Quantification and Use of Dynamic Bit-Significances in Communication Transmitters

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Abstract

We discuss a universal measure that allows to quantify the usually variable significance of each data bit at a particular bit position in successive data frames in a digital communication system. Calculation rules are given to allow for the computation of the significances of parity bits when the significances of the data bits are given. In simulation examples we show how to use the bit-significances for source-adaptive allocation of transmission energy.

1 Introduction

The data bits in digital communication systems either stem from a quantiser, which might be part of a multimedia source codec, or they represent information which is discrete in time and amplitude by nature, e.g. a compressed text. Further, digital data transmission is usually carried out by frames which are used to split the whole amount of data into many relatively small portions that are separately encoded and transmitted. This concept is driven by limitations on the delay and on the complexity.

The data bit at a particular position within a frame might sometimes be very important while in other frames it is less significant, i.e., the quality degradation of the source-decoder output signal due to a bit error is not always the same for a particular bit position.

It was shown in [1] that large performance improvements can be achieved by allocating transmission energy to the bits according to their *current* significance. In this paper we extend these basic ideas. We discuss a universally applicable bit significance measure, the "S-value", and state calculation rules for the S-avlues that will allow for the use of channel coding. Finally we present some simulation results that demonstrate the effectiveness of the new concept.

2 Bit Significance Measure

2.1 Significance of Quantiser Bits

The example in Figure 1 illustrates a typical case in which the significances of the data bits change with the locations of the continuous-valued input source samples x. We consider two realisations "A" and "B" of the source signal x, which is scalar quantised by the reproducer values $y_0, ..., y_7$; the mean squared error is used a quality criterion. The sample A is quantised by y_1 (the

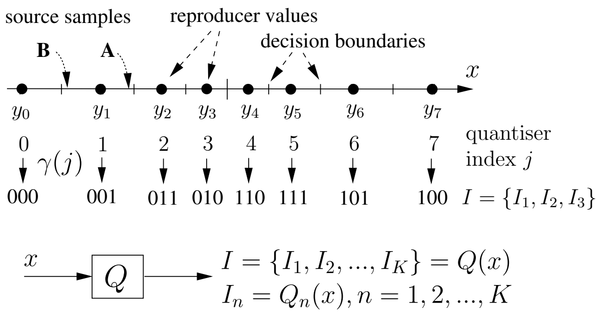

Figure 1: Three-bit ($K = 3$) scalar quantiser with Gray bit-mapping $\gamma()$.

nearest neighbour of x among the reproducer values) and the corresponding bit vector "001" is transmitted. At the decoder output an error in the middle bit will *not* lead to seriously larger distortion than the reproduction without any bit errors, because the reproducer value y_2 corresponding to the bit-combination "011" is also located close to the input sample A. The situation is different if we consider the sample B. Although B is also quantised by the reproducer value y_1, an error in the middle bit will now lead to much larger distortion. However, this time an error in the rightmost bit will cause only a small increase of the mean squared error. Hence, we have bits with strongly variable sensitivities, depending on the location of the unquantised source samples. This can be extended to vector quantisers, i.e., the argumentation is true in a rather general context.

Let us assume that $I = \{I_1, I_2, ..., I_K\} \in \mathcal{I}$ denotes a K-bit vector that represents the quantiser index j, and that $\mathcal{I} \doteq \{0,1\}^K$ is the set of all possible K-bit vectors. Moreover, we obtain the bit-vector I from the quantiser index j and by the bit-mapping $\gamma()$, i.e., $I = \gamma(j)$; in Figure 1 we use a Gray mapping as an example.

