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Numerical Investigation of the Hopf-Bogdanov-Takens Mode Interaction for a Fluid-conveying Tube

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Abstract

We investigate the dynamics after loss of stability of the downhanging configuration of a fluid conveying tube with a small end mass and an elastic support. By varying the fluid flow rate and the stiffness and location of the elastic support, different degenerate bifurcation scenarios can be observed. In this talk we investigate the bifurcating solution branches of the codimension 3 interaction between a Hopf bifurcation and a Takens-Bogdanov bifurcation. An elaboratory discussion of the primary and secondary solution branches was already given by Langford and Zhan. After reducing the system to the three-dimensional Normal Form equations we apply a numerical continuation procedure to locate the expected higher order bifurcation branches and possibly detect more complicated dynamics, like Shilnikov orbits.

Keywords: Fluid-conveying tube; Hopf bifurcation; Takens-Bogdanov bifurcation; Mode interaction; continuation method; Shilnikov szenario
2000 MSC: 34C23; 34C25; 34C37; 37G10; 37G15; 37G20

1. Introduction

The dynamics of fluid-conveying tubes has been studied intensively since the mid of the last century, see e.g. [1–5] due to various reasons: First the onset of oscillations of pipelines, pipes for hydropower plants and cooling towers poses an enormous risk to the environment and should be avoided in almost all circumstances. Second the rather simple mechanical model shows a very rich bifurcation structure and can therefore be considered as a toy problem to investigate various resonance scenarios, which might also occur in realistic structures. For example, Bajaj and Sethna ([4]) studied the three-dimensional problem and demonstrated, that the fluid flow might cause symmetry breaking of the rotational symmetric straight downhanging configuration. Depending on e.g. the wall thickness the bifurcating solution could be a standing or rotating wave.

If the pipe is elastically supported by a spring, also divergence bifurcations and different kinds of mode interaction can occur, leading to complicated dynamical behaviour. The investigation of the effect of a spring support on the

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stability of the straight configuration in [6] demonstrated two important effects: An additional fixation of the pipe might decrease its stability and viscous damping can play an important role and even destabilize the downhanging configuration.

Depending on the location of the elastic support, the author observed diffent types of mode interaction between the Hopf bifurcation and the divergence bifurcation: If the spring is attached close to the midspan, usually a Zero-Hopf bifurcation point occurs, where a real eigenvalue and a pair of purely imaginary eigenvalues simultaneously cross the imaginary axis for a certain value of the spring stiffness and the fluid flow rate. If however the spring is attached close to the free end of the tube, Takens-Bogdanov bifurcations occur: Along the Hopf bifurcation boundary the critical frequency ω decreases to zero and at the intersection with the divergence boundary a non-semisimple double zero eigenvalue occurs. In this contribution we want to explore the ongoing dynamics, when these two cases coalesce: At the Takens-Bogdanov bifurcation point we find also a pair of purely imaginary eigenvalues.

After investigating the primary and secondary solution branches, referring to the study by Langford and Zhan ([7]), we apply the continuation package MatCont ([8]) to explore the higher order bifurcation branches, which give rise to interesting dynamics, like homoclinic and heteroclinic orbits.

2. Mechanical Model and Linear Stability Investigation

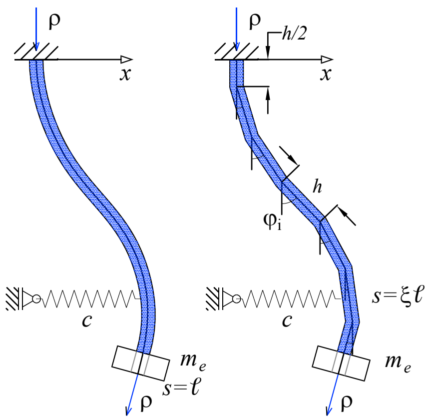

Fig. 1. Continuous and discretized model of a fluid-conveying tube with an end-mass.

