Exploring Non-Local Observables in Shock Wave Collisions

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Doktoratskolleg Particles and Interactions

Der Wissenschaftsfonds.

1. Introduction

Quark-Gluon Plasma in Heavy Ion Collisions

- **Heavy ion collision** experiments at RHIC and LHC produce a deconfied state of quarks and gluons, the so-called **quark-gluon plasma** (QGP).
- The QGP in these experiments behaves like a **strongly coupled liquid**, not like a weakly coupled gas.
- The plasma **thermalizes on a very short time scale** ($\approx 10^{-23}$ sec), which is theoretically not well understood yet.
- Due to the strong coupling **perturbative QCD is not suitable** to study the **quantum dynamics** of these collisions.

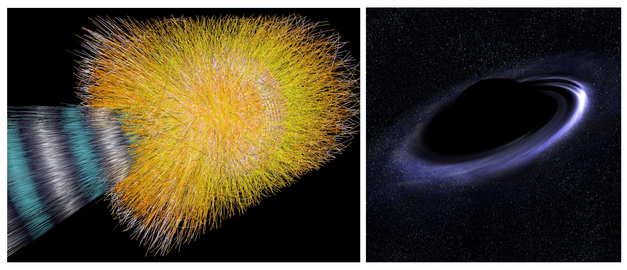

Figure 1: The formation and thermalization of a plasma in the field theory is mapped to the formation of a black hole in the gravity theory via the AdS/CFT correspondence.

AdS/CFT Correspondence [2]

- AdS/CFT maps strongly coupled supersymmetric Yang-Mills (SYM) theory in 4D to classical gravity on 5D anti-de Sitter (AdS5) space.
- We use the plasma in SYM theory as a **toymodel** for the experimentally realized QGP.
- **Thermalization** in the strongly coupled 4D SYM theory is mapped to **black hole formation** in AdS5.

2. Shock Wave Collisions in SYM

Full-Stopping

- **Wide shocks** stop each other in the collision before they **explode hydrodynamically**.
- Outgoing shocks are **slowed down**, **energy** and **pressure** are **positive**.

Transparency

- **Narrow shocks** pass each other almost **"transparently"** without loosing velocity.
- **Energy** and **pressure** can be **negative**.

Energy-Momentum Tensor

$$\langle T^{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\begin{pmatrix}\mathcal{E} & \mathcal{S} & 0 & 0\\ \mathcal{S} & \mathcal{P}_{\parallel} & 0 & 0\\ 0 & 0 & \mathcal{P}_{\perp} & 0\\ 0 & 0 & 0 & \mathcal{P}_{\perp}\end{pmatrix}$$

The **time evolution** of the EMT is extracted from a **numerical relativity simulation** of **colliding gravitational shock waves**. [5]

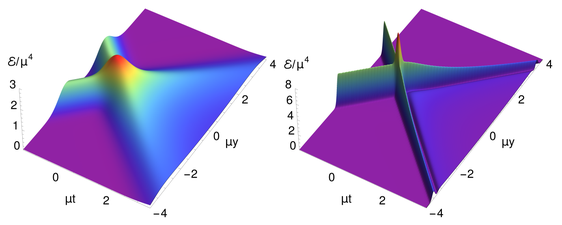

Figure 2: Two Lorentz contracted "nuclei" are modelled as Gaussian energy distributions in SYM heading towards each other at the speed of light. Energy density $\mathcal{E}$ for wide (left) and narrow (right) shocks.

3. Observables from AdS/CFT

AdS/CFT allows to compute **expectation values of observables** in 4D SYM theory **from purely geometric objects** in the 5D gravity theory such as the **metric**, **geodesics** and **minimal surfaces**.

Local Observables

- The **energy-momentum tensor** can be extracted from the **metric** $g_{\mu\nu}$ near the boundary.

$$\langle T^{\mu\nu}(x)\rangle = -\frac{2}{\det g}\frac{\delta S}{\delta g^{\mu\nu}(x)}$$

Non-Local Observables

- **Two-point functions** of gauge invariant operators $\mathcal{O}$ with large conformal weight Δ are given by the **length of geodesics** γ. [3]

$$\langle \mathcal{O}(t,\vec{x})\mathcal{O}(t,\vec{x}')\rangle \approx e^{-\Delta \mathrm{Length}(\gamma)}$$

- The **entanglement entropy** of a spatial region A is given by the **area of a minimal surface** Σ. [4]

$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A = \frac{\mathrm{Area}(\Sigma)}{4G_N}$$

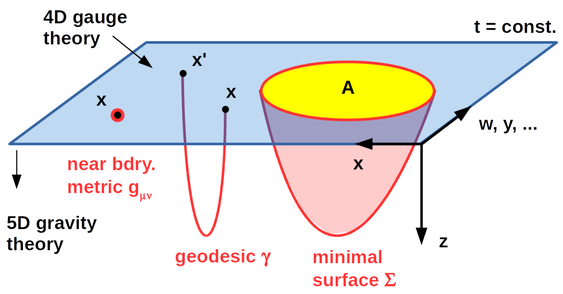

Figure 7: Geometric description of the EMT, 2PF and EE in terms of the near boundary metric, geodesics and extremal surfaces.

