

# Perturbation-based Distributed Beamforming for Wireless Relay Networks

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**Abstract**—This paper deals with distributed beamforming techniques for wireless networks with half-duplex amplify-and-forward relays. Existing schemes optimize the beamforming weights based on the assumption that channel state information (CSI) is available at the relays. We propose to use adaptive beamforming based on deterministic perturbations and limited feedback (1-bit) from the destination to the relays in order to avoid CSI at the relays. Two scalable perturbation schemes are considered and practical implementation aspects are addressed. Simulation results confirm that the proposed techniques closely approach optimum performance and have satisfactory tracking properties in time-varying environments.

## I. INTRODUCTION

### A. Background

Terminal cooperation in wireless networks has been recognized as a means to form virtual arrays that can realize spatial diversity in a distributed fashion. An important special case is distributed beamforming with half-duplex amplify-and-forward (AF) relays. The coherent AF scheme in [1] requires local channel phase information at the relays to achieve coherent phase combining with equal power at all relays. Beamforming with non-uniform power allocation (PA) under a sum power constraint [2, 3] and under individual relay power constraints [4] offers significant performance gains. However, optimal beamforming with PA places strong requirements regarding channel state information (CSI) at the relays. For centralized arrays with co-located antennas, this requirement has been circumvented by adaptive gradient beamforming techniques that iteratively adjust the beamforming weights using stochastic vector perturbations and limited feedback from the destination [5]. A related approach based on deterministic perturbations is presented in [6]. In a similar spirit, feedback-assisted distributed beamforming with phase perturbation in wireless networks was considered in [7] and extended to the multiuser context in [8]. However, both methods do not assume a relay setup and do not address distributed PA.

### B. Contribution and Organization of Paper

We consider perturbation-based beamforming (PB-BF) with 1-bit feedback in a relay network. Under the assumption of a sum power constraint, the relays use the feedback bit to adapt their beamforming weights in order to maximize either the signal-to-noise ratio (SNR) or the received signal power at the

destination. This approach does not require any CSI at the relays. Two different perturbation schemes are investigated, both of which are based on deterministic perturbation sets to avoid extensive signaling/feedback overhead. Within this context, we present a scalable protocol, discuss implementation aspects, and provide numerical performance comparisons. Simulation results corroborate that our approach can satisfactorily track time-varying channels in non-static environments. We note that in the context of wireless ad-hoc networks a related idea was touched upon in [9] without explicitly addressing the important practical problem of weight exchange.

The rest of the paper is organized as follows. Section II introduces the system model and Section III proposes perturbation-based distributed beamforming with 1-bit feedback. A comparison with optimum batch solutions is provided in Section IV. Section V discusses simulation results and conclusions are provided in Section VI.

## II. SYSTEM MODEL

We consider a perfectly synchronized wireless network with single antenna nodes where a single source  $\mathcal{S}$  communicates with a single destination  $\mathcal{D}$  via  $R$  half-duplex relays  $\mathcal{R}_i$ ,  $i = 1, \dots, R$  (cf. Fig. 1). The half-duplex constraint necessitates a two-hop protocol. In the first hop,  $\mathcal{S}$  transmits the signal  $\sqrt{P_s}s$  to the relays which receive

$$x_i = \sqrt{P_s}h_i s + w_i, \quad i = 1, \dots, R. \quad (1)$$

Here,  $s$  is the transmit symbol normalized as  $\mathbb{E}\{|s|^2\} = 1$  ( $\mathbb{E}\{\cdot\}$  denotes expectation),  $P_s$  denotes the average transmit power of  $\mathcal{S}$ ,  $h_i$  is the complex coefficient of the flat fading “backward” channel<sup>1</sup> between  $\mathcal{S}$  and  $\mathcal{R}_i$ , and  $w_i \sim \mathcal{CN}(0, N_0)$  denotes i.i.d. complex Gaussian noise. In the AF scenario considered, the second hop amounts to each relay transmitting a complex scaled version of the signal it has received, i.e.,

$$r_i = \alpha_i^* \lambda_i x_i, \quad \text{with } \lambda_i \triangleq \sqrt{\frac{P}{P_s |h_i|^2 + N_0}}. \quad (2)$$

Here, complex conjugation (superscript  $*$ ) of the beamforming weights  $\alpha_i$  will simplify notation later on, and  $\lambda_i$  is a power normalization factor such that the average relay power is  $\mathbb{E}\{|r_i|^2 | h_i\} = |\alpha_i|^2 P$ . The destination receives  $y = \sum_{i=1}^R g_i r_i + v$ , where  $g_i$  denotes the complex coefficient of the “forward” channel between  $\mathcal{R}_i$  and  $\mathcal{D}$ , and  $v \sim \mathcal{CN}(0, N_0)$

<sup>1</sup>Note that our discussion does not presume specific channel statistics.

