EFFICIENT OFDM CHANNEL ESTIMATION IN MOBILE ENVIRONMENTS BASED ON IRREGULAR SAMPLING

Peter Fertl and Gerald Matz

Institute of Communications and Radio-Frequency Engineering, Vienna University of Technology
Gusshausstrasse 25/389, A-1040 Vienna, Austria
phone: +43 1 58801 38942; fax: +43 1 58801 38999; email: peter.fertl@nt.tuwien.ac.at

ABSTRACT

We present a novel pilot-aided channel estimation method for OFDM packet transmissions over time-varying channels. The proposed channel estimator is based on 2-D irregular sampling techniques that allow for flexible training data arrangement. The resulting iterative conjugate gradient algorithm involves block Toeplitz matrices and thus has low computational complexity. We apply our scheme to a WLAN system by jointly exploiting preamble and continual pilot subcarriers (that are present for the purpose of frequency offset tracking) for channel estimation. Simulation results confirm the excellent performance of our method, which is thus an attractive add-on to WLAN and WiMAX receivers in high-mobility scenarios.

1. INTRODUCTION

Motivation. Orthogonal frequency-division multiplexing (OFDM) is an important broadband wireless communication scheme [1]. OFDM packet transmissions are currently used in wireless local area networks (WLAN) like IEEE 802.11a,g [2] and in broadband wireless access systems (e.g. IEEE 802.16a [3]). Moreover, it seems to be a promising technique for wireless access in vehicular environments (WAVE) (see the current draft of IEEE 802.11p [4]).

Coherent detection in such systems requires accurate channel state information (CSI) at the receiver. Usually, CSI is obtained by embedding training data into the transmit signal (known as pilot-symbol-aided channel estimation) and using a least squares (LS) [5] or a minimum mean-square error (MMSE) estimator [6]. In packet-oriented OFDM systems, the training data is usually given by a preamble consisting of one or two OFDM symbols at the beginning of each packet. In this paper we consider high-mobility scenarios where large Doppler frequencies induce rapid channel variations. In this case, state of the art channel estimators can provide reliable CSI only for very short packets (i.e., consisting only of a few data symbols after the preamble). Short packets are undesirable, however, since they increase the relative amount of training and thus reduce net data rate.

A more suitable approach for estimating time-varying channels scatters training symbols into the data stream. To recover the unknown channel coefficients, one-dimensional (1-D), double 1-D, and two-dimensional (2-D) MMSE filtering algorithms have been proposed [7–9]. However, this is satisfactory only for rectangular pilot lattices that obey the Nyquist criterion [10], thus prohibiting more flexible pilot arrangements. Furthermore, these MMSE filters require knowledge of the second order channel statistics and may have high computational complexity.

Contributions. The main contributions of this paper can be summarized as follows:

- We propose to use 2-D irregular sampling techniques for OFDM channel estimation that even work in the case where pilots are nonuniformly scattered in time and frequency.
- We develop an adaptation of the 2-D irregular sampling algorithm from [11] which results in a low-complexity channel estimator for fast time-varying channels.
- Finally, we present simulation results for a WLAN system where preamble and continual pilot carriers (usually present for the purpose of frequency offset tracking) are exploited for channel estimation.

The main advantages of our 2-D irregular sampling based channel estimation scheme are (i) enhanced flexibility regarding the pilot arrangement; (ii) excellent performance in terms of accurate CSI; (iii) low computational complexity which in contrast to existing techniques does not scale with the number of pilot symbols; (iv) apart from coarse estimates of the maximum delay and the maximum Doppler no other prior knowledge is required. We note that recently 1-D irregular sampling ideas have been independently proposed in a different channel estimation context in [12].