4. Geodesics in AdS

Finding geodesics is the central point in this work. [1]
Both, the **two-point function** and the **entanglement entropy** are calculated using the length of spacelike geodesics. They are found by solving the **geodesic equation** (1) numerically.

$$\ddot{X}^\mu + \Gamma^\mu{}_{\alpha\beta}\dot{X}^\alpha\dot{X}^\beta = -J\dot{X}^\mu, \quad s.t. \quad X^\mu|_{bdry} = (t, 0, \pm l/2) \tag{1}$$

$$\mathrm{Length}(\gamma) = \int_\gamma \mathrm{d}\sigma\sqrt{g_{\mu\nu}X^\mu(\sigma)X^\nu(\sigma)} \tag{2}$$

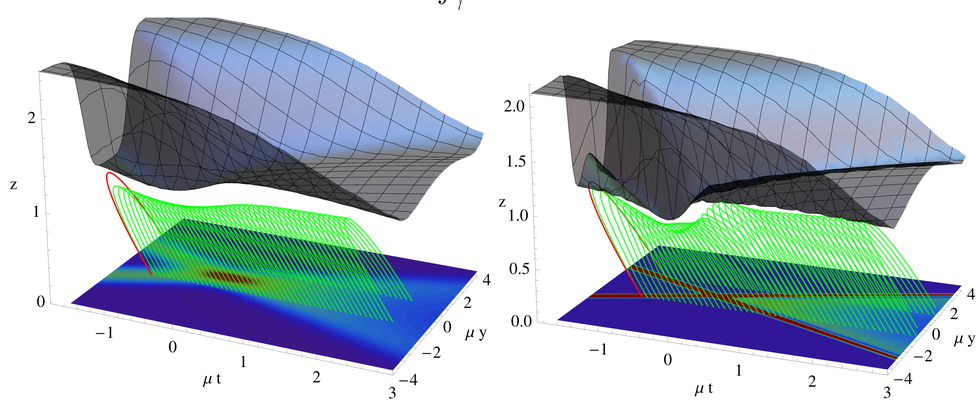

Figure 5: Black hole horizon (black), ansatz geodesics (red), time evolved geodesics (green) and energy contours at z=0 for wide (left) and narrow shocks (right).

5. Entanglement Entropy

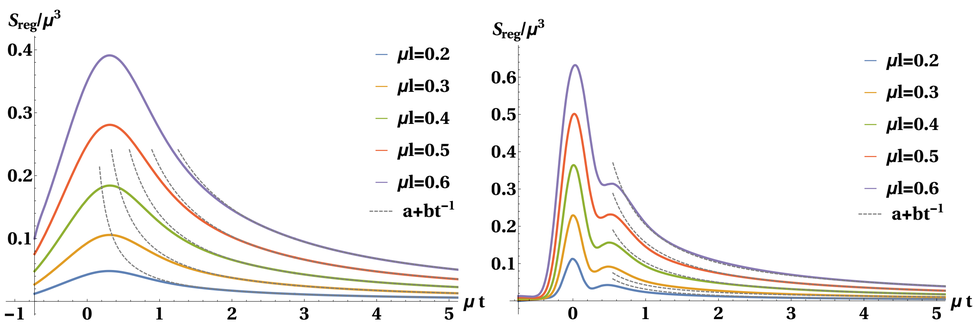

Figure 6: Time evolution of entanglement entropy for various system sizes L; wide shocks (left) and narrow shocks (right).

Time Evolution of Entanglement Entropy [1]

- As the **shocks enter the entangling region** the **entanglement entropy grows rapidly**.
- After this rapid initial growth follows a regime of **linear growth** approx. **until the shocks collide**.
- **Narrow shocks** reach a **global maximum close to the collision time** ($t = 0$), for **wide shocks** the maximum is **clearly delayed**.
- For the **narrow shocks** there is an additional **local minimum after the collision**, which does not appear for the wide shocks.

