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A FACTOR GRAPH APPROACH TO JOINT ITERATIVE DATA DETECTION AND CHANNEL ESTIMATION IN PILOT-ASSISTED IDMA TRANSMISSIONS

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ABSTRACT

We consider a pilot-assisted interleave-division multiple access (IDMA) system transmitting over block-fading channels. We describe this system in terms of a factor graph and use the sum-product algorithm to develop a receiver that performs joint data detection and channel estimation. Suitable approximations to the messages passed by the sum-product algorithm yield an implementation with a complexity that scales linearly with the number of users. Simulation results demonstrate large performance gains compared to classical receivers performing separate channel estimation and data detection.

***Index Terms*—** IDMA, multiuser detection, iterative receivers, factor graphs, sum-product algorithm

1. INTRODUCTION

Interleave-division multiple access (IDMA), recently proposed in [1], achieves user separation by means of user-specific interleavers combined with low-rate channel coding. The iterative IDMA multiuser detector derived in [1] assumes perfect channel state information (CSI) at the receiver. In practice, however, pilot-assisted channel estimation is usually employed to obtain (imperfect) CSI.

Here, we propose an iterative joint data detection and channel estimation algorithm for pilot-assisted IDMA (see [2, 3] for related work in the context of single-user systems). This algorithm is derived by applying the sum-product algorithm [4] to the factor graph corresponding to the overall system (cf. [5, 6]). A low-complexity implementation of the receiver is obtained by means of Gaussian approximations to the messages propagated through the factor graph.

The paper is organized as follows. The pilot-assisted IDMA system is described in Section 2. In Section 3, we construct the corresponding factor graph and derive the messages propagated through the graph. Finally, simulation results presented in Section 4 demonstrate the performance gains obtained with the proposed receiver.

2. PILOT-ASSISTED IDMA

We assume an uplink multiple-access scenario where M users transmit data synchronously to a base station, using IDMA transmitters shown in Fig. 1. The bit sequence of the mth user, $\mathbf{b}^m = (b_1^m \cdots b_K^m)^T$, is encoded into a binary codeword of length N using a serial concatenation of a terminated convolutional code and a low-rate repetition code. The codeword is interleaved by a user-specific interleaver $\pi^m(\cdot)$ and mapped to BPSK symbols $x_n^m \in \{-1, 1\}$. We write $\mathbf{x}^m = (x_1^m \cdots x_N^m)^T = \mathcal{C}^m(\mathbf{b}^m)$ for the combined effect of channel coding, interleaving, and BPSK mapping.

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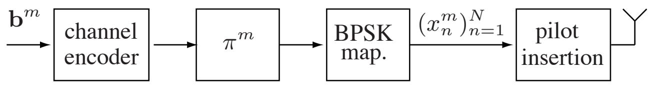


Fig. 1. IDMA transmitter for the mth user.

We assume block-fading channels that stay constant during "channel blocks" of length L. Let h_l^m be the channel (fading) coefficient of the mth user within the lth channel block. The h_l^m are assumed i.i.d. Gaussian with zero mean and unit variance, i.e., $h_l^m \sim \mathcal{N}(0, 1)$.

To enable channel estimation, any user m transmits L_p pilot symbols $p_{l,k}^m$, $k = 1, \ldots, L_p$, at user-specific (disjoint) pilot positions within the lth channel block. This leaves $L_x = L - ML_p$ instants per channel block for all users to transmit their BPSK data symbols. We will denote the data symbols also by $x_{l,k}^m := x_{(l-1)L_x+k}^m$, with $k = 1, \ldots, L_x$ and $l = 1, \ldots, N_b$, where $N_b := N/L_x$ is the number of blocks (N is assumed to be a multiple of L_x). The N receive values corresponding to data symbols are given by

$$r_{l,k} = \sum_{m=1}^{M} h_l^m x_{l,k}^m + w_{l,k}\,, \tag{1a}$$

for $k = 1, \ldots, L_x$, $l = 1, \ldots, N_b$, where $w_{l,k}$ denotes white Gaussian noise of variance σ_w^2. In contrast, the MN_bL_p receive values corresponding to the pilot symbols are given by

$$\tilde{r}_{l,k}^m = h_l^m p_{l,k}^m + w_{l,k}^m\,, \tag{1b}$$

for $k = 1, \ldots, L_p$, $l = 1, \ldots, N_b$, $m = 1, \ldots, M$.

