

# Optimal partitioning of a boiler-turbine unit for Fuzzy model predictive control

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**Abstract:** In order to effectively control the multi-input multi-output (MIMO) characteristics of a boiler-turbine unit Fuzzy model predictive control (FMPC) has been previously applied successfully. An important step in the FMPC design is the partitioning (also called the design of the antecedent rules). In this work it is shown that by exploiting the plant characteristics and by using suitable metrics of the nonlinearity a representative one-dimensional partition space can be defined. Furthermore, it is demonstrated that a simple partition into two linear models suffices to obtain a stable and performing control system. In order to achieve this simple FMPC a nonlinear stationary feed-forward decoupling is implemented. Simulation results demonstrate stability and performance of the resulting control system even for large reference steps. The methodology developed in this work is quite general and can be easily applied to the FMPC design for other nonlinear MIMO plants.

*Keywords:* Modeling and simulation of power systems, Nonlinear predictive control, Model predictive and optimization-based control, Intelligent control of power systems.

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## 1. INTRODUCTION

The boiler-turbine unit is the main part of a modern fossil-fuel-fired power plant which converts the fuel's chemical energy into mechanical and electrical energy. The main purpose of the control theoretical aspect is to regulate the power output and at the same time keeping the internal variables such as pressure and drum water level in a certain range Åström and Bell (2000).

As power plants are nonlinear multi-input multi-output (MIMO) systems, controlling such systems is complex and needs advanced control technologies. Therefore, model predictive controllers are utilized to handle constraints and to improve the performance of the boiler-turbine system for economic and safe plant operation. Model predictive control (MPC) is known as a versatile advanced process control, which is able to deal with the already mentioned issues, Camacho and Bordons (2004). Because of nonlinear effects in the boiler-turbine unit a Fuzzy modeling and control approach has been chosen by many previous authors, Li et al. (2012); Wu et al. (2014); Tan et al. (2005); Kong et al. (2015), leading to a Fuzzy MPC (FMPC), Roubos et al. (1999). Furthermore, because of couplings in the inner dynamics of the systems, a MIMO FMPC is given in this paper to demonstrate the features and flexibility of such tool in nonlinear control technologies, see Dalhoumi et al. (2010); Roubos et al. (1999). In contrast, a genetic algorithm design of a coupled PI-controller and a state feedback controller are presented in Dimeo and Lee (1995), which lead – in comparison to the mentioned MIMO FMPC – only to good steady-state results, but not to good results during the overall area of operation in general.

An optimal partitioning and state-feedback FMPC for a boiler-turbine unit is introduced and discussed in this work. For the approximation of the nonlinearities a local linear model network (LLMN) is chosen, where the membership functions are calculated as trapezoid functions. The choice of the partition variables is motivated by a unique representation without loss of information. The optimal partitioning is done using the  $\nu$ -gap metric and a specially tailored plant-specific criterion, leading to a one-dimensional partition space. The resulting FMPC is able to stabilize the boiler-turbine unit with only two local linear models (LLMs). This low dimensional partition and control is in strong contrast to previous works Li et al. (2012); Wu et al. (2014); Tan et al. (2005); Kong et al. (2015), where more LLMs are used. Furthermore, in contrast to mentioned approaches, in this work a state-tracking FMPC is implemented and presented. In the case of this special plant, the coupled outputs are automatically controlled to their specific reference values without explicit output-tracking.

In previous works both the choice of the partitioning variables as well as the partitioning methods have been less systematic: In Wu et al. (2014) all outputs and inputs are incorporated in the partitioning space (three inputs and three outputs). Therefore a six-dimensional partition space results. In addition, a Gustafson-Kessel clustering algorithm is employed to define the operating points for the LLMs in Wu et al. (2014). The authors present an obviously coarse plant representation with high computational cost. Therefore, the superiority of the proposed method in this paper is given by the one-dimensional partition space and the effective computation.

A simple approach is to use only the power output, Li et al. (2012), because expert knowledge tells that this is the most important variable. This allows for simple and intuitive choice of the LLMs, however, the possibly decisive influence of other state variables is lost. Li et al. (2012) utilize a more efficient approach for partitioning, where the  $\nu$ -gap metric is employed to discern between candidate LLMs. As only one partitioning variable (power  $E$ ) exists, a simple optimization algorithm is employed resulting in five LLMs. A hierarchical Fuzzy MPC is introduced in the solution of Kong et al. (2015), which incorporates both the plant-wide economic process optimization and regulatory process control. Furthermore, the implementation and closed-loop simulation results do not show the critical operation areas of the used plant.

