PRAMANA
— journal of
physics

Vol. 71, No. 4
October 2008
pp. 785–796

Non-classical neutron beams for fundamental and solid state research

H RAUCH
Atominstitut der Oesterreichischen Universitaeten, 1020 Wien, Austria
E-mail: rauch@ati.ac.at

Abstract. The curious dual nature of the neutron, sometimes a particle, sometimes a wave, is wonderfully manifested in the various non-local interference and quantum contextuality effects observed in neutron interferometry. Non-classical states may become useful for novel fundamental and solid state research. Here we discuss unavoidable quantum losses as they appear in neutron phase-echo and spin rotation experiments and we show how entanglement effects in a single particle system demonstrate quantum contextuality. In all cases of interactions, parasitic beams are produced which cannot be recombined completely with the original beam. This means that a complete reconstruction of the original state would, in principle, be impossible which causes a kind of intrinsic irreversibility. Even small interaction potentials can have huge effects when they are applied in quantum Zeno-like experiments. Recently, it has been shown that an entanglement between external and internal degrees of freedom exists even in single particle systems. This contextuality phenomenon also shows that a quantum system carries much more information than usually extracted. The path towards advanced neutron quantum optics will be discussed.

Keywords. Neutron interferometer; neutron optics; quantum optics; matter waves.

PACS Nos **03.75.Dg; 03.75.Be; 42.50.-p; 03.75.-b**

1. Introduction

Pure single particle effects exist in any neutron experiment due to the extremely low phase space density of neutron beams (10^{-14}). Coherent beam separations up to several centimetres can be achieved by perfect silicon interferometers and these beams can be influenced by nuclear, magnetic, and gravitational interactions, and also by topological effects. The status of neutron interferometry until 2000 has been summarized in [1]. It can be stated in a rather general form that the wave function behind the interferometer contains much more information than can be extracted by standard experimental techniques. In most cases one focuses on one parameter, but there are always additional effects which may be of interest in the discussion about basic quantum entanglement, irreversibility and decoherencing effects. The question of entanglement and decoherence becomes an essential issue for understanding the quantum mechanics and especially the quantum measurement process. Several books and review articles deal with this topic [2–4]. Here we will show that not only dissipative interactions cause an irreversible change of the wave function, but deterministic ones also can cause such a change.

The well-known Zeno phenomenon (paradox) can be taken as a characteristic example of a temporal or spatial evolution of a quantum system which is kept under frequent observation and which becomes frozen in the initial state [5–8], but shows an essentially different behaviour when realistic situations are considered [9–17]. The topic is closely related to the non-exponential decay of quantum states for very short times where a quadratic time dependence is expected and to so-called 'interaction-free' measurements [14,18].

Additionally it must be considered that quantum entanglement between external and internal degrees of freedom exists and as a consequence measurement of any observable influences the outcome of a later measurement of another observable, which denotes quantum contextuality [19–21].

2. Basic relations

The wave function for the beam in the forward direction (0) behind a Mach–Zehnder interferometer (figure 1) is given by the superposition of wave functions arising from the right and the left beam paths. They are transmitted-reflected-reflected (trr) and reflected-reflected-transmitted (rrt), respectively, and they are equal in amplitude and phase due to symmetry reasons ($\psi_{\mathrm{trr}} = \psi_{\mathrm{rrt}}$). When parts of the beams are exposed to an interaction, a phase shift occurs

$$\chi = \oint k_{\mathrm{c}} \mathrm{d}s, \tag{1}$$

where k_{c} denotes the canonical momentum of the neutrons along the beam paths. Conservative interactions cause a change of the kinematical momentum k, which can be described by an index of refraction (e.g., [1])

$$n = \frac{k}{k_0} = \sqrt{1 - \frac{\overline{V}}{E}}. \tag{2}$$

$\overline{V}$ denotes the mean interaction potential within the phase shifter which amounts to $\overline{V} = 2\pi\hbar^2 N b_{\mathrm{c}}/m$ for nuclear interaction and to $\overline{V} = \pm\mu B$ for magnetic interaction. m and μ denote the mass and the magnetic moment of the neutron, N the particle density and b_{c} the coherent neutron scattering length of the phase shifter. When a purely magnetic interaction is considered, eq. (2) describes the longitudinal Zeeman splitting.