Given the input source sample x, we quantify the distortion $d'(\tilde{I}_l = \xi \,|\, x)$ if a particular bit I_l, $l = 1, 2, ..., K$, is reconstructed with some value $\tilde{I}_l = \xi \in \{0,1\}$ by

$$d'(\tilde{I}_l = \xi \,|\, x) = (x - y_{\gamma^{-1}(\tilde{I})})^2 \ , \tag{1}$$

with the quantiser reproducer value y_L, the inverse bit mapping $L = \gamma^{-1}(\tilde{I})$, and the bit vector $\tilde{I} = \{\tilde{I}_1, \tilde{I}_2, ..., \tilde{I}_K\} \in \mathcal{I}$, for which $\tilde{I}_l = \xi$ and $\tilde{I}_n = Q_n(x)$ for $n = 1, 2, ..., l-1, l+1, ..., K$. The notation $Q_n(x)$ corresponds to the output bit number n of the quantiser (including the bit mapping $\gamma()$) given the continuous-valued source sample x. In other words: in (1) we use the reproducer value for reconstruction that has the same bit vector as the nearest neighbour of the input sample x, only excluding the bit position l for which we enforce the bit value $\xi \in \{0,1\}$ specified on the left-hand side of (1).

The definition (1) will lead to a value of $d'(\tilde{I}_l = \xi | x) = 0$, if the input sample x is exactly identical to one of the reproducer values *and* if the specified bit value ξ is equal to the corresponding bit value at the output of the quantiser. As we will define the significance of a bit by the inverse of the (normalised) distortion this would cause numerical problems if the distortion was close to zero. Therefore we modify (1) as follows:

$$d'(\tilde{I}_l = \xi \,|\, x) = (x - y_{\gamma^{-1}(\tilde{I})})^2 + \beta \tag{2}$$

with $0 < \beta \ll \sigma_x^2$, where σ_x^2 is the variance of the input samples (e.g. $\beta = 10^{-4} \cdot \sigma_x^2$).

Using (2) we define the normalised distortion

$$d(\tilde{I}_l = \xi \,|\, x) = \frac{d'(\tilde{I}_l = \xi \,|\, x)}{d'(\tilde{I}_l = 0 \,|\, x) + d'(\tilde{I}_l = 1 \,|\, x)} \ . \tag{3}$$

Hence, $0 < d(\tilde{I}_l = \xi \,|\, x) < 1$ and $d(\tilde{I}_l = 0 \,|\, x) + d(\tilde{I}_l = 1 \,|\, x) = 1$.

The significance of a reconstruction $\tilde{I}_l$ with some bit value ξ is inversely proportional to the (normalised) distortion (3). Therefore, we define the significance

$$q(\tilde{I}_l = \xi | x) \doteq \frac{1}{d(\tilde{I}_l = \xi | x)} \ . \tag{4}$$

As $0 < d(\tilde{I}_l = \xi | x) < 1$, the significance $q(\tilde{I}_l = \xi | x)$ will take values in the open interval $(1, +\infty)$.

Now we define the *logarithmic bit significance measure* (S-value) as the logarithmic[1] ratio of the bit significances (4) for a "0" and a "1" bit:

$$S(\tilde{I}_l \,|\, x) \doteq \log \frac{q(\tilde{I}_l = 0 \,|\, x)}{q(\tilde{I}_l = 1 \,|\, x)} \ . \tag{5}$$

[1]Throughout the paper, the natural log is used in this definition.

The S-value measures how sensitive the quality is in terms of a particular bit position. If the magnitude $|S(\tilde{I}_l \,|\, x)|$ is zero, this means it doesn't matter for the quality if the bit is reconstructed as "0" or "1". If the magnitude is large, the bit is very sensitive and the sign[2] of $S(\tilde{I}_l \,|\, x)$ determines whether the reconstruction of the bit should be "0" (when $S(\tilde{I}_l \,|\, x) > 0$) or "1" (when $S(\tilde{I}_l \,|\, x) < 0$).

2.2 Significance Measure for Data Bits

In a "pure" data transmission scenario (i.e., the data is discrete in time and the data samples take values from a finite set), time-varying significances of a particular bit-position in a packet might arise from transmitting in-band signalisation (e.g. to initiate handover to another mobile radio cell) as in GSM. The signalisation bits will usually be extremely sensitive to bit errors, as their erroneous interpretation will cause severe system failures, while in "normal" traffic the bit position is less important in which case we might be willing to accept a higher error probability in the reconstruction of the bits.

Another example is text compression: if a variable-length lossless coding scheme is applied, some bit errors might cause symbol insertions and deletions or complete loss of synchronisation, while an error in exactly the same bit position in another coded block might only cause a single symbol error at the decoder output: again we observe strongly variable bit-error sensitivities of the data bits.