In contrast to the cited references the proposed methodology provides clear criteria for the choice of the partition variables, exploits the known nonlinear dynamics for reduction of the partition space, and results in a minimal representation of fuzzy rules. Additionally, state- instead of output feedback is utilized for the control structure.

The remainder of this paper is organized as follows: Sec. 2 describes the boiler-turbine unit in detail. Sec. 3 establishes a order reduction algorithm, projection, and a LLMN approach by using the  $\nu_{\text{gap}}$  metric to define the operating points of the nonlinear system. The used TS-Fuzzy model of the boiler-turbine unit and the specific partitioning is introduced in Sec. 4. A state-tracking MIMO FMPC is discussed and designed in Sec. 5. Simulation results of the FMPC and a discussion to state-of-the-art FMPCs are illustrated in Sec. 6, and Sec. 7 summaries the results.

## 2. SYSTEM DESCRIPTION

The boiler-turbine described in this paper is a third order nonlinear model adopted from the authors of Tan et al. (2005), which originally has developed by K.J.Åström and R.D. Bell in 1987. The model is based on the nonlinear dynamic behavior of a 160 MV drum-type oil-fired power plant in Malmo, Sweden.

Assume that the nonlinear dynamic behavior of the system is given as:

$$\dot{x} = f(x, u), \quad (1a)$$

$$y = g(x, u), \quad (1b)$$

where  $f$  and  $g$  represents the state and output nonlinear equations. The dynamics of the boiler-turbine unit is given by:

$$\dot{x}_1 = \frac{dP}{dt} = -0.0018u_2x_1^{\frac{9}{8}} + 0.9u_1 - 0.15u_3, \quad (2a)$$

$$\dot{x}_2 = \frac{dE}{dt} = \frac{((0.73u_2 - 0.16)x_1^{\frac{9}{8}} - x_2)}{10}, \quad (2b)$$

$$\dot{x}_3 = \frac{d\rho}{dt} = \frac{((141u_3 - (1.1u_2 - 0.19x_1))}{85}. \quad (2c)$$

For eqs. (2a)-(2c)  $x_1 = P$  denotes drum pressure (kg/cm<sup>2</sup>),  $x_2 = E$  denotes electric power (MW), and the fluid density is given by  $x_3 = \rho$ . The manipulated variables of the system are defined as the valve position for the fuel mass

flow ( $u_1$ ), the steam to turbine flow ( $u_2$ ), and the feedwater flow to the drum ( $u_3$ ).

The output variables of the system are the drum pressure  $P$ , the electric power  $E$ , and the drum water level  $L$  which is calculated by using the following equations for the steam quality ( $\alpha_s$ ) and the evaporation rate ( $\eta_s$ , kg/s):

$$\alpha_s = \frac{(1 - 0.001538x_3)(0.8x_1 - 25.6)}{x_3(1.0394 - 0.0012304x_1)}, \quad (3a)$$

$$\eta_s = (0.85u_2 - 0.147)x_1 + 45.59u_1 - 2.51u_3 - 2.096. \quad (3b)$$

Thus, the system output can be formulated as:

$$y_1 = x_1 = P, \quad (4a)$$

$$y_2 = x_2 = E, \quad (4b)$$

$$y_3 = L = 0.05(0.13073x_3 + 100\alpha_s + \frac{\eta_s}{9} - 67.975). \quad (4c)$$

For the manipulated variables (system inputs) different constraints have to be fulfilled:

$$0 \leq u_1, u_2, u_3 \leq 1, \quad (5a)$$

$$-0.007 \leq \dot{u}_1 \leq 0.007 \quad (5b)$$

$$-2 \leq \dot{u}_2 \leq 0.02, \quad (5c)$$

$$-0.05 \leq \dot{u}_3 \leq 0.05. \quad (5d)$$

## 3. ANALYSIS OF MODEL DYNAMICS

The crucial parts of modeling for fuzzy controller design are 1) the choice of the partitioning variables and 2) the choice of the LLMs (partitioning). In this work a new, effective, and stabilizing partitioning strategy is introduced.

### 3.1 Stationary operating points and linearization

Setting  $\dot{x} = 0 = f(x_0, u_0)$  in (1a) leads to stationary operating points defined by  $x_0$  and  $u_0$ .

