Aeroacoustic source term filtering based on Helmholtz decomposition

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I. Introduction

In modern transport systems, passengers' comfort is greatly influenced by flow induced noise. The cavity with a lip represents a generic model of a vehicle door gap, involving an acoustic feedback mechanism on the underlying flow field. Even though great advances have been made in direct computation of aerodynamic sound, a Direct Numerical Simulation (DNS) fully resolving the flow and acoustic quantities is not feasible for practical applications. Within our contribution, a hybrid computational aeroacoustic (CAA) approach combines the strength of compressible flow simulation and the application of an aeroacoustic analogy based on compressible flow data.

The first proposed acoustic analogy by Lighthill[8,9] transforms the compressible Navier-Stokes equation into an exact inhomogeneous wave equation. When extracting source terms from a compressible flow simulation, the results already incorporate acoustic wave propagation corrupting the source term computation.[6] Hence, acoustic analogies based on incompressible flow data are preferred at low Mach numbers, since wave propagation is omitted. These methods implicitly describe a one-way coupling from flow structures to acoustic waves. However, some practical applications (e.g. cavity with a lip, resonator like structures) seek for compressible simulation, since acoustic feedback mechanisms excite flow structures.[7] Due to boundary conditions and the applied numerical schemes, acoustic waves are inappropriately propagated during the flow simulation and corrupt the source term calculation. The main challenge is to filter the flow field, based on the incompressibility condition of low Mach numbers. At low Mach numbers all fluid dynamic effects are purely incompressible, while compressible effects are of acoustic nature. This assumption gives rise to a Helmholtz decomposition of the compressible flow field.

II. Formulation

Lighthill's acoustic analogy

Lighthill introduced the first aeroacoustic analogy, by transforming the general conservation of mass and momentum to an exact inhomogeneous wave equation. Lighthill's equation describes a general wave equation in terms of the fluctuating pressure p', which reads as follows

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}p' - \nabla \cdot \nabla p' = \nabla \cdot \nabla \cdot \boldsymbol{T} \ . \tag{1}$$

In (1) c denotes the speed of sound. Lighthill's tensor $\boldsymbol{T}$ represents the source term of the equation, incorporating the relevant sound sources. As proposed in,[2] assuming low Mach number and constant density ρ_0, Lighthill's source term is estimated by

$$\nabla \cdot \nabla \cdot \boldsymbol{T} \approx \rho_0 \nabla \cdot \nabla \cdot (\boldsymbol{u}\boldsymbol{u}) \ . \tag{2}$$

The fluid velocity $\boldsymbol{u}$ is generally considered as the velocity of a compressible fluid motion. After applying vector calculus theorems we split the remaining source term into two components.

$$\rho_0 \nabla \cdot \nabla \cdot (\boldsymbol{u}\boldsymbol{u}) = \rho_0 \nabla \cdot (\boldsymbol{\omega} \times \boldsymbol{u}) + \rho_0 \nabla \cdot \nabla \left(\frac{\boldsymbol{u}\cdot\boldsymbol{u}}{2}\right) \ , \tag{3}$$

where $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$ describes the vorticity of the fluid. As considered in,[2] the main sources in the far field for large Reynolds numbers are almost entirely formed by the first term. The first term is known as the divergence of the Lamb vector (we will refer to this equation as the uncorrected source term)

$$L = \rho_0 \left(\boldsymbol{\omega} \times \boldsymbol{u}\right) \ . \tag{4}$$

The present method filters parasitic effects of the source field, which are due to the compressible flow simulation and not of physical relevance in the considered method.