3. ITERATIVE JOINT MULTIUSER DETECTION AND CHANNEL ESTIMATION

We will now use a factor graph framework to develop a low-complexity receiver performing joint channel estimation and multiuser detection. The proposed receiver is based on the MAP detector [5]

$$\hat{b}_i^m = \arg \max_{b_i^m \in \{0,1\}} p(b_i^m | \mathbf{r})\,. \tag{2}$$

Here, b_i^m is the ith information bit of the mth user, $\mathbf{r}$ is the full received sequence (consisting of all $r_{l,k}$ and $\tilde{r}_{l,k}^m$), and $p(b_i^m|\mathbf{r})$ denotes the posterior probability mass function of b_i^m.

3.1. Factor Graph

In what follows, let $\mathbf{b}$ denote the vector of length MK containing all information bits b_i^m and let $\mathbf{X} = (\mathbf{x}^1 \cdots \mathbf{x}^M)$ be the $N \times M$ matrix consisting of all BPSK data symbols x_n^m. Applying Bayes' rule, and assuming equally likely data bits b_i^m, we have

$$p(b_i^m|\mathbf{r}) = \sum_{\sim b_i^m} p(\mathbf{b}|\mathbf{r}) \propto \sum_{\sim b_i^m} f(\mathbf{r}|\mathbf{b}) \tag{3}$$

where $f(\mathbf{r}|\mathbf{b})$ is the conditional probability density function (pdf) of $\mathbf{r}$ given $\mathbf{b}$, $\sum_{\sim x}$ denotes summation over all unknown variables in the summand except x, and $\propto$ denotes equality up to factors irrelevant to the maximization in (2). Exploiting the one-to-one correspondence between $\mathbf{b}$ and $\mathbf{X}$, $f(\mathbf{r}|\mathbf{b})$ can be factored as

$$f(\mathbf{r}|\mathbf{b}) = \sum_{\mathbf{X}} f(\mathbf{r}|\mathbf{X}) \prod_{m=1}^{M} \mathrm{I}\big(\mathbf{x}^m = \mathcal{C}^m(\mathbf{b}^m)\big)\,,$$

where the indicator function $\mathrm{I}(\cdot)$ is one if its argument is true and zero otherwise. With $\mathbf{H}$ denoting the $M \times N_b$ matrix of all h_l^m, we have further $f(\mathbf{r}|\mathbf{X}) = \int f(\mathbf{r}|\mathbf{X},\mathbf{H})\, f(\mathbf{H})\, d\mathbf{H}$ with

$$f(\mathbf{r}|\mathbf{X},\mathbf{H}) = \prod_{l=1}^{N_b} \prod_{k_1=1}^{L_x} f(r_{l,k_1}|\mathbf{x}_{l,k_1},\mathbf{h}_l) \prod_{m=1}^{M} \prod_{k_2=1}^{L_p} f(\tilde{r}_{l,k_2}^m|h_l^m)\,.$$

Here, we used (1) and the definitions $\mathbf{x}_{l,k} = (x_{l,k}^1 \cdots x_{l,k}^M)^T$ and $\mathbf{h}_l = (h_l^1 \cdots h_l^M)^T$. Furthermore, $f(\mathbf{H}) = \prod_{l=1}^{N_b} \prod_{m=1}^{M} f(h_l^m)$.