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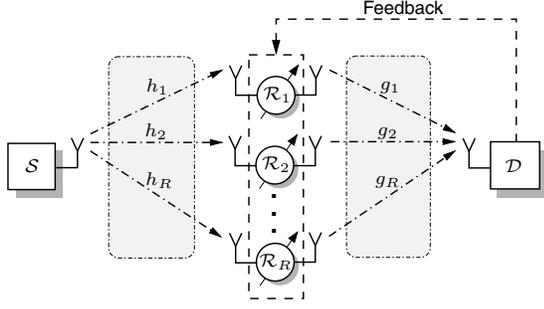


Fig. 1. Wireless relay network with feedback.

is complex Gaussian noise. Inserting (1) and (2) yields the compound channel model<sup>2</sup>

$$y = \xi s + \eta, \quad \text{with } \xi \triangleq \alpha^H \bar{\mathbf{h}}, \quad \eta \triangleq \alpha^H \bar{\mathbf{G}} \mathbf{w} + v. \quad (3)$$

Here,  $\bar{\mathbf{h}} \triangleq [\bar{h}_1 \dots \bar{h}_R]^T$  with  $\bar{h}_i \triangleq h_i g_i \lambda_i \sqrt{P_s}$ ,  $\bar{\mathbf{G}} \triangleq \text{diag}(\bar{g}_1, \dots, \bar{g}_R)$  with  $\bar{g}_i \triangleq g_i \lambda_i$ , and  $\mathbf{w} \triangleq [w_1 \dots w_R]^T$ . Since the weight vector  $\alpha \triangleq [\alpha_1 \dots \alpha_R]^T$  enters also the noise part in (3), it demands careful design to prevent noise amplification.

From (3), the average power corresponding to the signal part of  $y$  and the SNR at  $\mathcal{D}$  are respectively obtained as

$$P_{\mathcal{D}}(\alpha) \triangleq \mathbb{E}\{|\xi s|^2 | \bar{\mathbf{h}}\} = \alpha^H \bar{\mathbf{h}} \bar{\mathbf{h}}^H \alpha = |\alpha^H \bar{\mathbf{h}}|^2, \quad (4)$$

$$\rho(\alpha) \triangleq \frac{\mathbb{E}\{|\xi s|^2 | \bar{\mathbf{h}}\}}{\mathbb{E}\{|\eta|^2 | \bar{\mathbf{G}}\}} = \frac{1}{N_0} \frac{\alpha^H \bar{\mathbf{h}} \bar{\mathbf{h}}^H \alpha}{1 + \alpha^H \bar{\mathbf{G}} \bar{\mathbf{G}}^H \alpha}. \quad (5)$$

In the following, we will use  $\gamma(\alpha)$  as generic notation for our objective function, which can either be  $P_{\mathcal{D}}(\alpha)$  or  $\rho(\alpha)$ . The beamforming vector  $\alpha$  can be batch designed to maximize  $\gamma(\alpha)$  subject to a specific relay power constraint. We resort to two types of power constraints: Constraining the complex beamforming weights to  $|\alpha_i|^2 = 1$  (this amounts essentially to phase-matching at the relays [7]) ensures identical per-relay power  $\mathbb{E}\{|r_i|^2 | h_i\} = P$ . In contrast, the total sum power constraint  $\mathbb{E}\{\sum_{i=1}^R |r_i|^2 | h_1, \dots, h_R\} = P$  requires that the beamforming vector has unit Euclidean norm,  $\|\alpha\|^2 = 1$ . However, such batch designs entail stringent requirements regarding the CSI available to the relays (cf. Section IV).

