Joint Source-Channel Decoding by Channel-Coded Optimal Estimation

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Abstract — **Channel-Coded Optimal Estimation (CCOE) is an algorithm which combines source-statistics, bit-reliability informations at the channel output and the redundancy introduced by some channel code to compute the a-posteriori probabilities of the possibly transmitted source-encoder indices at the receiver. A recursive formula for the computation of the a-posteriori probabilities is derived and some special cases are given. The algorithm is generalized for the parallel transmission of several quantizer indices which commonly encode each source-signal vector. Simulation results are presented and the performances of several algorithms for source-channel decoding are compared.**

I. Introduction

Joint source-channel coding has become a major area of interest because the classical separation between source and channel coding has turned out to be not justified in practical systems with limited delay.

Channel decoding should be carried out with soft-decisions, i.e. the bit-reliability informations at the channel output should not be “quantized away” by hard decisions before channel decoding. In [4] a soft-output channel-decoding algorithm was stated which accepts soft-inputs and computes soft-output values that can be utilized in the following stages of the transmission system, e.g. by a second stage of channel decoding in a concatenated coding scheme or by a source decoder.

In [1] a source-decoding scheme was stated that accepts soft-bits and combines them with statistical information on the source-encoder indices to optimally estimate the decoder output signal. The technique can be used after soft-output channel decoding or without any channel coding at all. The estimation is carried out for *parameters* of the source codec rather than for single bits of the parameter-indices. Since a source encoder is principally unable to remove all the redundancy from a correlated source signal if the coding delay is limited, adjacent source-encoder indices and their bits are more or less correlated. This correlation is exploited to improve the performance of the transmission system in presence of noise on the channel.

In [5] an algorithm was stated that accepts soft input-values for soft-output decoding of a convolutional channel code but also a-priori knowledge about the source-encoder index-bits: Their dependencies are modeled by Markov-models.

The algorithm described in this paper is a trial to close a part of the gap between the two types of algorithms described above. On one hand, as in [1], the estimation of the source-decoder output signal is carried out parameter-based exploiting bit-reliability informations and the dependencies of the whole indices which are mostly stronger autocorrelated than their single bits. On the other hand a channel code is integrated into the estimation process, and it is not used as a separate preprocessing stage nor left away completely.

II. Channel-Coded Optimal Estimation

A. Simple model of a transmission system

Consider the problem of transmitting a source-signal vector U_k (k is the time-index for the vector with the dimension $N_U \in \mathbb{N}$) that could be a vector of waveform samples but also LPC-coefficients[1] of a speech codec or scaling factors of an audio codec. As depicted in figure 1 the source signal is source-encoded in a first step to reduce the bit-rate compared to the uncoded case. The

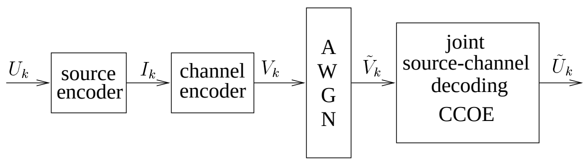

Fig. 1: Simple model of a transmission system

source encoder which can generally be thought of as quantizer issues one index I_k at each time k.

Each index I_k is represented by a limited number of $N \in \mathbb{N}$ bits. Since perfect[2] source-encoding is impossible in systems with limited delay some correlation remains between the transmitted indices if the source signal was correlated, which more or less is the case in most practical situations (e.g. coding of speech signals). In this paper the correlation of the indices is modeled by a first-order Markov-process. The Markov-model is described by the index-transition probabilities $P(I_k = \lambda \,|\, I_{k-1} = \mu)$, $\lambda, \mu = 0, 1, \ldots, 2^N - 1$, which are

[1] The LPC-coefficients (**L**inear **P**redictive **C**oding) describe the spectral shape of a source signal.

[2] perfect in the sense that adjacent indices I_{k-1} and I_k are completely independent

assumed to be known and constant for each $k = 1, 2, \ldots$ (stationary Markov-source).

