Robust long-distance quantum communication with atomic ensembles and linear optics

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Quantum communication deals with absolutely secure transfer of classical messages by means of quantum cryptography[1] or faithful transfer of unknown quantum states by means of quantum teleportation[2] between distant sites. The essential element for quantum communication is to create remote entangled pairs of high fidelity. The main difficulty to create such a distant entangled pair is that both photon loss and decoherence grow exponentially with distance. However, with the help of quantum repeater, it is demonstrated that both the entanglement sources and the time needed to create a single distant entangled pair of high fidelity could scale polynomially with the communication length[3,4]. Here we propose a robust quantum repeater by extending the original Duan-Lukin-Cirac-Zoller (DLCZ) scheme based on atomic ensembles and linear optics[5]. Our protocol entails the advantage of two photon interference[6,7,8], which is more robust than single photon interference used in the DLCZ scheme. In comparison with our previous proposal[8], the entangled memory qubits are manipulated across long distance, and local entanglement pairs are avoided. Our proposal provides an exciting possibility for realistic long distance quantum communication.

Quantum repeater is a crucial element for long distance quantum communication. In a seminal paper Duan *et al.* proposed a promising implementation of quantum repeater with the help of atomic ensembles and linear optics. It is shown that the atomic ensembles,

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used as local memory qubits, have collectively enhanced coupling to light even without the aid of high finess cavities[5,9]. To realize quantum repeater based on atomic ensembles and linear optics, significant progress has been achieved, including nonclassical correlated photon pairs in cold atoms[10] and in hot atomic vapor cells[11], controllable single-photon source from atomic ensembles[12,13,14], and entanglement of two remote cold atomic ensembles[15].

Unfortunately, the single photon interference utilized in the DLCZ scheme demands long distance phase stabilization within the photon wavelength, which is almost an impossible task for current technology. This drawback has been overcome by our recent proposal[8], where we proposed a fault-tolerant quantum repeater based on atomic ensembles and single photon sources. Local polarization-entangled pairs are prepared in advance and then connected to long communication length by entanglement swapping. The two photon interference[6,7] exploited in entanglement connection is 10^8 times more stable than single photon interference[8]. It is a robust and faithful implementation of quantum repeater and will pave the way to efficient long distance quantum communication.

In this paper, we propose an alternative quantum repeater based on atomic ensembles and linear optics. The new scheme also exploits the advantage of two photon interference, and thus is a robust implementation of quantum repeater. Different from our previous proposal, the preparation of local entangled pairs are not necessary. All the entanglement manipulation procedures, such as entanglement generation, connection and purification, are implemented across long distance.

The basic element for an atomic ensemble based quantum repeater is a pencil shaped atomic sample (Frensnel number $F = 1$) of N atoms with Λ level structure (see inset in Fig. 1). Initially, all the atoms are optically pumped to the ground state $|g\rangle$, after which the sample is illuminated by an off-resonant short laser pulse. A forward-scattered Stokes photon is emitted via spontaneous Raman transition and at the same time a collective atomic excitation is generated in the atomic ensemble. The two mode squeezed state between forward-scattered Stokes mode and collective atomic mode reads[5]

$$\psi = |0_a 0_s\rangle + \sqrt{\chi} S^\dagger a^\dagger |0_a 0_s\rangle + \chi \frac{(S^\dagger a^\dagger)^2}{2} |0_a 0_s\rangle, \qquad (1)$$

by neglecting higher order terms, where $|0_a\rangle = \bigotimes_i |g\rangle_i$ is the ground state of the atomic ensemble and $|0_s\rangle$ denotes vacuum state of Stokes photon. Here $a^\dagger$ is the creation operator of the Stokes mode and the collective atomic excitation operator is defined by