2. SYSTEM MODEL

Transmission Setup. We consider a cyclic prefix (CP) OFDM system with K subcarriers and total transmit bandwidth B. The duration of one OFDM symbol equals $(K + L_{\mathrm{CP}})T_s$ seconds where L_{CP} denotes the cyclic prefix length and $T_s = 1/B$ denotes the sampling interval. A packet consists

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of N consecutive OFDM symbols and can thus carry in total NK symbols which are obtained from the data bits via channel coding, interleaving, and mapping. The maximum delay of the channel is assumed to be smaller than the cyclic prefix length. While the channel may vary within a packet, it is assumed to be effectively constant within each OFDM symbol. With perfect synchronization, the corresponding system model is then given by

$$Y[n,k] = H[n,k]\, X[n,k] + Z[n,k]. \qquad (1)$$

Here, $n \in \{0,\ldots,N-1\}$ and $k \in \{0,\ldots,K-1\}$ are the symbol and subcarrier index, respectively; furthermore, $Y[n,k]$ and $X[n,k]$ are the receive and transmit symbols, respectively, $H[n,k]$ denotes the channel coefficients, and $Z[n,k]$ is additive white Gaussian noise with variance σ_Z^2.

Training Data. Within each packet, P of the NK symbols $X[n,k]$ in (1) are reserved as pilot symbols for training. The corresponding set of pilot positions is denoted as $\mathcal{P} = \{(n_p,k_p),\ p=1,\ldots,P\}$. Below we will see that the placement of the training symbols is extremely flexible. In particular, we explicitly allow for nonuniform pilot arrangements (details are described in Section 4).

Channel Model. We consider time-varying, frequency-selective Rayleigh fading channels that satisfy the assumption of wide-sense stationary uncorrelated scattering (WSSUS) [13]. The time-frequency domain coefficients $H[n,k]$ are related to the delay-Doppler spreading function $S[m,l]$ of the channel via a 2-D Fourier transform [14],

$$H[n,k] = \frac{1}{\sqrt{KN}} \sum_{m=0}^{M_\tau-1} \sum_{l=-\frac{M_\nu}{2}}^{\frac{M_\nu}{2}} S[m,l]\, e^{-j2\pi\left(\frac{mk}{K}-\frac{ln}{N}\right)}. \qquad (2)$$

Here, m, l denote discrete delay and discrete Doppler, respectively, and we assumed that $S[m,l]$ has (effective) support $[0, M_\tau-1] \times [-M_\nu/2, M_\nu/2]$ with M_τ denoting the channel's maximum delay spread and M_ν characterizing the maximum Doppler spread[1]. Since in practice $M_\tau \ll K$ and $M_\nu \ll N$, (2) implies that $H[n,k]$ is a 2-D lowpass function. For convenience, we denote the number of degrees of freedom of the channel (i.e., the number of non-zero elements of $S[m,l]$) as $M = M_\tau(M_\nu+1)$.

3. PROPOSED CHANNEL ESTIMATION

Basic Idea. As a first step, it is straightforward to obtain an LS channel estimate at the pilot positions according to

$$\hat{H}_{\mathrm{LS}}[n,k] = \frac{Y[n,k]}{X[n,k]} = H[n,k] + \frac{Z[n,k]}{X[n,k]}, \qquad (n,k) \in \mathcal{P}. \quad (3)$$

In view of the irregular pilot arrangement, the reconstruction of the channel coefficients $H[n,k]$, $[n,k] \notin \mathcal{P}$, via conventional interpolation methods is difficult if not impossible. However, (3) can be interpreted as a noisy nonuniform/irregular 2-D sampling of $H[n,k]$ (i.e., the sampling points do not constitute a regular lattice). This is the motivation for us to consider alternative approaches using irregular sampling techniques (e.g. [15]). Estimating the channel from the noisy irregular samples (3) then amounts to a reconstruction of the 2-D lowpass function $H[n,k]$. To achieve this goal, we adapt the so-called *ABC algorithm*[2] [11, 16] to our channel estimation setup. This choice is motivated by the fact that the ABC algorithm is a low-complexity iterative method that features excellent performance.

