Simultaneous Georeferencing of Aerial Laser Scanner Strips

Helmut Kager

Abstract:
This paper deals about discrepancies between overlapping laser scanner strips. These gaps can be eliminated to a great portion doing a simultaneous adjustment by least squares. An adjustment strategy is proposed for doing that: correcting exterior orientation elements recorded by dGPS and an IMU, as well as interior orientation elements concerning the Scanner-dGPS-IMU system.

Automated determination (measurement) of tie features (instead of tie points) is described. The distribution of control features (instead of control points) is discussed.

1. Introduction

Laser scanners are mounted in aircrafts for collecting 3D-data of the surface of the earth. Proceeding the flight path, the laser beam sent downwards is deflected rhythmically aside and scans the ground surface in a meandric or parallel pattern with a high pulse rate. Most such devices use the technique of run-time measurement: the distance to a ground point then is a function of the time gap between the pulse was sent and received.

The direction of the laser beam is given by some deflecting device like a rotating or oscillating mirror and some trigger causing discrete pulses. So, the device records polar co-ordinates of ground points in its own local co-ordinate system. The origin of this device coordinate system follows the flight path and its movement can be measured with dGPS (differential Global Positioning System) very precisely using the phase comparison method. Since coupled to the aircraft, the attitude of the device changes also during the flight and can be recorded with INS (Inertial Navigation System) – more exactly with an IMU (Inertial Measurement Unit).

The components GPS, IMU and laser scanner have to be synchronised; moreover, their relative - but constant - displacements have to be determined (calibration of eccentricities).

For transforming laser scanner strips into the national ground-survey co-ordinate system using dGPS and INS, we principally need only one ground reference station with known ground-survey coordinates. Moreover, we need also the form of the geoid. But, in practice, we should not be satisfied with that minimal solution because:
The form of the geoid is not sufficiently known in many regions.

The on-the-fly-initialisation for solving the GPS phase ambiguities nowadays is possible for fast moving objects like aircrafts with a r.m.s.e. of about ±10cm; this might result in errors of some dm. Usually, neighbouring precision of dGPS is better by one order of magnitude. The errors increase with the strip length. [1]

The attitudes as delivered from IMUs in use are prone to errors of about ±0.01gon resulting in ±16cm on the ground assuming 1000m relative flying height. Errors of IMU attitude also introduce some torsion of the laser scanner strips inducing errors in ground coordinates. Equally, IMU attitudes have a high neighbouring precision based on the gyros used; nevertheless, they show drifting phenomena. The resulting error effects might reach again some dm in the positions of ground points. [1]

System failure or system instabilities shall be mentioned also: e.g. the change of the set of available GPS satellites during a strip might cause some displacement; however, IMU data helps to bridge such critical gaps.

Last, but not least, the missing rigorous supervision of the whole measuring process has to be mentioned.

Instead of the minimal solution cited above (single ground reference station and geoid) the subsequent alternative is proposed which eliminates the shortcomings of the above:

- Use of more GPS ground reference stations surrounding the area of interest. This may (probably better) be achieved by a virtual reference station [7]. Supposing known ground-survey coordinates of all these ground reference stations, this also eliminates the (unknown) linear portion of the geoid’s undulation. The undulations of higher degree remain; they might be neglected for the usually relative small extent of practical projects.
- Some of the GPS ground reference stations may be replaced by ground reference points which can be “identified” somehow in the point clouds of the laser scanner strips (see 2.1). For planimetric fitting, roofs of buildings and/or prominent fault lines in the terrain are suitable, for height fitting, horizontal areas free of vegetation are recommended. In photogrammetric terminology, we call those reference points usually control points.
- Monitoring a many of plane and height discrepancies in the common areas of overlapping laser scanner strips and, therefrom, improvement of GPS-positioning and IMU-attitude data. Mathematically, this can be formulated with correction polynomials (of probably quite low degree) for the registered orientation elements as function of time: one strip – one polynomial. This procedure preserves the high neighbouring precision of both system components and copes with any drifting phenomena. The adjustment of all these sets of coefficients of the polynomials has to be done simultaneously for all strips of a block (key word: block adjustment by strips) – using the positions of corresponding points (features) in the overlapping areas as observations. Their residuals are to be minimised in the adjustment. A statistically better approach is the strategy to use original observations [2]: the polar coordinates recorded by the laser scanner; given position and attitude of the scanner, the Cartesian ground coordinates are (simple) functions of those recorded (\(v, \chi, \rho\))-values, i.e. nadir-angle \(v\), fore-sight angle \(\chi\) and distance \(\rho\).