Let us assume we are willing to tolerate a bit-error probability $p_l < 0.5$ in the reconstruction $\tilde{I}_l$ of a bit I_l with the realisation $i_l \in \{0,1\}$ that is known at the transmitter side. Based on this we can define a "distortion measure" according to

$$d(\tilde{I}_l = \xi | i_l) = \begin{cases} p_l & i_l = \xi \\ 1 - p_l & i_l \neq \xi \end{cases} \quad \text{with} \quad \xi \in \{0,1\} \ . \tag{6}$$

The interpretation of (6) is as follows: the value of the distortion measure is $1 - p_l$ if the reconstruction $\tilde{I}_l$ is different from the bit-value i_l, i.e., it is smaller than the Hamming distance, as we are willing to tolerate some bit-error probability p_l. If the reconstruction $\tilde{I}_l$ equals the original i_l the distortion measure is p_l (and not zero); in this way, we make it less important for the reconstruction to be error-free. If the tolerable bit-error probability is set to $p_l = 0.5$, the distortion measure has the same value in both cases, i.e., the reconstruction with "1" is as good as the reconstruction with "0". Note that in this case the bit is completely irrelevant and one should not waste power for its transmission.

The normalised significance $q(\tilde{I}_l = \xi | i_l)$ of a reconstruction $\tilde{I}_l$ with some bit value ξ is inversely propor-

[2]The definition (5) is such that the interpretation of the sign is consistent with the well-established log-likelihood ratios [2] used for soft-in/soft-out *de*coding at the *receiver side*.

tional to the distortion, which is now defined by (6):

$$q(\tilde{I}_l = \xi | i_l) \doteq \frac{1}{d(\tilde{I}_l = \xi | i_l)} \ . \qquad (7)$$

Similar as in (5) we define the *logarithmic bit-significance measure* (S-value):

$$S(\tilde{I}_l \mid i_l) \doteq \log \frac{q(\tilde{I}_l = 0 \mid i_l)}{q(\tilde{I}_l = 1 \mid i_l)} \ . \qquad (8)$$

If we insert (6) and (7) into (8) we obtain

$$S(\tilde{I}_l \mid i_l) = \begin{cases} \log(p_l/(1-p_l)), & i_l = 1 \\ \log((1-p_l)/p_l), & i_l = 0 \end{cases} \ , \qquad (9)$$

which can equivalently be written as

$$S(\tilde{I}_l \mid i_l) = \text{sign}(1 - 2 \cdot i_l) \cdot \log \frac{1-p_l}{p_l} \quad \text{for} \quad p_l < 0.5 \qquad (10)$$

with $i_l \in \{0, 1\}$. Note that again the sign of $S(\tilde{I}_l \mid i_l)$ determines the bit-value a reconstruction $\tilde{I}_l$ should have, while the magnitude $|S(\tilde{I}_l \mid i_l)| = \log \frac{1-p_l}{p_l} > 0$ describes the significance of the bit. If the tolerable error probability is $p_l = 0.5$, we obtain $\log \frac{1-p_l}{p_l} = 0$, i.e., the bit has no significance at all, while the value of $\log \frac{1-p_l}{p_l}$ is very large when p_l is close to zero.

3 Bit-Significance Algebra

3.1 Conversion between S-values, Significances and Distortions

From the definitions (4), (7) and the fact that the distances (3), (6) are normalised (i.e., they sum up to one over all possible $\xi \in \{0, 1\}$) we obtain

$$S(\tilde{I}_l \mid C) = \log \frac{d(\tilde{I}_l\!=\!1|C)}{d(\tilde{I}_l\!=\!0|C)} = \log \frac{d(\tilde{I}_l\!=\!1|C)}{1 - d(\tilde{I}_l\!=\!1|C)} = \log \frac{1 - d(\tilde{I}_l\!=\!0|C)}{d(\tilde{I}_l\!=\!0|C)} \qquad (11)$$

where C denotes the condition either on a continuous-valued source sample x or on a data bit i_l. We can solve (11) either for $d(\tilde{I}_l = 1 \mid C)$ or for $d(\tilde{I}_l = 0 \mid C)$; the results can be written in compact form as follows:

$$q(\tilde{I}_l = \xi \mid C) = \frac{1}{d(\tilde{I}_l = \xi \mid C)} = \frac{1 + e^{S(\tilde{I}_l \mid C)}}{e^{S(\tilde{I}_l \mid C) \cdot \xi}} \qquad (12)$$

with $\xi \in \{0, 1\}$. Hence, we can easily convert the normalised significances and distortions into an S-values and vice versa.