Since  $\rho$  is a pure integrator state  $\rho_0$  can be arbitrarily chosen, and (1a) guarantees that  $f(P_0, E_0) = 0$  holds. By exploiting the knowledge that the set-point for drum level  $L$  will always be  $L \equiv 0$ , the corresponding stationary value  $\rho_0$  can be computed from (4c) together with (3a) and (3b). This yields

$$\rho_0 = \rho_0(P_0, E_0) |_{L=0}. \quad (6)$$

Consequently, both states  $x_0 = x_0(P_0, E_0)$  and inputs  $u_0 = u_0(P_0, E_0)$  for stationary operating points only depend on  $(P_0, E_0)$ , see Fig. 1.

The linearized state equation  $\Delta x = F(\Delta x, \Delta u)$  of the plant (2c) for an operating point  $x_0 = [P_0, E_0, \rho_0]$  is given by

$$\frac{d\Delta P}{dt} = -0.0020u_{20}P_0^{\frac{1}{8}}\Delta P + 0.9\Delta u_1 - 0.0018P_0^{\frac{9}{8}}\Delta u_2 - 0.15\Delta u_3, \quad (7a)$$

$$\frac{d\Delta E}{dt} = [0.082u_{20} - 0.018]P_0^{\frac{1}{8}}\Delta P - 0.1\Delta E + 0.073P_0^{\frac{9}{8}}\Delta u_2, \quad (7b)$$

$$\frac{d\Delta \rho}{dt} = [0.002235 - 0.013u_{20}]\Delta P - 0.013P_0\Delta u_2 + 1.66\Delta u_3. \quad (7c)$$

In eqs. (7b)-(7c) the  $\Delta$ -variables are deviations from the respective operating point. Sets of typical operating points also defining  $\rho_0$  and  $L_0$  can be found in Wu et al. (2014); Li et al. (2012).

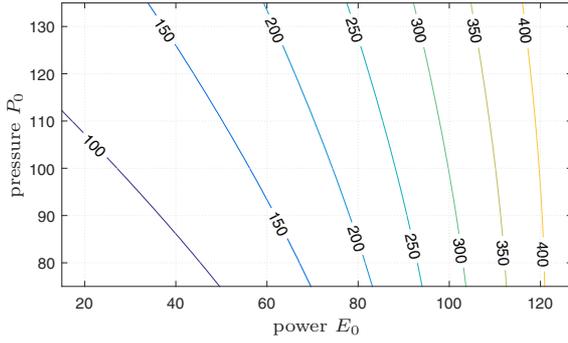


Fig. 1. Contour plot of  $\rho_0 = \rho_0(P_0, E_0)$  for stationary operating points with  $L \equiv 0$ .

### 3.2 Choice of partitioning variables

In a less systematic way the choice of the partitioning variables can be simply all outputs Tan et al. (2005), all outputs and inputs Wu et al. (2014), or the power output only Li et al. (2012).

In this work the knowledge on the system dynamics is exploited: In principle all outputs or states are candidates for partitioning variables, but  $L$  is omitted as  $L \equiv 0$  has been chosen. Furthermore, the inputs can be excluded since all stationary operating points are uniquely defined by  $(P_0, E_0)$ . Hence, only  $P$  and  $E$  constitute the partition space.

### 3.3 Partitioning

A useful measure to assess the validity of a single design model for a range of the nonlinear plant is the  $\nu$ -gap metric, see Du et al. (2009); Tan et al. (2004):

$$\nu = \nu(P_0, E_0), \quad 0 \leq \nu \leq 1. \quad (8)$$

This criterion has been applied for the selection of the LLMs in a simple iterative optimization in Li et al. (2012).

Any transfer function of Eqs. (7b)-(7c) with  $\Delta L$  as output has changing signs in the integrator gain (see the third subplot in Fig. 2); all other transfer functions do not change sign (see Fig. 2). Defining  $G(P_0, E_0) = \mathcal{L}[F(P_0, E_0)]$  the transfer function matrix of plant Eqs. (7b)-(7c), and taking  $G_{3,1}(P_0, E_0) = G_{u_1, y_3}(P_0, E_0)$  the transfer function from  $u_1$  to  $L$ , a partial fractional expansion gives:

$$G_{u_1, y_3}(P_0, E_0) = \frac{R(P_0, E_0)}{s} + \frac{B(s)}{A(s)} \quad (9)$$

The residual  $R(P_0, E_0)$  of the integrator is used to define an alternative criterion

$$\lambda(P_0, E_0) = \frac{c}{|R(P_0, E_0)| + c}, \quad c \in \mathbb{R}^+. \quad (10)$$

