



A non-conventional neutron polariser concept

G. Badurek^{a,*}, C. Hartl^b, E. Jericha^a

^aAtominstitut, Vienna University of Technology, Stadionallee 2, A-1020 Vienna, Austria

^bInstitute of High Energy Physics of the Austrian Academy of Sciences, Nikolsdorfergasse 18, A-1050 Vienna, Austria

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Abstract

Even in an idealised case conventional methods of neutron polarisation inevitably loose at least 50% of the intensity by removing one spin state from the incident beam. We demonstrate, however, that it should be possible to polarise a pulsed beam of slow neutrons without losing any particle. This so-called “Dynamical Neutron Polarisation” (DNP) method is based upon acceleration and deceleration, respectively, of the two neutron spin states during passage through an NMR-like arrangement of crossed static and radio-frequency magnetic fields, followed by spin-dependent spin rotation in a homogeneous precession field. Precise ramping of the strength of this precession field synchronously with a time modulated $\pi/2$ -spin turn device at the exit position allows to stop precession for all neutron wavelengths exactly at the moment where both spin states are aligned parallel, albeit on the cost of a tiny energy difference between them. We present a theoretical description of DNP which is essentially based upon a semi-classical spin-rotation formalism and present a typical result of a thorough investigation of the performance of such a non-conventional polariser with respect to a manifold of instrumental parameters.

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1. Introduction

Currently all standard polarisation techniques used in neutron instrumentation (e.g. polarised ^3He absorption spin filters, magnetic crystal Bragg reflection, totally reflecting supermirrors, birefringent magnetic prisms or lenses) are based upon some filtering procedure, i.e. the elimination of one spin component from the incident unpolarised particle ensemble. Dynamic Neutron Polarisation (DNP), however, unlike all other methods uses nothing else than magnetic fields for the polarisation process and should allow in principle to keep the full intensity, virtually without losing any particle [1–4].

In the case of a Continuous source emitting strictly Monoenergetic particles the concept of Dynamical Neutron Polarisation (CMDNP) is quite simple: The beam

traverses an NMR-like arrangement of a strong static and a perpendicular resonant radio-frequency field which causes an energetic splitting between the two neutron spin eigenstates. Leaving this splitting stage with slightly different velocities v_{\pm} , these two states hence accumulate different LARMOR precession angles during their subsequent passage through a static magnetic field. Precession is initiated at the field entrance by a $\pi/2$ -spin turn device which flips the spins into a plane perpendicular to the field. After travelling some distance these two states then inevitably become aligned parallel for a short moment. If the precession exactly at this moment is stopped by a second $\pi/2$ -spin turn device, which flips both spin states back into the field direction, the beam appears then to be fully polarised.

The importance of DNP lies, however, in the much more complex case of Polychromatic incident neutrons emerging from a Pulsed source (PP-DNP), where the polarisation condition needs to be fulfilled for a bandwidth of wavelengths. There one can exploit the fact that in an

*Corresponding author. Tel.: +43 1 58801 14129;

fax: +43 1 58801 14199.

E-mail address: badurek@ati.ac.at (G. Badurek).

idealised case of infinitely short source pulses all particles are emitted simultaneously at a time $t = 0$. If a neutron is allowed to travel a distance L before entering the splitting stage its wavelength can be uniquely identified at any position x_0 along the polariser axis by its arrival time t_0 via $\lambda = ht_0/(mx_0)$, where h is Planck's constant and m is the neutron mass. This means that all wavelengths can be polarised, provided the magnetic fields involved are ramped precisely as a function of time according to specific boundary conditions. Evidently, the longer the incident flight path L and hence the larger the spatial dispersion of the different wavelengths will be, the less degradation of the polariser's performance with increasing source pulse length will occur.

At first an overview will be given of how to determine possible time dependencies of the splitting and the precession field and of the final $\pi/2$ -spin turn device of a PPDNP facility. Finally, we present a typical result of extensive numerical simulations of such a kind of polariser assuming realistic experimental conditions. In both cases we use a semi-classical spin-rotation formalism, which allows to take into account technical details of the setup. A quantum mechanical wave packet description of DNP, verifying its physical soundness but leaving technical details out of consideration, is given separately [5].

