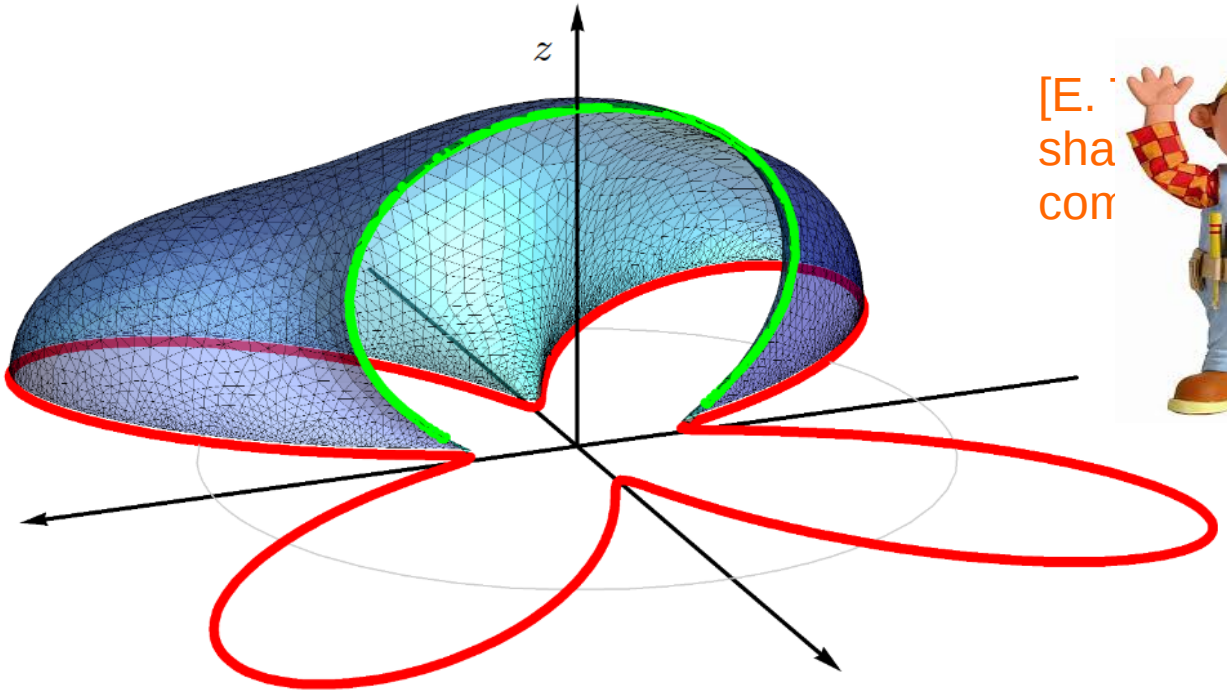
Holographic entanglement entropy

- In practice computing extremal co-dim. 2 hyper-surfaces is numerically involved.

$$\delta\mathcal{A} = \delta\left[\int d^3\sigma\sqrt{\det\left(\frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}g_{\mu\nu}\right)}\right]$$



- Can we somehow simplify our lives?



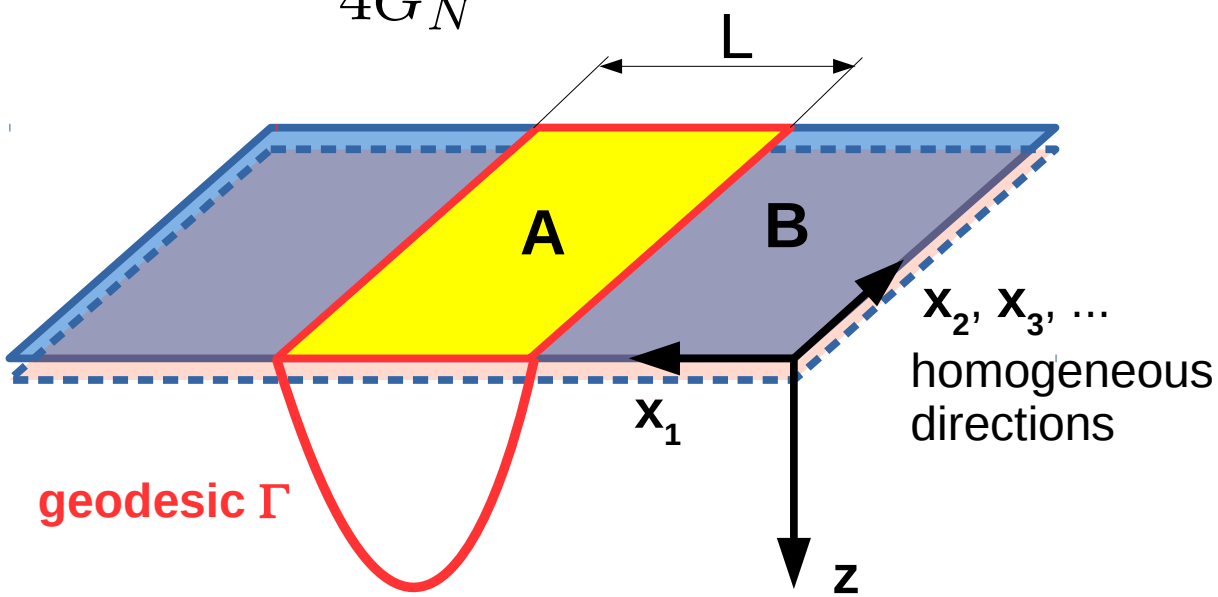
[E. mal surface for a star
sha region (red) in AdS4
con rface Evolver]

Entanglement entropy from geodesics

Consider a **stripe region** of **infinite extend** in **homogeneous directions** of the geometry. The **entanglement entropy** is prop. to the **geodesics length** in an **auxiliary spacetime**.

