MIMO RECEIVER DIVERSITY IN GENERAL FADING

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ABSTRACT

There have recently been a large number of papers that derive the diversity order of various, low complexity, suboptimal receiver structures for MIMO communications. In almost all analyses the MIMO channel is assumed to be i.i.d. Rayleigh fading. It is of interest to investigate how these results generalize to other fading models (e.g., correlated Ricean fading). We show in this paper that the diversity achieved by virtually any receiver is preserved within a very general class of fading models (including i.i.d. Rayleigh fading and correlated Ricean fading). This result obviates the need to recalculate the diversity of various receivers for different fading distributions in a piecemeal fashion.

Index Terms— Multiple-input multiple-output (MIMO), space-time block codes (STBC), receiver diversity, fading.

1. INTRODUCTION

Background. In the context of narrowband MIMO communications over block-fading channels, there have been several papers dealing with the problem of determining the diversity order obtained by a specific receiver structure [1–4]. In general terms, the diversity order of any particular communications scheme will of course also depend on the coding applied at the transmitter and on the statistical properties of the MIMO channel. Typically, the MIMO channel is assumed to be i.i.d. Rayleigh fading. Although there is no doubt that this assumption comes with a certain mathematical convenience, it may not be physically justified for the scenarios of interest [5].

For systems using space-time block codes (STBCs) and maximum-likelihood (ML) detection/decoding, it is well known that the diversity order is not sensitive to the i.i.d. Rayleigh fading assumption. In particular, the diversity order achieved over an i.i.d. Rayleigh fading channel is fully determined by the rank of the codeword difference matrices, and remains the same under (non-singular) correlated Rayleigh and Rician fading assumptions. This is relatively straightforwardly proven by analyzing the pairwise error probability (PEP), see e.g. [6] or [7] for the case of orthogonal STBCs. A similar result, obtained in [8], states that the optimal diversity-multiplexing tradeoff is unaffected by the assumption of correlated Rayleigh or Ricean fading.

Contributions. A natural question is thus whether these conclusions hold exclusively for optimal transmitters and receivers, and the purpose of this paper is to demonstrate that this is not the case. The main point we wish to make is that the diversity order of virtually any reasonable receiver does not strongly depend on the particular assumptions regarding the fading characteristics. This is true even when the detector considered is not able to achieve the full diversity offered by the channel and the coding. We give a mathematically precise statement to this effect in Proposition 1 (Section 2), where we show that the diversity order is constant over a class of channel distributions. The value of this result is that it straightforwardly extends several previous results on receiver diversity, derived under the i.i.d. Rayleigh assumption, to a large class of channel distributions. This is especially useful when the receiver of interest is not easily analyzed using some set of PEP events.

It should also be noted that although we make and prove our statements in a more general setting, we shall exemplify them by explicitly considering detectors for spatial multiplexing systems. The reason for this is not that spatial multiplexing is inherently more interesting, but rather that there exists a multitude of receivers previously proposed for this transmission strategy. In contrast, much of the research into general STBC schemes have focused on designs for which the ML detector exhibits a particularly simple structure (see e.g. [9]). Although our results apply to these cases as well, they are not as illustrative of the point we wish to make.

This work was supported by the STREP project No. IST-026905 (MASCOT) within the Sixth Framework Programme of the European Commission.

2. SYSTEM MODEL AND DEFINITIONS

System Model. We will explicitly consider the block fading MIMO channel model given by

$$\mathbf{Y} = \mathbf{H}\mathbf{C} + \mathbf{V}\,, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{n\times m}$ is the *channel matrix* containing the equivalent complex baseband gains (fading coefficients); $\mathbf{C} \in \mathcal{C} \subset \mathbb{C}^{m\times l}$ is the transmitted space-time block codeword drawn from a codebook $\mathcal{C}$; $\mathbf{Y} \in \mathbb{C}^{n\times l}$ is a matrix containing the received signals; and $\mathbf{V} \in \mathbb{C}^{n\times l}$ is additive Gaussian noise, assumed to be spatially and temporally white with variance σ^2 per complex component. The random channel matrix is assumed to be distributed according to some distribution μ, characterized by a probability density function (pdf) f_μ. The precise assumptions regarding μ will be made clear below. We will also assume that the transmitter has no channel state information, in which case the transmitted codeword $\mathbf{C}$ is statistically independent of $\mathbf{H}$.

