Unbalance Compensation of a Magnetically Levitated Rotor for the Whole Operating Range

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***Abstract*—Unbalance vibration of magnetically levitated rotors can lead to instability caused by saturation of the actuators. Further the rotor orbit can increase to not acceptable values. To overcome this problem of the presented paper an unbalance compensation for rotors with a high unbalance even in the low frequency range was developed. The controller consists of two different approaches based on generalized notch filters. The parameters of the filter are dependent on the angular frequency and can be easily evaluated off-line and stored in a look-up table or implemented as a linear approximation. The robustness for different angular velocity ranges is determined in this paper. Also the dependence of the stability on the parameter of the proposed compensation is illustrated. The proposed approach has advantages in terms of calculating the closed loop stability due to the linear nature and in terms of run time and complexity. Experimental results of the developed unbalance compensation on an industrial system are included.**

I. INTRODUCTION

The importance of active magnetic bearings (AMBs) is significantly increased in recent years. This fact is caused by several attractive advantages compared to classical bearing systems. AMBs have almost no friction, are wearless, have the ability for long-term high speed running and do not need lubricants. But one of the most significant advantages is the possibility to affect the mechanical properties during operation [1], [2]. This fact enables the reduction of unwanted vibrations, which occur during operation. In this context one of the most important reduction techniques is the minimization of the unbalance vibration. Unbalance occurs, if the principal axis of inertia of the rotor is not coincident with its axis of geometry. In most cases it is almost impossible to balance the rotor, because the unbalance distribution is changed during operation. With conventional ball bearings reaction forces occurs due to the unbalance. This forces causes unwanted vibrations, which are transmitted to the machine housing. In contrast with AMBs this reaction forces can be considered in the control loop. The consideration of the unbalance in the control loop is called unbalance control and can have the following different tasks:

- Rejection of synchronous bearing forces: The synchronous bearing current is approximately a quadratic function of the rotational speed. So an upper limit of the angular velocity of an AMB is the actuator saturation, because of the unbalance force. Therefore the aim of this compensation technique is to reject the synchronous bearing current. This can be done by making the controller blind for the unbalance frequency. With this unbalance control the upper limit caused by the angular velocity can be significantly increased.
- Rejection of the unbalance vibration: The aim is to reject the vibration due to the reaction forces of the unbalance and the housing. To get a suitable rejection the system needs high damping forces which can also lead to a saturation of the amplifiers. Thus this balancing technique is a converse approach to the first one, because of the high damping instead of no gain for the unbalance frequency.

A simple realization of unbalance control is the insertion of a Notch filter [3]. The main drawback of this method is that an open loop designed notch filter can destabilize the system. An observer based approach solved the problem of destabilization [4], but the computing time is quite high and an accurate model is required. Another realization is the adaptive feed forward method like it is described in [5] and [6]. The advantage of this approach is that destabilization of the system can only occur if the adaption process itself is unstable. But the adaption process is often complex and nonlinear and convergence cannot be proven in all cases. A phase stabilizing approach of a Notch filter based structures are shown in [8].

This paper is based on the approach of [7], where a generalized Notch filter is used for the rejection of the bearing forces. The Notch filter in this paper is implemented in a more general way and not using the common LTI representation. Therefore numerical problems in the narrow band are eliminated. Due to the linear nature of the resulting system, stability can easily be proven, what is a big advantage of this method. It should be noted that the unbalance information could also be used to track the actual angular velocity. This angular velocity could be used for different compensations, like the compensation of the impact of the gyroscopic effect on the rigid body modes, as it is demonstrated in [9].

The focus of this paper is the design of an unbalance controller which can be used in the whole speed range. The control strategy uses both tasks of unbalance compensation and is based on the generalized Notch filter approach. The controller is tested on an experimental system, which has a high unbalance even for low frequencies. Therefore such an approach is necessary in almost the whole speed range.