The above outline of a technique to improve the geometric quality of laser scanner data should give an idea how to overcome gaps between strip surfaces. Unfortunately, the proposed method requires access to the original data of the laser scanner: GPS, IMU, and Polar data as function of time. The laser scanner companies want to provide 3D-data for the
end-user – so, they want to provide “DTMs” (i.e. grids) resp. point clouds in the national ground-survey co-ordinate system, only; key word “user-friendly”. But this “end-product” is prone to having bias and is too late in the process-chain for elementary repair. Nevertheless, we have to stress the fact that our criticism is valid only for exploiting the full potential of laser scanner data: we want to get the few-cm-precision of the laser scanner also as accuracy of the end product.

Some provisori(ly (temporary) solution was proposed in [5]: it was based on raw 3D-data given in the national co-ordinate system strip by strip. Instead of correcting flight path (dGPS) and attitude data (IMU), we tried to compensate for the apparent XYZ-deformations by correction polynomials for individual strips of ground points. This procedure has the disadvantage that it copes merely with phenomena and does not assess the true problem. But it has the advantage that the necessary data is available to end-users.

Here we aim at a strict, highly automateable procedure minimizing 3D-gaps. Before going into adjustment details we have to discuss the determination of strip-tying features.

2. Determination of Strip-tying Features

The principle of strip-tying by features is shown in figure 1 using a special case. As we are not able to associate homologous points in the point-clouds created by Lidar (LIght Detection And Ranging), we have to recourse to simple geometric features like planes which can be derived from regions of Lidar-points. Such a plane-feature is an approximation of the tangent-plane of the underlying surface. So, we associate first order differentials of the surface and call them homologous features – a generalization of the well-known “homologous points” of standard photogrammetry. It should be mentioned here that the term “feature” also includes lines (straight or curved). But this aspect should not be followed here in detail, since a line can be conceived as intersection of planes (surfaces) and handled by these means.

---

![Figure 1: Principle of height block adjustment with laser scanner strips](image-url)
At some chosen ground position XY, a plane can be interpolated into every point-cloud of overlapping strips. Since the available orientation of the raw strips is relatively good, we can expect that the homologous features will also overlap.

### 2.1 Discussion of "homologous points" vs. "homologous planes"

A point has three coordinates - so, knowing them in 3D-space this point has no degrees of freedom. A tie-point, i.e. a point common to overlapping regions lets no (relative) degrees of freedom to the such tied regions.

A plane has two degrees of freedom - so, a point in one region can move in two independent directions with respect to the other region. A tie-plane, i.e. a plane common to overlapping regions lets also two degrees of freedom to the such tied regions. I.e., the such tied surfaces may shift relatively in two directions; the third direction (the surface's normal) is fixed (relatively!).

From these deliberations one can ask for equivalence conditions between homologous planes and homologous points. The answer is simply given by the fact: three intersecting planes yield a common point. So, we need three (neighboured) homologous planes to get the same(?) effect of tying as from one homologous point! And, with the restriction that the intersection angles are steep enough. (A point can be considered as intersection of three orthogonal planes: e.g. the three coordinate planes yield an optimal intersection.)

(The degrees of freedom discussed above only mean shifts in 3D, not rotations!)

Homologous plane features consist of regions of about 5 to 20m extension; for shortness, we call it a patch. See Figure 2.

The above deliberations also hold true for control points. We have to replace control points by control features: We determine geodetically four supporting points for one patch plane. See Figure 2. The fourth (superfluous) point serves for checking and over-determination purposes.