Receivers. We will consider the generic problem of detecting the transmitted codeword $\mathbf{C}$ under the assumption that the receiver of interest is allowed to form a decision $\hat{\mathbf{C}} \in \mathcal{C}$ based on the receive matrix $\mathbf{Y}$, on the channel matrix $\mathbf{H}$, and possibly on a-priori information regarding the noise variance σ^2. Such a receiver can be fully characterized by a (measurable) mapping

$$\varphi :\ \mathbb{C}^{n\times l} \times \mathbb{C}^{n\times m} \times \mathbb{R}^+ \mapsto \mathcal{C}\,, \qquad \hat{\mathbf{C}} = \varphi(\mathbf{Y}, \mathbf{H}, \sigma). \tag{2}$$

We emphasize that this characterization does not exclude noncoherent or training-based detectors, since $\varphi(\mathbf{Y}, \mathbf{H}, \sigma)$ may be independent of $\mathbf{H}$. However, the detector is not allowed to change its decision policy depending on the particular distribution μ, at hand. The complete set of mappings of the form (2) is slightly to broad to provide a useful characterization of the receivers of interest. In particular, we need to restrict attention to the class of receivers whose performance can be characterized as a function of the average signal to noise ratio (SNR). This requires that the code-word decisions $\hat{\mathbf{C}}$ remain unchanged when the channel matrix $\mathbf{H}$ and the noise $\mathbf{V}$ undergo common scalings, and give the following definition.

Definition 1 *A receiver is said to be **standard** if its output is invariant with respect to a common scaling of the inputs:*

$$\varphi(\mathbf{Y}, \mathbf{H}, \sigma) = \varphi(\kappa\mathbf{Y}, \kappa\mathbf{H}, \kappa\sigma) \tag{3}$$

for all $\kappa > 0$.

We stress that this property is not at all restrictive, i.e., virtually any reasonable detector will be standard. In the following, we discuss some well-known illustrative examples. Let us first consider the ML detector given by

$$\varphi_{\mathrm{ML}}(\mathbf{Y}, \mathbf{H}, \sigma) \triangleq \arg\min_{\hat{\mathbf{C}} \in \mathcal{C}} \|\mathbf{Y} - \mathbf{H}\hat{\mathbf{C}}\|^2 . \tag{4}$$

Since $\arg\min_{\hat{\mathbf{C}} \in \mathcal{C}} \|\kappa\mathbf{Y} - \kappa\mathbf{H}\hat{\mathbf{C}}\|^2 = \arg\min_{\hat{\mathbf{C}} \in \mathcal{C}} \kappa^2 \|\mathbf{Y} - \mathbf{H}\hat{\mathbf{C}}\|^2$, it is obvious that the minimizer of (4) remains unchanged under a common scaling of $\mathbf{H}$ and $\mathbf{Y}$. Thus the ML detector is a standard receiver. It can also be seen that if the linear minimum mean square error filter, $\mathbf{G} \triangleq (\mathbf{H}^{\mathrm{H}}\mathbf{H} + \sigma^2\mathbf{I})^{-1}\mathbf{H}^{\mathrm{H}}$ is applied to the received signal, the output $\mathbf{Z} = \mathbf{G}\mathbf{Y}$ will be invariant to common scalings of $\mathbf{H}$, $\mathbf{Y}$, and σ. Thus, any receiver whose decision is based on $\mathbf{Z}$ is standard. The same is true for output of the zero forcing (ZF) filter, given by $\mathbf{H}^{\dagger}\mathbf{Y}$ where $\mathbf{H}^{\dagger}$ denotes the pseudo-inverse of $\mathbf{H}$, and for ZF and MMSE decision feedback detectors, all of which are standard receivers.