$$\mathcal{A} = \int d^3\sigma \sqrt{\det\left(\frac{\partial X^\mu}{\partial \sigma^a}\frac{\partial X^\nu}{\partial \sigma^b} g_{\mu\nu}\right)} = \int dx_3 \int dx_2 \int d\sigma \sqrt{\Omega^2 g_{\mu\nu}\frac{\partial X^\mu}{\partial \sigma}\frac{\partial X^\nu}{\partial \sigma}}$$

$$S_A = const.\frac{\mathrm{Length}(\Gamma)}{4G_N} \qquad \tilde{g}_{\mu\nu} = \Omega(z,t,x_1)^2 g_{\mu\nu}$$

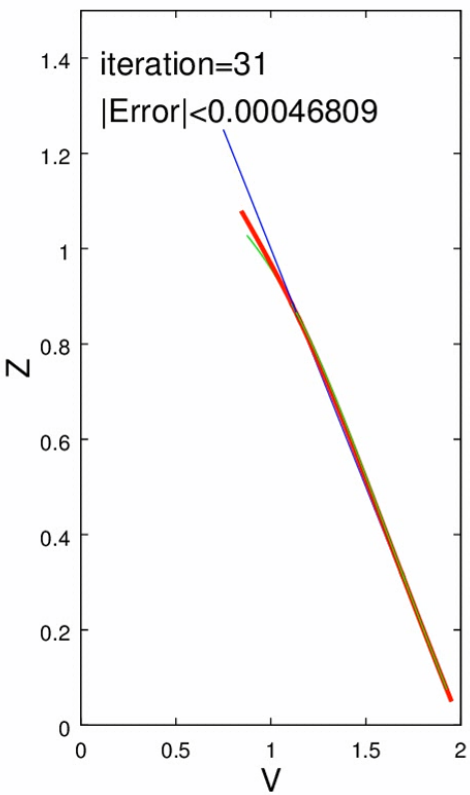


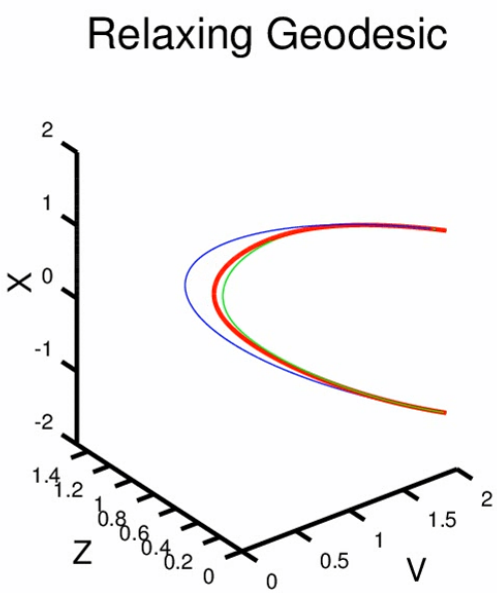
Numerics: relax, don't shoot!

Geodesic equation as **two point boundary value problem:**

$$\ddot{X}^\mu + \Gamma^\mu_{\alpha\beta}\dot{X}^\alpha\dot{X}^\beta = -J\dot{X}^\mu\,,$$

BC's: $X(\pm 1)^\mu \equiv (V(\pm 1), Z(\pm 1), X(\pm 1)) = (t, 0, \pm l/2)$





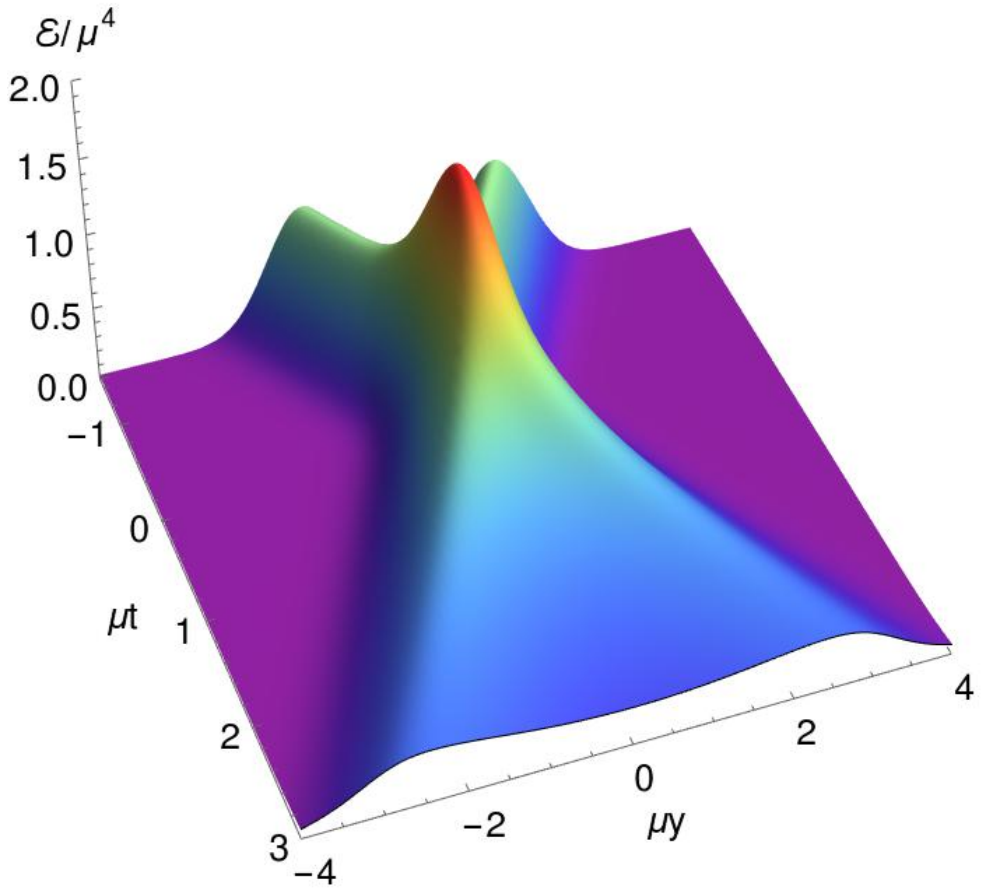
- There are two **standard numerical methods** for solving two point boundary value problems:
  [see Numerical Recipes]