Channel Reconstruction Method. According to (2), $H[n,k]$ is a 2-D trigonometric polynomial of degree $M_\tau \times (M_\nu+1)$ with coefficients $S[m,l]$. We are given noisy samples $\hat{H}_{\mathrm{LS}}[n_p,k_p]$ of $H[n,k]$. Channel estimation can now be formulated as the following LS problem,

$$\hat{H}[n,k] = \arg \min_{\tilde{H}[n,k]\in\mathcal{T}} \sum_{p=1}^{P} \left|\tilde{H}[n_p,k_p] - \hat{H}_{\mathrm{LS}}[n_p,k_p]\right|^2, \qquad (4)$$

where $\mathcal{T}$ denotes the subspace of 2-D trigonometric polynomials of degree $M_\tau \times (M_\nu+1)$ characterized by (2) (recall that $(n_p,k_p) \in \mathcal{P}$ are the P pilot positions). For a more compact formulation of the LS problem, we define the length-P vectors[3] $\tilde{\mathbf{h}} = [\tilde{H}[n_1,k_1],\ldots,\tilde{H}[n_P,k_P]]^T$ and $\hat{\mathbf{h}}_{\mathrm{LS}} = [\hat{H}_{\mathrm{LS}}[n_1,k_1],\ldots,\hat{H}_{\mathrm{LS}}[n_P,k_P]]^T$. The side-constraint $\tilde{H}[n,k] \in \mathcal{T}$ can be reformulated as $\tilde{\mathbf{h}} = \mathbf{V}\mathbf{s}$ with the length-M vector $\mathbf{s}$ and $P\times M$ double Vandermonde matrix $\mathbf{V}$ given by

$$[\mathbf{V}]_{p,q} = \frac{1}{\sqrt{KN}} e^{-j2\pi\left(\frac{mk_p}{K}-\frac{ln_p}{N}\right)}, \qquad q = m + \left[\frac{M_\nu}{2} + l\right] M_\tau + 1\,.$$
$$[\mathbf{s}]_q = S[m,l]\,,$$

With these definitions, (4) can be reformulated as

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \left\|\mathbf{V}\mathbf{s} - \hat{\mathbf{h}}_{\mathrm{LS}}\right\|^2.$$

This LS problem leads to the linear equations

$$\mathbf{V}\hat{\mathbf{s}} = \hat{\mathbf{h}}_{\mathrm{LS}} \qquad (5)$$

for the estimate $\hat{\mathbf{s}}$ of the unknown vector $\mathbf{s}$. After solving (5), $\hat{H}[n,k]$ can be calculated from $\hat{\mathbf{s}}$ (i.e., $\hat{S}[m,l]$) using (2).

The complexity of solving the double Vandermonde system (5) scales with the number of pilots P. A more efficient implementation, whose complexity is independent of P, can be obtained by pre-multiplying (5) with $\mathbf{V}^H$ (for a frame theoretical motivation see [17]). This results in

$$\mathbf{T}\hat{\mathbf{s}} = \hat{\mathbf{s}}_{\mathrm{LS}}\,, \qquad \mathbf{T} = \mathbf{V}^H\mathbf{V}\,, \qquad (6)$$

with the length-M vector $\hat{\mathbf{s}}_{\mathrm{LS}} = \mathbf{V}^H\hat{\mathbf{h}}_{\mathrm{LS}}$ and the $M\times M$ matrix $\mathbf{T}$ that has block Toeplitz stucture with Toeplitz blocks. Since $\mathbf{T}$ may have large condition number (see Section 4), solving (6) requires regularization. This can be achieved by using the iterative conjugate gradient (CG) method (see [18] for details) with early termination. CG typically features fast convergence and is particularly suited for Toeplitz systems.

[1] For simplicity, we assume that M_ν is even.

[2] Here, A, B, and C respectively stand for adaptive weights (not used in our case), block Toeplitz matrices, and conjugate gradient.

[3] Superscript ^{T} (^{H}) denotes (Hermitian) transposition.