### III. ADAPTIVE PERTURBATION-BASED BEAMFORMING

#### A. Transmission Principle

To avoid CSI at the relays, we study distributed beamforming using feedback-assisted adaptive weight perturbation. The idea underlying this approach is to maximize the objective function  $\gamma(\alpha)$  by adjusting the beamforming weights at the relays in an iterative manner using limited feedback (see Fig. 1). For co-located arrays with centralized processing similar ideas have been proposed in [5, 6].

Transmission happens in frames consisting of a training interval  $\mathcal{T}_p$  and a data interval  $\mathcal{T}_d$ . The relays use different beamforming weights to forward the training and data parts of each frame received from  $\mathcal{S}$  to  $\mathcal{D}$  according to (2). The idea

<sup>2</sup>Superscript  $T$  ( $H$ ) denotes (Hermitian) transposition;  $\text{diag}(x_1, \dots, x_m)$  is the  $m \times m$  diagonal matrix with diagonal elements  $x_1, \dots, x_m$ .

is to apply the currently best beamforming vector, denoted  $\alpha_k$  ( $k$  is the frame index), to the data while using a perturbed version  $\tilde{\alpha}_k$  of the beamforming vector for the training portion. The destination evaluates the effectiveness of the perturbed weights and checks whether or not the perturbation improved the objective function  $\gamma(\alpha_k)$ . It then provides the relays with one bit of feedback to indicate which beamforming vector shall be used to forward the data of the next frame.

In the proposed scheme, the weights  $\tilde{\alpha}_k$  are obtained by an additive perturbation from the data beamforming vector  $\alpha_k$ . While in a centralized setup the perturbation vectors can be chosen randomly for each frame, our distributed setup necessitates a deterministic vector set, collected in a  $R \times N$  matrix  $\mathbf{Q} = [\mathbf{q}_0 \dots \mathbf{q}_{N-1}]$ , from which the perturbation vector is picked in a cyclic fashion (cf. Section III-D).

In the following, we present two variants of the proposed PB-BF scheme and describe the individual steps in more detail. For this discussion, we assume that all channel coefficients remain constant during the weight adaptation process.

#### B. Take/Reject (T/R) Perturbation

The perturbed weights for the  $k$ th frame are computed as

$$\tilde{\alpha}'_k = \alpha_k + \mu \mathbf{q}_{k \bmod N}, \quad (6)$$

where  $\mu$  is a step-size parameter determining the adaptation rate, followed by proper normalization, i.e.,  $\tilde{\alpha}_k = \tilde{\alpha}'_k / \|\tilde{\alpha}'_k\|$  in case of a sum power constraint and  $\tilde{\alpha}_{k,i} = \tilde{\alpha}'_{k,i} / |\tilde{\alpha}'_{k,i}|$  for a per-relay power constraint. The weights  $\tilde{\alpha}_k$  are applied to the training sequence received at the relays, which is then forwarded to  $\mathcal{D}$ . At  $\mathcal{D}$ , the known training sequence and the receive signal are used to evaluate the performance of  $\tilde{\alpha}_k$  within  $\mathcal{T}_p$  according to the objective function, i.e.,  $\tilde{\gamma}_k = \gamma(\tilde{\alpha}_k)$ . Recall that  $\gamma(\cdot)$  represents either the received signal power  $P_{\mathcal{D}}(\cdot)$  in (4) or the SNR  $\rho(\cdot)$  in (5). The actual estimation of these quantities will be addressed in Section III-E.

The destination then compares  $\tilde{\gamma}_k$  to the performance  $\gamma_k = \gamma(\alpha_k)$  achieved with the beamforming vectors  $\alpha_k$  that up to this point performed best. If  $\tilde{\gamma}_k \leq \gamma_k$ ,  $\tilde{\alpha}_k$  does not perform better than  $\alpha_k$  and hence the relays should stick with  $\alpha_k$  for the data in the next transmission frame (“reject”  $\tilde{\alpha}_k$ ). Otherwise ( $\tilde{\gamma}_k > \gamma_k$ ), the beamforming vector  $\tilde{\alpha}_k$  improves on  $\alpha_k$  and should thus be used in the next frame to transmit the data (“take”  $\tilde{\alpha}_k$ ). This rationale can be implemented by letting  $\mathcal{D}$  provide the relays with a single bit of feedback, given by

$$c_k = \mathbf{u}(\tilde{\gamma}_k - \gamma_k),$$

where  $\mathbf{u}(\cdot)$  denotes the unit step function. Depending on the feedback bit, the relays update the data beamforming vector for the next frame as

$$\alpha_{k+1} = \begin{cases} \alpha_k, & \text{if } c_k = 0, \\ \tilde{\alpha}_k, & \text{if } c_k = 1. \end{cases}$$