Performance Metrics. An *error* is declared whenever a receiver decides on a codeword different from the transmitted one, i.e., if $\hat{\mathbf{C}} \neq \mathbf{C}$. The associated probability of error is given by

$$p_e(\mu, \sigma) \triangleq \mathrm{P}(\hat{\mathbf{C}} \neq \mathbf{C}) . \tag{5}$$

We find it illustrative to make the dependence on the channel distribution and noise variance explicit. The corresponding *diversity order* is defined by

$$d(\mu) \triangleq \lim_{\sigma^2 \to 0} \frac{\ln p_e(\mu, \sigma)}{\ln \sigma^2} \tag{6}$$

where the dependence on the distribution is again made explicit. The probability of error and the diversity order of course also depend very much on the codebook $\mathcal{C}$ and the receiver φ that are used, although we do not make this explicit in our notation.

Channel Models. The class of channel distributions that we shall consider are characterized by the following definition.

Definition 2 *A channel distribution μ is said to be **simple** if it has a density function $f_\mu(\mathbf{H})$ which is*

1. *continuous and strictly positive at $\mathbf{H} = \mathbf{0}$;*
2. *upper bounded according to*

$$f_\mu(\mathbf{H}) \leq K_\mu \exp(-\|\mathbf{H}\|^{a_\mu}) . \tag{7}$$

for some K_μ and $a_\mu > 0$.

The class of all simple distributions is denoted $\mathcal{S}$. In order to make the generality of this class of distributions clear, we point out that it includes the important special cases of Rayleigh or Rician fading with arbitrary (non-singular) correlation, i.e.,

$$\mathrm{vec}(\mathbf{H}) \sim \mathcal{N}_{\mathbb{C}}(\mathbf{m}, \mathbf{R})$$

with some mean vector $\mathbf{m}$ and correlation matrix $\mathbf{R} \succ \mathbf{0}$. Note also that $\mathbf{R}$ does not necessarily have to be Kronecker-structured [5]. Definition 2 does exclude, however, Nakagami fading channels[1] and Rayleigh fading channels with singular correlation ($\mathrm{rank}(\mathbf{R}) < mn$). Still, restricting attention to the class of simple distributions allows us to give a generally valid statement regarding the diversity of any standard receiver.

3. MAIN RESULT AND EXAMPLES

The main contribution of this work is captured by the following proposition (the proof is provided in Appendix A). In essence, it states that the diversity achieved with almost any receiver is not sensitive to assumptions regarding the fading distribution. However, it should be noted that Proposition 1 does not actually state *how* the diversity should be computed, only that once this is done for some code-receiver pair and simple distribution, the result holds universally for all simple distributions.

Proposition 1 *The diversity order of any standard receiver is constant within the set of simple distributions,*

$$\mu, \nu \in \mathcal{S} \quad \Longrightarrow \quad d(\mu) = d(\nu) .$$

This result directly applies to ML detection and to MMSE and ZF detectors (as well as their decision-feed back variants). We next consider some more interesting examples in order to illustrate the type of conclusions that follows by the proposition.

3.1. LLL Lattice-Reduction-Aided ZF Detector

The use of lattice reduction (LR) in order to improve the performance of standard suboptimal detectors, such as the ZF detectors, was suggested in [10]. Consider the system

$$\mathbf{y} = \mathbf{H}\mathbf{c} + \mathbf{v} \tag{8}$$

where the transmitted signal vector $\mathbf{c} \in \mathbb{Z}_{\mathbb{C}}^m$ where $\mathbb{Z}_{\mathbb{C}} = \mathbb{Z} + i\mathbb{Z}$ is the set of complex (Gaussian) integers. Note that (8) represents a special case of (1) obtained with $l = 1$. The key idea of LLL LR-aided ZF detection is to view $\mathbf{H}$ as the basis of an m-dimensional lattice in $\mathbb{C}^n$ and to perform a basis transformation via the LLL algorithm [11] prior to ZF detection. The detector output equals ($\mathbf{H}^{\dagger}$ denotes the pseudo-inverse of $\mathbf{H}$)

$$\hat{\mathbf{c}} = \varphi_{\text{LLL-ZF}}(\mathbf{y}, \mathbf{H}, \sigma) = \mathbf{T}\theta[\mathbf{T}^{-1}\mathbf{H}^{\dagger}\mathbf{y}] \tag{9}$$

where $\theta[\cdot]$ denotes rounding to the closest point in $\mathbb{Z}_{\mathbb{C}}^m$; furthermore, the matrix $\mathbf{T}$ is obtained from $\mathbf{H}$ via the LLL algorithm and provides a one-to-one mapping from $\mathbb{Z}_{\mathbb{C}}^m$ to $\mathbb{Z}_{\mathbb{C}}^m$.