The different kinematical momentum of the beam inside the phase shifter also causes a spatial shift Δ of the wave packets and the phase shift can also be written as

$$\chi = (1 - n)kD = \Delta \cdot k. \tag{3}$$

A realistic description of a neutron beam can only be achieved by a wave packet formalism, which, for a stationary and one-dimensional situation, is

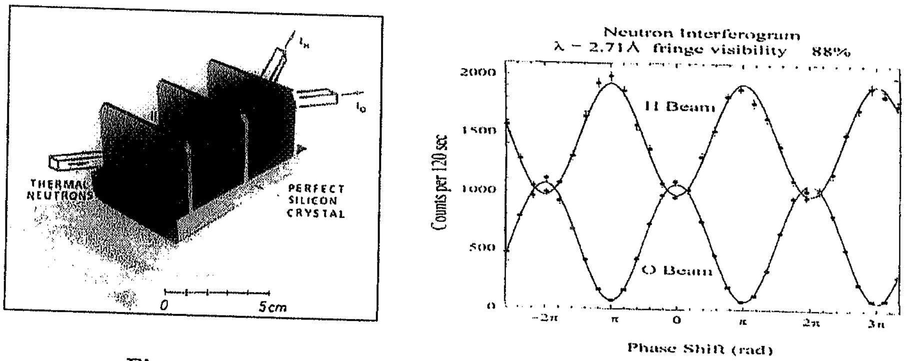

Figure 1. Sketch of a perfect crystal neutron interferometer and a typical interference pattern.

$$\psi(r) \propto \int a(k) \mathrm{e}^{ikr}\,\mathrm{d}k, \tag{4}$$

where $a(k)$ denotes the amplitude function, which is related to the momentum distribution function as $g(k) = |a(k)|^2$. It represents a coherent superposition of plane waves, which are non-local and which may be responsible for the non-local feature of quantum mechanics [22]. The interference pattern follows as

$$I(\Delta) \propto |\psi(0) + \psi(\Delta)|^2 = |\psi_{\mathrm{rrt}} + \psi_{\mathrm{trr}} \mathrm{e}^{i\Delta\cdot k}|^2 = 1 + |\Gamma(\Delta)| \cos(\Delta \cdot k), \tag{5}$$

where $\Gamma(\Delta)$ denotes the coherence function which is given by the autocorrelation function of the wave function (e.g., [23])

$$|\Gamma(\Delta)| = |\langle\psi(0)\psi(\Delta)\rangle| \propto \left|\int g(k)\ \mathrm{e}^{ik\Delta}\mathrm{d}k\right|. \tag{6}$$

$|\Gamma(\Delta)|$ can be determined by the measured visibility of the interference pattern at large phase shifts [24]. For Gaussian wave packets the characteristic widths Δ_{c} of the coherence function and the momentum spread of the packets δk fulfill the minimum Heisenberg uncertainty relation ($\Delta_{\mathrm{c}}\delta k = 1/2$) and

$$|\Gamma(\Delta)| = \exp[-(\Delta \cdot \delta k)^2/2]. \tag{7}$$

When several phase shifts act onto the beams χ or Δ are additive quantities (eqs (1) and (3)), which have been demonstrated in a dedicated phase echo experiment (figure 2) [25]

$$\chi = \chi_1 + \chi_2 \quad \text{or} \quad \Delta = \Delta_1 + \Delta_2. \tag{8}$$

In this experiment a large phase shift, i.e. larger than the coherence length, has been applied by a Bi and alternatively by a Ti phase shifter. The visibility of the interference pattern nearly disappeared completely, in agreement with eqs (5) and

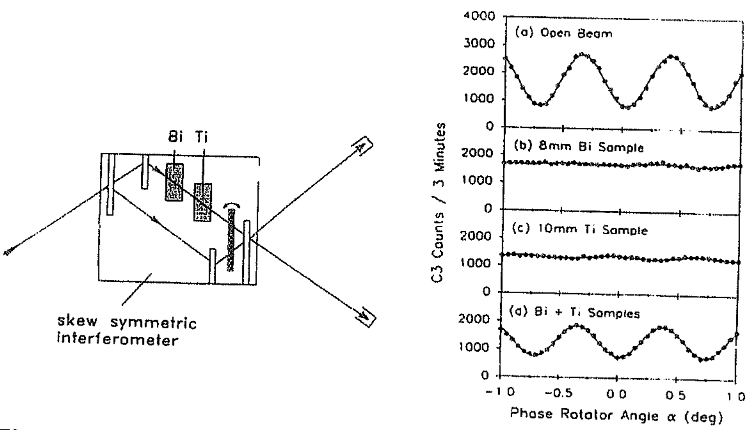

Figure 2. Phase-echo experiment using phase shifters with positive (Bi) and negative (Ti) coherent scattering lengths [25].

