On the rotation of rigid fibers in turbulent channel flow

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Abstract

We investigate the effect of local shear and turbulence anisotropy on the rotation of elongated rigid fibers. To this aim, statistics of the fiber angular velocity, Ω, are extracted from direct numerical simulation of turbulence coupled with Lagrangian fiber tracking. We show that elongation is important for fibers with small inertia ($St \leq 5$ in our flow-fiber combination), and that, in the strong shear region near the wall, fiber anisotropy adds to flow anisotropy to induce strong deviations on fiber rotation with respect to spherical particles.

Keywords: Wall turbulence, rigid fibers, rotation statistics, direct numerical simulation, Lagrangian tracking

1. Introduction

A possible approach for modeling rotation of fibers in turbulence is to describe the time evolution of fiber angular velocity as a stochastic (random walk) process in orientation space [1]: In analogy with the translational motion of tracers and particles in Homogeneous Isotropic Turbulence (HIT), one can hypothesize that the Lagrangian time series of angular velocity are characterized by a Gaussian probability distribution with Markovian properties. Based on this hypothesis, fiber rotation can be described within the theory of diffusion as a Ornstein-Uhlenbeck (OU) process [2], which is completely characterized by a statistically-stationary Gaussian distribution and autocovariance which takes the specific form of a negative exponential. This modeling approach assumes that both translational diffusion and rotational diffusion are homogeneous and isotropic. This assumption was proven acceptable for unbounded flows where dispersion is indeed stationary and Gaussian [3, 4, 5], and may be partly extended also to wall-bounded shear flows [6, 7], where Lagrangian velocity autocorrelations exhibit exponential decay if particles are not sampling near-wall regions of high turbulence anisotropy. All these studies, however, are concerned with spherical particles. Much less effort has been devoted to exploring the applicability of standard diffusion laws to non-spherical particles, as in the case of fibers: Previous modeling attempts focus on HIT [1, 8, 9], and thus neglect effects due to turbulence anisotropy. Recent studies [9, 10], however, show that orientation of elongated fibers is correlated with local velocity gradients and that the strength of this correlation is heavily influenced by fiber shape. These findings suggest that anisotropy of fibers adds to anisotropy of turbulence to deviate fiber behavior (in particular, rotation rates) from that predicted by stationary Gaussian dispersion models. In the work we examine the effect of local shear and flow anisotropy on the rotational dynamics of fibers with different elongation and inertia. This analysis is performed to assess the extent of diffusion laws application can be applied to describe fiber rotational dispersion in wall-bounded turbulence. More specifically, we are interested in assessing the possibility of modeling rotation as an OU process depending both on flow parameters (shear, anisotropy) and on fiber parameters (inertia, elongation).

2. Physical Problem and Numerical Methodology

The reference flow configuration is Poiseuille flow of incompressible, isothermal and Newtonian fluid in a plane channel at friction Reynolds number $Re_\tau = u_\tau h/\nu = 150$, with u_τ the friction velocity, ν is fluid viscosity and h is the channel half-height. We performed pseudo-spectral DNS, imposing periodic boundary conditions in the streamwise (x) and spanwise (y) directions and no-slip conditions at the walls. Time integration a 2nd-order Adams-Bashforth scheme for the non-linear terms and an implicit Crank-Nicolson scheme for the viscous terms. The channel size is $1885 \times 942 \times 300$ wall units (identified with the superscript "+" and obtained using ν and u_τ) in x, y and z, discretized with $128 \times 128 \times 129$ nodes.

Lagrangian fiber dynamics related to [11]. The translational equation of motion of an individual fiber is given by the linear momentum equation $\mathrm{d}\mathbf{u}_p/\mathrm{d}t = \mathbf{F}/m$, where $\mathbf{u}_p$ is fiber velocity, $\mathbf{F} = \mu\mathbf{K}(\mathbf{u}_{@p} - \mathbf{u}_p)$, with μ the fluid dynamic viscosity, $\mathbf{K}$ the resistance tensor and $\mathbf{u}_{@p}$ the fluid velocity at fiber position, is the total hydrodynamic drag force acting on the fiber (strictly valid for an ellipsoid under creeping flow conditions) and $m = \frac{4}{3}\pi a^3 \lambda \rho_p$ is fiber mass with a the the semi-minor axis, λ the aspect ratio of the ellipsoid, and ρ_p is fiber density. The resistance tensor $\mathbf{K}$ is expressed in the Eulerian frame of reference, $\mathbf{x} =< x, y, z >$. Two other Cartesian coordinate systems, both with origin at the fiber center of mass, are used to describe fiber motion: a Lagrangian frame of reference, $\mathbf{x}'$, and a co-moving frame of reference, $\mathbf{x}''$, with axes parallel to the inertial frame. The rotational motion of the fiber is governed by the following equation: $\mathrm{d}(\mathbf{I}\cdot\Omega')/\mathrm{d}t + \Omega' \times (\mathbf{I}\cdot\Omega') = \mathbf{N}'$, where Ω is fiber angular velocity, $\mathbf{I}$ is the moment of inertia tensor and $\mathbf{N}$ the torque acting on the fiber. The equations of fiber motion are solved using a mixed explicit/implicit differencing procedure [12]. The total tracking time in wall units is $t^+ = 3500$, with time step size equal to that of the fluid: 0.03 in wall units. The main simulation parameters are a, and the fiber response time [13]:

$$St = \frac{2(a^+)^2\rho_p}{9\rho}\,\frac{\lambda\ln(\lambda + \sqrt{^2 - 1})}{\sqrt{\lambda^2 - 1}}\ . \tag{1}$$

In this study, we have selected: $a^+ = 0.36$, $\lambda = 1$ (spherical particles), 3, 10, 50, and $St = 1, 5, 30, 100$. To ensure converged statistics, swarms of $200,000$ fibers are tracked for each particle category, assuming dilute flow and one-way coupling.