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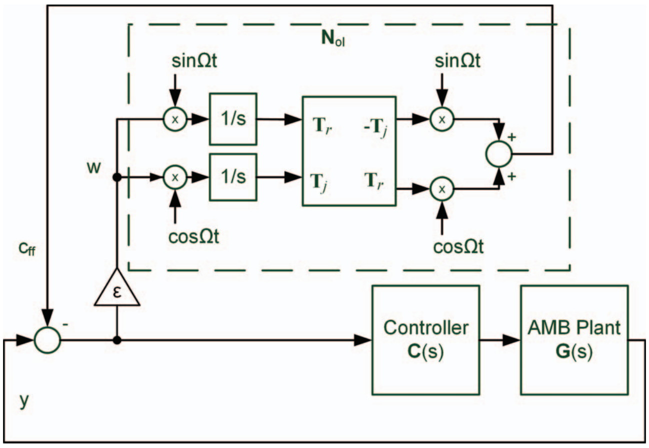

Fig. 1. Force free controller

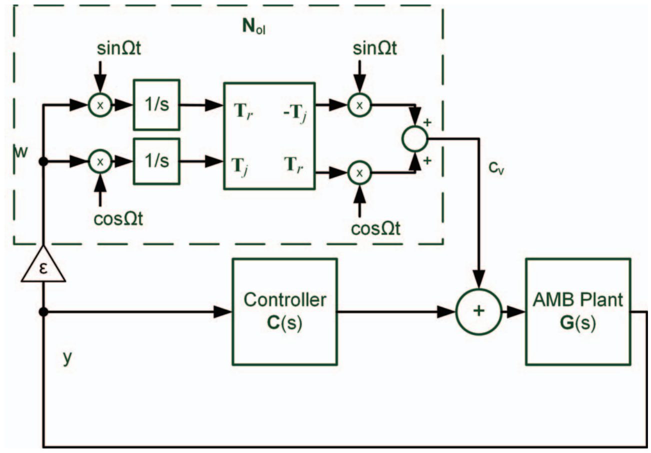

Fig. 2. Vibration controller

The contributions can be summarized as follows:

- In contrast to other publications where only the stability of the closed loop system is calculated, this paper also considers the robustness against possible model errors. This important fact shows which structure is better for a specific angular velocity range.
- The filters are developed regarding to speed tracking of the levitated rotor. Therefore the magnetic bearing controller can run independent of the motor controller.

II. Unbalance Controller

Unbalance controller have the the aim to reject the synchronous bearing force (force free controller) or to reject the synchronous bearing vibrations (vibration controller) caused by the unbalance of the rotor or anisotropic magnetic bearing characteristic. For the rejection of the synchronous bearing forces the controller should be "blind" for synchronous oscillation. The simplest method is the insertion of a Notch filter in the feedback path. However an open loop designed filter can destabilize the system and the LTI implementation shows problems in the narrow band case. The solution is another implementation of the Notch filter using a two modulation step approach, which is shown in Fig. 1. The closed loop system with $\mathbf{C}(s)$ and $\mathbf{G}(s)$ is assumed to be stable. For the controller $\mathbf{C}(s)$ a decoupling approach like in [9] is used. The plant is a 5-DOF active magnetic bearing, where the radial movement is described by the linearized equation

$$\begin{aligned}\mathbf{M}\ddot{\mathbf{x}} + \mathbf{G}(\omega)\dot{\mathbf{x}} + \mathbf{B}\mathbf{K}_s\mathbf{B}^T\mathbf{x} &= \mathbf{B}\mathbf{K}_i\mathbf{i} \\ \mathbf{y} &= \mathbf{C}\mathbf{x}\end{aligned} \tag{1}$$

with the mass matrix $\mathbf{M}$ the matrix of the gyroscopic effect $\mathbf{G}(\omega)$ the transformation matrix from the bearing coordinates to the center of gravity coordinates $\mathbf{B}$, the force to current factors $\mathbf{K}_i$ the position vector $\mathbf{x}$, the current vector $\mathbf{i}$, the matrix of the negative stiffness $\mathbf{K}_s$, the output transformation $\mathbf{C}$ and the output vector $\mathbf{y}$.