A channel code is applied to improve the performance of the transmission system in presence of noise on the channel. Generally we will *not* assume systematic channel codes and use the notation $V_k = \mathrm{CC}(I_k)$ for the computation of the codeword V_k with N_V bits from the source-encoder index I_k with N bits by some channel code.

In the model in figure 1 the codeword is transmitted over an AWGN-channel. As usual, coherently detected binary phase-shift keying is assumed for the modulation. For a single bit this scenario is depicted in figure 2. The

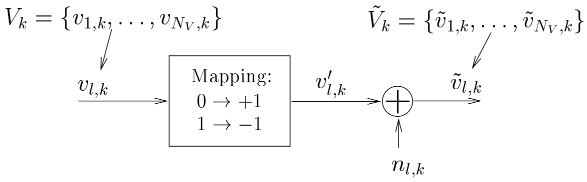

Fig. 2: Transmission model for a single bit $v_{l,k}$, $l = 1, 2, ..., N_V$, of the codeword V_k

"binary" bits $v_{l,k} \in \{0, 1\}$ are first mapped to "bipolar" bits $v'_{l,k} \in \{+1, -1\}$. The mapping is mathematically described by

$$v'_{l,k} = 1 - 2 \cdot v_{l,k} \quad . \tag{1}$$

After that a white Gaussian noise-sample $n_{l,k}$ is added to $v'_{l,k}$. The noise has zero mean and a variance of $\sigma_n^2 = \frac{N_0}{2E_s}$ where E_s is the energy that is used to transmit each bit (data bits and redundancy bits) and N_0 is twice the noise power spectral density $N_0/2$ on the (white noise) channel. The conditioned p.d.f. of the received value $\tilde{v}_{l,k}$ at the channel output given that the (bipolar!) bit $v'_{l,k}$ has been transmitted is therefore given by

$$p(\tilde{v}_{l,k}|v'_{l,k}) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-\frac{1}{2\sigma_n^2}(\tilde{v}_{l,k} - v'_{l,k})^2}, \quad \sigma_n^2 = \frac{N_0}{2E_s} \quad .$$

Using (1) we obtain

$$p(\tilde{v}_{l,k}|v_{l,k}) = \frac{e^{-\frac{1}{2\sigma_n^2}(\tilde{v}_{l,k}-1)^2}}{\sqrt{2\pi}\sigma_n} e^{-\frac{1}{2\sigma_n^2} \cdot 4 \cdot \tilde{v}_{l,k} \cdot v_{l,k}} \quad . \tag{2}$$

The joint conditioned p.d.f. of the vector $\tilde{V}_k$ to be received given that the codeword $V_k \in \{0, 1\}^{N_V}$ has been transmitted is given by the product

$$p(\tilde{V}_k|V_k) = \prod_{l=1}^{N_V} p(\tilde{v}_{l,k} \mid v_{l,k}) \tag{3}$$

since the channel noise is assumed to be white.

B. Problem formulation

Using the system model stated above, the problem now is to *estimate* the output signal-vector $\tilde{U}_k$ of the source decoder in such a way that the value of a suitable "overall distortion" is minimum. The SNR-value calculated from the components of the input vector U_k and the output vector $\tilde{U}_k$ is an appropriate error criterion (which has to be maximized) e.g. if the source signal represents a "waveform". In this case a mean-square estimator (MS) is a good choice for the computation of the decoder-output signal since it directly corresponds to the demand for maximum SNR. The mean-square estimator is given by [1]

$$\tilde{U}_k^{(MS)} = \sum_{\lambda=0}^{2^N-1} U_q(\lambda) \cdot P_{AP}(\lambda, k) \tag{4}$$

where $U_q(\lambda)$ is the entry with the number λ of the quantization table of the source encoder with 2^N "code-vectors" of dimension N_U. The summation (4) has to be carried out for each of the N_U components of the source-signal vector. The quantity $P_{AP}(\lambda, k)$ is called a-posteriori probability of the codeword $V_k^{(\lambda)}$ that is associated with the index $I_k = \lambda$. The a-posteriori probability is defined by

$$P_{AP}(\lambda, k) = P\big(V_k^{(\lambda)} \mid \tilde{V}_k, \tilde{V}_{k-1}, \tilde{V}_{k-2}, \ldots\big) \quad . \tag{5}$$