(8) as shown in figure 2. Bi and Ti cause opposite phase shifts due to a positive and negative coherent scattering length and, therefore, the interference pattern can be retrieved when both phase shifters are applied simultaneously and $\chi_1 + \chi_2 \cong 0$ [25]. These experiments can be improved when magnetic fields are used which do not cause any absorption or incoherent scattering effects. The analogy of the phase-echo concept to the spin-echo concept should be emphasized [26,27].

3. Quantum irreversibility effects

In the phase echo experiments, the question arises whether a complete retrieval of the interference pattern is feasible or not. First of all, it should be mentioned that any phase shift is caused by an interaction which changes the momentum of the neutron (eq. (2)) and such interaction potentials do not cause only a phase shift but also a back-reflection and/or back- and forth-reflections of parts of the wave function, as shown schematically in figure 3. One notices additionally that the phase shifts of the direct transmitted beam are additive whereas all other partial waves have much larger phase shifts and their amplitudes vary as well. A complete retrieval seems to become impossible [28]. The losses are at least in the order of $(\overline{V}/E)^2$ and can become much larger when resonance effects are considered as well (e.g. supermirrors) [29,30].

In this connection, quantum Zeno-like experiments have been proposed when neutrons cross a magnetic field many times within a perfect neutron resonator [9,15,17,31]. Due to unavoidable quantum losses, a complete freezing of the initial state becomes impossible. Such experiments can also be used to investigate a non-exponential behaviour of quantum state transitions.

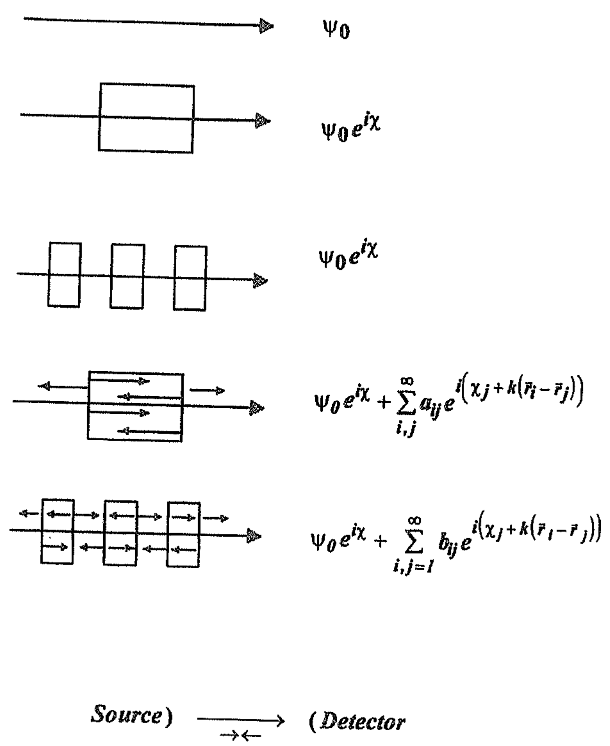

Figure 3. Approximate and complete wavefunctions when different phase shifters causing the same total phase shift are used.

4. Neutron post-selection experiments

Equations (5) and (7) show that the interference pattern at high interference order disappears when the phase shift becomes larger than the coherence length. When the interference in ordinary space disappears, a modulation of the momentum distribution appears. This is shown schematically in figure 4 [22]. In light optics [32,33] and more recently in neutron optics [34], it has been shown that interference can be retrieved when a momentum post-selection method is applied (figure 5). In this case, the wave packets in ordinary space become enlarged and overlap with the reference packet. This figure also shows that interferences can be observed with beam detectors No. $1, \ldots, n$, whereas a simultaneous measurement of the total count rate does not show any interference pattern. In terms of a wave-particle dualism, several authors describe the wave properties by the square of the fringe visibility (V^2) and the particle properties by diagonal terms of the density operator (P_{D}, which can also be seen as path distinguishability) [35–38]