Due to the unbalance vibration the sensor signals contains a synchronous sinusoidal part. The aim of the compensator $\mathbf{N}_{ol}$ is to generate a compensation signal with the same amplitude, phase and frequency like the unbalance vibration and subtract it from the sensor signal. Therefore the sensor signal is split up into two signals. One signal is multiplied by $\sin(\Omega t)$ and the other one by $\cos(\Omega t)$ to shift the frequency Ω down to zero. After this frequency shift the signal is integrated and shifted back to the frequency Ω using the multiplication with $\sin(\Omega t)$ and $\cos(\Omega t)$. The resulting signal c_{ff} is subtracted from the position signal. For the integration action the frequency Ω of the sensor signal is shifted to zero. Therefore the integration action eliminates the DC value. Because this DC value corresponds with the frequency Ω in the non modulated system, the synchronous signal part is cancelled from the sensor signal. To stabilize the system the entries from $\mathbf{T}_r$ and $\mathbf{T}_j$ are from great relevance. In literature, the multiplications with the trigonometric function are often replaced by transformation in rotating frames. [7] stated that this transformation has problems with oval orbits due to bearing anisotropies.

For the rejection of the synchronous bearing vibration the controller should add a synchronous force to the output of the controller. The resulting transfer function is an inverse Notch filter with the filter frequency Ω. However the same stability problem like for the force free controller occurs, if open loop designed filters are inserted into the feed back loop. Fig. 2, shows that the same structure N_{ol} as for the force-free controller can be used. Only the compensation signal c_v is added to the current instead of feed it back to the sensor signal. However the vibration controller in the current form cannot be used in combination with the speed tracking method of [10] (no communication with the motor controller is necessary), because the unbalance is completely compensated. Thus the integration action can be substituted by low pass filters.

The compensation signals c_{ff} or c_v are calculated from the unbalance controller N_{ol} with:

$$\mathbf{c}_{ff}(t) = [\sin(\Omega t)\mathbf{I}\ \cos(\Omega t)\mathbf{I}] \begin{bmatrix} \mathbf{T}_r & -\mathbf{T}_j \\ \mathbf{T}_j & \mathbf{T}_r \end{bmatrix} \cdot \int \begin{bmatrix} \sin(\Omega t)\mathbf{w}(t) \\ \cos(\Omega t)\mathbf{w}(t) \end{bmatrix} dt \quad (2)$$

To get the input output description, which is used for stability analysis, equation (2) is transformed in the s-domain.

$$\mathbf{N}_{ol} = \frac{1}{s^2 + \Omega^2} (s\mathbf{T}_r - \Omega \mathbf{T}_j) \quad (3)$$

From equation (3) an inverse notch characteristic can be confirmed, because if $s = j\Omega$ the denominator becomes zero. Nevertheless the two modulation Notch filter has some advantages compared to the LTI implementation:

- If ϵ (Fig.1) is been set to zero, the filter is an ideal feed forward compensation. A LTI implementation does not allow this useful operation.
- If the integrators are converged the output of the force-free controller c_{ff} shows the unbalance in rotating coordinates.
- For practical implementation the LTI Notch filter shows digitalization problems in the narrow band case. However this problem is cancelled mainly by the two modulation step implementation.

III. Controller Design and Stability Analysis of the Closed Loop System

To design an acceptable unbalance control strategy it is important to know the performance limitations of the different filters dependent on the angular velocity.

A. Force-Free Controller

To choose the parameter of the force free controller it is important to know, where the closed loop system is stable dependent on the compensation parameter. The poles of the closed loop system extended with the force free controller can be calculated with:

$$0 = \det[\mathbf{I} + \mathbf{S}(s)\mathbf{N}_{ol}(s)] \quad (4)$$

where $\mathbf{S}(s)$ is the sensitivity matrix of the stabilized system without unbalance compensation. For a stable system all poles have to be in the left side of the pole zero map. Thus the eigenvalues must have a positive damping factor. In the special case of a decoupled or slightly coupled system a decentralized unbalance controller can be used. The main advantage of such a controller is the much lower computing time. [9] and [10] also developed control structures to decouple magnetic bearing system. Therefore systems with high couplings can also use the decentralized control structure. Finally can be stated, that a decentralized structure can be implemented for many cases. To find optimal parameters the system is decoupled in translation and tilting movements. This decoupling is implemented in many control structures for magnetic bearings. For a decentralized implementation the matrices have a diagonal form $\mathbf{T}_r = \text{diag}(T_{r1}, ..T_{rn})$ and $\mathbf{T_j} = \text{diag}(T_{j1}, ..T_{jn})$ and the parameter for both tilting and both translation movements are the same. Instead of T_{rn}, T_{jn} the angle α_n is used for stability analysis. T_{rn}, T_{jn} and α_n full fills the following relations:

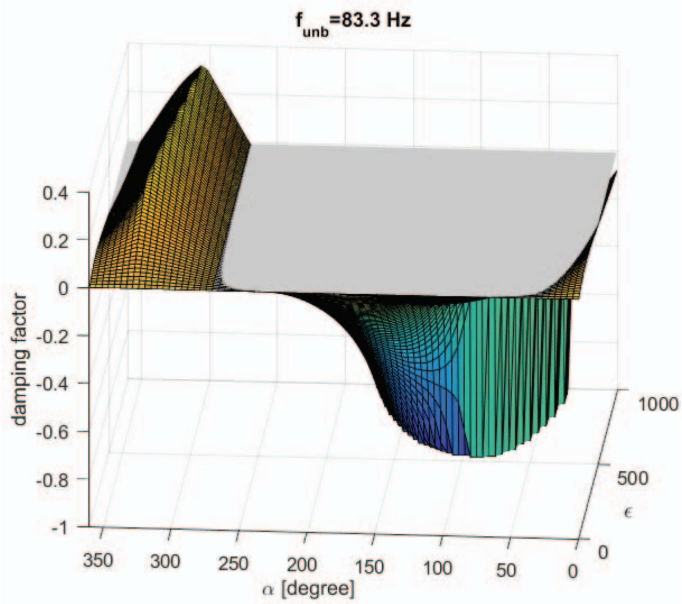

Fig. 3. Closed loop pole dependent on α and ϵ

$$T_{rn} = \cos(\alpha_n) \quad (5)$$

$$T_{jn} = \sin(\alpha_n) \quad (6)$$

Fig. 3 shows the damping ratio of one eigenvalues dependent on the parameter α_n and ϵ for an constant unbalance frequency of 83.3 Hz. The grey plane shows the boarder between an unstable and a stable eigenvalue. For a decoupled system there are two eigenvalues which are strong affected by the parameter of the unbalance controller. Both eigenvalues have the same dependence on the parameter in the stable range, because they are conjugate complex. Only for some unstable ranges two real poles result from the conjugate complex pole pair. Thus one resulting pole has a damping of -1 and the other one a damping of 1. This effect can be seen in the most of the following Figures. The stability of the system is mainly affected by the parameter α_n. This is the reason why ϵ_n and α_n are used instead of T_{rn} and T_{jn}. Otherwise the stability will be strong affected by two parameters T_{rn} and T_{jn} and therefore the design process will be more difficult. However too high values of ϵ_n decreases the bandwidth of the positive damping. If ϵ_n is even more increased the bandwidth of the stable solutions will be zero. But if the value of ϵ_n remains in a range where the bandwidth is big, the stability of the system can mainly be tuned by α_n. Fig. 4 shows the damping ratio of one eigenvalues dependent on the parameter α_n and the unbalance frequency f_{unb}. To illustrate the dependence even for low frequencies a logarithmic scale for the unbalance frequency axis was chosen. It can be seen that the bandwidth of positive damping increases for higher angular velocity ranges. It is theoretical possible to get a stable unbalance