It was recently shown under the assumption of i.i.d. Rayleigh fading that LLL LR-aided ZF detection achieves the maximal receive diversity n [1]. Furthermore, the LLL LR-aided ZF detector $\varphi_{\text{LLL-ZF}}$ is a standard receiver since $\mathbf{H}^{\dagger}\mathbf{y}$ in (8) is invariant under common scaling of $\mathbf{y}$ and $\mathbf{H}$ and the matrix $\mathbf{T}$ chosen by the LLL algorithm is invariant with respect to scaling of $\mathbf{H}$ [11]. Hence, Proposition 1 implies that the diversity order of $\varphi_{\text{LLL-ZF}}$ is n for any simple distribution.

[1] We note, however, that there are MIMO systems for which the diversity achieved under Nakagami and Rayleigh fading is indeed different.

3.2. Partial Equalization

In [2], a detector based on an idea termed partial equalization (PE) was proposed. The detector is applicable to any system of the form (8). It obtains an estimate of $\mathbf{c} \in \mathcal{C}$ based on the metric

$$\hat{\mathbf{c}} = \varphi_{\mathrm{PE}}^{\alpha}(\mathbf{y}, \mathbf{H}, \sigma) = \arg\min_{\mathbf{c}\in\mathcal{C}} (\mathbf{z}_{\mathrm{ZF}} - \mathbf{c})^{\mathrm{H}} (\mathbf{H}^{\mathrm{H}}\mathbf{H})^{1-\alpha} (\mathbf{z}_{\mathrm{ZF}} - \mathbf{c}), \tag{10}$$

where $\mathbf{z}_{\mathrm{ZF}} = \mathbf{H}^{\dagger}\mathbf{y}$ and $\alpha \in [0, 1]$ is a parameter that allows to trade performance versus complexity. Note that standard ML and ZF detectors are special cases of (10) corresponding to $\alpha = 0$ and $\alpha = 1$, respectively. When α is increased from 0 to 1, $\varphi_{\mathrm{PE}}^{\alpha}$ achieves a diversity that changes continuously from n (ML diversity) to $n - m + 1$ (ZF diversity). The key observation is that by increasing α, also the conditioning of $(\mathbf{H}^{\mathrm{H}}\mathbf{H})^{1-\alpha}$ is improved which in turn makes (10) easier to solve.

Assuming i.i.d. Rayleigh fading, it was shown in [2] that for $\alpha = \frac{1}{2}$ a diversity of

$$d = \frac{1}{2}n + \frac{1}{2}(n - m + 1), \tag{11}$$

midway between ML and ZF diversity, is obtained. Since $\mathbf{H}^{\dagger}\mathbf{y}$ is invariant under common scaling of $\mathbf{H}$ and $\mathbf{y}$, and since the minimizer of (10) is not affected by a scaling of $(\mathbf{H}^{\mathrm{H}}\mathbf{H})$, it follows that the PE receiver $\varphi_{\mathrm{PE}}^{\alpha}$ is standard. Proposition 1 thus allows us to conclude that the PE receiver $\varphi_{\mathrm{PE}}^{1/2}$ achieves the diversity in (11) for any simple fading distribution.

3.3. Further Examples

The fixed-complexity sphere decoder proposed in [12] achieves full diversity under the assumption of i.i.d. Rayleigh fading [3]. Since this receiver can be shown to be standard, the full-diversity result remains valid for any simple distribution. Further, it was rigorously established in [4] that (optimal) ordering will not increase the diversity order of the ZF decision feedback detector (which is a standard receiver). Again, the proof was given under the assumption of i.i.d. Rayleigh fading but, as a consequence of Proposition 1, holds true for any simple fading distribution.