Criterion (10) detects the magnitude of the integrator's residual normalized between 0 ( $|R(P_0, E_0)| \gg 0$ ) and 1 ( $|R(P_0, E_0)| = 0$ ). A comparison between the  $\nu$ -gap criterion and (10) is given Fig. 3; both criteria give almost identical results. More important, the set of plants where  $R(P_0, E_0) = 0$  is given by

$$R(P_0, E_0) = G_{u_1, y_3}(P_0, E_0) \cdot A(s)|_{s:A(s)=0} = 0, \quad (11)$$

and can be rearranged as an explicit function

$$E_0 = h(P_0). \quad (12)$$

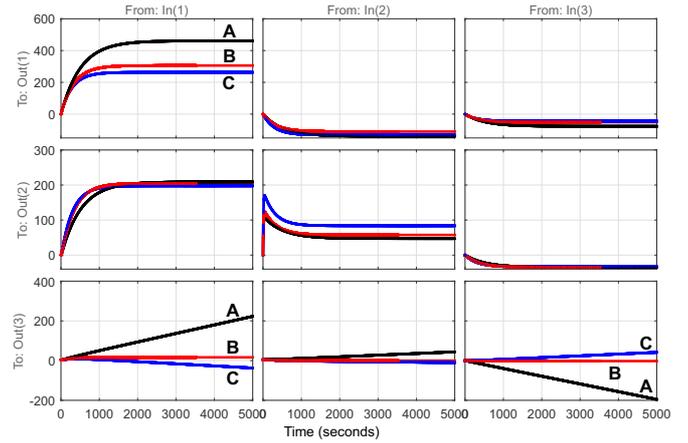


Fig. 2. Step response of the linearized system for three operating points  $(P_0, E_0)$ : A(86.4, 36.7), B(101, 78.4), C(130, 106). Rows correspond to outputs, columns to inputs.

Eq. (12) defines the maxima of both criteria where the integrator gain changes sign.

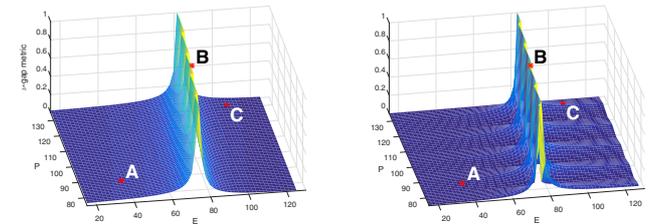


Fig. 3. Partitioning criteria. Left:  $\nu$ -gap value. Right: normalized inverse integrator residual. Red stars mark position and values of the three operating points A, B, and C.

The following assumption allows for a dimensional reduction of the partition space spanned by  $P$  and  $E$ :

*Assumption 1.* Let criteria  $c_1 = \nu(P_0, E_0)$  in Eq. (8) and  $c_2 = \lambda(P_0, E_0)$  in Eq. (10) be partially differentiable  $\forall (P_0, E_0) \setminus (E_0 = h(P_0))$ . Then both fulfill

$$c_i = a \Rightarrow |\nabla c_i| = b, \quad \text{with } a, b \in \mathbb{R}^+. \quad (13)$$

*Theorem 1.* Criteria  $c_i$  defined in Eq. (8) and Eq. (10) can be simplified to

$$\tilde{c}_i = \tilde{c}_i(\delta) \quad (14)$$

where  $\delta = \delta(P_0, E_0)$ ,  $|\frac{d\tilde{c}_i}{d\delta}| = |\nabla c_i|$ , and  $\delta$  is the normal distance between parallel contour lines  $E_0 = h(P_0)$  and  $c_i = a$  in the  $(P_0, E_0)$ -plane.

*Proof 1.* As  $\nu(P_0, h(P_0)) = \lambda(P_0, h(P_0)) = 1$  Assumption 1 implies that all contour lines  $c_i = a$  will be equidistant to  $E_0 = h(P_0)$  with distance  $\delta$ . Moreover, as  $|\nabla c_i| = b$  holds for each point along the contour,  $|\frac{d\tilde{c}_i}{d\delta}| = |\nabla c_i|$  holds.