3.2 Significance of Parity Bits

Now we consider two data bits i_1, i_2 that are channel-encoded by a single parity-check bit r, i.e.,

$$r = i_1 \oplus i_2 \ , \qquad (13)$$

where "$\oplus$" denotes the modulo-2 addition. The bits i_1, i_2 may either directly represent the information we want to transmit or they might have been generated by a quantisation of continuous-valued source samples (or parameters of a multimedia codec). For both cases the S-values were defined above.

The question is now what the S-value of the parity bit is, i.e., how important is that the reconstruction $\tilde{R}$ of the parity bit at the receiver side is correct? Of course, the significance of $\tilde{R}$ depends on what we do at the receiver. If we simply ignored the received parity bit it is completely insignificant, in which case channel coding would just be a waste of bit rate. We will assume, however, that we perform some decoding (maximum likelihood decoding or possibly some approximation) that will decide upon a valid channel-codeword. This means we will make sure at the receiver that the encoding rule $r = i_1 \oplus i_2$ will hold for the *reconstructed* bits, i.e., we will enforce $\tilde{R} = \tilde{I}_1 \oplus \tilde{I}_2$. Then, we may write

$$S(\tilde{R} \mid C) = \log \frac{q(\tilde{R} = 0 \mid C)}{q(\tilde{R} = 1 \mid C)} = \log \frac{q(\tilde{I}_1 \oplus \tilde{I}_2 = 0 \mid C)}{q(\tilde{I}_1 \oplus \tilde{I}_2 = 1 \mid C)} \ , \qquad (14)$$

where $C = \{C_1, C_2\}$ represents the information known at the transmitter, i.e., the values of the transmitted data bits (i.e., $C_l = i_l$) or the source samples (i.e., $C_l = x_l$).

Assuming independent[3] data bits, i.e., the reconstruction of one bit does not influence the significance of the other, we obtain

$$S(\tilde{R} \mid C) = \log \frac{q_1(0) \cdot q_2(0) + q_1(1) \cdot q_2(1)}{q_1(1) \cdot q_2(0) + q_1(0) \cdot q_2(1)} \ , \qquad (15)$$

with the abbreviation $q_l(\xi) \doteq q(\tilde{I}_l = \xi \mid C_l)$. In (15) we use a multiplication to join two component-significances that lead to either a "0" or a "1" for the parity bit. As for each case two possible combinations of the data bits exist, we have two summands in both the numerator and the denominator.

For brevity of notation we will omit the conditions C in what follows as long as there is no risk of confusion. We insert (12) into (15) which gives

$$S(\tilde{R}) = S(\tilde{I}_1 \oplus \tilde{I}_2) = \log \frac{1 + e^{S(\tilde{I}_1)} \cdot e^{S(\tilde{I}_2)}}{e^{S(\tilde{I}_1)} + e^{S(\tilde{I}_2)}} \ . \qquad (16)$$

It is well known from log-likelihood algebra [2] that (16) can be approximated by

$$S(\tilde{R}) \approx \text{sign}(S(\tilde{I}_1)) \cdot \text{sign}(S(\tilde{I}_2)) \cdot \min\big(|S(\tilde{I}_1)|, |S(\tilde{I}_2)|\big). \qquad (17)$$

[3] If the data bits originate from *one* quantiser index, this assumption would not be fulfilled. In practice, however, the quantiser index bits of large blocks of source samples are jointly channel-encoded. Hence, we can ensure by interleaving prior to channel encoding that the bits of one quantiser index are spread over the whole block of input data bits. With sufficiently long spread of the data bits at the input of, e.g., a convolutional encoder, all channel-encoder output bits will only depend on *one* bit of any quantiser index.