It contains all information (about the transmitted index) which is available at the receiver. It represents the probability that the index $I_k = \lambda$ has been transmitted given all the channel words (groups of received values) $\tilde{V}_k, \tilde{V}_{k-1}, \ldots$ that have been received up to the current time k. The codeword $V_k^{(\lambda)}$ is a hypothesis at the receiver of the (possibly) transmitted codeword that is associated with the index $I_k = \lambda$.

If the a-posteriori probabilities $P_{AP}(\lambda, k)$ are known other types of estimators can be also used if they correspond in a better way to the overall-quality criterion of the transmission. If, for instance, the index λ_M (or equivalently the codevector) with minimum error probability is required at the receiver the maximum a-posteriori estimator (MAP) is selected which is given by

$$P_{AP}(\lambda_M, k) \geq P_{AP}(\lambda, k), \; \lambda_M, \lambda = 0, 1, ..., 2^N - 1$$

and the corresponding receiver output equals

$$\tilde{U}_k^{(MAP)} = U_q\big(I_k = \lambda_M\big) \quad . \tag{6}$$

The problem now is to calculate the a-posteriori probabilities $P_{AP}(\lambda, k)$ using the known "quantities" and informations on the transmission system, i.e.

- the received channel-words represented by $\tilde{V}_k, \tilde{V}_{k-1}, \ldots$
- the p.d.f.s given by (2) and (3) (this includes the assumption of an AWGN-channel *and* the value of $\frac{E_S}{N_0}$).
- the channel code
- the transition probabilities $P(I_k = \lambda \mid I_{k-1} = \mu)$ of the Markov model for the source indices I_k.

C. Derivation of a recursive formula to calculate the a-posteriori probabilities

First (5) is rewritten as

$$P_{AP}(\lambda,k) = \frac{\overbrace{p\big(V_k^{(\lambda)},\tilde{V}_k,\tilde{V}_{k-1},\tilde{V}_{k-2},\dots\big)}^{=p_1(\lambda)}}{\sum_{\xi=0}^{2^N-1} \underbrace{p\big(V_k^{(\xi)},\tilde{V}_k,\tilde{V}_{k-1},\tilde{V}_{k-2},\dots\big)}_{=p_1(\xi)}} \quad (7)$$

where $p(.)$ denotes joint probability-density functions since the components of the vectors $\tilde{V}_k, \tilde{V}_{k-1},\dots$ take on unlimited numbers of real values. Using $p_1(\lambda) = p(V_k^{(\lambda)},\tilde{V}_k,\tilde{V}_{k-1},\tilde{V}_{k-2},\dots)$ as an abbreviation, the following reformulation holds:

$$p_1(\lambda) = p\big(\tilde{V}_k \mid V_k^{(\lambda)},\tilde{V}_{k-1},\tilde{V}_{k-2},\dots\big)\cdot p\big(V_k^{(\lambda)},\tilde{V}_{k-1},\tilde{V}_{k-2},\dots\big) \quad . \quad (8)$$

If the transmitted codeword $V_k^{(\lambda)}$ (channel input) at time k is assumed to be known (hypothesis!), the value of the conditioned probability density function $p(\tilde{V}_k|V_k^{(\lambda)},\dots)$ does not depend on the channel outputs at the times $k-1,k-2,\dots$ since the statistical properties of the observation at the channel output are completely defined by the hypothesis $V_k^{(\lambda)}$ and the properties of the noise on the channel. The latter was assumed to be "white"; so we can rewrite (8) as

$$p_1(\lambda) = p\big(\tilde{V}_k \mid V_k^{(\lambda)}\big)\cdot \underbrace{p\big(V_k^{(\lambda)},\tilde{V}_{k-1},\tilde{V}_{k-2},\dots\big)}_{=p_2(\lambda)} \ . \quad (9)$$

