# Fractal Geometry as an Approach to Quality in Architecture

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**Abstract.**

The paper asks whether there is a connection between quality in architecture and the characteristics of form that can be described by means of fractal geometry. Taking up Carl Bovill's ideas, the question will be dealt with in two respects. First the existence of fractal characteristics (self-similarity, ruggedness and iteration) is studied in works of architecture that are commonly regarded as outstanding examples such as buildings by Antoni Gaudí, Frank Lloyd Wright, Bruce Goff or Gerrit Rietveld. In the course of that process, various distances and levels of scale are examined.

The second approach is based on measuring the fractal dimension of architectural drawings. The 'box-counting method' is applied to different levels of scale. Factors influencing the different parameters of this method of measurement are studied. The hypothesis is put forward that a connection between the fractal characteristics of form and quality exists, which determines architectural rank. The paper also deals with the difficulties of verifying such a hypothesis.

**Historic Examples**

In architecture but also in city-planning, fractal characteristics can be found in more or less pronounced ways. Those examples which offer almost exactly the same elements on different scales form the exceptions. In analogy to the mathematically formed fractals, the octagon of the Castel del Monte for example can be treated as the initiator. This starting form is completed by additional octagons at its edges – this defines the generator. This rule of projection could be continued until infinity. The existing built object thus only offers the first step, whereas some Indian temples show a self-similarity with several iterations. In this case the basic form of the overall temple is repeated in a slightly modified way in different sizes on its surface. Thus a surface with a similar roughness is produced, independently of the scale used. A third example can be given by some Gothic windows. They are formed by a repeated overarching of two pointed arches by an additional one. The existing examples of the Gothic period show this repetition that is continued over a couple of scales.

**Variation of a Motive**

In architecture, there are in general no elements that are exactly the same as the whole. Self-similarity is then the variation of a motive on different scales. The character of the whole building – or the environment – is continued in several details.

What can this transformation look like, for example with Antoni Gaudí? For him the column is no longer just a vertical, straight, cylindrical Euclidean form. It rather follows the same rules as a tree that is formed by nature – from stem to branch and to the widely ramified boughs. The construction of columns in the Sagrada Familia looks like a wood made of stone. Antonio Gaudí not only brought nature into ornamental forms as the designers and architects of the Art Nouveau did.[1] He used the construction principles of nature, its underlying structure, e.g. the interrelations of the forces of a tree. This principle is not only true for the whole, but also for each detail.

The Colegio Teresiano by Antoni Gaudí, built in 1889, offers an example of self-similarity on the basis of the pointed arch. The straight variant is given by the pinnacle made of brick at the border of the roof. Inside follows the parabolic arch of the corridors and the window- and door-reveals. With simple tools and some variation of the basic form, Antoni Gaudí reached the impression of Spartan soberness and architectonical complexity at the same time.[2]

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01 Zerbst Rainer, *Antoni Gaudí* (1993), Benedikt Taschen Verlag GmbH, page 31

02 Zerbst Rainer, *Antoni Gaudí* (1993), Benedikt Taschen Verlag GmbH, page 94

**Details as a Reflection of the Whole**

A distant view gives a first impression of buildings. Coming closer, it depends on the respective building itself whether or not some more details of interest appear. As an example the view of Robie House by Frank Lloyd Wright offers more and more details on a lower scale with a repetition or variation of forms respectively. The building "remains" interesting. In addition to that, self-similarity helps us with regard to orientation. When Frank Lloyd Wright called for inspiration from nature, he did not copy nature but translated it into architecture. What he was looking for was the importance of a simple, specific form, which would be the expression of the building – the leitmotif. The whole and all other formal elements – as its reflection – are derived from this basic idea and basic form. The formal elements are held together in scale and character. [3]

Frank Lloyd Wright's prairie-houses are developed out of the conditions of the environment with regard to purpose, material and construction. The flat cubes are connected with nature by wide projecting roofs – an analogy with the broad plane of the prairie. The horizontality of the overall view of Robie House is carried on by the window-strip and the long stretches of stone, including the water table, copings and sills. This effect is underlined by the format of the bricks and the image of the joints. The mortar beds are stressed by cutting back, while the narrow header joints are of red-colored cement and brought flush. [4] So the iteration follows from the whole building down to the brick. Besides the angle of the living room prow is found in variations and on different scales all over the building. This ranges from the slope of the roof, over the glass designs of doors and windows, the pattern of the iron gates to the garage court down to the lamp designs. The four corners of the dinning room table take up stands, containing stained glass lamps, whose glass design is translated into the shelves for flower arrangements on top. Simple forms and rules, which are embodied in a basic idea, run through several iterations. So the cascade of interesting detail runs down to the design of the colored windows and the bricks respectively.

Another example for the fractal concept by Frank Lloyd Wright is "Fallingwater", built in 1935. The horizontal formation of the environment, the cubic riffs, is continued in the reinforced-concrete cantilever slabs, the terraces with parapets, down to the stratification of the masonry of stones. The building seems part of the rock formations. In addition to that, the eddy of the river is brought into the building by the waxed uneven stones on the floor that are mirroring the shades of the trees from outside.