Combining these expressions and inserting them into (3) yields

$$p(b_i^m|\mathbf{r}) \propto \sum_{\sim b_i^m} \int f(\mathbf{X},\mathbf{H},\mathbf{r}|\mathbf{b})\, d\mathbf{H} \tag{4}$$

with

$$f(\mathbf{X},\mathbf{H},\mathbf{r}|\mathbf{b}) = \prod_{l=1}^{N_b} \prod_{k_1=1}^{L_x} f(r_{l,k_1}|\mathbf{x}_{l,k_1},\mathbf{h}_l) \prod_{m=1}^{M} \mathrm{I}\big(\mathbf{x}^m = \mathcal{C}^m(\mathbf{b}^m)\big) \times \prod_{k_2=1}^{L_p} f(\tilde{r}_{l,k_2}^m|h_l^m)\, f(h_l^m)\,. \tag{5}$$

According to (1), $f(r_{l,k}|\mathbf{x}_{l,k},\mathbf{h}_l)$ and $f(\tilde{r}_{l,k}^m|h_l^m)$ are Gaussian distributions with variance σ_w^2 and respective mean $\mathbf{h}_l^T\mathbf{x}_{l,k}$ and $h_l^m p_{l,k}^m$.

A segment (for the first channel block) of the factor graph [4–6] corresponding to $f(\mathbf{X},\mathbf{H},\mathbf{r}|\mathbf{b})$ in (5) is depicted in Fig. 2. For simplicity, only the channel coefficient of user 1 within that block is shown. Applying the sum-product algorithm [4] to this factor graph yields an approximation (due to the existence of cycles in the factor graph) to the marginal (4) for all information bits b_i^m simultaneously.

3.2. Receiver Structure

Running the sum-product algorithm with parallel scheduling [6] on the factor graph in Fig. 2 leads to the receiver structure shown in Fig. 3. The block termed "soft multiuser detector" corresponds to the upper dotted boxes in Fig. 2. It receives soft information from the individual users' channel decoders [1], performs an update of this soft information using the current channel estimate, and passes the improved soft bits back to the decoders. These improved soft bits are also provided to channel estimation units (corresponding to the lower dotted box in Fig. 2) that calculate refined estimates of the channel coefficients. The per-user soft-channel decoding (consisting of the deinterleavers, soft channel decoders, and interleavers) corresponds to the blocks $\mathrm{I}(\mathbf{x}^m = \mathcal{C}^m(\mathbf{b}^m))$ in Fig. 2. When the sum-product algorithm is terminated, the signs of the *a posteriori* information bits computed by the channel decoder provide bit decisions approximating (2).

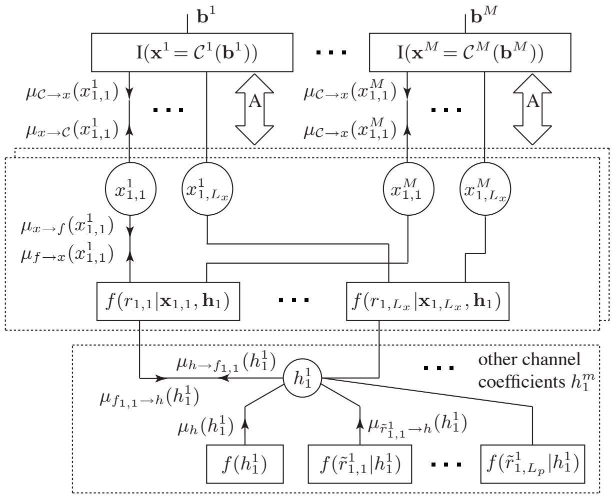


Fig. 2. Factor graph describing $f(\mathbf{X},\mathbf{H},\mathbf{r}|\mathbf{b})$ in (5) for the first channel block. The broad arrows denoted "A" indicate messages from/to other channel blocks ($l = 2, 3, \ldots$).

3.3. Messages

Iterative approximate computation of $\hat{b}_i^m$ in (2) via the sum-product algorithm requires calculation of the messages to be propagated along the edges of the factor graph in Fig. 2.