Function  $\tilde{c}_i$  can be readily constructed from Theorem 1 since  $\frac{d\tilde{c}_i}{d\delta}$  is known and due to Eq. (12)  $\tilde{c}_i(0) = 1$  holds. In Fig. 4 a numerical approximation of  $\nu(\delta)$  is shown. From Fig. 4 it is clear that a partition over  $\delta$  must respect the integrator's sign change at  $\delta = 0$  to achieve performance of the closed-loop, i.e. the partitions must switch at  $\delta = 0$ . In the boiler-turbine plant the integrator's sign change does not pose a direct problem for closed loop

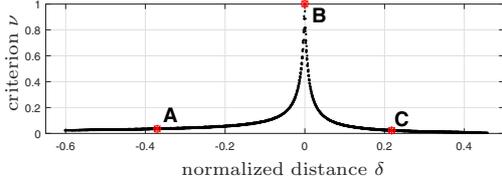


Fig. 4.  $\nu$ -gap criterion over normalized distance  $\delta$  between contour lines and  $\nu(0) = 1$ . Results for operating points *A*, *B*, and *C* also plotted.

stability; nevertheless, in general a value of  $\nu = 1$  can also indicate a sign change in the plant's gain which has severe implications for closed loop stability.

#### 4. FUZZY MODELING OF BOILER-TURBINE UNIT

Fuzzy models can be interpreted as logical models which use “if-then” rules to establish qualitative relationship among the variables in the model. In this work the Fuzzy rules are given as local linear models (LLM), which are two equivalent formulations, see Abonyi (2002).

In case of LLMs the choice of the partition space is the key point of this idea; it should describe the strongest nonlinearities in the process to obtain the best results, see Sec. 3. The resulting membership functions designed for the numerical approximation given in Fig. 4 are illustrated in Fig. 5. For each membership function  $\Phi_i$  a linear plant

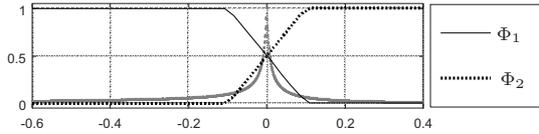


Fig. 5. Fuzzy rules for the plant. Membership functions  $\Phi_i$  are plotted over the partitioning variable  $\delta$ . The result of the  $\nu$ -gap criterion is plotted in gray.

according to eqs. (7b)-(7c) is defined using an operating point  $(P_{0,i}, E_{0,i})$ . The membership functions given in Fig. 5 are used for the controller design and the simulation results, which are discussed and illustrated in Sec. 6.

#### 5. MULTI-INPUT MULTI-OUTPUT (MIMO) FUZZY MODEL PREDICTIVE CONTROL

Standard MPC formulations are well known and given in e.g. Camacho and Bordons (2004). In this paper a local linear model was computed for nonlinear dynamics, see Sec. 4. To avoid non-convex optimization, a set of local linear models was extracted from a TS-Fuzzy model approach which is then utilized by the MPC algorithm, Abonyi (2002).

##### 5.1 Structure of Fuzzy model predictive control

The resulting MIMO Fuzzy MPC (FMPC) structure for the boiler-turbine unit is introduced in Fig. 6.

Each local linear model (LLM) is controlled by its own MPC. The state tracking FMPC controls the drum pressure  $x_1 = P$ , the electric power  $x_2 = E$ , and the fluid density  $x_3 = \rho$ , with the vector of manipulated variables  $\vec{u} \in \mathbb{R}^{3 \times 1} := [u_1, u_2, u_3]^T$ , thus, given as  $u_1 - u_3$ , which are optimized over a given prediction horizon  $n_p$ .

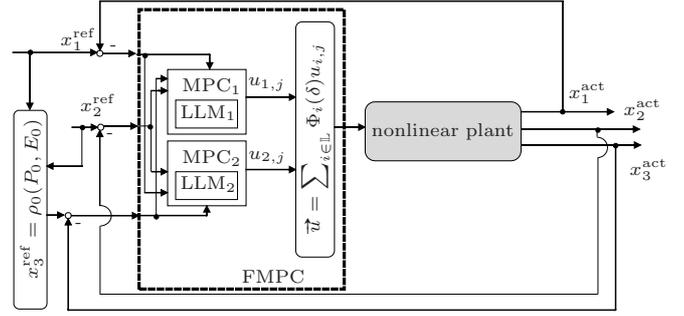


Fig. 6. Scheme of state-controlled FMPC,  $\forall i \in \mathbb{L} = \{1, 2\}$  and  $j \in \{1, 2, 3\}$ , where  $\mathbb{L}$  is the set of the LLMs used for designing the FMPC and  $j$  is the number of the specific system-input. Note that the discrete time-step  $k$  is not considered in this figure.