6. Two-Point Function

Time Evolution of Two-Point Functions [1]

- In-going **shocks destroy** the **initial correlations**.
- **During the collision**, **new correlations** are formed.
- For **wide shocks** the correlations start to **grow earlier** ($t < 0$).
- For **narrow shocks** the correlations start to **grow later** ($t = 0$) and **overshoot their initial values**.

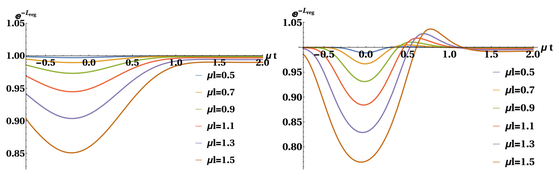

Figure 8: Time evolution of 2PF for various system sizes L; wide shocks (left) and narrow shocks (right).

7. Looking Behind the Horizon

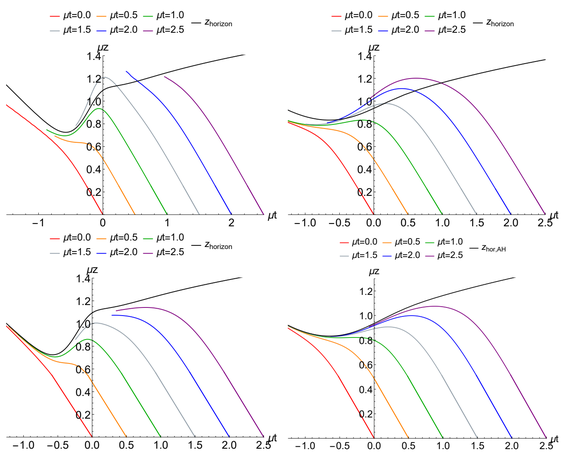

Figure 9: Geodesics probing the 2PF can reach behind the horizon (top), while extremal surfaces probing the EE cannot (bottom). [1]

8. Null Energy Condition (NEC)

The **NEC** is given by the projection of the **EMT** onto lightlike vectors.

$$\langle k^\mu k^\nu T_{\mu\nu}\rangle \geq 0 \quad \forall \quad k^2 = 0$$

- **Narrow shocks** can violate the **NEC**. [7]

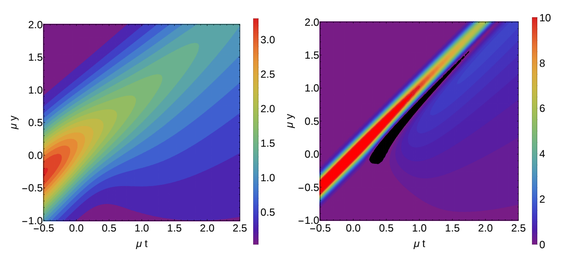

Figure 3: NEC for wide (left) and narrow (right) shocks. Violation in the black area.

9. Quantum NEC

The **quantum null energy condition (QNEC)** is conjectured to give an **upper bound** for this violation. [9]

$$\langle T_{\mu\nu}k^\mu k^\nu\rangle \geq \frac{1}{2\pi}S''$$

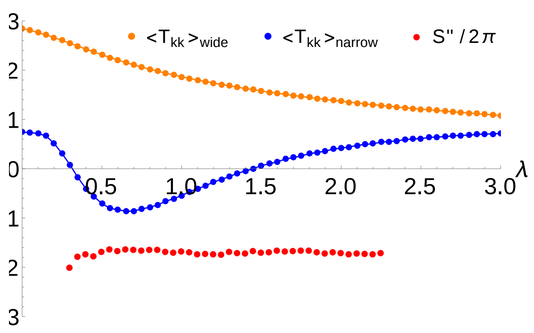

Figure 4: Preliminary results for the QNEC (red), compared with the NEC for wide (orange) and narrow (blue) shocks.

10. Summary

Conclusion

- We use collisions of **shock waves in SYM** theory as **toymodel for heavy ion collisions**.
- Within AdS/CFT **non-local observables** such as **two-point functions** and **entanglement entropy** can be computed from **geodesics** and **minimal surfaces** in the gravity theory.
- We study the **time evolution** of two-point functions and entanglement entropy and find **qualitatively different behavior for narrow and wide shocks**.
- **Narrow shocks** show **overshooting** in the **two-point function** and a **local minimum** in the **entanglement entropy after the collision**. These features do **not appear** in the **wide shocks**.

Outlook

- The **geometric probes** for the **two-point function reach behind the horizon**, while the probes for the **entanglement entropy do not**. We are currently investigating this further.
- For **narrow shocks** we observe a **violation of the NEC**.
  The question if the **conjectured QNEC** is satisfied, is currently under investigation and first (preliminary) results suggest that this is the case.

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12. References

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