The p.d.f. $p_2(\lambda)$ is now reformulated: First it is expanded by introducing the hypothesis $V_{k-1}^{(\mu)}$ for the transmitted codeword at time $k-1$ according to

$$p_2(\lambda) = \sum_{\mu=0}^{2^N-1} p\big(V_k^{(\lambda)},V_{k-1}^{(\mu)},\tilde{V}_{k-1},\tilde{V}_{k-2},\dots\big) \quad (10)$$

and after that it is rewritten:

$$p_2(\lambda) = \sum_{\mu=0}^{2^N-1} \overbrace{P\big(V_k^{(\lambda)} \mid V_{k-1}^{(\mu)},\tilde{V}_{k-1},\tilde{V}_{k-2},\dots\big)}^{P_3(\lambda,\mu)}\cdot p\big(V_{k-1}^{(\mu)},\tilde{V}_{k-1},\tilde{V}_{k-2},\dots\big) \quad . \quad (11)$$

The probabilities $P(V_k^{(\lambda)})$ of the codewords $V_k^{(\lambda)}$ and those of the source-encoder indices $I_k=\lambda$ are identical since the codewords V_k are deterministically computed from the indices I_k. Therefore we can also write

$$P_3(\lambda,\mu) = P\big(I_k=\lambda \mid I_{k-1}=\mu,\tilde{V}_{k-1},\tilde{V}_{k-2},\dots\big) \ .$$

For the generation of the indices I_k a first-order Markov-source was assumed. Therefore the index I_k at time k only depends on the output index I_{k-1} at the previous time $k-1$, i.e.

$$\begin{aligned} &P\big(I_k=\lambda \mid I_{k-1}=\mu, I_{k-2}=\mu_1,\dots\big) \\ &\quad = P\big(I_k=\lambda \mid I_{k-1}=\mu\big)\ , \\ &\quad \lambda,\mu,\mu_1,\dots \in \{0,1,\dots,2^N-1\} \end{aligned} \quad (12)$$

holds. The observation $\tilde{V}_{k-1}$ at the channel output does not increase the knowledge about I_k if the index I_{k-1} (or equivalently the associated codeword) is assumed to be known. Because of the Markov-property (12) the observations $\tilde{V}_{k-2},\dots$ do not influence the probability $P_3(\mu,k)$. Taking these facts into account the probability $P_3(\lambda,\mu)$ can be simplified to

$$P_3(\lambda,\mu) = P\big(I_k=\lambda \mid I_{k-1}=\mu\big) \quad . \quad (13)$$

This is equivalent to the index-transition probabilities of the Markov-source which are assumed to be known[3].

Using (13) in (11) we obtain

$$p_2(\lambda) = \sum_{\mu=0}^{2^N-1} P\big(I_k=\lambda \mid I_{k-1}=\mu\big)\cdot \underbrace{p\big(V_{k-1}^{(\mu)},\tilde{V}_{k-1},\tilde{V}_{k-2},\dots\big)}_{=p_4(\mu)} \quad . \quad (14)$$

The term $p_4(\mu)$ can be rewritten as

$$p_4(\mu) = P\big(V_{k-1}^{(\mu)} \mid \tilde{V}_{k-1},\tilde{V}_{k-2},\dots\big)\ \cdot\ p\big(\tilde{V}_{k-1},\tilde{V}_{k-2},\dots\big)$$

so we get

$$p_2(\lambda) = \sum_{\mu=0}^{2^N-1} P\big(I_k=\lambda \mid I_{k-1}=\mu\big)\cdot \underbrace{P\big(V_{k-1}^{(\mu)} \mid \tilde{V}_{k-1},\tilde{V}_{k-2},\dots\big)}_{=P_{AP}(\mu,k-1)}\cdot \underbrace{p\big(\tilde{V}_{k-1},\tilde{V}_{k-2},\dots\big)}_{A_{k-1}} \quad (15)$$

instead of (14). The second term is equal to the a-posteriori probabilities $P_{AP}(\mu,k-1)$ at the previous time $k-1$, and the last term A_{k-1} only depends on the observations at the channel output; so we can write

$$p_2(\lambda) = A_{k-1}\cdot \sum_{\mu=0}^{2^N-1} P\big(I_k=\lambda \mid I_{k-1}=\mu\big)\cdot P_{AP}(\mu,k-1)\ . \quad (16)$$

[3]In practice $P_3(\lambda,\mu)$ is measured for the source-encoder indices, stored and used as fixed data in the estimation process.