4. CONCLUSIONS

We have established that the receive diversity of most MIMO receivers is not strongly affected by particular assumptions regarding the channel's fading distribution. Although this fact was previously known for special cases of receivers and fading, we believe that it is not widely recognized in the literature on MIMO receivers that such a statement holds under surprisingly general terms. It should, however, also be stressed that we make no claim regarding the effect of the fading distribution on the coding (or rather decoding) gain of any particular receiver. This quantity could of course be sensitive to the particular distribution. Still, if one is mainly interested in diversity as a rough measure of a receiver's robustness against fading, our result implies that it is sufficient to analyze the case of i.i.d. Rayleigh fading.

A. PROOF OF PROPOSITION 1

We now prove that given any two simple distributions $\mu, \nu \in \mathcal{S}$, it follows that $d(\mu) = d(\nu)$ for any standard receiver. Interestingly enough, proving this general claim is easier than computing the actual value of $d(\mu)$ for specific receivers and fading distributions.

Let the *conditional* probability of error be defined according to

$$p_e(\mu, \sigma, \mathbf{C}) \triangleq \mathrm{P}(\hat{\mathbf{C}} \neq \mathbf{C} | \mathbf{C}).$$

The probability of error, defined by (5), is obtained according to

$$p_e(\mu, \sigma) = \sum_{\mathbf{C}\in\mathcal{C}} p_e(\mu, \sigma, \mathbf{C}) \mathrm{P}(\mathbf{C}).$$

Similarly, let the conditional diversity order be given by

$$d(\mu, \mathbf{C}) \triangleq \lim_{\sigma^2 \to 0} \frac{\ln p_e(\mu, \sigma, \mathbf{C})}{\ln \sigma^2}. \tag{12}$$

It is straightforward to prove that

$$d(\mu) = \min_{\mathbf{C}\in\mathcal{C}'} d(\mu, \mathbf{C}).$$

where the minimum is evaluated over

$$\mathcal{C}' \triangleq \{\mathbf{C} \in \mathcal{C} \mid \mathrm{P}(\mathbf{C}) > 0\}.$$

To establish Proposition 1 it is therefore sufficient to show that $d(\mu, \mathbf{C}) = d(\nu, \mathbf{C})$ for any $\mu, \nu \in \mathcal{S}$ and any $\mathbf{C} \in \mathcal{C}'$.

Let $\mathcal{E}_{\mathbf{C},\sigma}$ be the set of channel and noise realizations for which an error occurs, given that $\mathbf{C}$ was transmitted and the noise has power σ^2, i.e.,

$$\mathcal{E}_{\mathbf{C},\sigma} \triangleq \big\{ (\mathbf{H}, \mathbf{V}) \bigm| \mathbf{C} \neq \varphi(\mathbf{HC}+\mathbf{V}, \mathbf{H}, \sigma) \big\}. \tag{13}$$

The conditional probability of error can then be expressed as

$$\begin{aligned} p_e(\mu, \sigma, \mathbf{C}) &= \mathrm{P}((\mathbf{H}, \mathbf{V}) \in \mathcal{E}_{\mathbf{C},\sigma}) \\ &= \int_{\mathcal{E}_{\mathbf{C},\sigma}} f_\mu(\mathbf{H})\, f_\sigma(\mathbf{V})\, d\mathbf{H}\, d\mathbf{V} \end{aligned} \tag{14}$$

where the independence of $\mathbf{H}$ and $\mathbf{V}$ was used and where

$$f_\sigma(\mathbf{V}) = (\pi\sigma^2)^{-nl} \exp(-\|\mathbf{V}\|^2/\sigma^2)$$

denotes the probability density function of the noise.