Messages $\mu_{\mathcal{C}\to x}(x_{l,k}^m)$, $\mu_{x\to f}(x_{l,k}^m)$, and $\mu_{x\to\mathcal{C}}(x_{l,k}^m)$. For the code function nodes $\mathrm{I}(\mathbf{x}^m = \mathcal{C}^m(\mathbf{b}^m))$, the sum-product algorithm amounts to the BCJR algorithm for soft-decoding the convolutional code [4, 7], while the repetition code is soft-decoded by summing the appropriate bit log-likelihood ratios (LLRs). The extrinsic information computed by the channel decoder, expressed by LLR values $\xi_{l,k}^m \in \mathbb{R}$ for the BPSK symbols $x_{l,k}^m$, is converted into messages (beliefs) $\mu_{\mathcal{C}\to x}(x_{l,k}^m)$ according to

$$\mu_{\mathcal{C}\to x}(x_{l,k}^m) = \frac{\exp\big(\xi_{l,k}^m\,(x_{l,k}^m+1)/2\big)}{1+\exp(\xi_{l,k}^m)}\,, \quad x_{l,k}^m \in \{-1, 1\}\,.$$

The variable nodes $x_{l,k}^m$ in Fig. 2 just pass on all incoming messages, i.e., $\mu_{x\to f}(x_{l,k}^m) = \mu_{\mathcal{C}\to x}(x_{l,k}^m)$ and $\mu_{x\to\mathcal{C}}(x_{l,k}^m) = \mu_{f\to x}(x_{l,k}^m)$.

Messages $\mu_{f\to x}(x_{l,k}^m)$. The messages from the channel factor nodes back to the symbol variable nodes equal

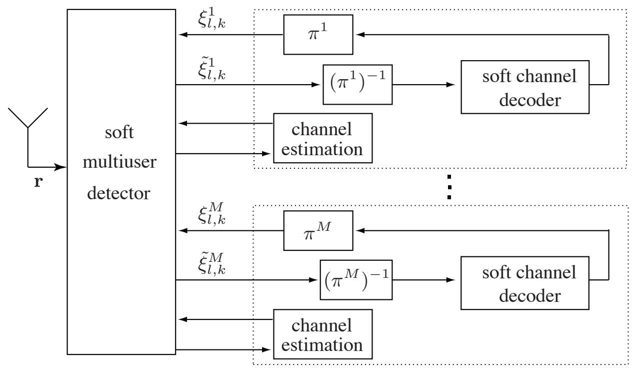


Fig. 3. IDMA receiver structure with joint data detection and channel estimation. ($\xi_{l,k}^m$ and $\tilde{\xi}_{l,k}^m$ are defined in Section 3.3.)

$$\mu_{f\to x}(x_{l,k}^m) \,=\, \sum_{\sim x_{l,k}^m} \int f(r_{l,k}|\mathbf{x}_{l,k},\mathbf{h}_l) \prod_{m_1=1}^{M} \mu_{h\to f_{l,k}}(h_l^{m_1}) \times \prod_{m_2\neq m} \mu_{x\to f}(x_{l,k}^{m_2})\, d\mathbf{h}_l\,. \quad (6)$$

The summation in (6) has 2^{M-1} terms and is thus exponentially complex in the number of users. To achieve linear complexity for the message update, we approximate the discrete messages $\mu_{x\to f}(x_{l,k}^m)$ by continuous messages of Gaussian form

$$\mu_{x\to f}(x_{l,k}^m) \,=\, \exp\!\left(-\frac{(x_{l,k}^m - a_{l,k}^m)^2}{2b_{l,k}^m}\right), \quad (7)$$

with mean $a_{l,k}^m = \tanh(\xi_{l,k}^m)$ and variance $b_{l,k}^m = 1-(a_{l,k}^m)^2$ (cf. [1]). Furthermore, the messages $\mu_{h\to f_{l,k}}(h_l^m)$ will also be modeled as Gaussian, i.e.,

$$\mu_{h\to f_{l,k}}(h_l^m) \,=\, \exp\!\left(-\frac{(h_l^m - \alpha_l^m)^2}{2\beta_l^m}\right), \quad (8)$$

with mean α_l^m and variance β_l^m to be determined later.