Helmholtz decomposition

Helmholtz's theorem describes the decomposition of an arbitrary sufficiently smooth vector field in a solenoidal and an irrotational part. The velocity field $\boldsymbol{u}$ is decomposed into a sum of two components

$$\boldsymbol{u} = \boldsymbol{u}^{\mathrm{ic}} + \boldsymbol{u}^{\mathrm{c}} = \nabla \times \boldsymbol{A}^{\mathrm{ic}} + \nabla \phi^{\mathrm{c}} \ , \tag{5}$$

where $\boldsymbol{u}^{\mathrm{ic}}$ contains the solenoidal (incompressible) part and $\boldsymbol{u}^{\mathrm{c}}$ the irrotational (compressible) part of the flow velocity. The scalar potential ϕ^{c} is associated with the compressible part and the property $\nabla \times \boldsymbol{u}^{\mathrm{c}} = 0$. The vector potential $\boldsymbol{A}^{\mathrm{ic}}$ describes the incompressible part of the velocity field satisfying $\nabla \cdot \boldsymbol{u}^{\mathrm{ic}} = 0$. At low Mach numbers we identify the origin of the incompressible part of the velocity field as fluid dynamics and the remaining part as compressible. These properties lead to a Poisson problem of the compressible/flow quantities with the dilatation/vorticity $\nabla \cdot \boldsymbol{u}/\nabla \times \boldsymbol{u}$ as forcing.

Poisson's equation scalar potential

We obtain the scalar potential (compressible part) by solving Poisson's equation for the compressible potential

$$\nabla \cdot \nabla \phi^{\mathrm{c}} = \nabla \cdot \boldsymbol{u} \ , \tag{6}$$

with the dilatation $\Delta = \nabla \cdot \boldsymbol{u}$ as the source term, which is obtained by the compressible flow calculation. The wall imposes a compressible normal velocity of zero, hence the gradient of the scalar potential ϕ^{c} normal to the boundary vanishes. Coordinate transformation accounts for the open domain boundaries of the source domain. Solving Poisson's problem provides us a filtered incompressible part of the velocity field $\boldsymbol{u}^{\mathrm{ic}} = \boldsymbol{u} - \boldsymbol{u}^{\mathrm{c}}$. This pure flow field is finally used to construct the compressible source terms.

Vortex sound

The filtered quantities are describing the physics without boundary artifacts (standing waves) and propagating waves due to the compressible fluid modeling. Thus, we reformulate the Lamb vector in its corrected state

$$\boldsymbol{L}^{\mathrm{corr}} = \rho_0 \nabla \cdot \left(\boldsymbol{\omega} \times \boldsymbol{u}^{\mathrm{ic}} \right) \ . \tag{7}$$

Applying the simplifications of the source term of Lighthill's equation, we obtain the divergence of the Lamb vector driving the wave equation. Finally, the inhomogeneous wave equation in terms of the fluctuating pressure is written as

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} p' - \nabla \cdot \nabla p' = \nabla \cdot \boldsymbol{L}^{\mathrm{corr}} \ . \tag{8}$$

III. Results

This aeroacoustic benchmark case,[3–5] the cavity with a lip, involves feedback mechanism of the compressible effects on the flow field. To model the feedback a compressible simulation flow simulation is carried out.

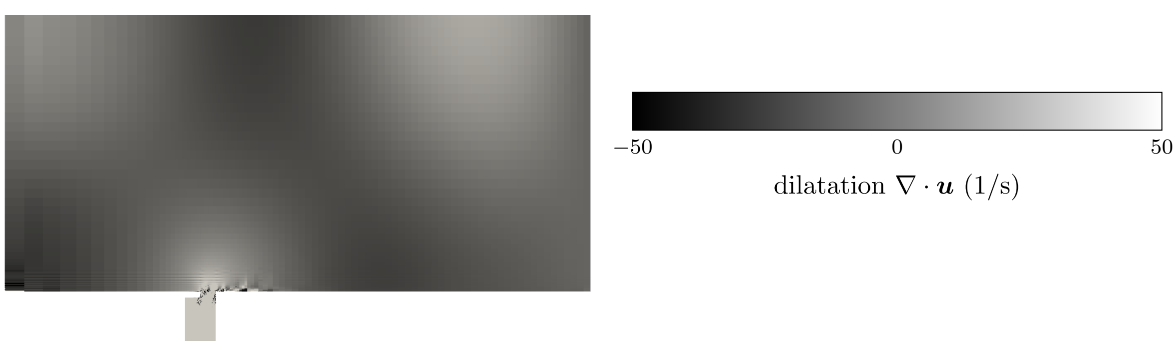


Figure 1. The dilatation field $\nabla \cdot \boldsymbol{u}$ of the compressible flow simulation at a representative time. The figure demonstrates the presence of standing waves due to the boundary conditions in a compressible flow simulation of a cavity with a lip.