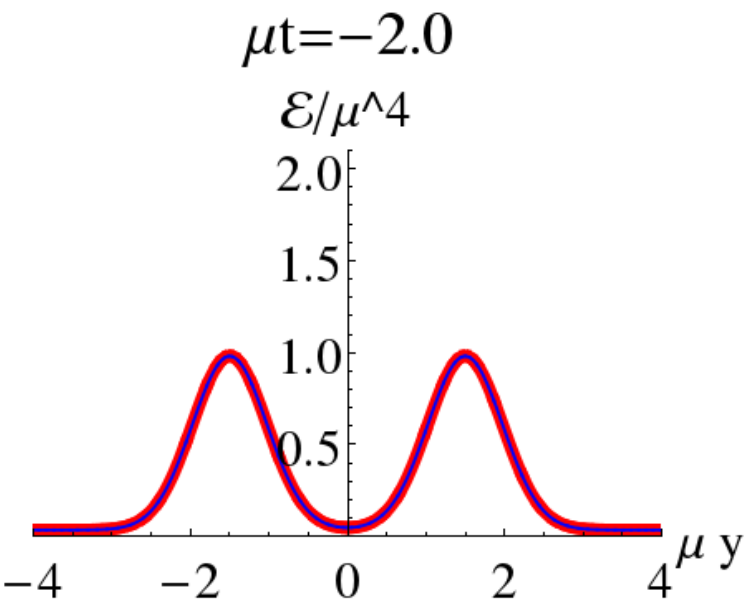
Shooting:
Very **sensitive to initialization** on **asymptotic AdS** spacetimes.

Relaxation:
Converges very **fast** if **good initial guess** is provided.

Holographic shock wave collisions

HIC is modeled by **two colliding sheets of energy** with **infinite extend in transverse direction** and **Gaussian profile** in **beam direction**. [Chesler-Yaffe 10]



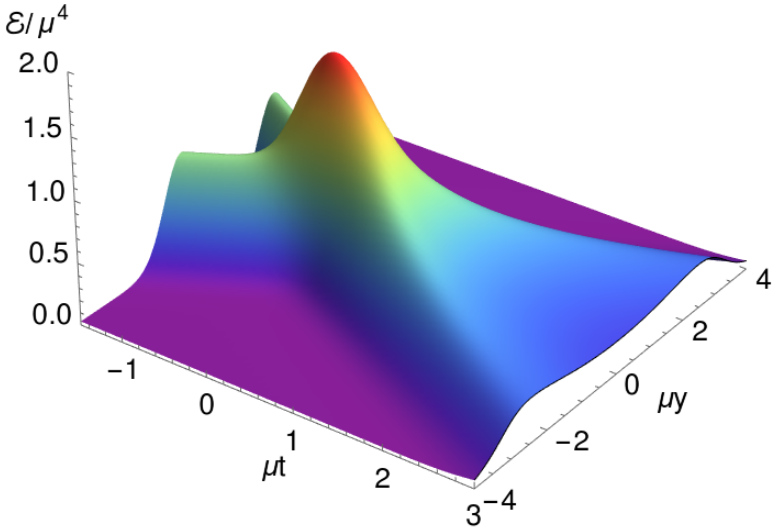


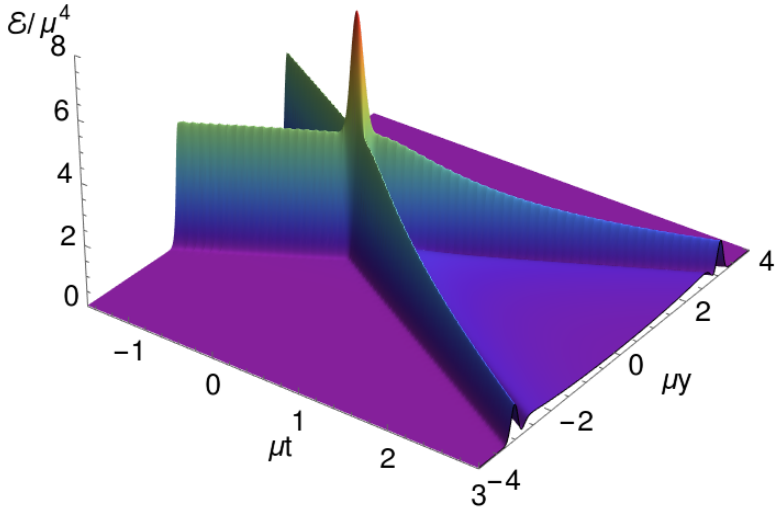
Wide vs. narrow shocks

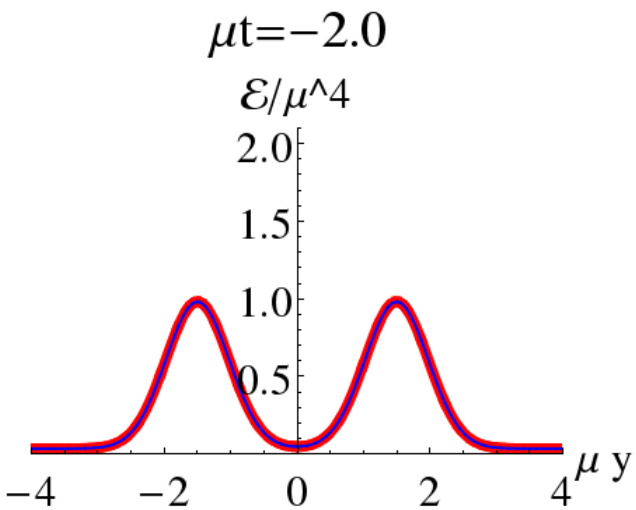
Two qualitatively different dynamical regimes