In the following, let $\mathbf{x}_{l,k}^{\sim m}$ denote the vector obtained by removing $x_{l,k}^m$ from $\mathbf{x}_{l,k}$, and similarly for $\mathbf{h}_l^{\sim m}$. Plugging the above Gaussian messages into (6), replacing the summations with respect to the $x_{l,k}^{m'}$, $m'\neq m$ with integrals, and picking an arbitrary user index $m'\neq m$, we obtain the approximation

$$\mu_{f\to x}(x_{l,k}^m) \,=\, \int I\big(\mathbf{x}_{l,k}^{\sim m'},\mathbf{h}_l^{\sim m'}\big) \prod_{m_1\neq m'} \mu_{h\to f_{l,k}}(h_l^{m_1}) \times \prod_{m_2\neq (m,m')} \mu_{x\to f}(x_{l,k}^{m_2})\, d\mathbf{x}_{l,k}^{\sim m,m'} d\mathbf{h}_l^{\sim m'}, \quad (9)$$

where

$$I\big(\mathbf{x}_{l,k}^{\sim m'},\mathbf{h}_l^{\sim m'}\big) = \int f(r_{l,k}|\mathbf{x}_{l,k},\mathbf{h}_l)\, \mu_{h\to f_{l,k}}(h_l^{m'})\, \mu_{x\to f}(x_{l,k}^{m'})\, dx_{l,k}^{m'}\, dh_l^{m'} = \int \exp\!\left(-\frac{(A-hx)^2}{2\sigma_w^2} - \frac{(h-\alpha_l^{m'})^2}{2\beta_l^{m'}} - \frac{(x-a_{l,k}^{m'})^2}{2b_{l,k}^{m'}}\right) dx\,dh$$

amounts to marginalization for user m'. (In the last line, we omitted all indices of $x_{l,k}^{m'}$ and $h_l^{m'}$ and set $A := r_{l,k} - \sum_{m''\neq m'} h_l^{m''} x_{l,k}^{m''}$ for simplicity.) Integration with respect to h gives

$$I\big(\mathbf{x}_{l,k}^{\sim m'},\mathbf{h}_l^{\sim m'}\big) \,=\, \int i(x)\, dx \quad (10)$$

with

$$i(x) = \sqrt{\frac{2\pi}{\frac{x^2}{\sigma_w^2}+\frac{1}{\beta_l^{m'}}}}\, \exp\!\left(-\frac{(A-\alpha_l^{m'}x)^2}{2(\sigma_w^2+\beta_l^{m'}x^2)}\right) \exp\!\left(-\frac{(x-a_{l,k}^{m'})^2}{2b_{l,k}^{m'}}\right).$$

For an approximate closed-form integration, we simplify $i(x)$ as follows. We have $a_{l,k}^{m'} = \tanh(\xi_{l,k}^{m'}) \in [-1,1]$ and $b_{l,k}^{m'} = 1-(a_{l,k}^{m'})^2 \in [0,1]$. After a sufficient number of iterations, $a_{l,k}^{m'} \to -1$ or $a_{l,k}^{m'} \to 1$ and $b_{l,k}^{m'} \to 0$. Hence, because of the second exponential factor, $i(x) \approx 0$ for x outside a small neighborhood of -1 or 1. Therefore, we can approximate $i(x)$ by setting $x^2 = 1$ in the square-root factor and in the denominator of the first exponent, which gives

$$i(x) \approx \sqrt{\frac{2\pi}{\frac{1}{\sigma_w^2}+\frac{1}{\beta_l^{m'}}}}\, \exp\!\left(-\frac{(A-\alpha_l^{m'}x)^2}{2(\sigma_w^2+\beta_l^{m'})}\right) \exp\!\left(-\frac{(x-a_{l,k}^{m'})^2}{2b_{l,k}^{m'}}\right).$$