$$P_{\mathrm{D}}^2 + V^2 = 1, \tag{9}$$

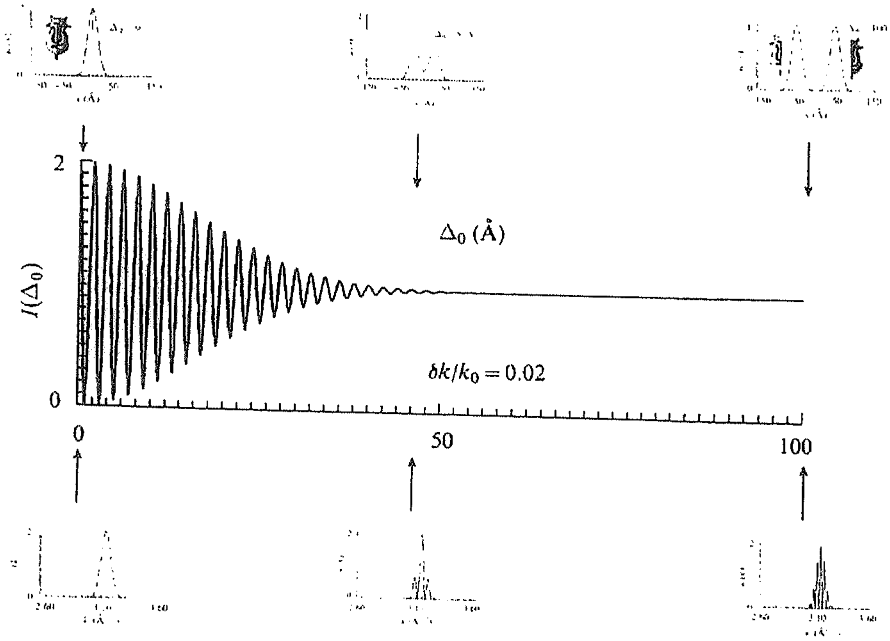

Figure 4. Schematic contrast reduction at higher interference order (middle) and wave packets in ordinary and momentum space at 0th, 50th and 100th interference order.

a relation which has been tested by means of a two-loop interferometer, where a distinct quantum state can be prepared with the first loop and analysed by the second one (figure 6). The measured values agree fairly well with the predictions [39–41].

5. Quantum engineered neutron states

The post-selection experiments have shown that non-classical neutron states can be produced by means of interference experiments, e.g. Schroedinger cat-like states where the neutron is dislocated at two local separated regions. Such states are most clearly described by means of Wigner functions [42–44]. The Wigner quasi-distribution function is given by

$$W_s(k,x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{e}^{ikx'}\psi^*\left(x+\frac{x'}{2}\right)\psi\left(x-\frac{x'}{2}\right)\,\mathrm{d}x' \tag{10}$$

and has the features that a spatial integration gives the momentum distribution and a momentum integration the spatial distribution.

A negative part of the Wigner function indicates a non-classical state. Quantum state reconstruction experiments based on quadrature measurements of position and momentum yield the Wigner function or the density operator of a quantum state