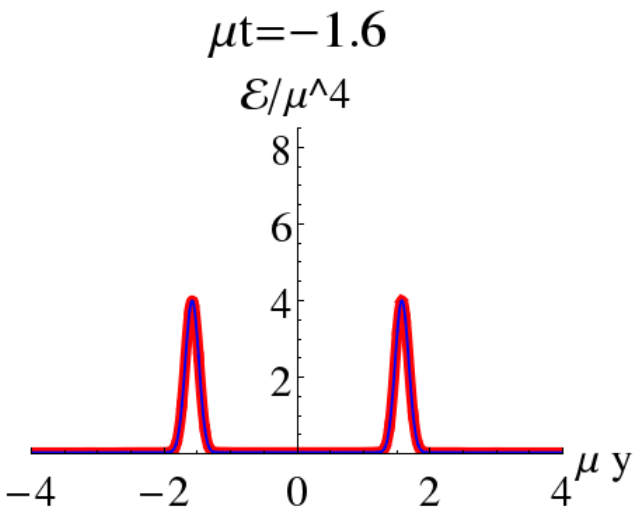
[Solana-Heller-Mateos-van der Schee 12]

- **Wide** shocks (~RHIC): **full stopping**
- **Narrow** shocks (~LHC): **transparency**







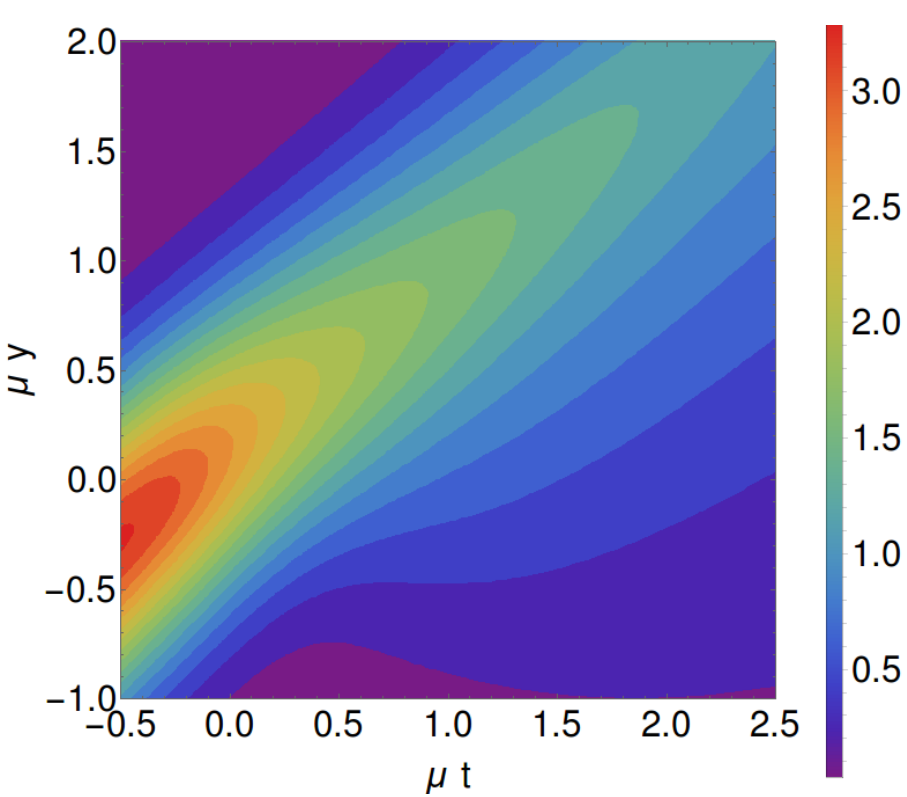


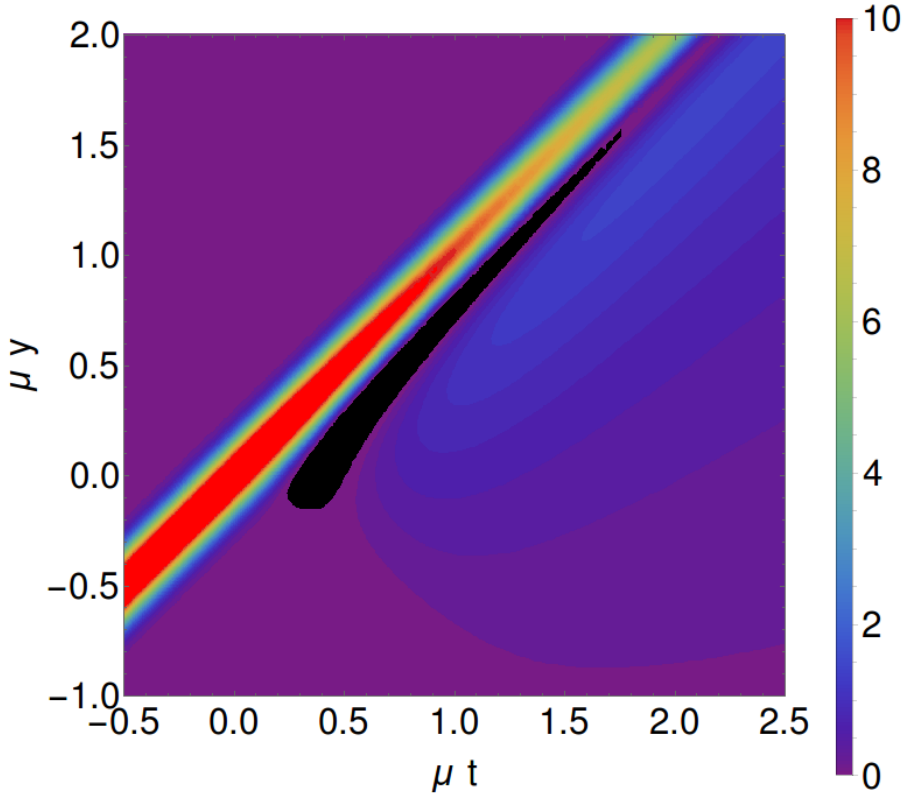
Violation of the null energy condition

"Well behaved" **classical theories satisfy** the **null energy condition** (NEC)