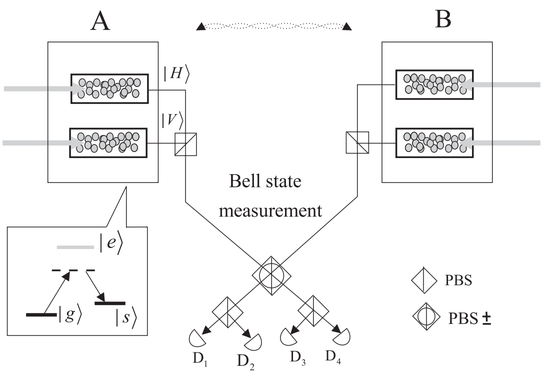

FIG. 1: **Setup for entanglement generation between sites A and B**. Forward-scattered Stokes photons, generated by an off-resonant write laser pulse with Rabi frequency Ω, detuning $\triangle$, via spontaneous Raman transition, are subject to a Bell-state measurement at the middle point. The Stokes photons generated at the same site are assumed to have different polarization i.e., $|H\rangle$ and $|V\rangle$. PBS (PBS$_\pm$) reflects photons with vertical polarization $|V\rangle$ ($|-\rangle$-polarization) and transmits photons with horizontal-polarization $|H\rangle$ ($|+\rangle$-polarization), where $|\pm\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)$. After passing through the PBS$_\pm$ and PBS successively, the Stokes photons are detected by single photon detectors. A coincidence count between single photon detectors D$_1$ and D$_4$ (D$_1$ and D$_3$) or D$_2$ and D$_3$ (D$_2$ and D$_4$) will project the four atomic ensembles into the complex entangled state $|\psi\rangle_{AB}$ up to a local unitary transformation. The inset shows the atomic level structure, with the ground state $|g\rangle$, metastable state $|s\rangle$, and excited state $|e\rangle$ of decay rate γ.

$S^\dagger = \frac{1}{\sqrt{N}} \sum_i |s\rangle_i \langle g|$, with $|s\rangle$ the metastable atomic state. The excitation probability fulfills $\chi = d_0 \gamma_s T \ll 1$, where $d_0 \gg 1$ is the optical depth and T is the pulse length of write laser. Here $\gamma_s = \gamma \frac{\Omega^2}{\triangle^2}$ is the effective decay rate, with γ the decay rate of the excited state $|e\rangle$, and Ω and $\triangle$ denotes Rabi frequency and detuning from the excited state of the write laser, respectively[16].

The entanglement generation setup is shown in Fig. 1. Considering two distant sites A and B, we assume each site has two atomic ensembles as memory qubits. The distance between the two sites satisfies $L_0 \leq L_{att}$, where L_{att} is the channel attenuation length. The two atomic ensembles at each node are excited simultaneously by a write laser pulse. It is safe to assume the Stokes photons generated from the two atomic ensembles at the same site have orthogonal polarization state, e.g., $|H\rangle$ and $|V\rangle$. The whole light-atom system can be described by a product state $|\psi\rangle_t = |\psi\rangle_{u_A}|\psi\rangle_{d_B}|\psi\rangle_{u_A}|\psi\rangle_{d_B}$, where $|\psi\rangle_{u_A}$, $|\psi\rangle_{d_B}$, $|\psi\rangle_{u_A}$ and $|\psi\rangle_{d_B}$ are light-atom state generated via spontaneous Raman transition given by Eq. (1) and are distinguished by subscript u, d (up and down) and A, B. The Stokes photons generated from the two sites are directed and subject to a Bell state measurement at the middle point. The coincidence count between two of the four detectors (Bell-state measurement) will project the two memory qubits at the two sites into a complex entangled state. For instance, when there is a coincidence count between D$_1$ and D$_4$, the memory qubits are projected into a superposition state described by

$$|\psi\rangle_{AB} = \frac{S^\dagger_{u_A}S^\dagger_{u_B} + S^\dagger_{d_A}S^\dagger_{d_B}}{2} + \frac{S^{\dagger 2}_{u_A} + S^{\dagger 2}_{u_B} - S^{\dagger 2}_{d_A} - S^{\dagger 2}_{d_B}}{4}|vac\rangle_{AB}, \tag{2}$$

by neglecting high-order terms. The first term is a maximally entangled state, while the second term is a two excitation state in which one of the four atomic ensembles has two excitations. The experiment is repeated until one coincidence count is registered and the success probability is of the order $O(\chi^2 e^{-L_0/L_{att}})$ by considering the channel attenuation.