We find that the significance of the parity bit is mainly determined by the lowest significance among the data bits. The preferable bit-value for the parity bit is determined by the sign of $S(\tilde{R})$ that is given by the product of the signs of the data bits: this is a direct consequence of the modulo-2 encoding rule (13).

3.3 Examples

Let us assume we have two data bits with the values $i_1 \in \{0,1\}$ and $i_2 \in \{0,1\}$ for which we are willing to accept the (symmetric) error probabilities $p_1 < 0.5$ and $p_2 < 0.5$ in their reconstructions. From (10) we know the formula for the S-values which we insert into (16) to find the S-value of the reconstruction $\tilde{R}$ of the parity bit. The result is

$$S(\tilde{R}) = \log \frac{1 + \left(\frac{1-p_1}{p_1}\right)^{i_1^{\pm}} \cdot \left(\frac{1-p_2}{p_2}\right)^{i_2^{\pm}}}{\left(\frac{1-p_1}{p_1}\right)^{i_1^{\pm}} + \left(\frac{1-p_2}{p_2}\right)^{i_2^{\pm}}} \qquad (18)$$

with the abbreviation $i_l^{\pm} = 1 - 2 \cdot i_l$. If all possible cases $i_l \in \{0,1\}, l = 1,2$, are evaluated we find

$$S(\tilde{R} \mid r) = \text{sign}(1 - 2 \cdot r) \cdot \log \frac{1 - p_r}{p_r} \qquad (19)$$

with

$$p_r = p_1 + p_2 - 2 \cdot p_1 \cdot p_2 \ . \qquad (20)$$

and

$$r = i_1 \oplus i_2 \ . \qquad (21)$$

Note that as long as $p_1, p_2 < 0.5$ we will also have $p_r < 0.5$ so $\log \frac{1-p_r}{p_r} > 0$. Moreover, (19) fits into the template (10) which was the general result for a bit for which we are willing to accept some error probability (p_r in this case).

Example 1: $p = p_1 = 10^{-3} = p_2$ We obtain $p_r = 2 \cdot p - 2 \cdot p^2 \approx 2 \cdot 10^{-3}$, i.e., the tolerable error probability of the parity-check bit is larger than for each individual data bit.

Example 2: $p = p_1 = 1/2 = p_2$ We obtain $p_r = 2 \cdot p - 2 \cdot p^2 = 0.5$. This is consistent with our expectations as it means that the reconstruction of the parity bit is completely insignificant as the same holds for the data bits.

Example 3: $p_1 = 10^{-2}, p_2 = 10^{-4}$ We find $p_r = 10^{-2} + 10^{-4} - 2 \cdot 10^{-2} \cdot 10^{-4} \approx 10^{-2}$. This means that the tolerable error probability of the parity bit almost equals that of the "weaker" bit (still $p_r > p_1$ and $p_r > p_2$). This is again consistent with our expectations as it means that a weak data bit "spoils" the parity check bit, even if the other data bit is very important. This is also reflected by the "min"-operation in (17). If we consider the reconstruction of an important data bit it is obvious that we will not be able to greatly benefit from the parity check if we tolerated a high error probability in the other data bit.

3.4 Generalisations

Note that the S-values for parity-check equations involving more than two data bits can be computed by a repeated, iterative application of (19) and (20).

Moreover, the S-value of a quantiser bit can be converted into a tolerable bit error probability p_l by use of (3) and (6):

$$p_l \doteq \min\left(d(\tilde{I}_l = 0|x), d(\tilde{I}_l = 1|x)\right) \qquad (22)$$

Hence, the calculation rules, the examples in Section 3.3 and all generalisations are true for both "real" data bits and quantiser bits.

4 Energy Allocation in Binary Transmission using S-Values

After computing the S-values by (5) or (10), we will use them to allocate transmission energy[4] E_l to the individual bits. A simple approach is to linearly distribute the energy, with the S-Values as weighting factors, but one may also use more general rules such as

$$E_l = \frac{|S(\tilde{I}_l|x)|^{\alpha}}{\frac{1}{K}\sum_{l=1}^{K} |S(\tilde{I}_l|x)|^{\alpha}} \cdot E_s \qquad (23)$$

with $\alpha > 0$; for $\alpha = 1$ this includes the linear case. In (23), E_s is the given *average* energy for each bit and K is the number of bits for which the joint energy allocation is to be applied.