Figure 2: Examples of three tying patches equivalent to one tying point; respective three control patches equivalent to one control point provided different exposition of the patch-set.
2.2 The Patch-finding Mission

We use chronological data of the Lidar-strips, since this data-structure preserves topology to a high degree whereas a point-cloud has to be considered topologically unstructured. The usual procedure on giving a point-cloud again a topology is triangulation (e.g. Delaunay [6]). But this is time consuming and in the XY-domain sometimes wrong (e.g. a point on the wall might appear inside the eaves of a house).

Since we want to use original data, i.e. unfiltered data, we don't want to use a regular (desirable), but interpolated (regrettable), grid.

Proposition: A topology in the domain of time and nadir-angle as seen from the trajectory is free of loops. (There is one exception: due to pitch-caused "over-scanning" the scanner may "look back" for a while, scanning parts of the ground three times until regaining its usual attitude. This happens seldom and the such generated data may be eliminated easily – during setting up the topology – to grant our proposition.)

For different types of laser scanners we consider in short the topological properties of the recorded point sequence. “topology” in this context defines the neighbourhood relations of points as to “span” the underlying surface in some useful (approximate) sense.

The topology of a laser scanner with push-broom fibre-optics can be mapped to a matrix grid. The topology of a laser scanner with rotating mirror can also be mapped to a matrix-like grid where the scan-lines fill the rows from left (e.g.). The topology of a laser scanner with oscillating mirror can be mapped to also a matrix-like grid where the scan-lines fill the rows alternately from left and right.

Since drop-outs of (single) measurements may occur, the such mapped columns might jump (with respect to Cartesian space) when filling the rows uncritically.

So, we don't use a matrix-approach but the - in this case - superior "vector of vector" approach: We have a vector of rows (i.e. scan-lines); such a row contains a vector of scanner points (i.e. the measurements at a point of time, itself being a vector of attributes);

The topology is then given by the rows and - between (timely) neighboured rows - by the monotony of nadir-angles; this yields - on demand - also a simple triangulation between rows. Another advantage is the fact that the strip-files may be processed simply sequentially keeping a relatively short vector of rows in memory. On the other hand it limits the size of recognizable patches.

This actual vector of rows is called “row-buffer”. We search patch-candidates in the row-buffer.

A patch-candidate is now a (tilted) plane supported by a region of laser-scanner points matching a vector of criteria: it

- is above the surrounding (if we search for a roof)
- is planar within some tolerance (e.g. standard deviation 0.04m)
- has minimal steepness (if we search for a roof)
- has not too many outliers (due to chimney, dormer, etc.)
- has minimal count of supporting points (not too small).
- etc.

Adjustment with data-snooping of a general plane with scan-lag compensation [8] is used to determine patch-candidates in the current row-buffer. So, we get for every strip a list of patch-candidates including quality measures.

A patch is then represented by

- a patch identifier (containing the strip identifier)
- its reference point (chosen centre of the used points of the region; to be kept constant in adjustment)
- its normal vector incl. accuracy
- its shift along the normal incl. accuracy
- scan-lag compensation incl. accuracy
- four anchor points circumscribing the region: each bearing the attributes: time $t$, polar coordinates nadir angle $\nu$, fore-sight $\chi$, distance $\rho$ to the adjusting plane; they represent the many of original polar points and will be used in adjustment as observations (so saving computing time)
- other statistics, etc.

When the row-buffer is worked off, its first row is replaced by the next row as read in from the chronological scanner file becoming logically the last row. So we get a moving (along the trajectory) row-buffer which is administrated as circular list.

This first run through the data gives for every strip an independent list of "normalized" patch-candidates.

In a second run, for every strip (the subset of overlapping strips of) these lists (accordingly sorted) are used as seeds for determining the respective homologous patch-candidate. So, an original patch-candidate may get no, one, or more partners. Any strip produces now a second list of homologous "normalized" patch-candidates. The structure is the same as above. In the first run patch identifiers are created, in the second run they are merely used. Accidental duplication of patch identifiers is prohibited as one can see easily.

It is noteworthy to stress the fact that all these homologous patch-candidates bearing the same patch identifier are of equal rights with concern of adjustment theory since their fundamental argument is merely the same reference point; no correlations between the observations of different strips are introduced.

The second run has an additional criterion in determining the plane: compatibility of normal vectors.