$$T_{\mu\nu}k^\mu k^\nu \geq 0\,, \quad k_\mu k^\mu = 0\,.$$

- In **quantum theories** the NEC **can be violated**. [Epstein 65]
- In **narrow shock wave collisions** the **null energy condition** (NEC) is **violated** in some region in the forward light cone **shortly after the collision**.



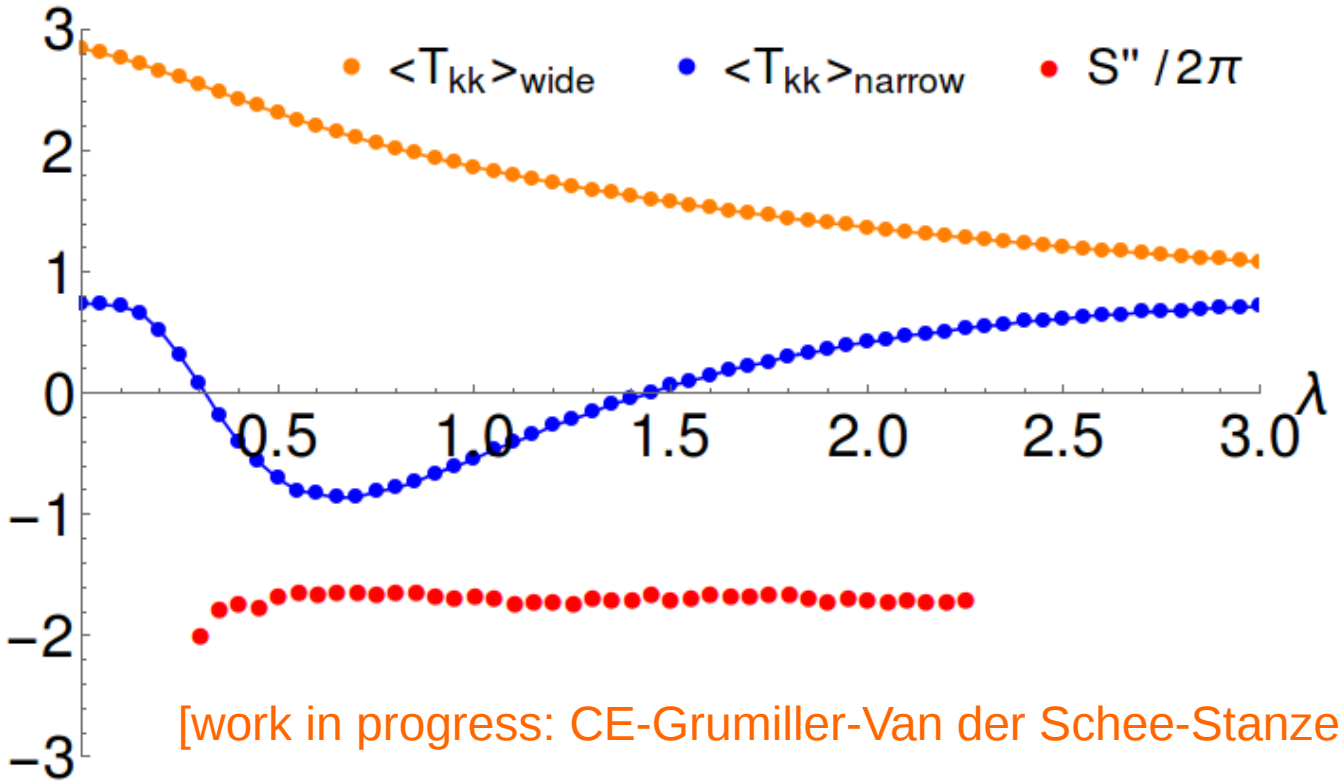


The quantum null energy condition is (preliminarily) fulfilled

Recently the **quantum null energy condition** (QNEC) was proposed [Bousso 15]