In this work it is assumed that the drum level  $y_3 = L$  is equal to zero ( $L \equiv 0$ ) in the operating stationary points, thus, the state tracking FMPC is able to control  $L$  to zero in the steady-state operation. This is achieved by setting  $x_3^{\text{ref}} = \rho_0(P_0, E_0)$  utilizing the result from (6). Note that this is effectively a stationary nonlinear feed-forward decoupling of  $L$  from  $P$  and  $E$ . Furthermore, feed-back of  $\rho$  can be implemented efficiently even if  $\rho$  is not measurable. Utilizing (3a)-(3b) and (4c)  $x_3$  can be computed from known inputs and measured outputs  $x_1$  and  $x_2$ . As all of these are algebraic relations, no state estimator is necessary.

The global plant is the simulated nonlinear continuous system given by Eqs. (2a)-(2c) and Eqs. (4a)-(4c). Note that the FMPC actually acts like a parallel connection of linear MPCs with output-blending, which effectively constitutes a nonlinear controller, Tanaka and Wang (2004). The number of LLMs to reproduce the nonlinear effects of the boiler-turbine unit is given by two, therefore, two parallel MPCs are output-blended (one MPC for each chosen operating point), see Sec. 4. The overall blended output (see Chap. 2.1 Tanaka and Wang (2004)) is defined as

$$\vec{u}^k = [u_1^k, u_2^k, u_3^k]^T = \sum_{i \in \mathbb{L}} \Phi_i(\delta)^k u_{i,j}^k \in \mathbb{R}^{3 \times 1}. \quad (15)$$

The Fuzzy trapezoid membership functions  $\Phi_i^k(\delta)$ , calculated with the normalized distance  $\delta$  introduced in Sec. 3, are given for the Fuzzy input vector depending on all LLMs,  $\forall i \in \mathbb{L} = \{1, 2\}$ ,  $j$  is the number of dedicated input, and  $k$  denotes the discrete time-step.

##### 5.2 Optimization algorithm of FMPC

The control objective in this research is given by minimizing a weighted summation of all FMPCs (so-called plantwide cost function). The overall optimization problem is given as:

$$J^* = \min_{\Delta \vec{u}} \sum_{i \in \mathbb{L}} \Phi_i J_i^* \quad \forall i \in \mathbb{L}. \quad (16)$$

In the second step the  $i$ th cost function  $J_i^*$  for each state-tracking MPC can be formulated as:

$$J_i^* = \min_{\Delta \vec{u}_i} J_i(\vec{u}_i) \quad \forall i \in \mathbb{L}, \quad (17a)$$

where

$$\begin{aligned}
 J_i(\vec{u}_i^{k,k+n_p-1}) &= \\
 &= \sum_{k=0}^{n_p-1} [(\vec{x}^{\text{ref},k} - \vec{x}^{\text{act},k})' Q_{3 \times 3} (\vec{x}^{\text{ref},k} - \vec{x}^{\text{act},k}) \\
 &\quad + \Delta \vec{u}^{k'} R_{3 \times 3} \Delta \vec{u}^k] \quad (17b)
 \end{aligned}$$

subject to

$$u_{i,\min} \leq u_i \leq u_{i,\max}, \quad (17c)$$

$$\Delta u_{i,\min} \leq \Delta u_i \leq \Delta u_{i,\max}, \quad (17d)$$

Where  $n_p$  denotes the prediction horizon and “ $'$ ” denotes transpose. Note that the  $\vec{x}^{\text{ref},k}$  are external reference values for each time-step  $k$  and are considered to be known (see Fig. 6).

Furthermore,  $Q_{3 \times 3} \in \mathbb{R}^{3 \times 3}$  is a positive semi-definite weighting matrix, and  $R_{3 \times 3} \in \mathbb{R}^{3 \times 3}$  is a positive definite weighting matrix. The weighting matrices and constraints are set to be equal for all LLMs. The structure of the discrete time state-space model with sampling time  $T_s = 10$  is given in the following state-space formulation:

$$\vec{x}^{k+1} = A_i^k \vec{x}^k + B_i^k \vec{u}^k, \quad (18a)$$

$$\vec{y}^{k+1} = C_i^k \vec{x}^k + D_i^k \vec{u}^k, \quad (18b)$$

where  $i$  represents the matrices linearized in the dedicated operating point  $(P_0, E_0)$ . The operating points in this approach are chosen as:  $(P_0^{\text{LLM}_1}, E_0^{\text{LLM}_1}) = (100, 75)$  and  $(P_0^{\text{LLM}_2}, E_0^{\text{LLM}_2}) = (110, 90)$ . Note that the  $C$  matrix is set to the identity matrix  $C := \mathbb{I}_{3 \times 3}$ , because of the state-tracking FMPC.