Having these two sets of lists of normalized patches, they serve as input for the adjustment programme. Patches which have no partner are cancelled.

### 3. Block Adjustment by Strips of Laser Scanner Observations

In the following, we describe our actual method of simultaneously fitting laser scanner strips in 3D. The capitalized terms in the following refer to notions used in ORIENT [3].

The basic observations for simultaneous 3D-fitting:
- The polar coordinates $\nu, \chi, \rho$ of the anchor points of the patches in the overlapping areas of laser scanner strips as delivered from the patch finding mission above (the cross bar indicates the observation property). The accuracy of such a polar point observation is estimated from the scanner characteristics (for the angles) and from the (redundant)
measurement process (adjustment of plane for the distance). They are stored in POLAR-rooms. All polar observations of one strip are stored in one POLAR-room.

- Ground coordinates $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ of control points which are measured geodetically terrestrially (total station and GPS) on some of the patches as proposed in the previous section. We recommend also to measure four points for a patch to give it also directional support. See figure 2 for an example. They are stored in CONPOI-rooms.

- The fictitious observations that all ground points of a patch lie in the same (global) plane. The accuracy of such a plane-point was estimated in the adjustment of the patch’s plane. All points of one patch are stored in one GESTALT-room. They stem from both runs of the patch finding mission and from control point measurements. This is the essential tying information between strips and reference frame.

- The shift-coefficients $\alpha_i, \beta_i, \gamma_i$ of all (individual) strips honouring their zero-expectation. The subscript $i$ indicates the exponent of time $t$ in the polynomial term. They are stored in ADPAR=OBS-rooms. Their accuracy is chosen as to handle eventual rank-deficiencies (preventive regularization).

- The tilt-coefficients $\omega_i, \varphi_i, \kappa_i$ of all (individual) strips honouring their zero-expectation. The subscript $i$ indicates the exponent of time $t$ in the polynomial term. They are stored in ADPAR=OBS-rooms. Their accuracy is chosen as to handle eventual rank-deficiencies (preventive regularization).

The basic **observed constants** for simultaneous 3D-fitting:

- The GPS $\mathbf{X}_0, \mathbf{Y}_0, \mathbf{Z}_0$ and IMU $\varpi_0, \varphi_0, \kappa_0$ measurements for the involved POLAR-points mentioned above. They are stored in GPSIMU-rooms parallel to the POLAR-rooms. Every polar point has one entry here with $t$ as common key.

The **unknowns** of the adjustment process are:

- Ground coordinates $\hat{\mathbf{X}}, \hat{\mathbf{Y}}, \hat{\mathbf{Z}}$ for all the tie-(anchor-)points of patches and control points mentioned above. They are stored in the REFSYS -room.

- The shift-coefficients $a_i, b_i, c_i$ of all strips (common or individual). The subscript $i$ indicates the exponent of time $t$ in the polynomial term. They are stored in ADPAR -rooms. The terms of order $i=0$ handle GPS-shift, those with $i=1$ can handle GPS-drift (i.e. shift change linearly with time).

- The tilt-coefficients $\omega_i, \varphi_i, \kappa_i$ of all strips (common or individual). The subscript $i$ indicates the exponent of time $t$ in the polynomial term. They are stored in ADPAR -rooms. The terms of order $i=0$ handle IMU-index errors; $i=1$ can handle change of index errors linearly with time (i.e. IMU-drift).

- Common rotations $\omega_0, \varphi_0, \kappa_0$ handle bore-sight alignment, i.e. differential rotation of IMU with respect to the Lidar-device. They are stored in a ROTPAR -room.

- The shift-coefficients $c_{i0}$ of all planes describing a patch. They are stored in ADPAR -rooms.

- Optionally, the tilt-coefficients $c_{10}, c_{01}$ of all planes describing a patch. They are stored in ADPAR -rooms. They can handle wrong tilt of patch planes caused by misalignment of the IMU.