Now, using (16) and (9) in (7), we obtain

$$P_{AP}(\lambda,k) = M_k \cdot \overbrace{p\big(\tilde{V}_k \mid V_k^{(\lambda)}\big)}^{\text{channel term, given by (3)}} \cdot \sum_{\mu=0}^{2^N-1} \underbrace{P\big(I_k=\lambda \mid I_{k-1}=\mu\big)}_{\text{trans. probabilities, Markov-source}} \cdot P_{AP}(\mu,k-1) \qquad (17)$$

for $\lambda = 0, 1, \ldots, 2^N - 1$. The factor M_k, which corresponds to the denominator in (7), normalizes the sum in (17) in such a way that $P_{AP}(\lambda, k)$ is a true probability that sums up to 1 over all indices λ. The factor A_{k-1} cancels out because it is constant for each k and it appears in the numerator and the denominator of (17).

The formula (17) is a recursion that allows the computation of the a-posteriori probabilities $P_{AP}(\lambda, k)$ using only known quantities and all the values of the a-posteriori probabilities at the previous time. For the initialization of the recursion at time $k = 0$ the unconditioned probability distribution of the indices can be used, if no other a-priori information is available, i.e.:

$$P_{AP}(\lambda,-1) = P(I=\lambda)\ , \quad \lambda = 0,1,...,2^N-1\ . \qquad (18)$$

To sum it up, joint source-channel decoding by Channel-Coded Optimal Estimation is carried out by the following steps:

- initialize the a-posteriori probabilities by (18)
- compute $P_{AP}(\lambda, k)$ by (17) for each time k
- estimate the output signal by an appropriate estimator

The formula (17) has been first stated in [3] for systematic channel codes (section II.D) but no derivation could be given there due to limited space. Recently a similar formula has been stated in [2]. It is more general in the sense that also channel codes can be used which compute the codewords from the unquantized source-signal (and not from the quantizer output bits).

D. Special cases of Channel-Coded Optimal Estimation

1) No channel coding If no channel coding is applied, then $V_k = I_k$ holds, and this reduces (17) to

$$P_{AP}(\lambda,k) = M_k \cdot p\big(\tilde{I}_k \mid I_k=\lambda\big) \cdot \sum_{\mu=0}^{2^N-1} P\big(I_k=\lambda \mid I_{k-1}=\mu\big) \cdot P_{AP}(\mu,k-1) \quad . \qquad (19)$$

This formula has first been stated by Fingscheidt and Vary [1] with the difference that the a-posteriori probability (5) is conditioned on the hard-decided received bit-vectors $\hat{I}_k$ which are directly derived from the two possible signs of each component of the received channel-words $\tilde{I}_k$ using (1). However, on an AWGN-channel the a-posteriori probabilities are equal in both versions. In the following the decoding technique that uses (19) to compute the a-posteriori probabilities is denoted by "Optimal Estimation" (OE).

2) Systematic channel codes If a systematic channel code (SCC) is used, the codewords can be denoted by

$$V_k^{(\lambda)} = \{I_k=\lambda, C_k = \text{SCC}(I_k=\lambda)\} \quad . \qquad (20)$$

The $N_C = N_V - N$ redundancy-bits of the channel code are located in the bit-vector C_k. Using this notation in the recursion (17) we obtain

$$P_{AP}(\lambda,k) = M_k \cdot \overbrace{p\big(\tilde{C}_k \mid C_k = \text{SCC}(I_k=\lambda)\big)}^{\text{channel-"probability" of } I_k=\lambda \text{ due to the redundancy bits}} \cdot \overbrace{p\big(\tilde{I}_k \mid I_k=\lambda\big)}^{\text{channel-"probability" of } I_k=\lambda \text{ due to the index bits}} \cdot \sum_{\mu=0}^{2^N-1} P\big(I_k=\lambda \mid I_{k-1}=\mu\big) \cdot P_{AP}(\mu,k-1)\ . \qquad (21)$$