2. DNP-setup theory

Though we will focus our attention to the case of pulsed polychromatic neutrons, we consider the monochromatic configuration also, since both setups have much in common. The horizontal polariser axis can be separated

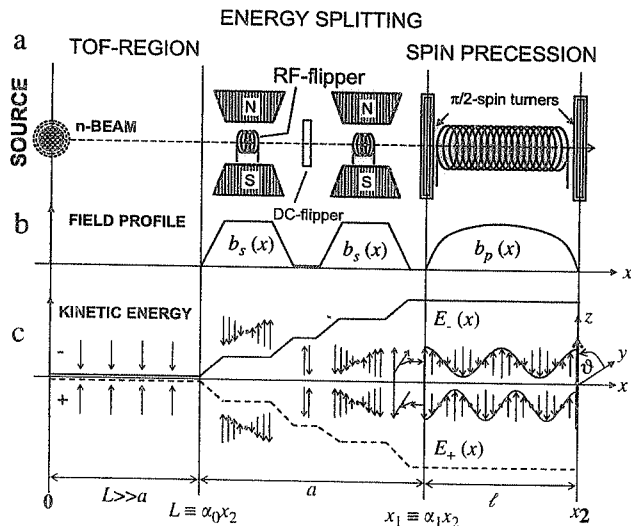


Fig. 1. Functional scheme of a DNP-facility. (a) xz -view of the apparatus. (b) Spatial profiles of the energy splitting field ($\parallel \hat{z}$) and the spin precession field ($\parallel \hat{x}$). (c) Position dependent neutron energy and neutron spin for the two spin states (\pm : initially up or down). The spin motion on passage through the twin-stage energy splitting system is shown in \hat{z} -projection, whereas the spin motion in the precession region is projected onto the xz -plane. $\vartheta = \pi/2$ denotes the resulting spin exit angle.

into three regions (Fig. 1). Behind the source (position $x = 0$) neutrons drift along a distance L , pass the energy splitting system (length a), and finally cover a distance ℓ , the region of spin precession and alignment.

We assume a radially homogeneous vertical energy splitting field in \hat{z} -direction, time and space dependencies shall be separable: $\vec{B}_s(t, x) = B_s(t)b_s(x)\vec{e}_z$. The spatial field profile $b_s(x)$ is determined by the magnet geometry and shall be zero outside $L < x < x_1 = L + a$. Only for a polychromatic pulsed beam the amplitude $B_s(t)$ has to be time-dependent and must be generated through accordingly driven coil currents. The oscillating field of an RF-flipper located at the centre of the static splitting magnet inverts the neutron spin states with the consequence that the splitting of the kinetic energy which occurs at the positive field slope is not cancelled again at the subsequent negative exit gradient, but is doubled instead. Again, for polychromatic particles the RF-flipper must operate equally well for each wavelength. This can be achieved either by proper variation of the RF-field amplitude with time or by using adiabatic gradient-type [6] resonance flippers. It is important to notice that the energetic splitting can be increased by using a sequence of magnet stages. However, in this case additional wavelength-independent flip devices (e.g. DC current sheets [7]) have to be placed in the regions of (very) low field between successive magnet stages in order to recover the appropriate initial spin orientation. To simplify our considerations we assume that both types of flippers for all wavelengths are operating with an efficiency of 100%.

The spin precession magnetic field with time-dependent amplitude (in the case of PPDNP) $\vec{B}_p(t, x) = B_p(t)b_p(x)\vec{e}_x$ is produced by a solenoid that is aligned along the beam. The profile $b_p(x)$ shall be zero outside the interval $x_1 < x < x_2 = x_1 + \ell$, so that we need not care about otherwise necessary adiabatic changes of the field direction from vertical to longitudinal orientation. Therefore the first $\pi/2$ -turner is virtually irrelevant, though in reality it is required to realise such an "instantaneous" reorientation of the spin relative to the field direction. The second $\pi/2$ -turner located at the end of the precession region (position $x = x_2$) needs somewhat more consideration. Its task is to stop LARMOR precession when the two initially anti-parallel spin states have accumulated a precession angle difference in the yz -plane of $(2n + 1)\pi$ ($n = 0, \pm 1, \dots$). This means that their "spin exit angle", characterised by the polar angle $\vartheta_{\pm}(t)$ in the yz -plane with respect to the \hat{y} -axis, is identical.

However, only in the monochromatic case alignment takes place along a fixed direction whereas for a PPDNP setup its orientation is different for each wavelength. Therefore a careful synchronous variation of both the strength and the orientation of the field inside the second spin turn device is required.