We consider a plane tube, which is clamped at the upper end $s = 0$ and carries a point mass m_e at the lower end $s = \ell$. At the intermediate point $s = \xi\ell$ an elastic spring of stiffness c is attached to the tube. A homogeneous fluid of velocity ϱ enters the pipe at the upper end and leaves it at the lower end. According to [5,9] the non-dimensional linearized differential equations and boundary conditions read

$$\ddot{x} + x'''' + \alpha\dot{x}'''' + \varrho^2 x'' + 2\sqrt{\beta}\varrho\dot{x}' - ((m_1 + \gamma(1-s))x')' = 0, \tag{1a}$$

$$x(0) = x'(0) = 0, \tag{1b}$$

$$x''(1) = 0, \tag{1c}$$

$$(x''' + \alpha\dot{x}''')(1) = m_1 x'(1), \tag{1d}$$

$$(x''' + \alpha\dot{x}''')(\xi^+) - (x''' + \alpha\dot{x}''')(\xi^-) = -cx(\xi), \tag{1e}$$

where $(\cdot)'$ and $\dot{(\cdot)}$ denote the derivatives w.r.t. the scaled arclength s and time t, respectively, α denotes the viscous material damping, β is the ratio of the fluid mass to the total mass of the fluid and the tube, m_1 is proportional to the weight of the end mass m_e and γ is the (non-dimensional) weight of the fluid-filled tube per unit length. The presence of the spring leads to a jump of the cross-sectional force $Q = -(x''' + \alpha\dot{x}''')$ at $s = \xi$.

2.1. Stability Boundaries for the Trivial Solution

It can be shown easily, that the trivial state is asymptotically stable for zero flow rate. If ϱ is increased, either a pair of complex eigenvalues will cross the imaginary axis, or a zero eigenvalue will occur. By inserting $x(s,t) = \exp(\sigma t)x_0(s)$ into (1), we obtain a boundary value problem for the eigenfunction $x_0(s)$, which has to be augmented by some scaling condition to obtain a non-zero mode. For the divergence bifurcation $\sigma = 0$ we may simply state the condition

$$x_0''^2(0) + x_0'''^2(0) = 1, \tag{2}$$

whereas for a Hopf bifurcation with $\sigma = i\omega$ the situation is a little bit less trivial: First we split x_0 into its real and imaginary part and rescale the imaginary part:

$$x_0(s) = u(s) + i\omega v(s), \tag{3}$$

which yields the system

$$-\omega^2 u + u'''' + \varrho^2 u'' - ((m_1 + \gamma(1-s))u')' - \alpha v'''' - 2\sqrt{\beta}\varrho v' = 0, \tag{4a}$$

$$-\omega^2 v + v'''' + \varrho^2 v'' - ((m_1 + \gamma(1-s))v')' + \omega^2\left(\alpha u'''' + 2\sqrt{\beta}\varrho u'\right) = 0. \tag{4b}$$

As scaling conditions we choose

$$v''^2(0) + v'''^2(0) = 1, \qquad u''(0)v''(0) + u'''(0)v'''(0) = 0. \tag{5}$$

Furthermore we replace ω^2 by a new unknown parameter η. With these rescalings the system (4) remains regular, when ω converges to 0 and yields $v(s)$ as eigenfunction for $\eta = 0$ and $u(s)$ as generalized eigenvector in that case. Of course, also the dynamical boundary conditions have to be restated accordingly.

The critical flow rate ϱ and the quantity η are regarded as "free parameters", which satisfy the trivial differential equations $\varrho' = 0$ and $\eta' = 0$.

The boundary value problem is solved by the multiple shooting algorithm Boundsco ([10]). In order to obtain the stability boundary in (c, ϱ)-space we apply the continuation routine Hom ([11]), which is very efficient in tracing out loops. For control purposes we also compare the results with the eigenvalue calculation for the discretized model shown in Fig. 1.

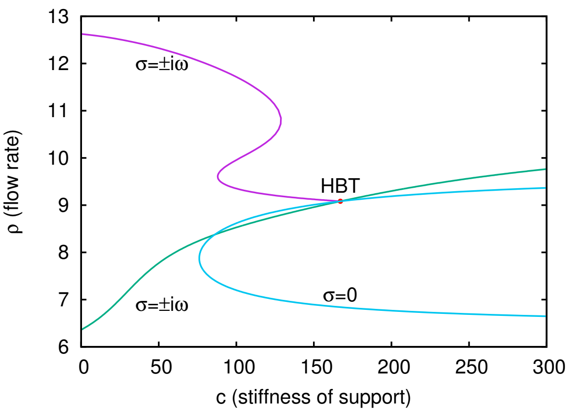

Fig. 2. Stability boundary in the (c, ϱ) parameter plane for a fluid-conveying tube for $\xi = \xi_c \approx 0.8747$. The stability boundaries with a zero eigenvalue ($\sigma = 0$) and with purely imaginary eigenvalue ($\sigma = \pm i\omega$) intersect at the Hopf-Bogdanov-Takens ("HBT") interaction.