Stopping Criterion. Early termination of the CG iterations requires specification of a stopping criterion. The normalized mean square error (MSE) of the channel estimate $\hat{H}_r[n,k]$ obtained after r CG iterations is

$$\epsilon_r = \frac{\sum_{n=0}^{N-1}\sum_{k=0}^{K-1}|\hat{H}_r[n,k] - H[n,k]|^2}{\sum_{n=0}^{N-1}\sum_{k=0}^{K-1}|H[n,k]|^2} \tag{7}$$

(note that in contrast to (4) summation here is over all symbols in a packet). Usually, ϵ_r decreases initially and reaches a minimum for a certain optimum iteration number r_{opt}. For $r > r_{\text{opt}}$, ϵ_r increases again due to noise enhancement caused by the poor condition number of $\mathbf{T}$. In practice, r_{opt} cannot be determined since $H[n,k]$ in (7) is not available. As an alternative, we adopt a stopping criterion from [19]. The CG iterations will be terminated as soon as either

$$\left\|\hat{\mathbf{h}}_r - \hat{\mathbf{h}}_{\text{LS}}\right\|^2 \le \gamma P, \qquad \gamma = \frac{\sigma_X^2\sigma_H^2}{\sigma_Z^2}, \tag{8}$$

or a certain maximum number of iterations have been performed. Here, $\hat{\mathbf{h}}_r$ is a length-P vector containing $\hat{H}_r[n,k]$, $(n,k) \in \mathcal{P}$, and $\sigma_X^2 = \mathcal{E}\{|X[n,k]|^2\}$ and $\sigma_H^2 = \mathcal{E}\{|H[n,k]|^2\}$. Note that all quantities in (8) are either provided by the channel estimation stage or can be straightforwardly estimated. Numerical experiments showed that (8) tends to be overly pessimistic in the sense that a few extra iterations decrease the MSE slightly further.

Algorithm Summary and Complexity. In the following, we summarize the individual steps of the overall channel estimation scheme and assess their computational complexity.

1) Pre-processing: Compute the matrix $\mathbf{T}$ using a 2-D fast Fourier transform (FFT) requiring $O(NK\log(NK))$ operations (see [11] for details). This can be done without explicit use of $\mathbf{V}$. If the pilot arrangement is the same for all packets, $\mathbf{T}$ can be pre-computed and stored in memory.

2) Initialization: Calculate the LS estimate $\hat{\mathbf{h}}_{\text{LS}}$ and perform a 2-D FFT to obtain $\hat{\mathbf{s}}_{\text{LS}}$ requiring $O(NK\log(NK))$ operations [11].

3) CG Iteration: Each CG iteration calculates an approximate solution $\hat{\mathbf{s}}_r$ of (6) which involves a multiplication by $\mathbf{T}$. This matrix-vector multiplication can be implemented efficiently using 2-D FFTs since $\mathbf{T}$ has Toeplitz-block-Toeplitz structure (see [20]). This requires only $O(M\log(M))$ operations, i.e., the complexity of one CG iteration scales with the maximum delay and Doppler spread but *not* with the number of pilots.

4) Post-processing: Determine $\hat{H}_r[n,k]$ from $\hat{\mathbf{s}}_r$ using again an $O(NK\log(NK))$ 2-D FFT. Terminate if (8) is satisfied, otherwise go to step 3.

4. DESIGN ASPECTS

Pilot Arrangement and Weighting. A large condition number of $\mathbf{T}$ degrades the CG convergence and the channel estimation error. In order for $\mathbf{T}$ to be invertible, $P \ge M_\tau(M_\nu+1)$ and an appropriate pilot arrangement is required. The analysis in [11] indicates that the condition number grows with increasing maximum pilot spacing, maximum delay, and maximum Doppler. Poor condition numbers are thus encountered with clusters (i.e., accumulations) in the pilot pattern since this results in a larger maximum pilot separation than in the case where the pilots are distributed more evenly. Our simulations indicated that in such situations the overall performance may degrade. However, since the pilot density is high within the clusters, locally channel reconstruction remains reliable.