The term $p\big(\tilde{C}_k \mid C_k = \text{SCC}(I_k=\lambda)\big)$ introduces additional information about the data bits because the code-constraints do not allow each possible bit-combination as a codeword. So if the redundancy bits were received with low error-probabilities (large absolute received values), then a data-bit combination which might be highly probable with respect to the source-correlation still can have a low a-posteriori probability because the redundancy bits that are associated with these data-bits have small channel-"probabilities", i.e. the received hard-decided redundancy-bits are different from the redundancy-bits which are associated with the data-bits under consideration.

3) Assumption that the source-indices are independent If the source-indices are independent and equally distributed, the following equation holds:

$$P\big(I_k=\lambda \mid I_{k-1}=\mu\big) = P(I_k=\lambda) = \frac{1}{2^N} \quad \forall\ \lambda,\ \mu\ . \qquad (22)$$

This reduces (17) to the non-recursive equation:

$$P_{AP}(\lambda,k) = p\big(\tilde{V}_k \mid V_k^{(\lambda)}\big) \Big/ \sum_{\xi=0}^{2^N-1} p\big(\tilde{V}_k \mid V_k^{(\xi)}\big) \quad . \qquad (23)$$

If a MAP-estimator is used to calculate the receiver output using the probabilities (23) this is equal to classical maximum-likelihood channel decoding. In this case the denominator in (23) does not have to be computed since it is constant for each k. In the following this technique is denoted by "CCmap" because only the channel code is exploited to compute the "a-posteriori" probabilities. An MS-estimator can also be used which will possibly yield better results than a MAP-estimator in terms of the "overall"-quality of transmission ("CCms"-technique).

The specializations in this subsection reveal that low-complexity channel decoding algorithms which are available for several types of channel codes and which approximate maximum-likelihood channel decoding cannot be used in the *general* case of Channel-Coded Optimal Estimation because (22) does not hold.

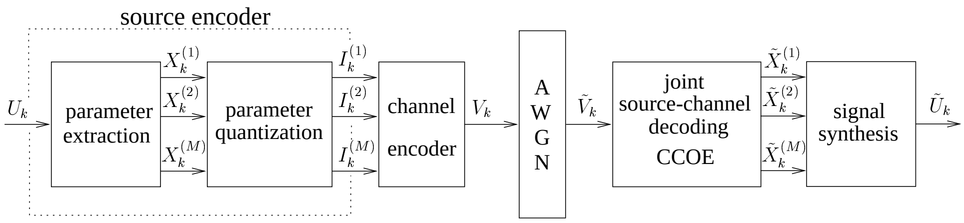

Fig. 3: Model of a transmission system with M source indices

$$P_{AP}(\lambda_1, ..., \lambda_M, k) = M_k \cdot p\big(\tilde{V}_k \mid V_k^{(\lambda_1,...,\lambda_M)}\big) \cdot \sum_{\mu_1=0}^{2^{N_1}-1} \cdots \sum_{\mu_M=0}^{2^{N_M}-1} P\big(I_k^{(1)}=\lambda_1, ..., I_k^{(M)}=\lambda_M \mid I_{k-1}^{(1)}=\mu_1, ..., I_{k-1}^{(M)}=\mu_M\big) \cdot P_{AP}(\mu_1, ..., \mu_M, k-1) \quad (24)$$

III. Generalizations of CCOE

In practice a source-signal is often source-encoded in two steps. First some (say $M \in I\!N$) parameter-vectors $X_k^{(1)}, ..., X_k^{(M)}$ are computed for each source signal-vector U_k. A parameter-vector could consist of LPC-coefficients, but the mean power could also be a (scalar) parameter. The parameter vectors are quantized, and M indices are issued that are channel-encoded and transmitted to the receiver. This scenario is depicted in figure 3. What we would like to do now is to use Channel-Coded Optimal Estimation in a "vectorized" fashion.