The vector  $\alpha_{k+1}$  will be the basis for the next perturbation according to (6). The destination  $\mathcal{D}$  performs the corresponding update  $\gamma_{k+1} = \max\{\tilde{\gamma}_k, \gamma_k\}$ . This process continues in an iterative manner. During the first frame, the scheme is

initialized with  $\tilde{\alpha}_0 = \alpha_0$  (by setting, e.g.,  $\alpha_0 = [1 \dots 1]^T / \sqrt{R}$  in case of a sum power constraint) and  $\gamma_0 = 0$ .

### C. Plus/Minus (P/M) Perturbation

T/R perturbation has the advantage that performance never deteriorates, i.e.,  $\gamma(\alpha_{k+1}) \geq \gamma(\alpha_k)$ . On the other hand, in many cases the perturbation (6) will not yield an improvement, which entails  $\alpha_{k+1} = \alpha_k$  and hence slow adaptation. We next discuss an alternative perturbation scheme with faster adaptation rate. Here, the training interval  $\mathcal{T}_p$  is split into two halves  $\mathcal{T}_p^+$  and  $\mathcal{T}_p^-$  for which different perturbed beamforming vectors are used, i.e.,<sup>3</sup>

$$\tilde{\alpha}_k^+ = \frac{\alpha_k + \mu \mathbf{q}_{k \bmod N}}{\|\alpha_k + \mu \mathbf{q}_{k \bmod N}\|}, \quad \tilde{\alpha}_k^- = \frac{\alpha_k - \mu \mathbf{q}_{k \bmod N}}{\|\alpha_k - \mu \mathbf{q}_{k \bmod N}\|}. \quad (7)$$

The destination  $\mathcal{D}$  then measures the performance of  $\tilde{\alpha}_k^+$  and  $\tilde{\alpha}_k^-$  by evaluating the objective function according to  $\tilde{\gamma}_k^+ = \gamma(\tilde{\alpha}_k^+)$  and  $\tilde{\gamma}_k^- = \gamma(\tilde{\alpha}_k^-)$  within  $\mathcal{T}_p^+$  and  $\mathcal{T}_p^-$ , respectively. While in principle we could pick the beamforming weights corresponding to the maximum of  $\tilde{\gamma}_k^+$ ,  $\tilde{\gamma}_k^-$ , and  $\gamma_k = \gamma(\alpha_k)$  (the performance of the current data beamforming vector), 1-bit feedback can only support binary choices. Hence,  $\alpha_k$  will be discarded in any case.  $\mathcal{D}$  broadcasts the feedback bit  $c_k = u(\tilde{\gamma}_k^- - \tilde{\gamma}_k^+)$  to the relays, indicating whether the ‘‘plus’’ perturbation  $\tilde{\alpha}_k^+$  or the ‘‘minus’’ perturbation  $\tilde{\alpha}_k^-$  performs better. In the next frame, the relays use the beamforming vector

$$\alpha_{k+1} = \begin{cases} \tilde{\alpha}_k^+, & \text{if } c_k = 0, \\ \tilde{\alpha}_k^-, & \text{if } c_k = 1. \end{cases}$$

Although P/M perturbation shows typically faster adaptation than T/R, sometimes both perturbations in (7) deteriorate the performance with respect to  $\gamma_k$ . Thus, P/M performance may fluctuate continually. Furthermore, only half of the training interval can be used to estimate each of  $\tilde{\gamma}_k^+$  and  $\tilde{\gamma}_k^-$ .

### D. Perturbation Set

Vector normalization of the weights in (6) and (7) ensures that the sum power constraint is satisfied, but requires that each relay knows all elements of the beamforming vector. Hence, a *stochastic* gradient algorithm with random perturbation vectors (as in [5]) cannot be applied to relay networks; this would require to exchange all weights among the relays, thus imposing a tremendous signaling overhead. Rather, we propose to use a matrix  $\mathbf{Q}$  of *deterministic* perturbation vectors (cf. [6]) known to each relay. This allows each relay to keep track of *all* beamforming weights and to perform vector normalization locally. Reasonable choices for the deterministic perturbation matrix are  $\mathbf{Q} = [\mathbf{F}, j\mathbf{F}]$  (i.e.,  $N = 2R$  vectors) for P/M perturbation and  $\mathbf{Q} = [\mathbf{F}, j\mathbf{F}, -\mathbf{F}, -j\mathbf{F}]$  ( $N = 4R$ ) for T/R perturbation; here,  $\mathbf{F}$  is an  $R \times R$  unitary matrix. We observed that choosing  $\mathbf{F}$  as discrete Fourier transform (DFT) matrix provides good performance under various conditions.