We note that one way to deal with large variations of the pilot density is to include adaptive weights w_p such that isolated pilots receive larger weights (see [11, 16] for details). This turns (4) into a weighted LS problem $\min(\tilde{\mathbf{h}} - \hat{\mathbf{h}}_{\text{LS}})^H\mathbf{W}(\tilde{\mathbf{h}} - \hat{\mathbf{h}}_{\text{LS}})$ with $\mathbf{W} = \text{diag}\{w_1, \ldots, w_P\}$, which in turn improves condition number and speeds up CG convergence.

Delay and Doppler Spread. The proposed channel reconstruction algorithm presupposes knowledge of the channel's maximum discrete delay spread M_τ and maximum discrete Doppler spread, characterized by M_ν. However, the choice of M_τ and M_ν is not very critical. In particular, slightly too large values do not degrade estimation accuracy a lot, provided that the pilot arrangement fulfills the Nyquist criterion [11]. The maximum discrete delay can be computed as $M_\tau = \lceil \tau_{\max} B \rceil$, where $\tau_{\max}$ is the maximum continuous-time delay in seconds. Alternatively, a worst-case choice for M_τ is provided by the length of the cyclix prefix. Determining the (effective) discrete Doppler spread is more difficult since the Doppler spectrum is significantly smeared out in the case of short packets. A reasonable rule of thumb is given by

$$M_\nu = 2\left\lceil \frac{\nu_{\max}}{B}\,(K + L_{\text{CP}})\,N \right\rceil. \tag{9}$$

Here, $\nu_{\max}$ is the maximum Doppler frequency in Hertz, given by $\nu_{\max} = f_c\omega_{\max}/c_0$, with $\omega_{\max}$ denoting the maximum terminal velocity, f_c being the carrier frequency and c_0 denoting the speed of light.

5. APPLICATION EXAMPLE

We apply our channel estimation scheme to an IEEE 802.11a system [2] to illustrate its performance via numerical simulations (the ideas and results can be straighforwardly carried over to the OFDM air interface of 802.16a [3]). The rationale is to use our channel estimator as an add-on that allows operation of the system over rapidly time-varying channels by exploiting the continual pilot subcarriers present for frequency offset tracking purposes.

Simulation Setup. The 802.11a standard operates with $K = 64$ subcarriers (of which 12 serve as guard and DC carriers), bandwidth $B = 20$ MHz, and carrier frequency $f_c = 5$ GHz. We used a rate 1/2 convolutional code, a block interleaver, 16-QAM modulation, and a CP length of $L_{\text{CP}} = 16$ samples. We propose to use the preamble and the four continual pilots prescribed by the standard jointly as training sym-

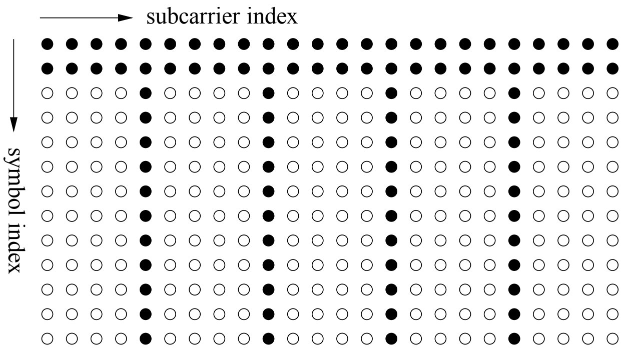

Fig. 1. *Schematic illustration of the chosen 802.11a training symbol arrangement (•) composed of the preamble (first two OFDM symbols) and four continual pilot subcarriers.*

bols for channel estimation. The resulting pilot arrangement is shown schematically in Fig. 1. Unless stated otherwise, we chose a packet length of $N = 60$ OFDM symbols, which amounts to $P = 336$ training symbols per packet.