The concept of Channel-Coded Optimal Estimation as developed in section II is more general than it appears at a first glance because the index I_k is nothing but a combination of bits, so we can simply set $I_k = \{I_k^{(1)}, ..., I_k^{(M)}\}$ where the new indices $I_k^{(j)}$, $j = 1, ..., M$, are the source-encoder outputs of the new, more realistic transmission system in figure 3. Introducing this into (17) we obtain (24) (see above) as the new formula to compute the joint a-posteriori probabilities

$$P_{AP}(\lambda_1, ..., \lambda_M, k) = P\big(V^{(\lambda_1,...,\lambda_M)} \mid \tilde{V}_k, \tilde{V}_{k-1}, ...\big)$$

for all the indices. Cross-correlations (if any) between the source-encoder indices $I_k^{(j)}$ are exploited in (24).

A MAP-estimator would be given by maximizing the probabilities $P_{AP}(\lambda_1, ..., \lambda_M, k)$ over all possible index-combinations. It is easy to see that this would be impossible for a practical system like GSM-full-rate speech transmission where 260 bits are issued by the source-encoder: If we knew the a-posteriori probabilities we would have to maximize over 2^{260} bit-combinations - and this is the low-complexity part since we first would have to compute the probabilities.

If a MAP-estimator is used, we will get that combination of indices which has been transmitted by the encoder with the highest probability. As mentioned above, an MS-estimator yields the best results in terms of SNR (of U_k and $\tilde{U}_k$) if only one index is transmitted that is issued by a quantizer for waveform-samples. If the source-signal vector is decomposed into parameters which are not of the "waveform-type" (e.g. LPC-coefficients), it is no longer guaranteed that MS-estimators for the parameter-vectors will yield the optimum overall SNR-performance. Still MS-estimators will often lead to better results than MAP-estimators.

In general, special estimators for the source-encoder parameters would have to be developed that fulfill the requirement for optimum performance with respect to SNR (or a criterion that better matches human perception). These estimators could also use the a-posteriori probabilities that are computed by the recursion (24). The problem of finding perceptually optimal estimators e.g. for a speech codec is unsolved, but it is not a subject of this paper where the focus lies on the computation of the a-posteriori probabilities.

IV. Simulation Results

A Gaussian random signal was correlated by a low-pass filter with the transfer function $H(z) = \frac{(1-a)\cdot z}{z-a}$, $a = 0.9$, to generate the source-signal U_k. A 5-bit optimal scalar quantizer was used as a source-encoder and a 5-bit CRC (Cyclic Redundancy Check) as a channel code. The codewords were transmitted over the AWGN-channel and the OE-technique and the CCOE-technique were applied, both with MAP- and MS-estimators. The CCms/map-techniques can be derived from CCOEms/map by *not* exploiting the source-statistics as stated in section II.D. The CCmap-algorithm is equivalent to maximum-likelihood channel decoding.

For each algorithm the SNR-value $(U(k), \tilde{U}(k))$ over E_b/N_0, the ratio of the energy E_b per transmitted *data*-bit and twice the noise power spectral density $N_0/2$ on the AWGN-channel, is plotted in figure 4: The CCOE-techniques work as good as or better than all other algorithms for all channel conditions under consideration if the same type of estimator is used. For "good" channels ($E_b/N_0 > 7$ dB) the CCOE-algorithms rely on the parity bits, i.e. channel errors are corrected mainly by the channel code. Therefore the performance of CCOEms/map is similar to CCms/map, and it is slightly superior to the OE-algorithms that do not exploit channel coding.