## 6. SIMULATION RESULTS

In this section simulation results of controlling the nonlinear boiler-turbine unit with the optimally partitioned FMPC are presented. The chosen sampling time  $T_s = 10$  s for all simulations. Furthermore, identical weights and horizons have been used (prediction horizon  $N_p = 15$  ( $\doteq 150$  s), control horizon  $N_c = 5$  ( $\doteq 50$  s)).

Note that the use of more LLMs would result in a better overall performance, however, the purpose of this simulation section is to illustrate that already with two LLMs a satisfactory performance can be obtained.

### 6.1 Closed-loop performance of FMPC

In Fig. 7 the functionality of the FMPC is illustrated, where the area of the change of sign is slowly transitioned, which poses a big challenge for any controller. Nevertheless, in spite of only two LLMs the proposed FMPC can safely perform the transition.

These results are in contrast to Tan et al. (2005); Li et al. (2012); Wu et al. (2014); Kong et al. (2015), where only steps are presented which require the FMPC to control the plant in the problematic area for a much smaller duration. Related units in the plots are given by (see also Sec. 2):  $y_1 = x_1 = P$  denotes drum pressure in kg/cm<sup>2</sup>,  $y_2 = x_2 = E$  denotes electric power in MW, and  $y_3 = L$  is the drum water level in kg/s.

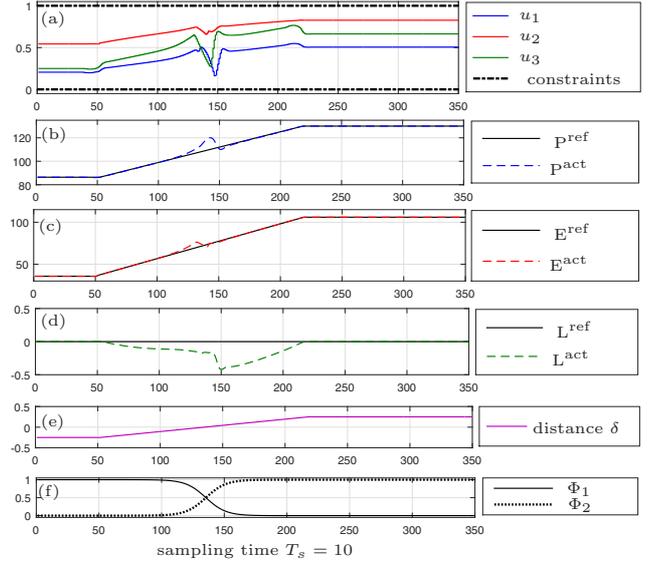


Fig. 7. Performance of FMPC for a ramp starting at sampling time 50.

The ramp’s reference values in Fig. 7 actually connect typical operating points of the plant. In subplot (a) of Fig. 7 the manipulated variables and the constraints are presented. It is obvious that the FMPC can follow the ramp during state-tracking with no active constraints in the manipulated variables. In subplots (b)-(d) the system outputs are illustrated. The benefit of state-tracking is illustrated in subplot (d) because the third output  $L$  is automatically controlled to zero. Fig. 7(e) shows the partition variable  $\delta$ , and the resulting membership functions are given in subplot (f). In Fig. 8 the resulting nonlinear trajectory in the original partition space  $(P, E)$  and the projection of the ridge where  $\delta = 0$  are shown.

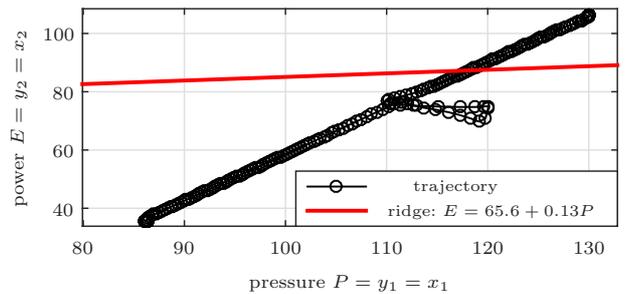


Fig. 8. Trajectory for the ramp transition given in Fig. 7.