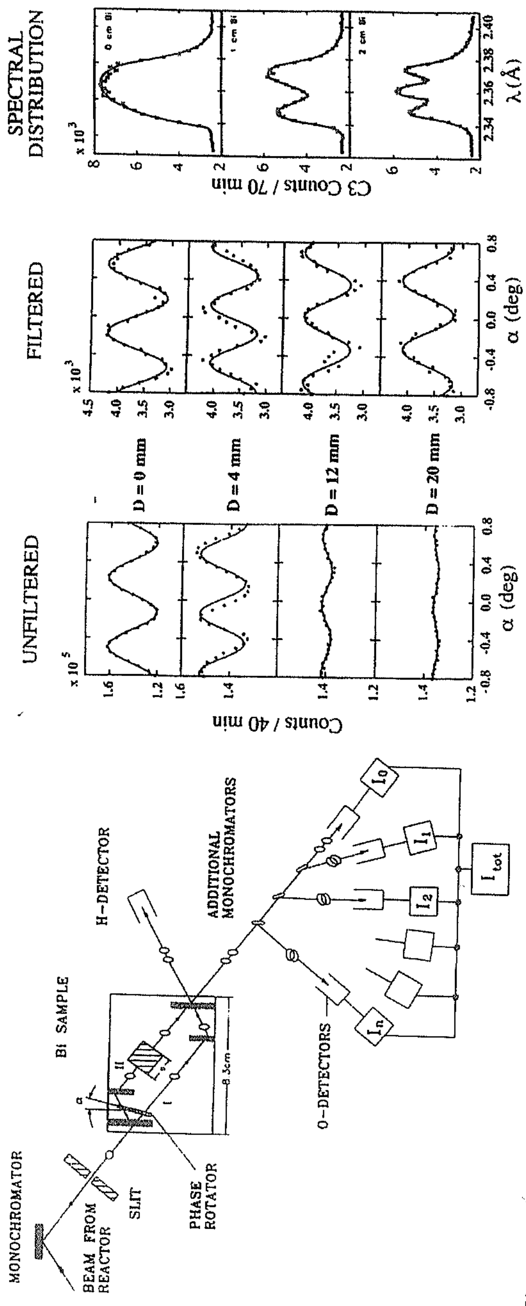

Figure 5. Sketch of a momentum post-selection set-up and interference pattern of the overall beam (left, detector I_0); the filtered beam (middle, detector I_n) and the related momentum distribution by rocking the analyser crystal [34].

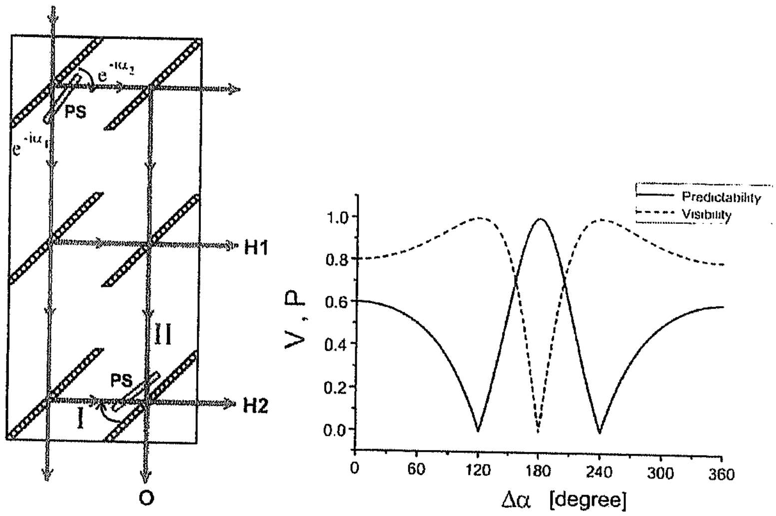

Figure 6. Double loop interferometer used to measure wave-particle properties and sketch of the experimental results [40].

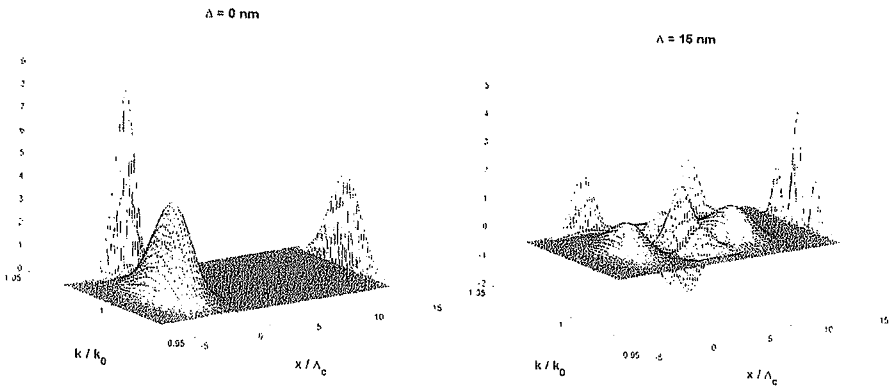

Figure 7. Interferometric Wigner functions at zero- and higher interference order.

[45–47]. An example of a neutron spin quantum state reconstruction experiment can be found in [48]. Figure 7 shows the Wigner function at low and high interference order [44]. When any fluctuations and parasitic side effects are included, a complete retrieval by means of phase-echo methods becomes impossible (see §3). This has been treated in detail for neutron spin-echo systems which are also interference systems of two beams in the forward direction but with beams split by the Zeeman effect [49].