By the assumption that μ is simple it follows that there are constants $\Gamma, \gamma > 0$ for which

$$\|\mathbf{H}\| \leq \gamma \quad \Rightarrow \quad f_\mu(\mathbf{H}) \geq \Gamma > 0.$$

In particular, $f_\mu(\mathbf{H})$ may be lower bounded according to

$$f_\mu(\mathbf{H}) \geq \Gamma\,[1 - \chi_\gamma(\mathbf{H})], \qquad \text{with } \chi_\gamma(\mathbf{H}) = \begin{cases} 1, & \|\mathbf{H}\| > \gamma \\ 0, & \|\mathbf{H}\| \leq \gamma. \end{cases}$$

It follows from (7) that $f_\nu(\sigma^{-\epsilon}\mathbf{H}) \leq K_\nu$ where $\epsilon > 0$ is an arbitrary constant and hence

$$f_\mu(\mathbf{H}) \geq \Gamma\,[1 - \chi_\gamma(\mathbf{H})]\, K_\nu^{-1}\, f_\nu(\sigma^{-\epsilon}\mathbf{H}). \tag{15}$$

By inserting (15) into (14) it follows that

$$\begin{aligned} p_e(\mu, \sigma, \mathbf{C}) &= \int_{\mathcal{E}_{\mathbf{C},\sigma}} f_\mu(\mathbf{H})\, f_\sigma(\mathbf{V})\, d\mathbf{H}\, d\mathbf{V} \\ &\geq \Gamma K_\nu^{-1} (I_1 - I_2) \end{aligned} \tag{16}$$

with

$$I_1 \triangleq \int_{\mathcal{E}_{\mathbf{C},\sigma}} f_\nu(\sigma^{-\epsilon}\mathbf{H})\, f_\sigma(\mathbf{V})\, d\mathbf{H}\, d\mathbf{V}, \tag{17a}$$

$$I_2 \triangleq \int_{\mathcal{E}_{\mathbf{C},\sigma}} \chi_\gamma(\mathbf{H})\, f_\nu(\sigma^{-\epsilon}\mathbf{H})\, f_\rho(\mathbf{V})\, d\mathbf{H}\, d\mathbf{V}. \tag{17b}$$

Consider first the integral I_1 in (17a). We perform the change of variables $\tilde{\mathbf{H}} = \sigma^{-\epsilon}\mathbf{H}$ and $\tilde{\mathbf{V}} = \sigma^{-\epsilon}\mathbf{V}$. Note also that $d\tilde{\mathbf{H}} = \sigma^{-2\epsilon nm}\, d\mathbf{H}$ and $d\tilde{\mathbf{V}} = \sigma^{-2\epsilon nl}\, d\mathbf{V}$. It further follows by (3) (with $\kappa = \sigma^{-\epsilon}$) that

$$\varphi(\mathbf{HC}+\mathbf{V},\mathbf{V},\sigma) = \varphi(\tilde{\mathbf{H}}\mathbf{C}+\tilde{\mathbf{V}},\tilde{\mathbf{V}},\sigma^{1-\epsilon})\,.$$

Together with (13) this implies

$$(\mathbf{H},\mathbf{V}) \in \mathcal{E}_{\mathbf{C},\sigma} \quad \Leftrightarrow \quad (\tilde{\mathbf{H}},\tilde{\mathbf{V}}) \in \mathcal{E}_{\mathbf{C},\sigma^{1-\epsilon}}\,.$$

Thus, I_1 is equivalently given by

$$I_1 \,=\, \sigma^{2\epsilon n(m+l)} \int_{\mathcal{E}_{\mathbf{C},\sigma^{1-\epsilon}}} f_\nu(\tilde{\mathbf{H}})\, f_\sigma(\sigma^\epsilon \tilde{\mathbf{V}})\, d\tilde{\mathbf{H}}\, d\tilde{\mathbf{V}}\,.$$

Since

$$\begin{aligned}\sigma^{2\epsilon nl}\, f_\sigma(\sigma^\epsilon\tilde{\mathbf{V}}) &= (\pi\sigma^{2(1-\epsilon)})^{-nl}\, \exp\big(-\|\tilde{\mathbf{V}}\|^2/\,\sigma^{2(1-\epsilon)}\big)\\ &= f_{\sigma^{1-\epsilon}}(\tilde{\mathbf{V}})\,,\end{aligned}$$