**Variation of the Fractal Characteristics**

Some of Bruce Goff's buildings contain characteristics of fractal geometry, for example Eugene Bavinger House near Norman, Oklahoma, built in 1950. The floor plan is based on a curve that has the form of the self-similar Cephalopode Nautilus. The units of the Nautilus follow the structure of a logarithmic spiral curve. This curve is called self-similar because the angles of the tangents are equal in all points. The Joe Price Studio in Bartlesville, Oklahoma built in 1956, shows another example of fractal architecture by Bruce Goff. All shapes include equivalent triangles on different scales. The angle of 60 degree or the multiple of it is repeated on different scales. The conversation 'pit' is hexagonal, the walls of the music-room are triangular and the ceiling of this room consists of a three-cornered decoration for better sound quality through reflection. [5]

The fractal concept is not only used in architecture that may belong to the so-called "organic" architecture. It can also be found in buildings that belong to the smooth modern style. Schröder House by Gerrit Rietveld, built in 1924, as an example of the de Stijl, is the result of assembling elements of lines and disks of different functions, characteristics and scales. The main elements of large flat, white disks primarily form the spatial and constructive structure of the building. Then some smaller flat disks, the linear elements as strong horizontal and vertical accents and finally windows, doors and railing follow. This combination is mirrored inside by the furniture.

**Fractal Dimension – Box-Counting Method**

The examples shown in this paper illustrate very clearly the differences between "fractal" architecture and the mathematically formulated ones. First of all, parts of the whole are, like in nature, no identical, scaled down images of the object. Furthermore, fractal characteristics in architecture are limited to a range of certain scales.

The buildings listed up in this paper nevertheless do not only offer some kind of fractal characteristics but also belong to architecture with some kind of quality. If self-similarity is maintained over certain scales, the building itself remains interesting over several levels. Then if fractal characteristics, self-similarity in particular, are a first indicator of quality in architecture, is it possible to measure this quality?

03 Bovill Carl, *Fractal Geometry in Architecture and Design* (1996), Birkhäuser Bosten, pages 127-128

04 Hoffmann Donald, *Frank Lloyd Wright's Robie House* (1984), Dover Publications, Inc., page 42

05 Jencks Charles, *The Architecture of the Jumping Universe* (1995), Academy Editions, page 44

Facades are more than the one-dimensional lines which mark them. As a rule, they consist of cuts such as windows and doors, indentations and ledges, structured surfaces and various details on different scales. They rather own a broken, fractal dimension, which can be measured by the box-counting method.

**Quality and Fractal Dimension**

In case of the examples described above, it is conspicuous that they offer a similar ruggedness on different scales. Fractal dimension can be treated as the expression of the degree of roughness, which shows how much texture an object has. With the help of the box-counting dimension this degree can be measured from one scale to the next. First investigations have shown that Robie House by Frank Lloyd Wright offers a relatively constant fractal dimension over several scales – '*Robie House Measurement 01*': $D_{(1/8\text{-}1/16)}$=1.70, $D_{(1/16\text{-}1/32)}$=1.73, $D_{(1/32\text{-}1/64)}$=1.66.

Some more measurements of Robie House offer different results. The reason for that can be found in the influence of various parameters. First of all the kind of preparation of plans is important; the measurement depends on the thickness of the lines used and on the presented details, the detail richness. Another factor of influence forms the position of the specific grid used in relation to the image. The smallest and the largest box size and the reduction factor play a different role. By means of these parameters, the distance and the quickness of the approach is given. Both factors can modify the results – '*Robie House Measurement 05*': $D_{(1/5\text{-}1/10)}$=1.53, $D_{(1/10\text{-}1/20)}$=1.60, $D_{(1/20\text{-}1/40)}$=1.82, $D_{(1/40\text{-}1/80)}$=1.49).

The approach to a building normally takes place continually. So the reduction factor has to be chosen in line with that. Reducing the steps of approach the resulting fractal dimension is strongly fluctuating – '*Robie House Measurement 13b*': $D_{(1/32\text{-}1/48)}$=2.02, $D_{(1/48\text{-}1/64)}$=1.12, $D_{(1/64\text{-}1/96)}$=1.85. Thus we can expect that certain architectonic elements and the materials used are effective only on specific scales. Similar roughness across certain scales can be proved by average values of specific scales. If these results remain equally high, also in relation to the mean value over all scales, as some reference, then a first statement can be given about the development of the character in form of its roughness. What is important in this connection is the lower and uppermost box-size, which is defined by the whole building itself and the smallest presented detail.

**Conclusions with Regard to Fractal Dimension**

Nature contains self-similarity and this is what really makes it fascinating from the large to the small. In comparison to architecture this is also the concept that makes Gothic cathedrals, rural houses and examples of the so-called organic architecture so interesting and diversified. A certain continuity of the measured 'box-counting' dimension across several scales can be treated as a first indicator that the building under view remains interesting over these scales. The reason lies in the emerging of new details – which means information – approaching the building. This continuity also offers some kind of recognition factor from the whole to the detail. Moreover, integrating the environment into the building, which is true for "Fallingwater", achieves a strong combination between building and nature. But the height of the fractal dimension neither tells us something about the quality of a building nor does it allow conclusions about the form at all.

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