With a double loop interferometer even more complicated and advanced non-classical states can be produced and measured. Figure 8 shows such a double loop interferometer and the related Wigner function at the exit of such a device [44]. A

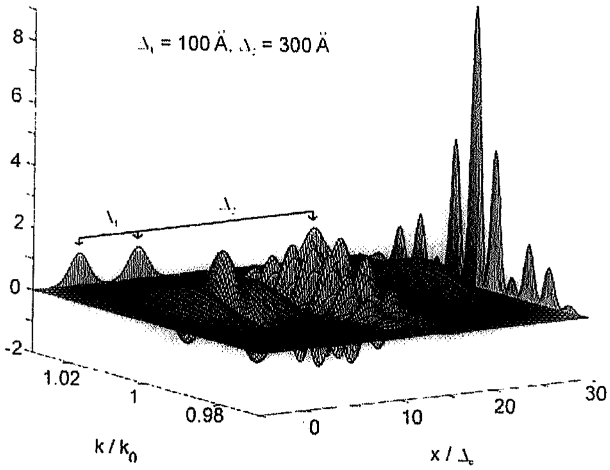

Figure 8. Triple humped Wigner function behind a double loop interferometer.

triple humped spatial wave function can be produced which observes the physical situation at a position r within a sample at time $t_0, t_1 = \Delta_1/v$ and $t_2 = (\Delta_1 + \Delta_2)/v$, i.e. it provides the basis of measuring condensed matter triple correlation functions.

6. Contexuality or self-entangled neutron states

Entanglement of pairs of photons or material particles is a well-known phenomenon (e.g. [50,51]). Contextuality means a quantum entanglement between different degrees of freedom in a single particle. In a related neutron experiment we made a joint measurement of the commuting observables of the spin and of the beam path through the interferometer [52]. The related entangled state can be produced within the interferometer when a polarized incident beam is split coherently into the two beam paths (I and II) and the spin in one beam path is rotated by Larmor precession to the $-y$ and in the other beam path to the $+y$ direction (figure 9). The entangled state reads as

$$|\psi\rangle = (|\rightarrow\rangle \otimes |\mathrm{I}\rangle + |\leftarrow\rangle \otimes |\mathrm{II}\rangle). \tag{11}$$

In this case Bell-like inequalities can be formulated, where the expectation values can be measured when the phase shift χ between the beams and the spin rotation angle α are chosen as given in the formulas

$$\begin{aligned} & -2 \leq S \leq 2, \\ & S = E(\alpha_1, \chi_1) + E(\alpha_1, \chi_2) - E(\alpha_2, \chi_1) + E(\alpha_2, \chi_2), \\ & E(\alpha, \chi) = \frac{N(\alpha, \chi) + N(\alpha + \pi, \chi + \pi) - N(\alpha, \chi + \pi) - N(\alpha + \pi, \chi)}{N(\alpha, \chi) + N(\alpha + \pi, \chi + \pi) + N(\alpha, \chi + \pi) + N(\alpha + \pi, \chi)}. \end{aligned} \tag{12}$$

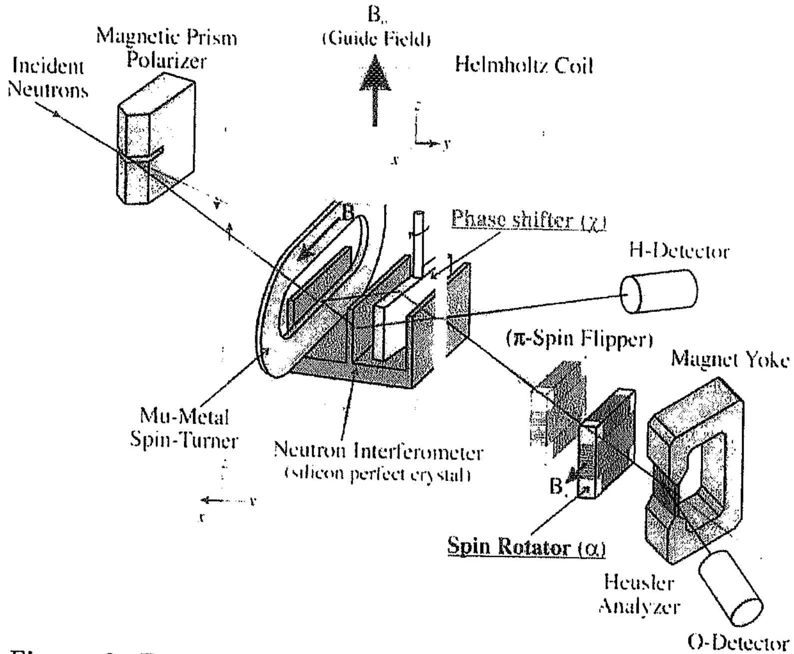

Figure 9. Experimental set-up for the production of spin-path entangled neutron states used for the proof of quantum contextuality [52].