$$\langle T_{\mu\nu} k^\mu k^\nu \rangle \geq \frac{1}{2\pi} S'' \,.$$

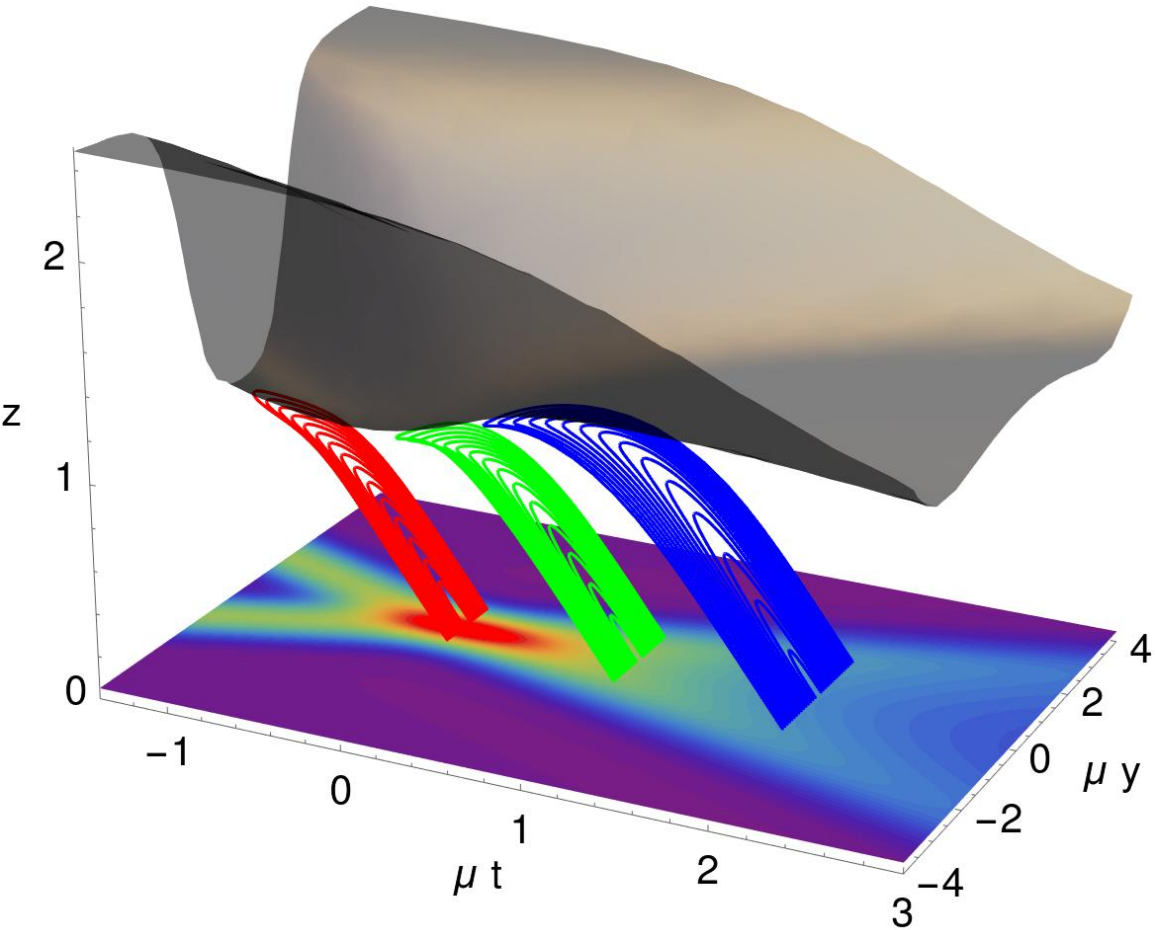
- Our **preliminary results** suggest that the **QNEC is fulfilled**.

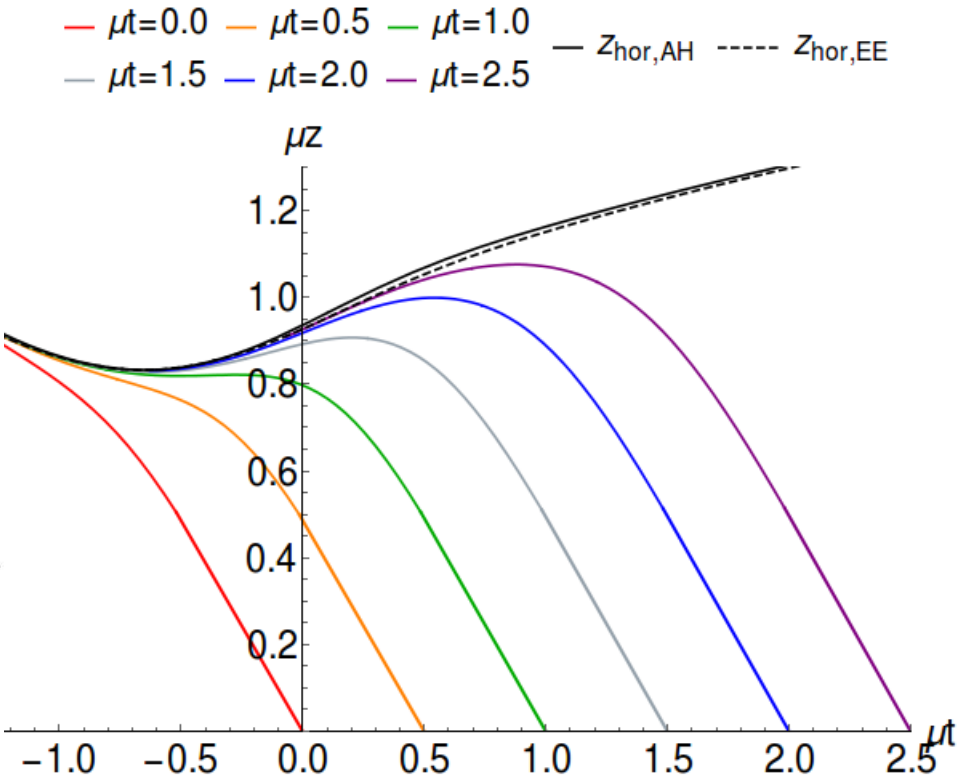


[work in progress: CE-Grumiller-Van der Schee-Stanzer]

Extremal surfaces

- In the case of **entanglement entropy** the **extremal surfaces do not reach beyond the AH**.
- If there is something like an **entanglement entropy horizon** it is **very close to the AH**.