This can be integrated in closed form (cf. (10)), yielding

$$I\big(\mathbf{x}_{l,k}^{\sim m'},\mathbf{h}_l^{\sim m'}\big) \,\propto\, \exp\!\left(-\frac{\big(r_{l,k} - (\mathbf{h}_l^{\sim m'})^T \mathbf{x}_{l,k}^{\sim m'} - \alpha_l^{m'} b_{l,k}^{m'}\big)^2}{2(\sigma_w^2+\beta_l^{m'}+\beta_l^{m'} b_{l,k}^{m'})}\right).$$

Inserting into (9) and repeatedly applying the above type of approximation to the marginalizations for the other users $\neq m$, we obtain

$$\mu_{f\to x}(x_{l,k}^m) \,\propto\, \exp\!\left(-\frac{\big(r_{l,k} - \alpha_l^m x_{l,k}^m - \sum_{m'\neq m} \alpha_l^{m'} a_{l,k}^{m'}\big)^2}{2\gamma_{l,k}^m}\right), \quad (11)$$

with $\gamma_{l,k}^m := \sigma_w^2 + \sum_{m'} \beta_l^{m'} + \sum_{m'\neq m} (\alpha_l^{m'})^2 b_{l,k}^{m'}$. Converting $\mu_{f\to x}(x_{l,k}^m)$ into an LLR value yields

$$\tilde{\xi}_{l,k}^m \,=\, \log\frac{\mu_{f\to x}(x_{l,k}^m = 1)}{\mu_{f\to x}(x_{l,k}^m = -1)} \,=\, \frac{\big(r_{l,k} - \sum_{m'\neq m} \alpha_l^{m'} a_{l,k}^{m'}\big)\, \alpha_l^m}{2\gamma_{l,k}^m}\,.$$

This LLR value is propagated to the corresponding channel decoder, as depicted in Fig. 3. It is interesting to observe that in the special case of perfect CSI, i.e., $\alpha_l^m \to h_l^m$ and $\beta_l^m \to 0$, the LLR becomes

$$\tilde{\xi}_{l,k}^m \,=\, \frac{\big(r_{l,k} - \sum_{m'\neq m} h_l^{m'} a_{l,k}^{m'}\big)\, h_l^m}{2\big(\sigma_w^2 + \sum_{m'\neq m} (h_l^{m'})^2 b_{l,k}^{m'}\big)}\,,$$

which is the linear multi-user detector derived in [1].

Messages $\mu_{f_{l,k}\to h}(h_l^m)$. Using (7), (8) and approximations similar to those that led to (11), we obtain

$$\mu_{f_{l,k}\to h}(h_l^m) \,=\, \sqrt{\frac{2\pi}{\frac{(h_l^m)^2}{\gamma_{l,k}^m}+\frac{1}{\beta_l^m}}}\, \exp\!\left(-\frac{(a_{l,k}^m)^2 (h_l^m - \nu_{l,k}^m)^2}{2(\gamma_{l,k}^m + (h_l^m)^2 \beta_l^m)}\right), \quad (12)$$

with $\nu_{l,k}^m := \big(r_{l,k} - \sum_{m'\neq m} \alpha_l^{m'} a_{l,k}^{m'}\big)/a_{l,k}^m$. At the variable node h_l^m (cf. node h_1^1 in Fig. 2), all incoming messages—i.e., $\mu_{\tilde{r}_{l,k}^m\to h}(h_l^m)$ from the pilot symbol function nodes $f(\tilde{r}_{l,k}^m|h_l^m)$, $\mu_h(h_l^m)$ from the *a priori* function node $f(h_l^m)$, and $\mu_{f_{l,k}\to h}(h_l^m)$ from the channel factor nodes $f(r_{l',k'}|\mathbf{x}_{l',k'},\mathbf{h}_{l'})$, $(l',k')\neq(l,k)$—are used to compute the message $\mu_{h\to f_{l,k}}(h_l^m)$ (see Fig. 2). To do this efficiently, we again use a Gaussian approximation for $\mu_{f_{l,k}\to h}(h_l^m)$. Indeed, since after a sufficient number of iterations $(a_{l,k}^m)^2 \to 1$, $(h_l^m)^2 \beta_l^m \ll \gamma_{l,k}^m$, and $(h_l^m)^2 \ll \gamma_{l,k}^m$, (12) is approximated (up to a constant factor) as