It is obvious that this state is not a maximally entangled state across the two sites due to the unwanted two excitation term. Fortunately, the unwanted two excitation term will be purified automatically when further entanglement swapping are applied to extend the communication length, as we will show below.

Entanglement swapping setup is depicted in Fig. 2. Let us consider four distant communication sites (A,B) and (C,D) and assume that we have created complex entangled states $|\psi\rangle_{AB}$ and $|\psi\rangle_{CD}$ between sites (A,B) and (C,D), respectively. The two memory qubits at site B and C are illuminated simultaneously by read laser pulse. We also assume that the retrieved anti-Stokes photons at the same site have different polarization, i.e., $|H\rangle$ and $|V\rangle$. The retrieved anti-Stokes photons are subject to a Bell state measurement at the middle point between B and C (see Fig. 2). In ideal case, if the retrieve efficiency is 100% and high-efficiency single photon detectors based on atomic ensembles are used to distinguish between one and two photons[17,18], the coincidence count between two of the detectors, i.e., D$_1$ and D$_4$, will prepare the two memory qubits at site A and D into a maximally entangled state $|\phi^+\rangle_{AD} = (S^\dagger_{u_A}S^\dagger_{u_D} + S^\dagger_{d_A}S^\dagger_{d_D})/\sqrt{2}|vac\rangle_{AD}$. It is interesting to find that the unwanted terms with two excitation in any atomic ensemble are automatically washed out after entanglement swapping. Therefore a maximally entangled pair between the memory qubits across sites (A,D) is generated by entanglement connection and the communication length

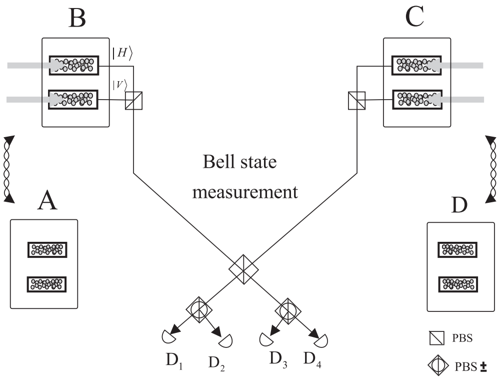

FIG. 2: **Setup for entanglement connection between sites A and D via entanglement swapping**. Complex entangled state have been prepared in the memory qubits between sites (A,B) and (C,D). The memory qubits at site B and C are illuminated by near resonant read laser pulse, and the retrieved anti-Stokes photons are subject to another Bell-state measurement at the middle point. The anti-Stokes photons at the same site are assumed to have different polarizations $|H\rangle$ and $|V\rangle$. After passing through PBS and PBS$_\pm$ successively, the anti-Stokes photons are detected by single photon detectors. Coincidence count between D$_1$ and D$_4$ (D$_1$ and D$_3$) or D$_2$ and D$_3$ (D$_2$ and D$_4$) are registered. The memory qubits will be projected into an effective maximally entangled state ρ_{AD} up to a local unitary transformation.

has been extended to $3L_0$.

However, for realistic atomic ensembles the retrieve efficiency is determined by the limited optical depth of the atomic ensemble[19], and current single photon detectors is incapable of distinguishing between one and two photons. Assume the retrieve efficiency and transmission efficiency η, the detector efficiency η_1 for one photon and η_2 for two photons, through some calculations one can find that the Bell-state measurement will project the memory qubits into an effectively maximally entangled state

$$\rho_{AD} = p_2\rho_2 + p_1\rho_1 + p_0\rho_0, \tag{3}$$

with

$$\begin{aligned} p_2 &= \frac{\eta^2\eta_1^2}{16p_c}, \\ p_1 &= [\frac{\eta^2(1-\eta)\eta_1^2}{8} + \frac{\eta^3}{16}(\frac{\eta_1\eta_2}{2} + \eta_1^2)]/p_c, \\ p_0 &= [\frac{\eta^3}{16}(1-\eta)(\frac{1}{2}\eta_1\eta_2 + \eta_1^2) + \frac{\eta^2(1-\eta)\eta_1^2}{16} + \frac{\eta^2}{8}(\frac{1}{4}\eta_2^2 + \eta_1^2)]/p_c, \end{aligned} \tag{4}$$