The **adjustment** is expected to **minimise** the following quantities by **least squares**: 

• The residuals of observed polar points $\mathcal{P}, \mathcal{X}, \mathcal{P}$ in the strips.
• The residuals of control points $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ with respect to patch planes.
• The offset of the adjusted ground points from the adjusted global patch plane.
• The polynomial shift-coefficients $\alpha_i, \beta_i, \gamma_i$ - since they are expected to have zero-values (corresponding to correct GPS data). This yields relatively small values of the correction polynomials ([2], p37).
• The polynomial drift-coefficients $\omega_i, \varphi_i, \kappa_i$ - since they are expected to have zero-values (corresponding to correct IMU data). This yields relatively small values of the correction polynomials ([2], p37).

The incorporation of the polynomial coefficients $a_i, b_i, c_i$ and $\omega_i, \varphi_i, \kappa_i$ into the LSQ minimum condition is called "preventive regularisation". The term regularisation comes from the definition of a “regular matrix”, i.e. a full-rank matrix, i.o.w. an invertible matrix. Alike, a singular normal equation matrix has to be made regular before a solution may be obtained. Such singularities may occur in our context when:

- Not enough ground control information is available (datum problem),
- Not enough deformation control information is available (degree of polynomial problem due to over-parameterisation),
- Bad distribution of ties resp. high correlation between adjacent strips due to weak ground control (typical polynomial oscillations).

ORIENT has built in a regularisation on the fly; i.e. when a singularity occurs (solving the normal equation system), a fictitious observation for the affected unknown will be generated allowing the decomposition process to continue. This is done automatically – the user is informed via protocol to let him make up his opinion about the validity of the results.

We have also to take care of getting rid of wrong hypotheses $\alpha_i, \beta_i, \gamma_i = 0$ or $\omega_i, \varphi_i, \kappa_i = 0$: Gross error detection by data snooping is recommended for that. Testing of significance of the $a_i, b_i, c_i, \omega_i, \varphi_i, \kappa_i$, and $c_{1,0}, c_{0,1}$ is also a must.

### 4. Minimal Distribution of Ground Control Points

We suppose that Lidar-strips have a similar geometric behaviour as strips in DGPS-supported aero-triangulation. We have to cope with deficiencies of the kinematic GPS as drift and even jumps on turns. In the meanwhile – as long as no exhaustive tests (simulations) are performed we suggest ground control to overcome the phenomena. The background of the following figure 3 is discussed in [2].
5. Block Montafon

This block, covering Gargellental and Garneratal in the region Montafon of Vorarlberg, stretches in altitude from 880m to 2875m, so spanning 2000m in height extent. So, this block had to be flown in two missions, one of them covering the valley regions with 24 strips the other one the superior areas with 52 strips. 4 of the 24 were cross-strips, and 3 of the 52. Mainly in the crossing strips tie positions were selected according figure 2 and then plane patches were searched for in every overlaying strip automatically. Acceptance criteria for tie patches were: more than 12 points with a standard deviation less than 5cm from the adjusting plane. Since the flown data had been clipped by the vendor at the project limits, a lot of strips lost their crossing partner. For these strips extra tie points had to be determined. Altogether, 1002 such plane-patches were used; the many, 340 of them occurred in 5 strips, 6 of them even in 15 strips, but also 244 only in 2 strips. Only 4 patches showed up as mismatch and had to be evicted by error detection methods. Additionally, the LVA Feldkirch hat prepared 42 ground patches (supported by 170 points on roofs in easily accessible areas) in a height range from 850m to 2114m. These control patches were found in up to 14 strips. Moreover, 18 patches on football fields were also used as height control. The adjustment of all these mentioned observations was done to determine GPS-shift and IMU-misalignment of
each of the two flight missions; moreover, experiments with GPS-shift and IMU-misalignment individually for every strip were undertaken using preventive regularisation. The analysis of the variants is still in progress.

6. Résumé

For high demands in accuracy – not mere precision – we need some ground control. The ideal configuration of control points is not yet known. With high probability the same procedure as used for GPS-supported aerotriangulation ([2], p157, fig B5.3-5) can be recommended: i.e. control points in the corners of a block together with cross-strips at the ends of the block. These cross-strips may be replaced by chains of height control points at the ends of the block.

The area of interest should be extended by about one strip-width to grant consistency of the strip-sewing.

Quality control of a block is necessary: graphic representations of discrepancies is a must to detect any system anomalies.

References