An important property of the rule (23) is that $E_l = 0$ if $|S(\tilde{I}_l|x)| = 0$, i.e., no energy is allocated to insignificant bits. Another essential property of (23) is that the average energy stays the same, i.e., $\frac{1}{K}\sum_{l=1}^{K} E_l = E_s$.

This allocation of energies will not be "optimal" in any strict sense. If quantiser bits are transmitted, one could, in principle, state an optimisation problem and try to solve it, e.g., by standard variational techniques such as Lagrange multipliers. The drawback of this approach is that the problem can usually only be solved numerically with high complexity [1]. Hence, our goal with the definition of S-values is to state a feasible general means of how to quantify bit significances and to perform an energy allocation in the sense of a good practical solution that, once the S-values are known, no longer depends on the specific details of the source data. The most suitable allocation rule for a particular application (e.g., the parameter α in (23)) will be determined by simulations.

If pure date bits are tranmsitted and their number (i.e., the bit rate) is fixed but the channel cannot carry this amount of information, it is impossible to reconstruct the data bits at the receiver without errors. If the channel quality is very low it might even be impossible to reconstruct them with the given tolerated bit error rates.

[4] The energy E_l for a bit is related to the power P_l by $P_l = E_l/T$, with T the constant bit transmission period.

In the latter case the energy allocation rule will achieve a best-effort solution, which means that the very significant data bits might exhibit a larger bit error rate than tolerated but still it will be lower than for the less significant bits.

5 Simulation Results

Some simulations results are given in this section for the quantisation and transmission of uncorrelated Gaussian source samples x with the variance $\sigma_x^2 = 1$. The source samples are optimally scalar quantised with fixed rate (Lloyd-Max quantiser [3]) by $K = 3$ bits. The transmission of the quantiser bits is carried out over a binary-input (BPSK modulation) Additive White Gaussian Noise (AWGN) channel with the average "channel SNR" E_s/N_0.

Figure 2 shows, for each bit of a Gray bit mapping (see illustration in Figure 1), the S-values that are obtained from (5) for $-3 < x < 3$ with $\beta = 10^{-4}\sigma_x^2$ in (2). A comparison with the natural binary bit mapping is given in Figure 3. In both figures the locations of the reproducer values are indicated by "$\times$" and the bit mappings used for transmission are given as well.

Figures 2 and 3 confirm that the S-values indeed measure the significances of the bits: the S-values are large in magnitude if the x-value is close to a reproducer value and the bits that change at the decision boundaries in the middle between two reproducer values have S-values close to zero. A positive sign of the S-value indicates that the reconstruction of this bit should be "0" while a negative sign indicates a "1".

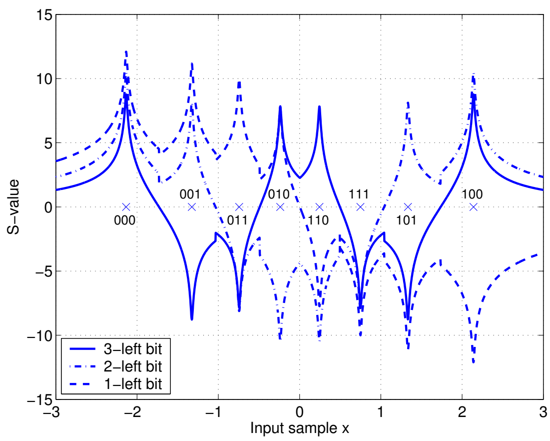

Figure 2: S-values $S(\tilde{I}_l|x), l = 1, 2, 3$, for a three-bit scalar Lloyd-Max quantiser for a Gaussian source; Gray bit mapping; $\beta = 10^{-4}\sigma_x^2$ in (2); the quantiser reproducer values are marked by "$\times$", the bit mapping is included as well.

For the Gray bit mapping and based on the S-values in Figure 2, Figure 4 shows the resulting energy distribution according to (23) for $\alpha = 1.0$.