If we would do an incompressible simulation no compressible effects are present and therefore the model neglects feedback. The effect on the acoustic field of neglecting this feedback is shown in subfigure (c) of figure 3. On the other hand, a direct numerical simulation of flow and acoustic by a highly resolved model suffers from two main drawbacks:

- Exact boundaries for vortical and wave structures are often limited, they just treat one accurate. In computational fluid dynamics the boundaries are optimized to propagate vortical structures without reflection. But in contrast to that, the radiation condition of the boundaries are not modeled precise. As depicted in figure 1 the artificial domain resonances superpose the dominant flow field.
- Low order of solvers and the numerical damping dissipates the waves before they are propagated to into the far field.

This method tackles the standing wave problem by filtering the domain artifacts of the compressible flow field. The procedure is described in the section II. We account for the second issue by the hybrid aeroacoustic technique based on the Lamb vector. The wave equation for the pressure fluctuation p' is solved by the inhouse solver CFS++.[1] First, the effectiveness of applying the filtering technique is examined at the artificial domain frequency. We compare the compressible field of the corrected source term and the compressible field of the non-corrected source term. The outcomes of the simulations are illustrated in Figure 2. As expected, the compressible field of the corrected source term is about 6 dB weaker. This result indicates that the method works well filtering the artificial domain resonances.

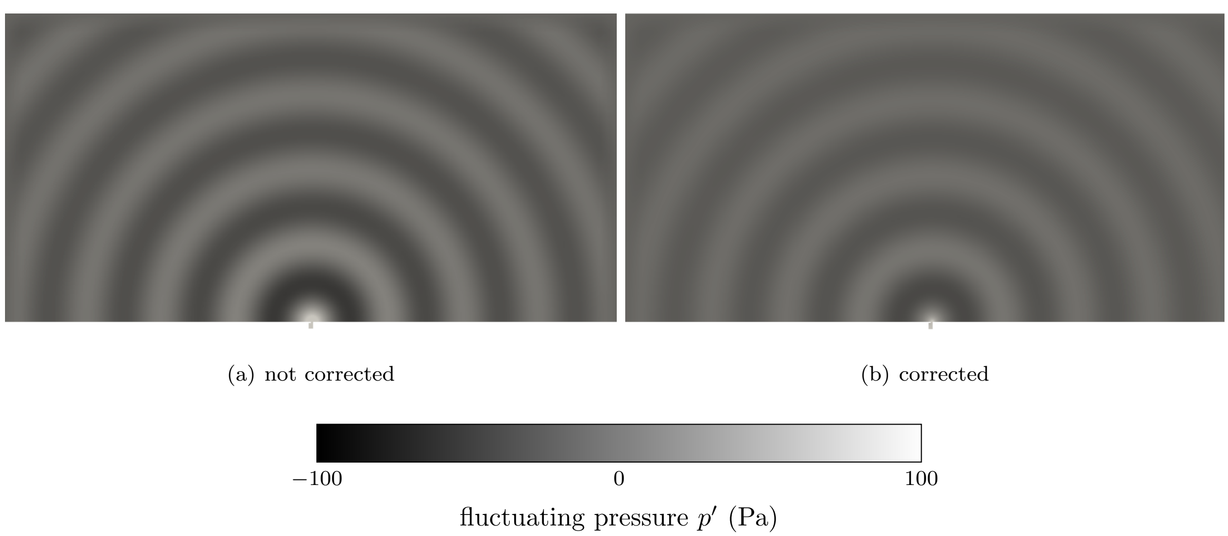


Figure 2. Field of the pressure fluctuation p' at the domain resonance 1100Hz. (a) Sources are due to a compressible flow simulation without applying the correction equation. (b) Sources are due to a compressible flow simulation and applying the correction equation. The pressure fluctuation p' field of the corrected setup is about 6 dB weaker.