Note that with the per-relay power constraint, element-wise normalization does not require knowledge of all weights at each relay, thus allowing also for stochastic perturbations.

<sup>3</sup>These expressions are valid for the sum power constraint. With a per-relay power constraint, weight normalization has to be performed element-wise.

### E. Channel, Power, and SNR Estimation

We next discuss the estimation of the receive signal power  $P_{\mathcal{D}}(\alpha)$  in (4) and the SNR  $\rho(\alpha)$  in (5) which are used as performance measures, as well as the estimation of the compound channel  $\xi$  in (3) required for coherent detection.

In the following, we omit the frame index  $k$  and denote the pilot sequence within a transmission frame as  $s_p[n]$ ,  $n \in \mathcal{T}'$ . The destination can then compute the maximum likelihood (ML) estimate of the compound channel as

$$\hat{\xi} = \frac{\sum_{n \in \mathcal{T}'} y[n] s_p^*[n]}{\sum_{n \in \mathcal{T}'} |s_p[n]|^2}. \quad (8)$$

Using (8), the ML estimates of receive signal power and SNR can be obtained as

$$\hat{P}_{\mathcal{D}} = |\hat{\xi}|^2, \quad \hat{\rho} = \frac{|\hat{\xi}|^2}{\frac{1}{|\mathcal{T}'|} \sum_{n \in \mathcal{T}'} |y[n] - \hat{\xi} s_p[n]|^2}. \quad (9)$$

For T/R perturbation, (8) and (9) are evaluated using  $\mathcal{T}' = \mathcal{T}_p$ . After each weight update the destination stores the channel estimate and uses it for data detection of the subsequent frames till the next update occurs. With P/M, (8) and (9) are calculated twice in each frame with  $\mathcal{T}' = \mathcal{T}_p^+$  and  $\mathcal{T}' = \mathcal{T}_p^-$ . The channel estimate corresponding to the better beamforming vector is then kept for data detection in the next frame. Alternatively, an approximate ML estimate for the channel coefficient  $\xi$  can be obtained by evaluating (8) over the whole training interval ( $\mathcal{T}' = \mathcal{T}_p$ ) within the same frame (cf. [5]), provided that the step-size  $\mu$  is chosen sufficiently small and  $|\mathcal{T}_p^+| = |\mathcal{T}_p^-|$ .

### F. Birth and Death of Relays

Our deterministic perturbation approach is scalable in that it can be easily adapted to deal with the situation where relays enter (‘‘birth’’) or leave (‘‘death’’) the network, even in the case of a sum power constraint. We assume that the maximum number of relays is  $R_{\max}$ , of which  $R \leq R_{\max}$  are active and can exchange information with  $\mathcal{D}$  but not with each other. In essence, the destination and each relay keep track of all the active relays. The relays can then compute the required vector norm locally. Additionally, all relays know their ‘‘identity’’ (index  $i$ ), which is fixed and enables them to pick their corresponding beamforming weight. If a relay  $\mathcal{R}_{i_0}$  drops out, it informs  $\mathcal{D}$  which in turn broadcasts the relay index  $i_0$  to the remaining relays using  $\log_2(R_{\max})$  bits. These relays then exclude the corresponding beamforming/perturbation weight from the update process. If a new relay enters the system, it contacts  $\mathcal{D}$  which in turn broadcasts  $R_{\max}$  bits to indicate to all relays (also to the new one) which relays are active. Since the new relay cannot know the current beamforming weights of the other relays, the weight adaptation process needs to be re-initialized in this case.

In the case of a per-relay power constraint, element-wise weight normalization allows that the relays only need to track their own weights. This renders a birth-and-death protocol particularly easy, since relays can enter or leave the system completely arbitrarily without informing the other relays.