As required, the bits with S-values close to zero are

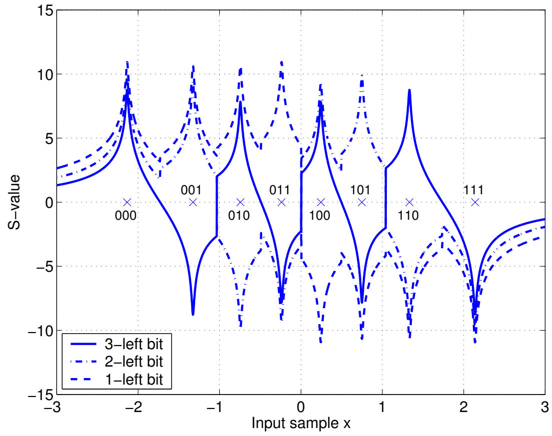

Figure 3: S-values $S(\tilde{I}_l|x), l = 1, 2, 3$, for a three-bit scalar Lloyd-Max quantiser for a Gaussian source; natural binary bit mapping; $\beta = 10^{-4}\sigma_x^2$ in (2); the quantiser reproducer values are marked by "$\times$", the bit mapping is included as well.

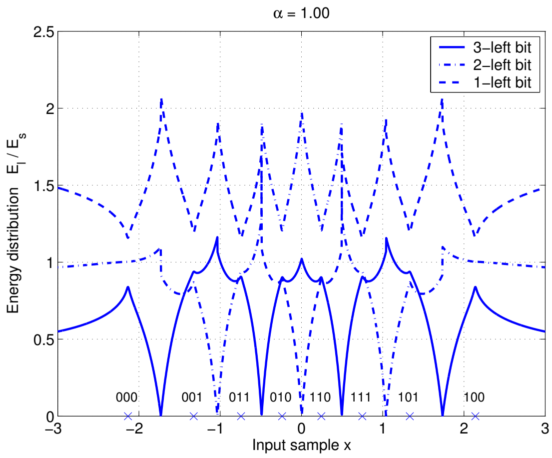

Figure 4: Energy distribution according to (23) with $\alpha = 1.00$ based on the S-values in Figure 2 (Gray bit mapping).

allocated very small energy, while the signficant bits will get up to twice as much energy than spent "on average".

In Figure 5 we present some simulation results. As a performance measure we use the signal-to-noise ratio (SNR) of the source samples x and their reconstructions $\tilde{x}$ at the decoder output. We compare the conventional transmitter (without source-adaptive energy allocation) with three schemes using S-values for energy allocation with $\alpha = \{0.5, 1.0, 2.0\}$ in (23). A simple hard-decision decoder was used, in which bit decisions are taken from the signs of the sampled matched-filter outputs. The resulting bit vectors were converted into quantiser indices via the inverse bit mapping $\gamma^{-1}()$ and the corresponding reproducer values were used a as source reconstruction $\tilde{x}$. In the region of interest, i.e., for SNRs of 10 dB or higher (more than 2/3 of the max. SNR), we observe a gain of about 1 dB in E_s/N_0 (which directly turns into

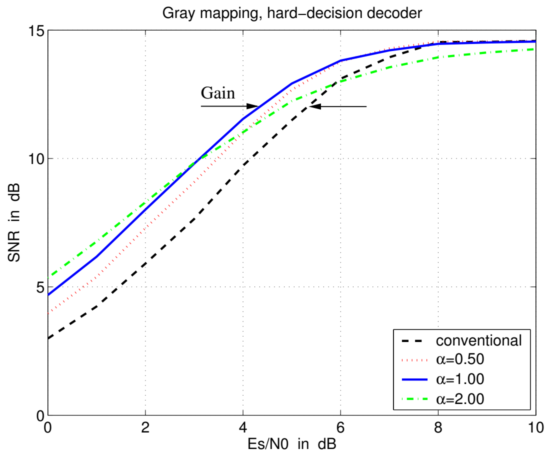

Figure 5: Performance in terms of source SNR; energy allocation by S-values with $\alpha = \{0.5, 1.0, 2.0\}$; comparison with a conventional transmitter; Gray mapping; conventional hard-decision receiver.

a gain in average transmission power) for the Gray mapping. (The gain for the natural binary mapping is somewhat smaller.). The energy distribution with $\alpha = 1.0$ in (23), i.e., linear averaging, turns out to be a good choice.