$$\mu_{f_{l,k}\to h}(h_l^m) \,\approx\, \exp\!\left(-\frac{(h_l^m - \nu_{l,k}^m)^2}{2\gamma_{l,k}^m}\right).$$

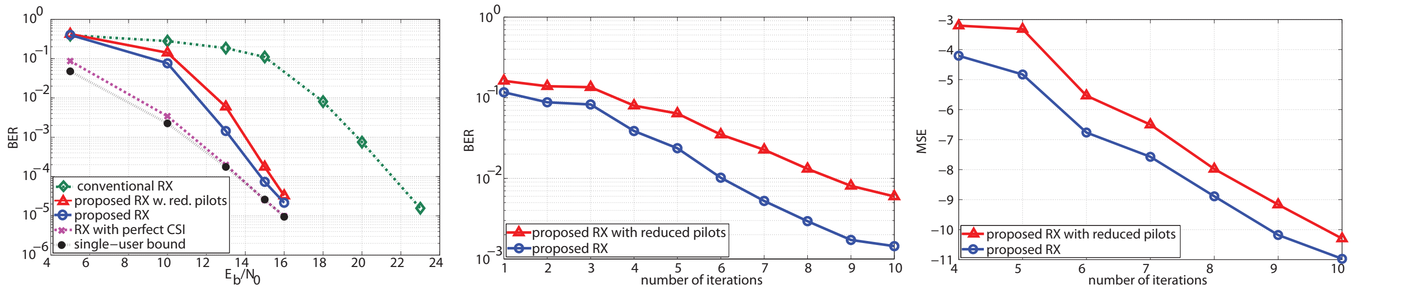
Messages $\mu_{\tilde{r}_{l,k}^m\to h}(h_l^m)$ and $\mu_h(h_l^m)$. Because $\tilde{r}_{l,k}^m$ conditioned on h_l^m is Gaussian with mean $h_l^m p_{l,k}^m$ and variance σ_w^2, the message from the pilot symbol function node is $\mu_{\tilde{r}_{l,k}^m\to h}(h_l^m) = \exp\big(-(\tilde{r}_{l,k}^m - h_l^m p_{l,k}^m)^2/2\sigma_w^2\big)$, which can be written as a Gaussian in h_l^m:

$$\mu_{\tilde{r}_{l,k}^m\to h}(h_l^m) \,=\, \exp\!\left(-\frac{(h_l^m - \tilde{r}_{l,k}^m/p_{l,k}^m)^2}{2\sigma_w^2/(p_{l,k}^m)^2}\right).$$

Furthermore, because $h_l^m \sim \mathcal{N}(0,1)$, the message from the *a priori* distribution function node of h_l^m is

$$\mu_h(h_l^m) \,=\, \exp\big(-(h_l^m)^2/2\big)\,.$$

Messages $\mu_{h\to f_{l,k}}(h_l^m)$. The message $\mu_{h\to f_{l,k}}(h_l^m)$ can be obtained as the product of the incoming messages (cf. Fig. 2):



(a) (b) (c)

Fig. 4. Performance of different IDMA receivers for $M=4$ users: (a) average BER versus E_b/N_0 after 10 iterations, (b) average BER versus number of iterations at $E_b/N_0 = 13$ dB, (c) MSE of estimated channel coefficients versus number of iterations at $E_b/N_0 = 13$ dB.