#### IV. COMPARISON WITH OPTIMAL BEAMFORMING

We next compare optimal batch beamforming designs with adaptive PB-BF. The former requires each relay having either local CSI (i.e., each relay's own back- and forward channel) or global CSI (i.e., all channels) available, whereas PB-BF exploits limited feedback to avoid CSI at the relays.

##### A. Optimal Batch Designs

*Equal Gain Combining (EGC).* Maximizing  $P_{\mathcal{D}}(\alpha)$  or  $\rho(\alpha)$  under a per-relay power constraint yields the beamforming weights  $\alpha_i = \bar{h}_i/|\bar{h}_i| = h_i g_i/|h_i g_i|$  that amount to coherent combining [1]. This scheme requires that each relay knows the phase of its backward and forward channel.

*Power Maximization under Sum Power Constraint (P-SP).* Optimizing  $P_{\mathcal{D}}(\alpha)$  in (4) under a sum power constraint amounts to maximizing  $|\alpha^H \bar{\mathbf{h}}|$  subject to  $\|\alpha\|^2 = 1$ . Via the Cauchy-Schwarz inequality, the solution is obtained as  $\alpha_i = \bar{h}_i/\|\bar{\mathbf{h}}\|$  requiring global CSI at  $\mathcal{R}_i$ . Alternatively, if global CSI is available at  $\mathcal{D}$ , each relay needs only local CSI and feedback of  $\|\bar{\mathbf{h}}\|$  from  $\mathcal{D}$ . This shows that the relays optimally allocate their transmit power to match the current local fading coefficients while performing coherent combining.

*SNR Maximization under Sum Power Constraint (S-SP).* The beamforming vector  $\alpha$  can also be chosen to maximize the SNR  $\rho(\alpha)$  in (5) [2, 3]. Under the sum power constraint  $\|\alpha\|^2 = 1$  this can be shown to lead to a generalized eigenvalue problem whose solution is ( $\mathbf{I}$  is the identity matrix) [3]

$$\alpha = \frac{(\mathbf{I} + \bar{\mathbf{G}}\bar{\mathbf{G}}^H)^{-1}\bar{\mathbf{h}}}{\|(\mathbf{I} + \bar{\mathbf{G}}\bar{\mathbf{G}}^H)^{-1}\bar{\mathbf{h}}\|}. \quad (10)$$

Again this essentially requires either global CSI at the relays or local CSI with feedback of  $\|(\mathbf{I} + \bar{\mathbf{G}}\bar{\mathbf{G}}^H)^{-1}\bar{\mathbf{h}}\|$  from  $\mathcal{D}$ . In contrast to P-SP, (10) also accounts for noise amplification.

##### B. Comparison with PB-BF Schemes

It can be shown that  $\rho(\alpha)$  and  $P_{\mathcal{D}}(\alpha)$  have only a global maximum (unique up to phase ambiguity) under both power constraints and this maximum is achieved by the corresponding optimal batch design. The proposed PB-BF schemes aim to maximize  $\rho(\alpha)$  or  $P_{\mathcal{D}}(\alpha)$ , and indeed approach their optimal counterparts (cf. Section V). EGC can be approximated by PB-BF using element-wise normalization and the objective function  $\gamma(\alpha)$  chosen as received signal power  $P_{\mathcal{D}}(\alpha)$ . P-SP and S-SP performance can be approached using  $\gamma(\alpha) = P_{\mathcal{D}}(\alpha)$  and  $\gamma(\alpha) = \rho(\alpha)$  as objective function, respectively, and vector normalization of the beamforming weights. The PB-BF schemes can be implemented via T/R or P/M perturbation.

#### V. SIMULATION RESULTS

We next investigate a network with  $R = 3$  relays via numerical simulations; we will refer to the PB-BF schemes by adding the prefix 'PB-' to the corresponding batch design. For a fair comparison, all schemes use the same total relay power  $\bar{P}$  (in (2) we thus have  $P = \bar{P}/R$  under a per-relay power constraint and  $P = \bar{P}$  under a sum power constraint). The source  $\mathcal{S}$  transmits BPSK symbols with transmit power  $P_s = \bar{P}$

and the destination  $\mathcal{D}$  employs an ML detector. We further assume error- and delay-free 1-bit feedback, and employ a deterministic perturbation set based on a  $3 \times 3$  DFT matrix.