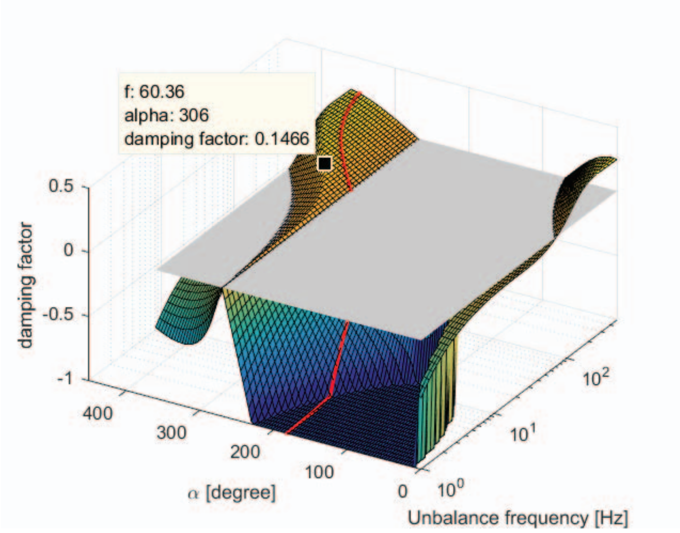

Fig. 4. Closed loop pole dependent on α and the rotation frequency f_{unb}

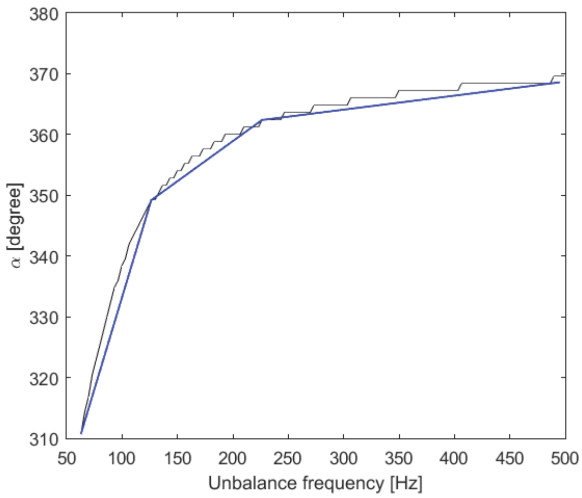

Fig. 5. Optimal angle α in the sense of damping

compensation for the whole angular velocity range. However from the practical point of view the compensation is only realizable for a minimum bandwidth. Because the system cannot be modelled exactly and a minimum of robustness is required for an industrial application. 60 Hz was chosen as starting unbalance frequency of the force free control to have a high enough robustness in this paper. The red line shows the damping factor, if the parameter T_{rn} and T_{jn} are chosen as the inverse of the sensitivity function, like it was proposed by [7]. However this method shows only stable solutions for higher frequencies for the present system. For implementation the parameters with the highest resulting damping according to Fig. 4 dependent on the unbalance frequency were used. This function of the maximum damping is linearized for three different unbalance frequency areas in the operating range according to Fig. 5. Thus an simple and robust implementation on a digital signal processor is achievable. Summarizing can be stated that the force free controller is very robust for high frequencies. However this unbalance rejection structure is not preferable for low frequencies in industrial applications.

B. Vibration Controller

In some cases the unbalance for low frequencies is already too high. Thus an compensation in the low speed range could be necessary. To explore for which parameter the vibration controller is robust and shows a good performance, the damping value of one pole is plotted dependent on different parameters. The closed loop poles of the system extended with the damping controller are given by:

$$0 = \det\left(\mathbf{I} + \mathbf{G}(s)\left[\mathbf{C}(s) + \mathbf{N}_{ol}(s)\right]\right) \tag{7}$$

Fig. 6 and Fig. 7 shows the damping value dependent on ϵ and α for two different unbalance frequencies f_{unb}. To illustrate a sufficient range the ϵ axes is plotted using a logarithmic scale. It can be seen that the maximum achievable damping factor decrease with increasing unbalance frequency. For 1 Hz the

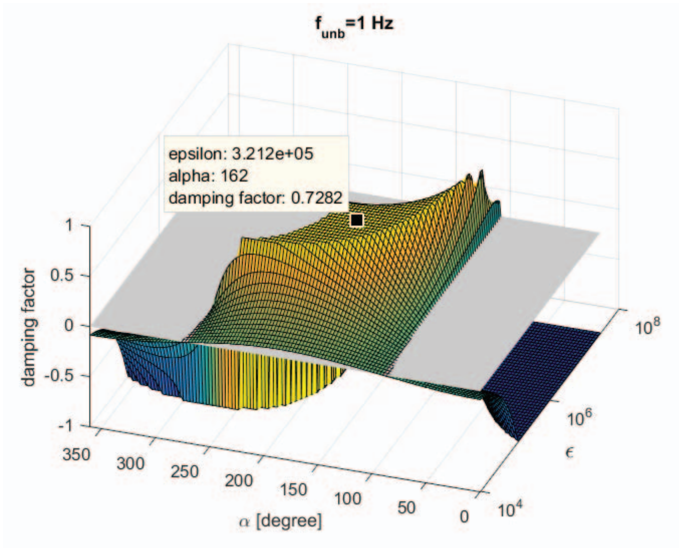

Fig. 6. Closed loop pole with vibration control dependent on α and ϵ with $f_{unb} = 1$ Hz