it follows that

$$\begin{aligned} I_1 \,&=\, \sigma^{2\epsilon nm} \int_{\mathcal{E}_{\mathbf{C},\sigma^{1-\epsilon}}} f_\nu(\tilde{\mathbf{H}})\, f_{\sigma^{1-\epsilon}}(\tilde{\mathbf{V}})\, d\tilde{\mathbf{H}}\, d\tilde{\mathbf{V}}\\ &=\, \sigma^{2\epsilon nm}\, p_e(\nu,\sigma^{1-\epsilon},\mathbf{C})\,, \end{aligned} \tag{18}$$

where the last equality is obtained by comparing to (14). Note also that due to (18) it follows that

$$\lim_{\sigma^2\to 0} \frac{\ln I_1}{\ln \sigma^2} = \epsilon nm + (1-\epsilon)\, d(\nu,\mathbf{C})\,. \tag{19}$$

where $d(\nu,\mathbf{C})$ is defined according to (12).

The integral I_2 in (17b) can be bounded according to

$$\begin{aligned} I_2 \,&\triangleq\, \int_{\mathcal{E}_{\mathbf{C},\sigma}} \chi_\gamma(\mathbf{H})\, f_\nu(\sigma^{-\epsilon}\mathbf{H})\, f_\sigma(\mathbf{V})\, d\mathbf{H}\, d\mathbf{V}\\ &\leq\, \int \chi_\gamma(\mathbf{H})\, f_\nu(\sigma^{-\epsilon}\mathbf{H})\, f_\sigma(\mathbf{V})\, d\mathbf{H}\, d\mathbf{V}\\ &=\, \int \chi_\gamma(\mathbf{H})\, f_\nu(\sigma^{-\epsilon}\mathbf{H})\, d\mathbf{H}\\ &=\, \sigma^{2\epsilon nm} \int \chi_\gamma(\sigma^\epsilon\tilde{\mathbf{H}})\, f_\nu(\tilde{\mathbf{H}})\, d\tilde{\mathbf{H}}\\ &\leq\, \sigma^{2\epsilon mn} \int_{\|\tilde{\mathbf{H}}\|\geq\gamma\sigma^{-\epsilon}} K_\nu\, \exp(-\|\tilde{\mathbf{H}}\|^{a_\nu})\, d\tilde{\mathbf{H}}\,, \end{aligned} \tag{20}$$

where the last inequality follows by (7). Since the value of the integral in (20) decays exponentially with decreasing σ it follows that

$$\lim_{\sigma^2\to 0} \frac{\ln I_2}{\ln\sigma^2} = \infty\,.$$

By comparing with (19), and assuming that $d(\nu,\mathbf{C}) < \infty$, it follows that I_2 tend to zero at a strictly faster rate than I_1. In particular, this implies that there exists σ' such that for $\sigma \leq \sigma'$ there is $I_2 \leq \frac{1}{2}I_1$, or equivalently

$$I_1 - I_2 \,\geq\, \tfrac{1}{2} I_1\,. \tag{21}$$

Inserting (21) into (16) and using (18) yields

$$p_e(\mu,\sigma,\mathbf{C}) \,\geq\, \tfrac{1}{2}\Gamma K_\nu^{-1}\sigma^{2\epsilon nm}\, p_e(\nu,\sigma^{1-\epsilon},\mathbf{C})\,,$$

for $\sigma \leq \sigma'$ and hence it follows that

$$d(\mu,\mathbf{C}) \,\leq\, \epsilon nm + (1-\epsilon)\, d(\nu,\mathbf{C})\,.$$

Since $\epsilon > 0$ was arbitrary, we conclude that

$$d(\mu,\mathbf{C}) \,\leq\, d(\nu,\mathbf{C})\,. \tag{22}$$

Note also that this conclusion is trivially satisfied if $d(\nu,\mathbf{C}) = \infty$.

Since no particular constraints were imposed on μ and ν other than that they be simple distributions, the argument above can be repeated with the roles of μ and ν interchanged, resulting in the inequality

$$d(\mu,\mathbf{C}) \,\geq\, d(\nu,\mathbf{C})\,.$$

Combined with (22) this finally implies

$$d(\mu,\mathbf{C}) = d(\nu,\mathbf{C})\,,$$

and thus concludes the proof. □

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