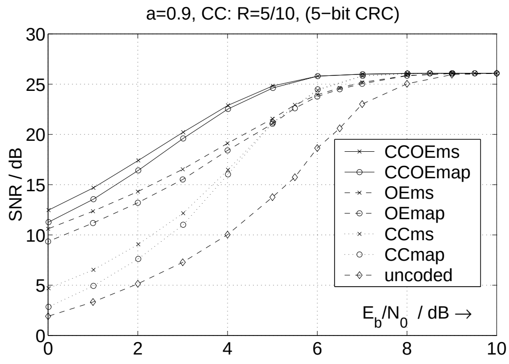

Fig. 4: Performances for a strongly correlated source signal ($a = 0.9$) coded by a 5-bit optimal scalar quantizer, channel coding by a 5-bit CRC (code rate R=5/10).

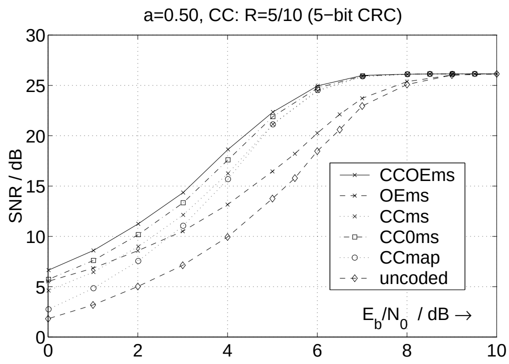

Fig. 5: Performances for a moderately correlated source signal ($a = 0.5$) coded by a 5-bit optimal scalar quantizer, channel coding by a 5-bit CRC (code rate R=5/10).

If the channel quality is bad, the CCOE-algorithms put more weight on the information that is based on the correlation of the source-signal. Consequently the performance of CCOE is similar to that of the OE-algorithms. The quality is much better than for the CC-algorithms that do not exploit the source statistics.

The strongest gains of the CCOE-algorithms over the others are achieved for moderately corrupted channels (E_b/N_0-values around 2..6 dB). All algorithms work better and mostly much better compared to a system where only hard decisions are taken for the data bits at the channel output (curve labeled "uncoded").

The performances of the OE-techniques and of the "uncoded" case are independent of the channel code. The algorithms that exploit the channel code (CCOE, CC) work significantly worse if a less powerful channel code is used but still their performance is better than in the systems that do not use the channel code.

It is interesting to study the effect of lower source-correlation. The results of appropriate simulations are depicted in figure 5. The curves for MS-estimators are depicted because the latter achieve the best system performance (see figure 4). To compare the algorithms with classical maximum-likelihood channel decoding the results for the CCmap-technique are also plotted.

In addition to the CCOE-algorithms the CC0ms-technique was included which exploits only the probability-distribution of the source-encoder indices but *not* the autocorrelation. The notation "CC0ms" corresponds to the use of "zeroth-order" knowledge on the source-statistics in contrast to additional first-order knowledge (in form of the first-order Markov-model) which is used in the CCOE-algorithms. The CC0ms-technique has the same performance as the CCOEms-algorithm if an uncorrelated source-signal is transmitted.

For a moderately autocorrelated signal (figure 5) the CCOE-technique works better than CC0ms and much better than CCms. The OE-algorithm works better than CCms for E_b/N_0 below about 2 dB. The CC-techniques and the uncoded transmission have the same performances as in figure 4 since they do not exploit the source-statistics and the same channel code is used.

If the source is strongly autocorrelated CCOEms works much better than CC0ms, because the autocorrelation allows a good prediction of the source signal. For moderately and strongly corrupted channels OEms therefore also works much better than CC0ms.

V. Conclusions

Channel-Coded Optimal Estimation (CCOE) was stated as an algorithm for joint source-channel decoding. Source-statistics, bit-reliability informations and the redundancy introduced by a channel code are commonly exploited to estimate the output signal at the receiver of a transmission system. A recursive formula was derived that allows the computation of the a-posteriori probabilities of the possibly transmitted indices which are used in the estimators for the output signal.

The performance of CCOE was stated and compared with other schemes of source-channel decoding. Overall CCOE works best of all algorithms that were compared. The disadvantage lies in the memory requirements and the complexity of CCOE especially if realistic systems are considered with several indices transmitted in parallel. Therefore future work will focus on the complexity reduction.

VI. References

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