where $p_c = \frac{\eta^2\eta_1^2}{16} + \frac{\eta^2(1-\eta)\eta_1^2}{8} + \frac{\eta^3}{16}(\frac{\eta_1\eta_2}{2} + \eta_1^2) + \frac{\eta^3}{16}(1-\eta)(\frac{1}{2}\eta_1\eta_2 + \eta_1^2) + \frac{\eta^2(1-\eta)\eta_1^2}{16} + \frac{\eta^2}{8}(\frac{1}{4}\eta_2^2 + \eta_1^2)$ is the success probability. Here $\rho_2 = |\phi^+\rangle_{AD}\langle\phi^+|$ is a maximally entangled state, $\rho_1 = \frac{|u_A\rangle\langle u_A| + |d_A\rangle\langle d_A| + |u_D\rangle\langle u_D| + |d_D\rangle\langle d_D|}{4}$ is a maximally mixed state in which only one of the four atomic ensembles has one excitation and ρ_0 is the vacuum state that none of the atomic ensembles are excited.

It is easy to see that the effectively maximally entangled states ρ_{AD} between sites A and D can be purified automatically to a maximally entangled state in the entanglement based quantum cryptography schemes. When we implement quantum cryptography via Ekert91 protocol[20] between sites A and D using the effective maximally entangled state, only the first term ρ_2 will contribute to a coincidence count between the two detectors and be registered after classical communication. The maximally mixed state term ρ_1 and vacuum term ρ_0 have no contribution to experimental results, and ρ_{AD} is equivalent to the Bell state $|\phi^+\rangle_{AD} = (S^\dagger_{u_A}S^\dagger_{u_D} + S^\dagger_{d_A}S^\dagger_{d_D})/\sqrt{2}|vac\rangle_{AD}$.

Note that both the entanglement generation and entanglement swapping procedures take advantage of two photon interference and are implemented across long distance L_0. There is no need to create local entangled pairs and the communication length has been extended to $3L_0$ after the entanglement swapping. The effectively entangled pair can be extended to longer distance by further entanglement swapping introduced in our previous proposal[8], where the probability p_2 that the left memory qubits have an maximally entangled pair will not decrease with communication distance during entanglement connection process.

The effectively maximally entangled memory qubits across distant sites extended via entanglement connection may be imperfect due to decoherence. The entangled term ρ_2 is no longer a maximally entangled state, but a mixed entangled state of fidelity F. For simplicity, we assume $\rho_2 = F|\phi^+\rangle_{AD}\langle\phi^+| + (1-F)|\psi^+\rangle_{AD}\langle\psi^+|$, where $|\psi^+\rangle_{AD} = (S^\dagger_{u_A}S^\dagger_{d_D} + S^\dagger_{d_A}S^\dagger_{u_D})/\sqrt{2}|vac\rangle_{AD}$ is another maximally entangled Bell state. The mixed entangled state ρ_2 can be purified by linear-optics entanglement purification protocol[21,22], after which one will get an effectively entangled state of fidelity $F' = \frac{F^2}{F^2+(1-F)^2}$. To get a remote entangled pair, nesting entanglement purification is needed[3,4]. The time and resources consumed to implement the nesting entanglement purification for the effective maximally entangled state scales polynomially with distance as we discussed thoroughly in our previous work[8].

In summary, we proposed an alternative robust quantum repeater by extending the DLCZ protocol. The new scheme overcomes the main drawback of the DLCZ scheme by exploiting the advantage of two photon interference. Compared with our previous proposal, the creation of local entangled pair is avoided, and entanglement generation, connection and purification are all implemented over long distance. Our proposal opens up an exciting possibility of implementing quantum repeater and will shed new light on realistic long distance quantum communication.