In Fig. 9 a large step in the reference values for the states is presented (minimal to maximal admissible operating points). It is illustrated, that constraints are correctly handled by the FMPC, see subplot (a) in Fig. 9. Note that the strict rate constraints defined in (5b) -(5d) are also active. Furthermore, the transition between the Fuzzy models is handled without problems, although the use of only two LLMs leads to some transient deviations. In subplot (b) of Fig. 9 it is illustrated that the constrained FMPC works well and with good performance. In addition, also the drum level  $L$ , given in Fig. 9(d) is controlled to zero, although the FMPC utilizes state-tracking instead of output tracking. The resulting trajectory of the step in the

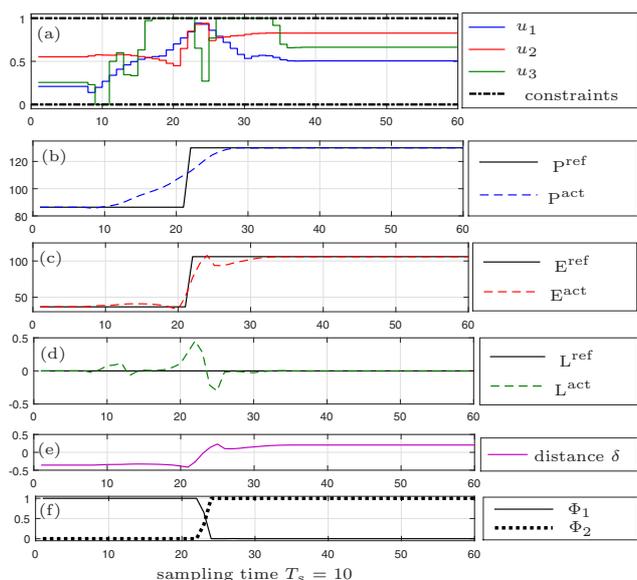


Fig. 9. Performance of FMPC for a step at sampling time 21.

partition space is presented in Fig. 10. In Fig. 10 it is shown that the FMPC handles the transition from one reference value to the other well. Note that the design operating points in this approach are chosen as:  $(P_0^{LLM1}, E_0^{LLM1}) = (100, 75)$  and  $(P_0^{LLM2}, E_0^{LLM2}) = (110, 90)$  which do not coincide with the reference values of the MPCs. The step in Fig. 9 for the pressure  $P$  is given from  $86 \text{ kg/cm}^2$  to  $130 \text{ kg/cm}^2$ , and the step of energy is given from  $36 \text{ MV}$  to  $106 \text{ MV}$ . These steps are far away from the design operating points for the MPCs. These facts clearly demonstrate the effectiveness of the specific FMPC with an optimal partitioning.

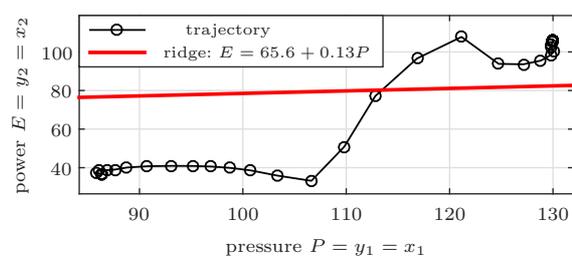


Fig. 10. Trajectory for the step response given in Fig. 9.

## 7. CONCLUSION AND OUTLOOK

A optimal partitioning of a boiler-turbine unit for Fuzzy model predictive control (FMPC) has been presented in this work. A tailored choice of partitioning variables, a partitioning criterion, and a stationary feed-forward decoupling allow for a minimal set of Fuzzy MPCs. Note that the presented methodology can be readily applied to other plants. Especially in the case that some plant gains change signs, the  $\nu$ -gap criterion will detect the problem and provide a suitable stabilizing partitioning strategy. In contrast to existing methods both, the one-dimensional partition space, and the resulting flexible effective MIMO FMPC are unique selling points in this field.

In future work, the presented controller for this plant will be extended with the cooperative FMPC which has been introduced in Killian et al. (2015). The performance of this control and the flexibility in case of actuator fault/drop has been illustrated in Killian and Kozek (2016). Another possible extension is the development of a nonlinear dynamic feed-forward decoupling for the level  $L$  thus improving the transient behaviour.

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