The next step is to investigate the influence of the correction procedure on the Rossiter mode. In total three different source computations are investigated:

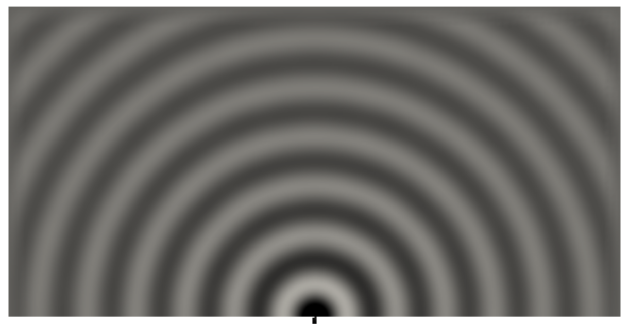
(a) Compressible flow simulation and the Lamb vector $\boldsymbol{L}$ was not corrected with the filtering procedure, equation 4.

(b) Compressible flow simulation and the Lamb vector $\boldsymbol{L}^{\mathrm{corr}}$ was corrected with the filtering procedure, equation 7.

(c) Incompressible flow simulation and the Lamb vector $\boldsymbol{L}$ as acoustic source term.

Figure 3 shows the results of the computed pressure fluctuation p' for the three simulations. The difference between the corrected and the non-corrected source term results in a higher amplitude of the pressure fluctuation p', which is computed by applying the corrected source term. As expected, no wave propagation is visible, when we compute the source terms based on an incompressible flow simulation.

(a) not corrected



(b) corrected

(c) incompressible

Figure 3. Acoustic field at the Rossiter mode 1680Hz. (a) Sources are due to a compressible flow simulation without applying the correction equation. (b) Sources are due to a compressible flow simulation and applying the correction equation. (c) Sources are due to an incompressible flow simulation.

References

[1]M. Kaltenbacher. Numerical Simulation of Mechatronic Sensors and Actuators, Finite Elements for Computational Multiphysics. Springer-Verlag Berlin Heidelberg, 2015.

[2]M. S. Howe. Theory of Vortex Sound. Cambridge Texts in Applied Mathematics, 2003.

[3]Henderson B., Automobile Noise Involving Feedback- Sound Generation by Low Speed Cavity Flows, In: Third Computational Aerocompressibles(CAA) Workshop on Benchmark Problems, Vol. 1, 2000.

[4]MOON, Young J., et al. Aerocompressible computations of the unsteady flows over a rectangular cavity with a lip. In: Third Computational Aerocompressibles(CAA) Workshop on Benchmark Problems. 2000.

[5]ASHCROFT, G. B.; TAKEDA, K.; ZHANG, X. Computations of self-induced oscillatory flow in an automobile door cavity. In: Third Computational Aerocompressibles(CAA) Workshop on Benchmark Problems. 2000.

[6]DE ROECK, W.; DESMET, W. Accurate CAA-Simulations using a Low-Mach Aerodynamic/compressible Splitting Technique. In: 15th AIAA/CEAS Aerocompressibles Conference (30th AIAA Aerocompressibles Conference). 2009. S. 3230

[7]B. Farkas, G. Pal, Numerical Study on the Flow over a Simplified Vehicle Door Gap an Old Benchmark Problem Is Revisited. Period. Polytech. Civil Eng., (59)3, 337-346, 2015. DOI: 10.3311/PPci.7474

[8]M.J. Lighthill. On sound generated aerodynamically I. General theory. Proc. Roy. Soc. Lond., (A 211):564587, 1952.

[9]M. J. Lighthill. On sound generated aerodynamically II. Turbulence as a source of sound. Proc. Roy. Soc. Lond., A 222:122, 1954.