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[1] Gisin, N., Ribordy, G., Tittel, W. & Zbinden, H. Quantum cryptography. *Rev. Mod. Phys.* **74**, 145-195 (2002).

[2] Bennett, C.H. *et al.* Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Phys. Rev. Lett.* **73**, 3081-3084 (1993).

[3] Briegel, H.-J., Dür, W., Cirac, J.I. & Zoller, P. Quantum repeaters: The role of imperfect local operations in quantum communication. *Phys. Rev. Lett.* **81**, 5932-5935 (1998).

[4] Dür, W., Briegel, H.-J., Cirac, J.I. & Zoller, P. Quantum repeaters based on entanglement purification. Phys. Rev. A **59**, 169-181 (1999).

[5] Duan, L.-M., Lukin, M., Cirac, J.I., & Zoller, P. Long-distance quantum communication with atomic ensembles and linear optics. *Nature* **414**, 413-418 (2001).

[6] Feng, X.-L. Zhang, Z.-M., Li, X.-D., Gong, S.-Q. & Xu, Z.-Z. Entangling distant atoms by interference of polarized photons. *Phys. Rev. Lett.* **90**, 217902 (2003).

[7] Simon, C. & Irvine, W.T.M. Robust long-distance entanglement and a loophole-free Bell test with ions and photons. *Phys. Rev. Lett.* **91**, 110405 (2003).

[8] Chen, Z.-B., Zhao, B., Schmiedmayer, J., & Pan, J.-W. Fault-tolerant quantum repeater with

atomic ensembles and linear optics. Preprint available at http://xxx.lanl.gov/quant-ph/0609151 (2006).

[9] Kozhekin, A.E., Mølmer, K., & Polzik, E. Quantum memory for light. Phys. Rev. A **62**, 033809 (2000).

[10] Kuzmich, A. *et al.* Generation of nonclassical photon pairs for scalable quantum communication with atomic ensembles. *Nature* **423**, 731-734 (2003).

[11] van del Wal, C.H. *et al.* Atomic memory for correlated photon states. *Science* **301**, 196-200 (2003).

[12] Eisaman, M.D. *et al.* Electromagnetically induced transparency with tunable single-photon pulses. *Nature* **438**, 837-841 (2005).

[13] Matsukevich, D.N. *et al.* Deterministic single photons via conditional quantum evolution. *Phys. Rev. Lett.* **97**, 013601 (2006).

[14] Chen, S. *et al.* A deterministic and storable single-photon source based on quantum memory. Preprint available at http://xxx.lanl.gov/quant-ph/0607036 (2006).

[15] Chou, C.W. *et al.* Measurement-induced entanglement for excitation stored in remote atomic ensembles. *Nature* **438**, 828-832 (2005).

[16] André, A. *Nonclassical States of Light and Atomic Ensembles: Generation and New Applications*, PhD thesis, Harvard University (2005).

[17] James, D.F.V. & Kwiat, P.G. Atomic-vapor-based high efficiency optical detectors with photon number resolution. *Phys. Rev. Lett.* **89**, 183601 (2002).

[18] Imamoḡlu, A. High efficiency photon counting using stored light. *Phys. Rev. Lett.* **89**, 163602 (2002).

[19] Gorshkov, A.V., André, A., Fleischhauer, M., Sørensen, A.S., & Lukin, M.D. Optimal storage of photon States in optically dense atomic media. Preprint available at http://xxx.lanl.gov/quant-ph/0604037 (2006).

[20] Ekert, A.K., Quantum cryptography based on Bell's theorem. *Phys. Rev. Lett.* **67**, 661-663 (1991).

[21] Pan, J.-W., Simon, C., Brukner, C. & Zeilinger, A. Feasible entanglement purification for quantum communication. *Nature* **410**, 1067-1070 (2001).

[22] Pan, J.-W., Gasparoni, S., Ursin, R., Weihs G. & Zeilinger, A. Experimental entanglement purification of arbitrary unknown states. *Nature* **423**, 417-422 (2003).