A Rayleigh fading channel with wide-sense stationary uncorrelated scattering (WSSUS) [14] was simulated according to [21], using an exponential delay profile with maximum delay spread of 200 ns (corresponding to $M_\tau = 4$) and a Jakes Doppler profile [22] with various maximum Doppler frequencies. The discrete Doppler spread M_ν was determined according to (9).

The receiver performed zero-forcing equalization using the estimated channel and channel decoding. The pilot set was seen to result in satisfactory CG convergence, thus adaptive weights were not taken into account. We further observed that the number of CG iterations depends only weakly on SNR, packet length N, Doppler spread M_ν, and delay spread M_τ. Typically between 10 and 15 iterations were performed. However, the specific pseudo-regular pilot structure allows reliable channel estimation only for channels with maximum delays not larger than the number of continual pilots, $M_\tau \leq 4$. All results shown were obtained by averaging over at least 80 000 packets.

Performance versus SNR. Fig. 2 shows bit error rate (BER, top) and MSE (bottom) versus SNR achieved with conventional preamble-based LS channel estimation (labeled 'preamble') and with our channel estimation scheme. Two curves are shown for our method: one uses the proposed stopping criterion (8) ('proposed - SC') for the CG algorithm, the other uses optimum CG termination at minimum MSE ('proposed - OT'). The receiver velocity was fixed at 100 km/h. It is seen that our proposed scheme significantly outperforms preamble-based LS estimation since the latter fails to track channel variations within a packet. Our method even closely approaches ideal performance using the true channel coefficients ('ideal'). Furthermore, it is seen that the stopping criterion has slightly poorer MSE than optimum termination; however, the corresponding BER curves are virtually identical.

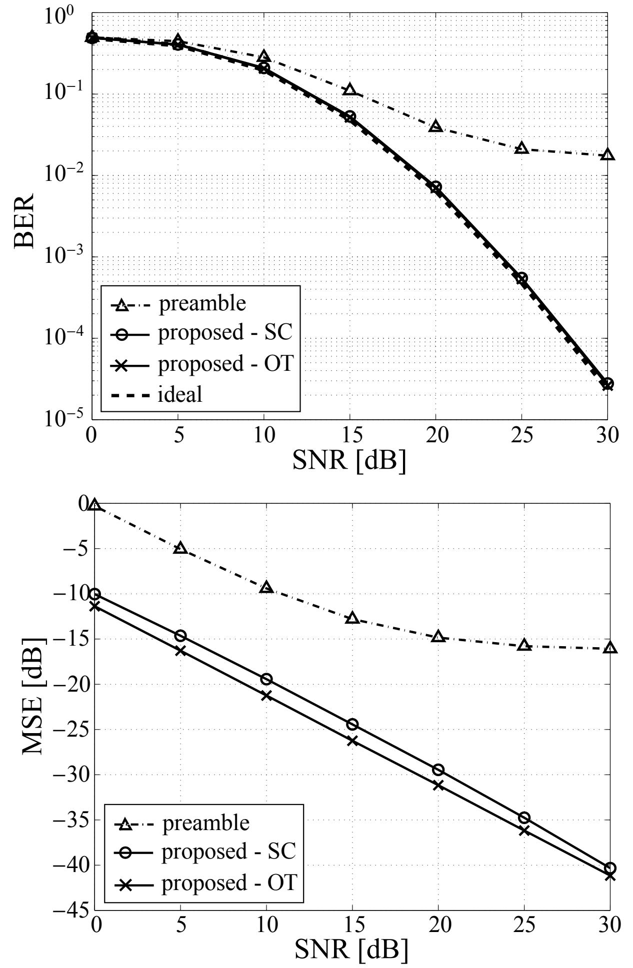

Fig. 2. *Performance comparison of proposed method with preamble-based LS estimator and with perfect CSI: BER (top) and MSE (bottom) versus SNR at 100 km/h.*

Impact of Receiver Velocity. We next investigate the dependence of the BER on the receiver velocity (equivalently, Doppler frequency) at a fixed SNR of 20 dB (see Fig. 3). It is seen that the performance achieved with preamble-based estimation degrades quickly with increasing velocity. In contrast, our method maintains almost constant performance close to perfect channel knowledge, even at very high velocities.