In Figure 6 we present performance results for the same transmission scheme as in Figure 5, but now with a conventional soft-decision decoder. Such a receiver, for an uncorrelated source and an AWGN channel model with binary input, coherent detection and without knowledge of source-adaptive energy allocation is known (e.g. [1]) to be given by

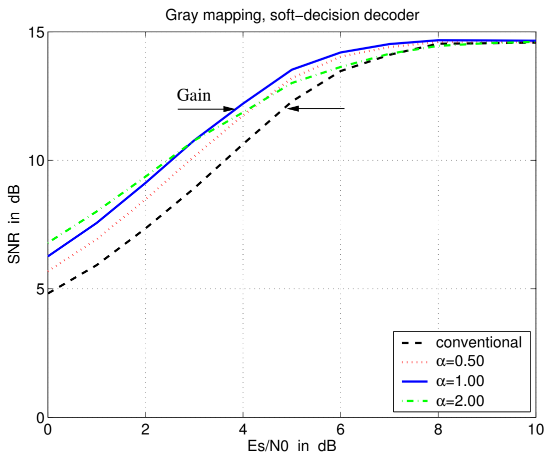

Figure 6: Performance in terms of source SNR; energy allocation by S-values with $\alpha = \{0.5, 1.0, 2.0\}$; comparison with a conventional transmitter; Gray mapping; conventional soft-decision receiver.

$$\tilde{x} = \sum_{I \in \mathcal{I}} y_{\gamma^{-1}(I)} \cdot \Pr(I \mid \tilde{z}_1, ..., \tilde{z}_K) \; . \tag{24}$$

The quantity $\tilde{x}$ represents the reconstruction of the input sample x at the decoder output and $y_{\gamma^{-1}(I)}$ denotes the quantiser reproducer value that corresponds — via the bit mapping $\gamma()$ — to the K-bit vector $I = \{I_1, ..., I_K\}$. Moreover, $\tilde{z}_l$ denotes the received sampled matched-filter output for the transmitted data bit $I_l \in \{0, 1\}$, $l = 1, ..., K$. The a-posteriori probability $\Pr(I \mid \tilde{z}_1, ..., \tilde{z}_K)$ can be computed by (Bayes rule)

$$\Pr(\underbrace{I_1, ..., I_K}_{= I} \mid \tilde{z}_1, ..., \tilde{z}_K) = A \cdot \Pr(I) \cdot \prod_{l=1}^{K} p(\tilde{z}_l \mid I_l) \; , \tag{25}$$

where the factor A is a constant that ensures that the probabilities $\Pr(I \mid \tilde{z}_1, ..., \tilde{z}_K)$ sum up to one over all possible $I \in \mathcal{I}$; $\Pr(I)$ is the known (or easily measurable) probability of the reproducer value which is transmitted by the bit vector I and $p(\tilde{z}_l \mid I_l)$ is the probability density function of the channel which for the binary-input AWGN model is given by

$$p(\tilde{z}_l \mid I_l) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp(-\frac{1}{2\sigma_n^2}(\tilde{z}_l - z_l)^2)\Big|_{z_l = 1 - 2 \cdot I_l} \; . \tag{26}$$

Note that the source-adaptive energy allocation based on the S-values by (23) will lead to different transmission energies E_l for the bits. As this energy allocation is unknown at the receiver, we use a conventional soft-decision decoder which has only knowledge about the *average* channel quality given by E_s/N_0. This leads to the noise-variance $\sigma_n^2 = \frac{N_0}{2E_s}$ in the channel model (26).

Figure 6 indicates that the conventional soft-decision decoder (24) gives a gain of somewhat less than 1 dB over the hard-decision decoder for all transmission systems. The relative gain by source-adaptive energy allocation using S-values is again around 1 dB in E_s/N_0 (average transmission power).

6 Conclusions

In this paper we described the S-values as a new measure for the signficances of data bits and we derived calculation rules for the S-values of parity bits, given the S-values of the data bits. The S-values can be used for bitwise allocation of transmission energy. We presented simulation results for a simple system model which show that significant gains can be achieved by the new concept.

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