The maximal violation of this inequality due to quantum mechanics happens for the following parameters: $\alpha_1 = 0$, $\alpha_2 = \pi/2$, $\chi_1 = \pi/4$ and $\chi_2 = -\pi/4$ and amounts to $S = 2\sqrt{2} = 2.82$. The first experiment in this direction yielded $S = 2.051 \pm 0.019$ indicating a violation of the Bell inequality and contradicting classical hidden variable theories [52].

More recently, related measurements dealt with the Kochen–Specker theorem [53] and the Mermin inequalities [54] which showed even a stronger violation of classical hidden variable theories [55]. Thus different degrees of freedom in a single particle have to be considered as entangled which opens new possibilities when such states are used in spectroscopy since a measurement on one degree of freedom influences the outcome of a subsequent measurement of the other commuting observable.

7. Discussion

Due to their variety of interactions neutrons are proper tools for test experiments in quantum mechanics. It has been shown that any interaction of a neutron beam with a potential produces unavoidable parasitic beams which are characteristic for this kind of interaction. Multiple potential barriers do not produce only an additive effect of losses, but also show enhanced losses due to various resonance effects. Therefore, the Zeno phenomenon, which is based on a repetitive interaction and observation of a quantum system, has to be discussed in a new light including unavoidable quantum losses. In this respect, the imperfect Zeno phenomenon and the limit of a non-completely frozen state for an infinite number of stages can be seen as a fingerprint of quantum mechanics. In the limiting case, when the coherence length exceeds the dimension of the spin rotation stages the evolution time has to be replaced by the passage time of the packet. In this case, the survival probability would also tend to zero. Various shapes of wave packets can be produced which can provide a basis for the measurement of higher order condensed matter correlation functions. For the first time an entanglement in a single particle system has been achieved which provided the verification of quantum contextuality. The proved single particle entanglement may have consequences for many particle entanglement experiments as well.

Acknowledgements

Very fruitful discussions with Yuji Hasegawa, Martin Suda and Stefan Filipp are gratefully acknowledged. This work has been supported by the Austrian Science Foundation (projects P13332-PHY and F-1513).

References

[1] H Rauch and S A Werner, *Neutron interferometry: Lessons in experimental quantum mechanics* (Oxford University Press, 2000)
[2] D Giulini, E Joos, C Kiefer, J Kupsch, L-O Stamatescu and H D Zeh, *Decoherence and the appearance of a classical world in quantum theory* (Springer Verlag, Berlin, 1996)
[3] M Namiki, S Pascazio, H Nakazato, *Decoherence and quantum measurement* (World Scientific, Singapore, 1997)
[4] M B Mensky, *Quantum measurements and decoherence* (Kluwer Academic Press, Dordrecht, 2000)
[5] B Misra and E C G Sudershan, *J. Math. Phys.* **18**, 756 (1977)
[6] L Fonda, G C Ghirardi and A Rimini, *Rep. Progr. Phys.* **41**, 587 (1978)
[7] D Home and M A B Whitaker, *Phys. Lett.* **A173**, 327 (1993)
[8] E Joos, Chap. 3 in [2]
[9] H Nakazato, M Namiki, S Pascazio and H Rauch, *Phys. Lett.* **A199**, 27 (1995)
[10] A Venugopalan and R Ghosh, *Phys. Lett.* **A204**, 11 (1995)
[11] A Pati, *Phys. Lett.* **A215**, 7 (1996)
[12] W H Itano, D J Heinzen, J J Bollinger and D J Wineland, *Phys. Rev.* **A41**, 2295 (1990)
[13] A Peres, *Am. J. Phys.* **48**, 931 (1980)
[14] P Kwiat, H Weinfurter, T Herzog, A Zeilinger and M Kasevich, *Phys. Rev. Lett.* **74**, 4763 (1995)
[15] S Pascazio, M Namiki, G Badurek and H Rauch, *Phys. Lett.* **A179**, 155 (1993)
[16] K Machida, H Nakazato, S Pascazio, H Rauch and S Yu, *Phys. Rev.* **A60**, 3448 (1999)
[17] Z Hradil, H Nakazato, M Namiki, S Pascazio and H Rauch, *Phys. Lett.* **A239**, 333 (1998)
[18] A C Elitzur and L Vaidman, *Found. Phys.* **23**, 987 (1993)
[19] A Peres, *Phys. Lett.* **A151**, 107 (1990)