Impact of Packet Length. Finally, we analyze the influence of the packet length on the MSE at an SNR of 20 dB and a receiver velocity of 100 km/h (see Fig. 4). The MSE of preamble-based LS estimation increases rapidly with increasing N because more channel variations occur within longer packets. For the proposed channel estimator the MSE decreases with increasing packet length. This can be explained by the fact that with long packets the pilot set is more and more dominated by the regular structure of the continual pilots while the preamble becomes negligible. This decreases the condition number of the matrix $\mathbf{T}$ and thereby reduces the MSE.

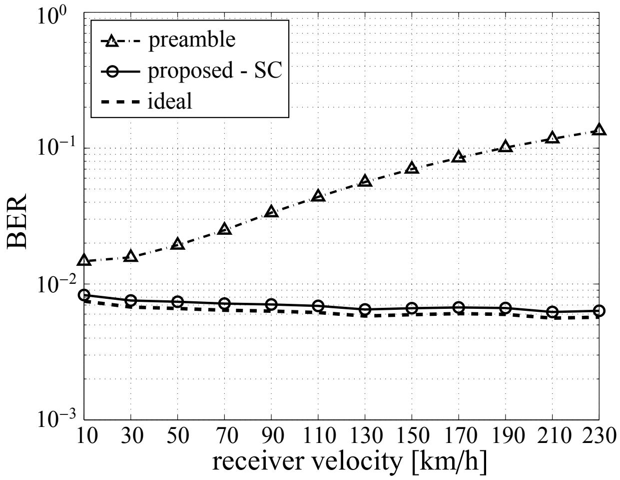

Fig. 3. *BER versus receiver velocity at 20 dB SNR.*

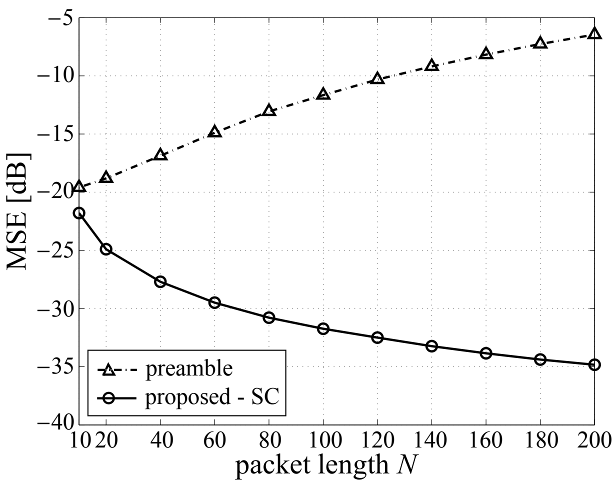

Fig. 4. *MSE versus packet length at an SNR of 20 dB and a velocity of 100 km/h.*

6. CONCLUSIONS

We introduced a novel 2-D channel estimation scheme for OFDM packet transmissions over fast time-varying channels. The proposed method uses irregular sampling techniques that allow for very flexible pilot arrangements. The resulting algorithm can be implemented efficiently using CG iterations and FFTs. We applied our algorithm to a WLAN system by considering preamble and continual pilots as joint training data. Simulation results show that the BER performance achieved with our scheme is close to that using perfect CSI even for very high Doppler frequencies where preamble-based channel estimation fails completely. However, in this scenario only moderate delay spreads are supported. We conclude that this approach is attractive as an add-on to current WLAN and WiMAX receivers in high-mobility scenarios. The application of the proposed method to multi-user channel estimation in IEEE 802.16e OFDMA systems is discussed in [23]. Due to its low computational complexity, our approach appears even more promising in the context of MIMO-OFDM.

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