##### A. Idealized Scenario

In this scenario, all channels are static i.i.d. Rayleigh fading with different path loss, i.e.,  $h_i, g_i \sim \mathcal{CN}(0, d_i^{-2})$  with  $d_i = 1, 3, 5$ . Each relay perfectly knows its backward channel (used in (2)), and  $\mathcal{D}$  has perfect knowledge of the compound channel  $\xi$  and the performance measures  $P_{\mathcal{D}}(\alpha)$  and  $\rho(\alpha)$ . Unless stated otherwise,  $\bar{P}/N_0 = 18$  dB.

*Convergence Behavior.* For the case of PB-S-SP using P/M and T/R perturbation with step size  $\mu = 0.1$  and  $\mu = 0.3$ , Fig. 2(a) shows the evolution of the receive SNR  $\rho_k = \rho(\alpha_k)$  (normalized by the maximum receive SNR) versus the frame index  $k$  for one channel realization. It is seen that with T/R  $\rho_k$  is nondecreasing and reaches almost optimal performance; a larger step size results in faster convergence but also in a larger gap to the optimum. Similar observations apply to P/M, which converges significantly faster than T/R, but features continual fluctuations whose amplitude increases with the step size.

For a systematic assessment of the convergence rate of PB-S-SP (with  $\mu = 0.1$ ), Fig. 2(b) shows the empirical cumulative distribution function (cdf) of the normalized SNR gap that remains after a certain number of frames (shown as curve labels). The cdfs were obtained with  $10^5$  fading realizations. P/M converges considerably faster than T/R. To achieve an SNR gap of less than 4.3% in 91% of the cases, P/M and T/R respectively require 40 and 70 iterations. However, after a large number of frames, T/R on average features a considerably smaller SNR gap than P/M. Our simulations also revealed that a larger number of relays leads to slower convergence; for space reasons, the corresponding curves cannot be shown here.

*BER Performance.* Fig. 2(c) plots bit-error rate (BER) versus nominal SNR  $\bar{P}/N_0$  (in dB) for the batch designed beamforming schemes EGC, P-SP, S-SP, and their perturbation-based counterparts. In each simulation run, only the frames after convergence of the PB-BF schemes were taken into account for the BER evaluation (again  $\mu = 0.1$ ). As a reference, we include an AF scheme that uses uniform PA and no coherent combining (labeled 'no BF').

It can be seen that all PB-BF performance curves are almost indistinguishable from those of their corresponding batch designs and offer significant gains over the no-BF case (e.g., 8 dB SNR improvement at a BER of  $10^{-2}$ ). SNR optimization (PB-S-SP) is seen to outperform power optimization (PB-P-SP) at high SNR. In fact, PB-S-SP and S-SP are the only schemes to achieve a diversity larger than 1. Power optimization under a sum power constraint (PB-P-SP) and under a per-relay power constraint (PB-EGC) perform almost identically; in fact, PB-P-SP appears to suffer from noise amplification at high SNR.

##### B. Realistic Scenario

We next use independent, time-varying flat fading channels with Jakes Doppler profile and the same path loss model as in Section V-A. Furthermore, the destination uses (8) and (9) to