maximum damping is 0.7282, but for 200 Hz the maximum achievable damping factor is only 0.1897. However Fig. 7 shows, that this high damping factor cannot be reached for experimental systems, because the robustness is very low. If a small model error occur the system will get unstable. The damping value for a sufficient robust compensation is in a range of 0.05 for an unbalance frequency of 200 Hz. For 1 Hz the damping value of 0.7282 can be reached for experimental systems. But also in this case for industrial application the damping will be chosen lower to get a higher robustness of the system. It should be noted that this damping of the eigenvalues has nothing in common with the damping of the rigid body modes. Therefore it is possible to have an sufficient unbalance control even with a damping factor of 0.05. Another interesting fact is that the bandwidth of the stable solutions is more or

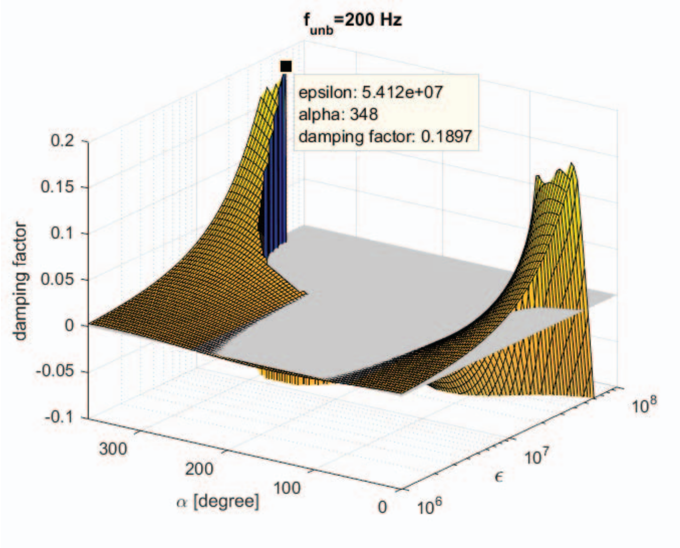

Fig. 7. Closed loop pole with vibration control dependent on α and ϵ with $f_{unb} = 200$ Hz

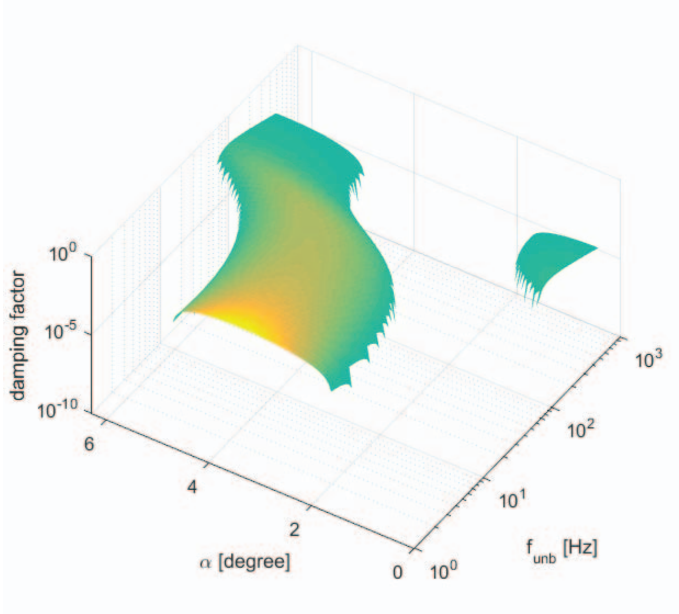

Fig. 8. Closed loop pole with vibration control dependent on α and f_{unb}

less the same for different speeds in the case of an vibration controller. For the force free controller this is not the case. The value ϵ for a sufficient damping must however be increased for higher frequencies.