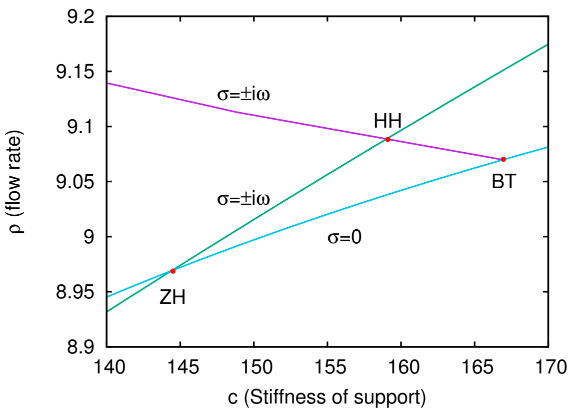

Fig. 3. Stability boundaries for $\xi = 0.8763$: The Hopf-Bogdanov-Takens intersection splits into a Bogdanov-Takens bifurcation ("BT"), a Hopf-Hopf interaction ("HH") and a Zero-Hopf interaction ("ZH"). In the triangular region between these co-dimension 2 bifurcation points the trivial state is stable.

For $\xi = \xi_c \approx 0.8747$ the stability boundary is shown in Fig. 2. For large values of c the trivial state loses stability first by a divergence bifurcation, if the flow rate is increased; for soft springs a flutter instability occurs. The second curve with purely imaginary eigenvalues lies above the first one and meets the $\sigma = 0$ boundary in a Bogdanov-Takens bifurcation. At $\xi = \xi_c$ also the lower Hopf bifurcation boundary runs through this point.

If ξ is increased slightly, the three-fold bifurcation point splits up into three bifurcation points of co-dimension 2: We find a Bogdanov-Takens bifurcation with a double non-semisimple zero eigenvalue, a Hopf-Hopf interaction and a Zero-Hopf interaction. In the triangular region between the connecting stability boundaries the trivial state is asymptotically stable.

If ξ were decreased, the BT and ZH points would survive, with the ZH occurring in an otherwise linearly unstable region. The Hopf-Hopf interaction would vanish, because the small critical eigenfrequency would become imaginary.

2.2. Normal Form Equations for the Hopf-Bogdanov-Takens Interaction

At the HBT interaction the Jordan Normal Form for the critical variables is given by

$$\mathbf{J}_0 = \begin{pmatrix} i\omega & 0 & 0 & 0 \\ 0 & -i\omega & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{6}$$

Since the model has a reflection symmetry about the vertical axis, the equations of motion contain only odd powers. In order to calculate the bifurcation equations up to third order, the Center Manifold approximation for the non-critical modes becomes trivial and the linearly unfolded system is given by

$$\dot{r} = (\lambda + c_1 r^2 + c_2 x^2) r, \tag{7a}$$

$$\dot{x} = y, \tag{7b}$$

$$\dot{y} = (\mu + c_3 r^2) x + (\nu + c_4 r^2) y + c_5 x^3 + c_6 x^2 y, \tag{7c}$$

where λ, μ and ν are the mathematical unfolding parameters, which for the considered choice of parameter values are given by

$$\begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix} = \begin{pmatrix} 1.45718 & -0.0103 & -58.20 \\ -178.759 & 0.6755 & -1331.6 \\ 8.0665 & 0.017 & 63.33 \end{pmatrix} \begin{pmatrix} \varrho - \varrho_c \\ c - c_c \\ \xi - \xi_c \end{pmatrix}. \tag{8}$$

For the rescaled cubic coefficients c_i the following values are obtained:

$$c_1 = 1, \qquad c_2 = 2.16093, \qquad c_3 = -266.391, \qquad c_4 = 9.23651, \qquad c_5 = -1, \qquad c_6 = 3.40442.$$

It should be noted, that in the Normal Form equations the angular variable $\varphi = \arg(z)$ has disappeared and the equations are independent of the frequency ω at the HBT point.