$$\mu_{h \to f_{l,k}}(h_l^m) = \mu_h(h_l^m) \prod_{k_1=1}^{L_p} \mu_{\tilde{r}_{l,k_1}^m \to h}(h_l^m) \prod_{\substack{k_2=1 \\ k_2 \neq k}}^{L_x} \mu_{f_{l,k_2} \to h}(h_l^m) . \tag{13}$$

Because all factors are Gaussian (partly due to the approximations above), $\mu_{h \to f_{l,k}}(h_l^m)$ is Gaussian as well. This is consistent with our previous Gaussian assumption in (8), i.e., $\mu_{h \to f_{l,k}}(h_l^m) = \exp\big(-(h_l^m - \alpha_l^m)^2/2\beta_l^m\big)$. The mean α_l^m and variance β_l^m can now be calculated from (13) (cf. [4]).

Scheduling. The proposed receiver uses parallel message scheduling [6], which means that the messages of all M users at the input of the multiuser detector are updated by the channel decoders simultaneously, and are used to calculate the messages for all users at the output of the multiuser detector concurrently. For improved performance, we update the messages $\mu_{h \to f_{l,k}}(h_l^m)$ (using the messages $\mu_{f_{l,k} \to h}(h_l^m)$) only after the third iteration.

4. SIMULATION RESULTS

We next demonstrate the performance of the proposed receiver algorithm. We simulated a pilot-assisted IDMA system with $M = 4$ users, each transmitting $K = 256$ information bits. The channel code is a serial concatenation of a terminated rate-$1/2$ convolutional code (code polynomial $[2\ 3]_8$) and a rate-$1/4$ repetition code; the overall code rate is thus $1/8$. The channel block length is $L = 50$.

Fig. 4(a) shows the average bit error rate (BER) obtained with different iterative receivers versus the signal-to-noise ratio (SNR) E_b/N_0. The curves correspond to (i) the proposed receiver employing a single pilot symbol per user and channel block (i.e., $L_p = 1$) or (ii) employing a single pilot symbol per user only in every fifth channel block; (iii) a "genie" iterative receiver with perfect CSI (cf. [1]); (iv) a conventional receiver that separately estimates the channel coefficients by means of a pilot-based least-squares estimator and then uses these channel estimates for iterative data detection; and (v) the single-user bound (with perfect CSI). In all cases, 10 iterations were performed. It is seen that our scheme gains about 7 dB of SNR compared to the conventional receiver and remains within about 1 dB of the genie receiver. Reducing the number of pilots in our system to one per five channel blocks merely results in an SNR penalty of less than 1 dB.

In Fig. 4(b), the BER for our receiver is shown versus the number of iterations at a fixed SNR of $E_b/N_0 = 13$ dB. The two curves correspond to the use of one pilot symbol per channel block or per five channel blocks. The impact of the number of pilots on the BER is clearly visible. However, both curves converge after 9–10 iterations.

Finally, Fig. 4(c) depicts the mean square error (MSE) of the channel estimates versus the number of iterations, again at $E_b/N_0 = 13$ dB and using one pilot symbol per channel block or per five channel blocks. The MSE is seen to decrease significantly in both cases. However, fewer pilots result in a slower MSE decrease, which is responsible for the slight SNR penalty observed in Fig. 4(a).

5. CONCLUSION

We proposed an iterative receiver with joint data detection and channel estimation for pilot-assisted IDMA transmisssion. The receiver structure was derived by applying the sum-product algorithm to the factor graph of the system. Using Gaussian message approximations, we developed an efficient implementation whose complexity scales linearly with the number of users. Simulation results showed significant performance improvements over classical receivers.

The system considered here can be extended in various ways. The message approximations can be refined, leading to more complex message updates but better performance. Studying this complexity-performance tradeoff is an interesting topic for further research. Extensions to MIMO-IDMA systems with spatial multiplexing [8] and to more general channel models are also possible.

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