Fig. 8 shows the damping factor of one eigenvalue dependent on f_{unb} and α for a linear increasing ϵ. Because the maximum achievable damping factor decreases with increasing unbalance frequency, the damping and the unbalance frequency axis is plotted using logarithmic scales. Therefore only positive damping factors are shown. The stability is mainly affected by the parameter α, what is almost equal to the force free controller. However the stability margin is sufficiently high for the whole speed range. Thus the damping controller can be used for every operating point. Only the maximum achievable damping decreases with increasing unbalance frequency. Finally can be stated that the vibration controller is robust for the whole angular velocity range. Only the damping will be significantly reduced. This is caused by the double integration behaviour of the closed loop system above the rigid body modes. However this unbalance rejection structure works well for low frequencies in industrial applications. But the usage can also be extended for high frequencies. If a pump has almost a high unbalance for low frequencies the vibration controller is a good choice as alternative to the force free control. For higher frequencies the vibration controller requires more and more current. Therefore a change to the force free controller could be useful.

IV. Experimental results

This section gives experimental results of the proposed unbalance structures obtained from a turbo molecular pump. The mechanical air-gap of the magnetic bearings are 120 μm and the maximum current is 2 A. Fig 9 shows that the pump has already a high unbalance amplitude A_{unb} at low angular velocities (red line) without compensation. As stated in the previous sections the force free controller is not very robust in this low frequency ranges. Thus a vibration controller is used in the low frequency range. However this unbalance controller cannot be used for very high speeds, because the actuator will saturate. For higher frequencies a force free controller is required. Therefore the angular velocity limit due to the unbalance force is significantly increased. The angular velocity is estimated using an unbalance observer according to [10]. As a consequence the unbalance must not be eliminated completely. This can be realized by exchanging the integrator by low pass filters. It is not beneficial to switch directly between both unbalance controller, because of the convergence time. It is rather better to run both structures in parallel for a sufficient switching time. In Fig. 9 the unbalance is not compensated in the lowest angular velocity range up to 2200 rpm. To retain a high enough distance to the actuator, the vibration controller is switched on and the unbalance vibration is reduced to a defined value. However the current increases severe. At about 4000 rpm the force free controller is switched on and run in parallel with the vibration controller. For angular velocities over 4300 rpm the vibration controller is switched of. The unbalance peak at 4300 rpm is caused by the switching action and the defined convergence speeds of both controller. The unbalance amplitude is more then a factor 4 lower as the mechanical air gap in the tested speed range. If the angular velocity or the angle is measured by an external device, almost the whole unbalance can be eliminated. But in the case of turbo molecular pumps the vibration nulling has not a high priority.

Fig. 10 shows the switching on action of the vibration controller. To show the performance of the control structure the compensation is switched on at 2800 rpm. The convergence speed of the vibration controller is about 0.3 s. There is also