3. Investigation of the Bifurcation Equations

The system (7) was already studied by Langford and Zhan ([7]). Its steady state solutions correspond either to steady states (for $r = 0$) or periodic solutions of the original equations. The following primary solution branches are found to appear close to the bifurcation point:

1. Fast oscillation: $r \neq 0$, $x = y = 0$. Bifurcation equation $\lambda + c_1 r^2 = 0$. It is supercritical for $c_1 < 0$; secondary bifurcations occur for $\mu + c_3 r^2 = 0$, and at $\nu + c_4 r^2 = 0$, if $\mu + c_3 r^2 < 0$. Along the ray $(\lambda, \mu, \nu) = -(c_1, c_3, c_4)x^2$ a secondary Bogdanov-Takens bifurcation occurs.
2. Static buckling: $x \neq 0$, $r = y = 0$. Bifurcation equation $\mu + c_5 x^2 = 0$. It is supercritical for $c_5 < 0$; secondary bifurcations occur for $\lambda + c_2 x^2 = 0$ (fast oscillations around the buckled state) and for $\nu + c_6 x^2 = 0$ (slow oscillation). Along the ray $(\lambda, \mu, \nu) = -(c_2, c_5, c_6)x^2$ a secondary Hopf-Hopf interaction takes place.

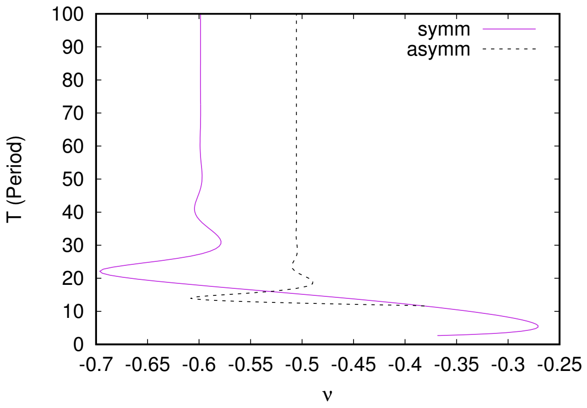

Fig. 4. Bifurcation diagram for the symmetric branch of solutions, which bifurcates at the secondary Hopf bifurcation.

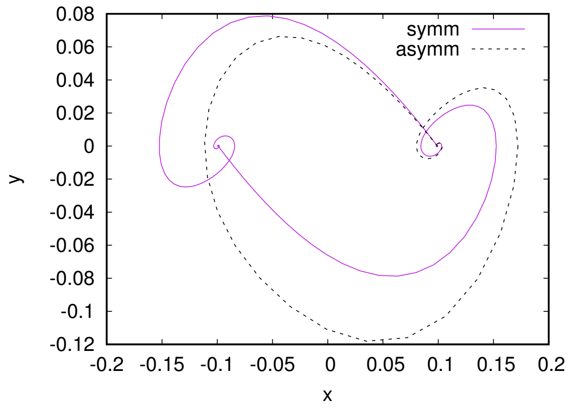

Fig. 5. Phase portrait of the symmetric and asymmetric Shilnikov orbits

3. Slow oscillation: For $\mu < 0$ a Hopf bifurcation occurs along the ray $\nu = 0$. It is supercritical, if $c_6 < 0$. This large periodic solution in the Takens-Bogdanov bifurcation is well studied in the literature (see e.g. [12]). Following a branch of solutions along a circle around the origin in the (μ, ν)-plane, a fold bifurcation ("Limit Point Cycle") can occur and the branch converges to a figure 8 double homoclinic orbit. In our model this solution can also bifurcate out of the $r = 0$ plane, if

$$\lambda + \frac{c_2}{T} \int_0^T x^2(t)dt > 0,$$

where T denotes the period of the slow oscillation.

Next we look at the secondary solution branch of steady states, which connects the buckled state and the fast rotation: It is determined by the bifurcation equations

$$\lambda + c_1 r^2 + c_2 x^2 = 0, \qquad y = 0, \qquad \mu + c_3 r^2 + c_5 x^2 = 0$$

and exists in the sector spanned by the vectors $-(c_1, c_3)r^2$ and $-(c_2, c_5)x^2$ in the (λ, μ)-plane. To find the stationary solutions, we need only solve a linear 2×2 system for r^2 and x^2. Their stability is governed by the Jacobian, evaluated at the stationary solution

$$\mathbf{J} = \begin{pmatrix} 2c_1 r^2 & 2c_2 rx & 0 \\ 0 & 0 & 1 \\ 2c_3 rx & 2c_5 x^2 & \nu + c_4 r^2 + c_6 x^2 \end{pmatrix}. \tag{9}$$

The explicit formula for the eigenvalues involves a lengthy cubic equation and doesn't provide sufficient insight into the evolution of the eigenvalues. Close to the primary branches, if one of the two variables x and r is very small, it is easy to find good estimates, but for general solutions we prefer to obtain the eigenvalues numerically.