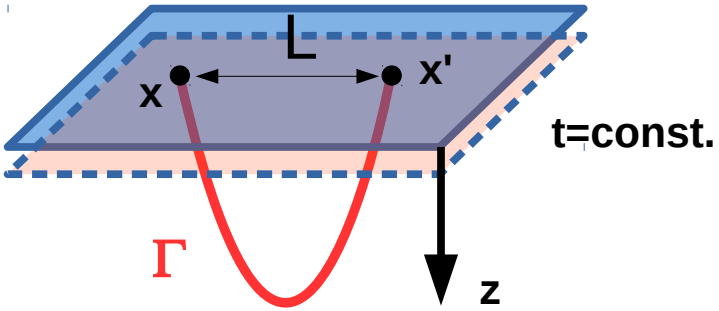


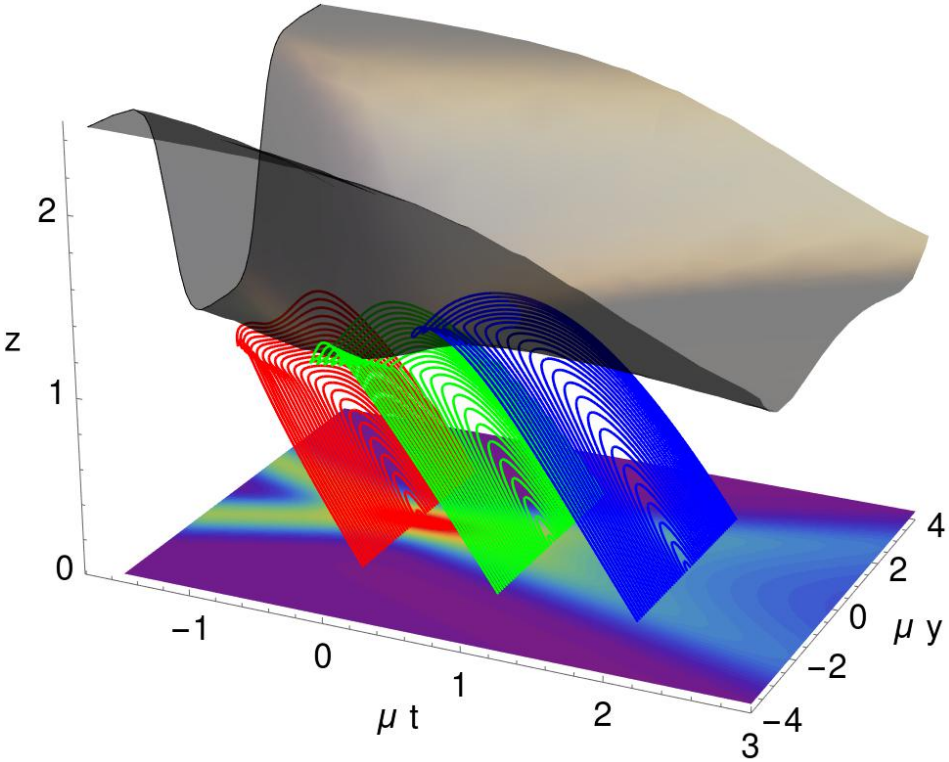


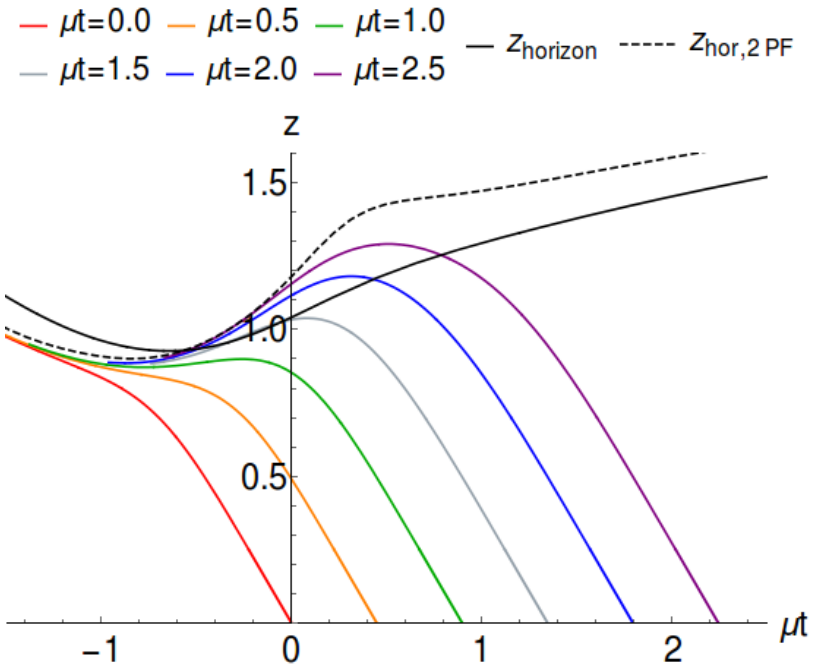
2-point functions from geodesics

Two point functions for operators **O(t,x)** of **large conformal weight Δ** can be computed form the **length of geodesics**. [Balasubramanian-Ross 00]

$$\langle \mathcal{O}(t,x)\mathcal{O}(t,x')\rangle \propto e^{-\Delta \mathrm{Length}(\Gamma)}$$