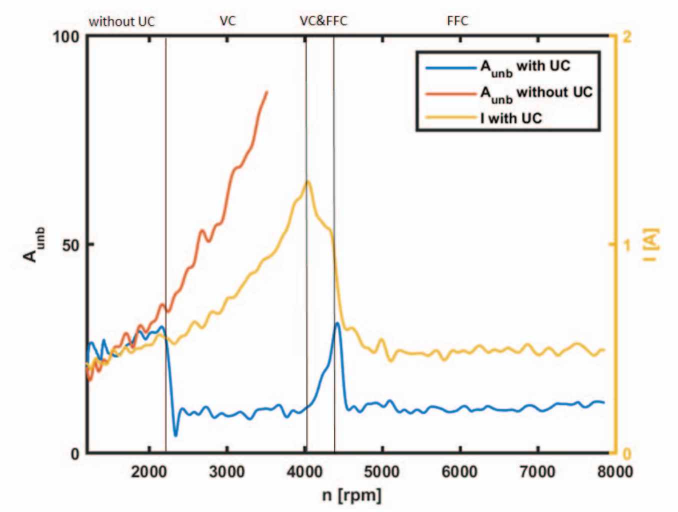

Fig. 9. Unbalance amplitude using force free and vibration control

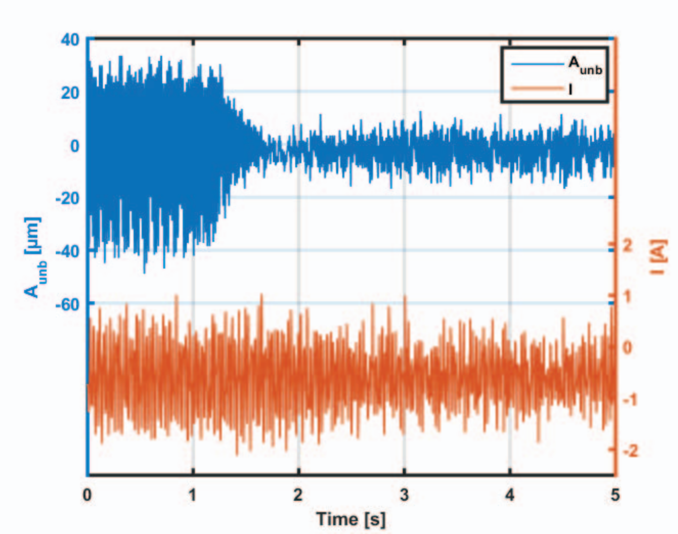

Fig. 10. Unbalance amplitude vibration control in the time domain

a part of the synchronous vibration left due to the unbalance observer. However, the current is almost constant before and after the vibration controller is switched on. Finally can be stated that the synchronous vibration is significantly reduced but the resulting current is almost constant. This fact proves a good damping effect in low speed range.

V. Conclusion

In this paper an unbalance control strategy for an active magnetic bearing system with a high unbalance amplitude in the low speed range was developed and tested. The unbalance control consists of two different approaches for different speeds (vibration control and force free control). For both unbalance structures stability, performance and robustness analysis were made. From the analysis can be concluded that the designed force free controller cannot be used for industrial applications in the whole operating range, because of the very low robustness. However the robustness increases with increasing speed. The vibration controller can be used for the whole speed range. But the vibration cancellation requires high currents and an upper limit of the angular velocity has to be considered, because of a possible saturation of the actuator. The resulting system uses the vibration controller in the low speed range where the force free control has a low robustness. In the range where the force free controller has a sufficient robustness the compensation system switches from the vibration controller to the force free controller. The performance of the developed control structure is tested on a turbo molecular pump.

References

[1] H. Bleuler, *Decentralized Control of Magnetic Rotor Bearing Systems*, PhD thesis, Federal Institute of Technology Zürich, 1984.

[2] G. Schweitzer, *Magnetic Bearings Theory, Design, and Applications to Rotating Machinery*, Springer Berlin, 2009, ISBN: 978-3-642-00496-4.

[3] C.R. Knospe, *Stability and performance of notch filter controllers for unbalance response*, Int. Symp. Magn. Suspension Technol.,NASA Langley Research Center, Hampton, VA, NASA Conf, Pub.3152, 1991.

[4] N.K. Rutland, P.S. Keogh and C.R. Burrows *Comparison of controller design for attenuation of vibration in a rotor-bearing system under synchronous and transient condition*, 4th International Symposium on Magnetics Bearings, Zürich, 1994.

[5] N. Taguchi, T. Ishimatsu, S.J. Woo and C. Ghler *Unbalance compensation of magnetic bearings*, IECON, Beregna, 1994.

[6] Z.-H. Liu, K. Nonami, and Y. Ariga, *Adaptive Unbalanced Vibration Control of Magnetic Bearing Systems with Rotational Synchronizing and Asynchronizing Harmonic Disturbance*, JSME International Journal - Series C, vol. 45, no. 1, pp. 142-149, 2002.

[7] R. Herzog, P. Bühler,, C. Gähler and R. Larsonneur, *Unbalance Compensation Using Generalized Notch Filters in the Multivariable Feedback of Magnetic Bearings*, IEEE Transactions on Control Systems Technology, September 1996.

[8] T. Nakamura, S. Wakui,, C. Gähler and Y. Nakamura, *A Phase Stabilization Method for Unbalance Vibration Control of Five-axes Active Magnetic Bearing Systems*, 2014 IEEE/ASME International Conference on Advanced Intelligent Mechatronics, Besanon, July 2014.

[9] M. Hutterer, M. Hofer, M. Schrödl, *Decoupled Control of an Active Magnetic Bearing System for a High Gyroscopic Rotor*, IEEE International Conference on Mechatronics, 2015

[10] M. Hutterer, M. Hofer, M. Schrödl, *Selfsensing Unbalance Rejection and Reduction of the Gyroscopic Effect for an Active Magnetic Bearing System*, 10th Asian Control Conference, 2015.