3.1. Numerical observations

In order to explore the behaviour of the predicted solution branches, the continuation package Matcont ([8]) is applied, which admits the detection of all kinds of static and periodic solutions and their possible bifurcations.

Starting off with the unstable fast rotation $r_0 = 0.2$, $\lambda = -0.2^2$ and fixing $\mu = 5$, such that $\mu + c_3 r_0^2 < 0$, the parameter ν was decreased from 0 down to $\nu_c = -c_4 r_0^2$, where a secondary Hopf bifurcation occurs.

Matcont follows the branch of symmetric periodic orbits, which originate in this Hopf bifurcation and finds the blue curve displayed Fig. 4: The period increases along the branch, while the distinguished parameter ν oscillates about a fixed value. Along the branch also other bifurcation points are found, like a "Branch Point Cycle" (BPC), where an asymmetric periodic orbit bifurcates from the first branch and also converges to a homoclinic orbit. The two homoclinic orbits are shown in Fig. 5: While the asymmetric orbit is a true (reversed) Shilnikov orbit, the symmetric orbit consists of two heteroclinic trajectories, which connect two saddle points. In both cases the trajectories approach the saddle point along a stable eigenvector and leave it along an unstable spiral.

Along the asymmetric branch also periodic doubling bifurcations occur; these are not shown in Fig. 4, because the bifurcating solutions are almost indistinguishable from the asymmetric branch.

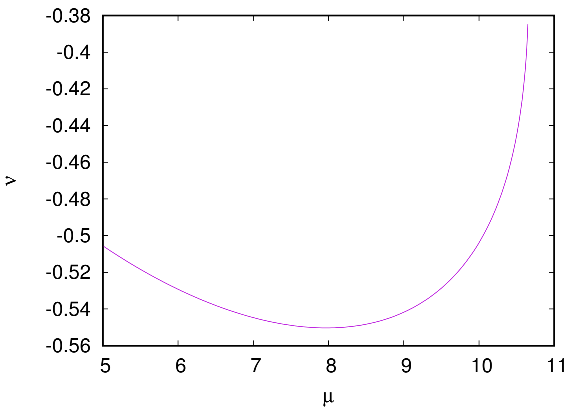

Fig. 6. Branch of long period orbits in the (μ, ν)-parameter plane for fixed $\lambda = -0.04$

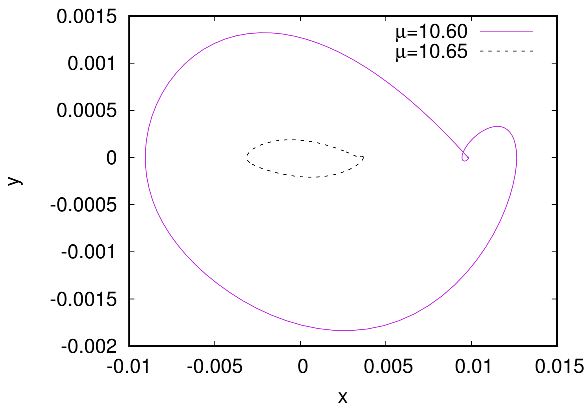

Fig. 7. Phase portrait of small periodic orbits with large periods close to the secondary BT bifurcation

Next we want to know, how the numerically obtained Shilnikov orbit is related to the low level bifurcation branches in the system. For this purpose we fix a large period T and follow the path of long periodic orbits in the (μ, ν) plane, still keeping λ constant. As shown in Fig. 6, this family of periodic solutions converges to the secondary Bogdanov-Takens bifurcations along the branch of fast oscillations $r \neq 0$. (For $r = 0.04$ the BT point lies at $(\mu, \nu) = (10.6556, -0.3695)$ for the chosen coefficients c_i.) The small periodic orbits close to the BT point are displayed in Fig. 7. Given this information, it should be possible, to prove also analytically the birth of a Shilnikov orbit in a Hopf-Bogdanov-Takens interaction.

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