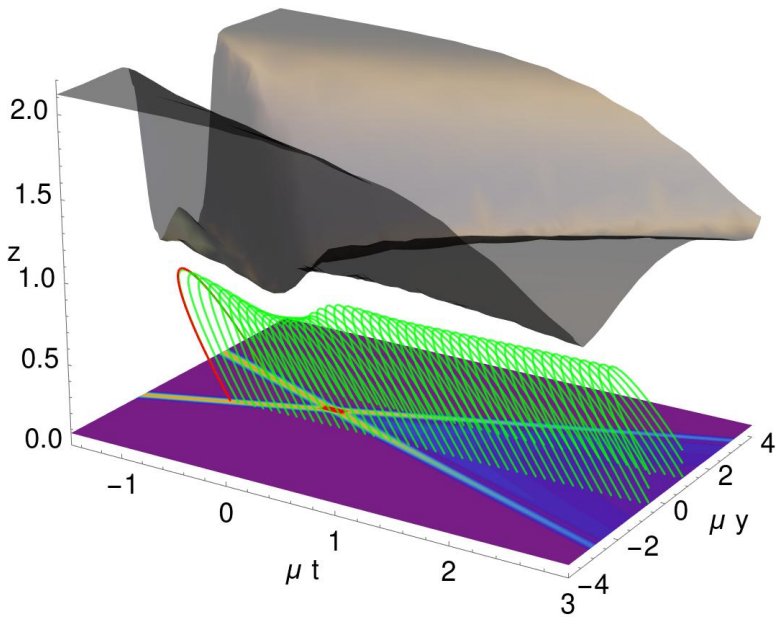


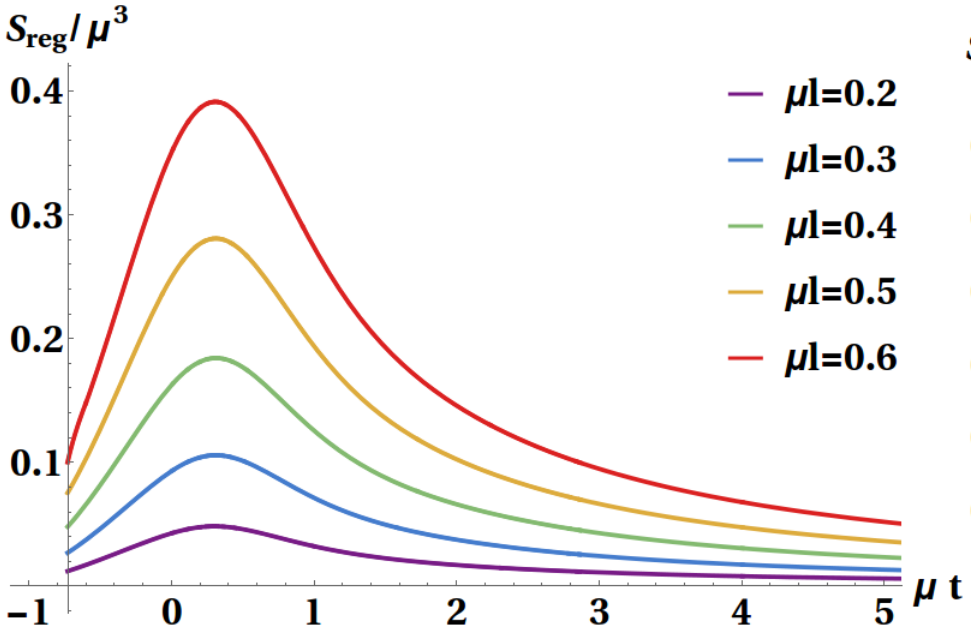


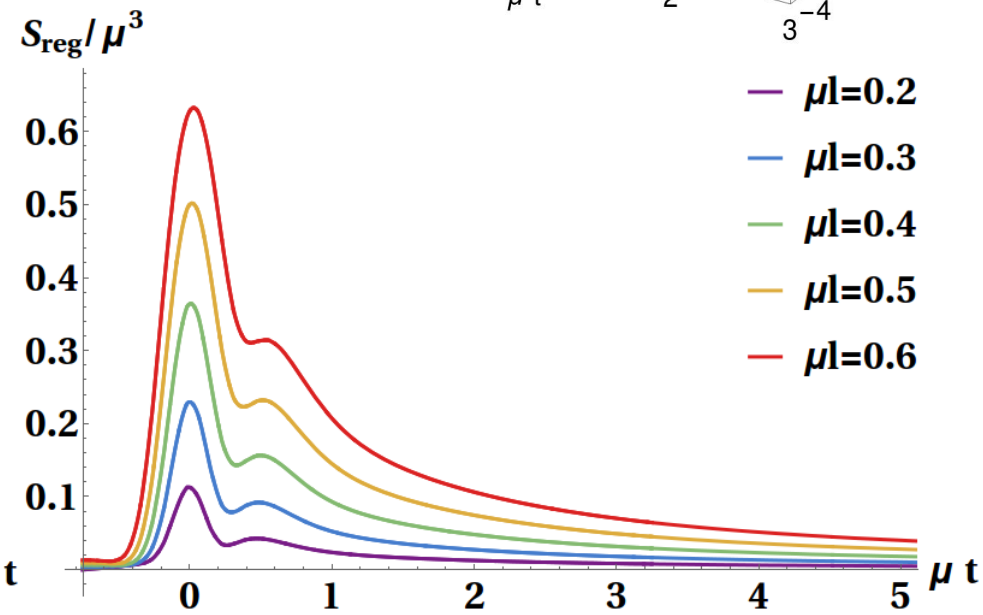
Time evolution of entanglement entropy

Characteristic behavior:

- **Rapid initial growth** when the shocks enter the entangling region.
- **Linear growth** when the shocks start to overlap.
- **Post collisional regime** which is a **featureless fall off for wide shocks** and shows an **additional shoulder for narrow shocks**.
- **Late time regime:** polynomial fall off very close to 1/t.







Summary

- **AdS/CFT** allows to study the **real time dynamics** of **strongly coupled QFT's** by solving the IVP of (classical) **supergravity** theories.
- The **NEC can be violated** in holographic shock wave collisions, but the **QNEC seems to be fulfilled**.
- **Entanglement entropy** may serve as an **order parameter** for the **full stopping–transparency transition**.
- Interestingly the **2-point functions contain information from behind the apparent** horizon, where the **entanglement entropy does not**.

Work in progress

- Improve the quality of the QNEC simulation.
  [CE-Grumiller-Van der Schee-Stanzer]
- **Going beyond supergravity**: string corrections, semi-holography, …
  [CE-Mukhopadhyay-Preiss-Rebhan-Stricker]