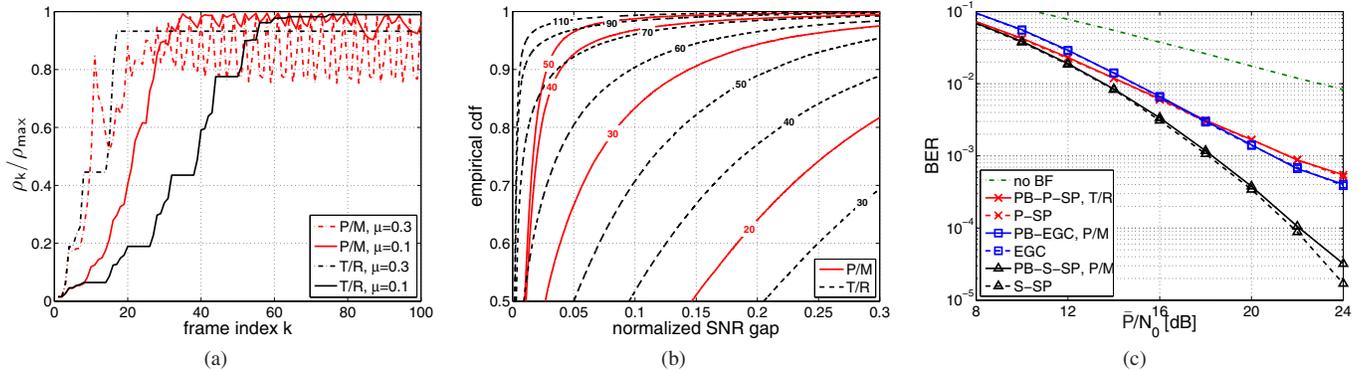


Fig. 2. PB-BF performance for idealized scenario with static channels in a 3-relay network: (a) example for SNR evolution at  $\mathcal{D}$  ( $\bar{P}/N_0 = 18$  dB), (b) cdf of the SNR gap after a fixed number of frames ( $\bar{P}/N_0 = 18$  dB,  $\mu = 0.1$ ), and (c) BER versus nominal SNR ( $\mu = 0.1$ ).

estimate the compound channel, the received signal power, and the instantaneous SNR. To this end, each transmission frame contains  $|\mathcal{T}_p| = 10$  pilot symbols in addition to  $|\mathcal{T}_d| = 40$  data symbols. The normalization in (2) is achieved by measuring the receive power at the relays during one frame.

We analyze the tracking capabilities of P/M perturbation (with  $\mu = 0.1$  and  $\mu = 0.5$ ) in terms of BER versus normalized Doppler frequency (i.e., Doppler in Hertz times frame length in seconds) for  $\bar{P}/N_0 = 22$  dB (see Fig. 3). In general, the BER degrades with increasing Doppler. At high Doppler frequencies the relay weights cannot be adapted fast enough to the channel variations (note that in practice, the feedback delay will add on top of this). Moreover, if there are channel variations within a frame, the compound channel and the objective function cannot be estimated accurately. Even at low Doppler, there is an order of magnitude BER penalty for PB-S-SP (cf. Fig. 3 with  $\mu = 0.1$  and Fig. 2(c) at  $\bar{P}/N_0 = 22$  dB).

We observe that at low Doppler frequencies a small step size ( $\mu = 0.1$ ) performs better whereas at higher Doppler frequencies a larger step size ( $\mu = 0.5$ ) is advantageous since it allows quicker adjustment of the relay weights. Note that with  $\mu = 0.1$ , PB-S-SP loses its entire performance advantage over PB-P-SP at high Doppler frequencies.

In time-varying scenarios, T/R suffers from the fact that the beamforming weights are not updated when the channel quality gets worse. This can be circumvented by building a forgetting factor into the performance measure  $\gamma_k$ .

## VI. CONCLUSION

We have investigated scalable perturbation-based distributed beamforming protocols in wireless relay networks that exploit 1-bit feedback to approach the optimal beamforming weights in an adaptive manner while avoiding CSI at the relay nodes. We used a deterministic perturbation set to optimize either received signal power or SNR at the destination under per-relay or sum power constraints. The best performance was observed with SNR as objective function under a sum power constraint. At high SNR, equal gain combining appears to be preferable over power optimization under a sum-power constraint. In time-varying environments, the proposed perturbation schemes

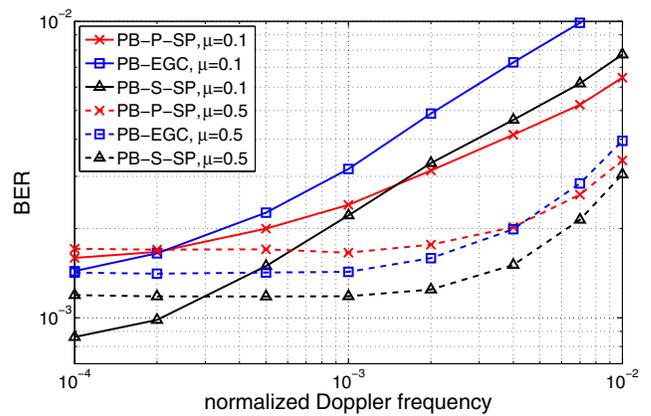


Fig. 3. BER versus normalized Doppler frequency for various PB-BF schemes using P/M perturbation under a realistic scenario ( $\bar{P}/N_0 = 22$  dB).

require a careful choice of the step-size parameter and the transmission frame length.

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