[20] N D Mermin, *Phys. Rev. Lett.* **65**, 3373 (1990)
[21] N D Mermin, *Rev. Mod. Phys.* **65**, 803 (1993)
[22] H Rauch, *Phys. Lett.* **A173**, 240 (1993)
[23] L Mandel and W Wolf, *Optical coherence and quantum optics* (Cambridge University Press, 1995)
[24] H Rauch, H Wölwitsch, H Kaiser, R Clothier and S A Werner, *Phys. Rev.* **A53**, 902 (1996)
[25] R Clothier, H Kaiser, S A Werner, H Rauch and H Wölwitsch, *Phys. Rev.* **A44**, 5357 (1991)
[26] F Mezei, *Z. Physik* **255**, 146 (1972)
[27] G Badurek, H Rauch and A Zeilinger, in: *Neutron spin echo* edited by F Mezei, *Lect. Notes Phys.* (Springer Verlag, Berlin, 1980) Vol. 128, p. 136
[28] H Rauch, *J. Phys. Conf. Series* **36**, 431 (2006)
[29] F Mezei, *Commun. Phys.* **1**, 81 (1976)
[30] V F Sears, *Acta Crystallogr.* **A39**, 601 (1983)
[31] M R Jaekel, E Jericha and H Rauch, *Nucl. Instrum. Methods* **A539**, 335 (2005)
[32] L Mandel and E Wolf, *Rev. Mod. Phys.* **37**, 231 (1965)
[33] G S Agarwal and D F V James, *J. Mod. Opt.* **40**, 1431 (1993)
[34] D L Jacobson, S A Werner and H Rauch, *Phys. Rev.* **A49**, 3196 (1994)
[35] D M Greenberger and A Yasin, *Phys. Lett.* **128**, 391 (1988)
[36] B-G Englert, *Phys. Rev. Lett.* **77**, 2154 (1996)
[37] P Ghose, *Testing quantum mechanics on new grounds* (Cambridge University Press, 1999)
[38] C Miniatura, C A Muller, Y Lu, G Q Wang and B G Englert, *Phys. Rev.* **A76**, 022101 (2007)
[39] M Zawisky, M Baron and R Loidl, *Phys. Rev.* **A66**, 063608 (2002)
[40] M Zawisky, *Found. Phys. Lett.* **17**, 561 (2004)
[41] S S Afshar, E Flores, K F McDonalds and E Knoesel, *Found. Phys.* **37**, 295 (2007)
[42] E P Wigner, *Phys. Rev.* **40**, 749 (1932)
[43] L Mandel and E Wolf, *Optical coherence and quantum optics* (Cambridge University Press, 1995)
[44] M Suda, *Quantum interferometry in phase space* (Springer, Berlin, 2006)
[45] M Freyberger, S H Kienle and V P Yakovlev, *Phys. Rev.* **A56**, 195 (1997)
[46] U Leonhardt, *Measuring the quantum state of light* (Cambridge University Press, 1997)
[47] P W Schleich, *Quantum interferometry in phase space* (Wiley-VCH, Berlin, 2001)
[48] Y Hasegawa, R Loidl, G Badurek, S Filipp, J Klepp and H Rauch, *Phys. Rev.* **A76**, 052108 (2007)
[49] H Rauch and M Suda, in: *Neutron spin-echo spectroscopy* edited by F Mezei, C Pappas, T Gutberlet (Springer, Berlin, 2003) p. 133
[50] J S Bell, *Physics* **1**, 195 (1964)
[51] R A Bertlmann and A Zeilinger (eds.), *Quantum [un]speakables* (Springer, Berlin, 2002)
[52] Y Hasegawa, R Loidl, G Badurek, M Baron and H Rauch, *Nature (London)* **425**, 45 (2003)
[53] S Kochen and E P Specker, *J. Math. Mech.* **17**, 59 (1967)
[54] N D Mermin, *Rev. Mod. Phys.* **65**, 803 (1993)
[55] Y Hasegawa, R Loidl, G Badurek, M Baron and H Rauch, *Phys. Rev. Lett.* **97**, 230401 (2006)