2.1. CMDNP: continuous monochromatic beam

Neglecting the tiny influence of the low fields between the strong splitting magnets the total spin-dependent

energy splitting for a system consisting of N stages is $E_{\pm} = E_0 \pm 2\mu NB_s$ ($\mu = \gamma\hbar/2 = -1.913\mu_N$ neutron magnetic moment, γ gyromagnetic ratio, μ_N nuclear magneton) [8]. The corresponding velocity splitting of the two spin states is then given as $v_{\pm} = v_0(1 \pm \varepsilon/2)$, with the splitting parameter $\varepsilon = 4\mu NB_s/mv_0^2 \ll 1$ (v_0 initial neutron speed). Spin up neutrons enter the precession region with *down* orientation, i.e. an angle $-\pi/2$ in the yz -plane, initial spin *down* neutrons with an angle $+\pi/2$. Using the local LARMOR frequency $\omega_p(x) = |\gamma|B_p b_p(x)$ the corresponding spin exit angles are then found as

$$\vartheta_{\pm} = \mp \frac{\pi}{2} + \int_{x_1}^{x_2} \frac{dx}{v_{\pm}} |\gamma| B_p b_p(x). \quad (1)$$

From the spin alignment condition $\vartheta_+ - \vartheta_- = 0 \pmod{2\pi}$ one obtains the following relationship between splitting and precession field:

$$NB_s \cdot B_p(b_p)\ell \cdot \frac{1}{v_0^3} = \mathcal{A} = \text{const.} \quad (2)$$

with $\mathcal{A} \equiv \pi m/(4\gamma\mu)$ and $(b_p) = (1/\ell) \int_{x_1}^{x_2} dx' b_p(x')$. For $\lambda = 5 \text{ \AA}$, $NB_s \cdot B_p(b_p)\ell \approx 0.37 \text{ T}^2 \text{ m}$ is required. Notice, however, that in spite of the reciprocal relation between B_s and B_p it is advisable to choose a large splitting field rather than a high precession field, because this allows to use a larger incident wavelength spread $\Delta\lambda/\lambda_0 \propto NB_s\lambda_0^2$. With $\lambda_0 = 0.5 \text{ nm}$ and $NB_s = 20 \text{ T}$ one must fulfill the condition $\Delta\lambda/\lambda_0 \leq 10^{-3}$.

2.2. PPDNP: pulsed polychromatic beam

In PPDNP the splitting field $B_s(t)b_s(x)\vec{e}_z$ for a neutron with initial velocity \tilde{v}_0 causes an accumulation of energy changes $dE = \pm\mu B_s(t(x))|b'_s(x)| dx$, with $t(x) \approx x/\tilde{v}_0$ in case of small splitting. The absolute value $|b'_s(x)|$ is taken because every time when the field gradient changes sign the spin has been inverted previously by a π -flipper. The (small)total relative energy shift as a function of position is then

$$\varepsilon(\tilde{v}_0, x) = \frac{2\mu}{m\tilde{v}_0^2} \int_L^x dx' B_s\left(\frac{x'}{\tilde{v}_0}\right) |b'_s(x')| \quad (3)$$

resulting in a final splitting $\tilde{v}_{\pm} = \tilde{v}_0[1 \pm \varepsilon(\tilde{v}_0)/2]$ with $\varepsilon(\tilde{v}_0) \equiv \varepsilon(\tilde{v}_0, x_1)$.

But instead of tracing the two spin states for a given initial velocity \tilde{v}_0 , here it is necessary to consider two neutrons emitted at time $t=0$ with *different* velocities $v_{0+} \approx v_{0-} \approx x_2/t \equiv v_0$. They reach the polariser end position x_2 at the *same* time $t = L/v_{0\pm} + \int_L^{x_1} dx/v_{\pm}(x) + \ell/v_{\pm}$. What are these initial velocities? In terms of the splitting parameter $\varepsilon(v_{0\pm}, x)$ the arrival time in first order approximation is

$$t = \frac{L}{v_{0\pm}} + \int_L^{x_1} \frac{dx}{v_{0\pm}} \left(1 \mp \frac{\varepsilon(v_{0\pm}, x)}{2}\right) + \frac{\ell}{v_{0\pm}} \left(1 \mp \frac{\varepsilon(v_{0\pm})}{2}\right). \quad (4)$$

Within in the splitting function ε the velocity difference is negligible and $v_{0\pm}$ can thus be replaced by the zero order

term x_2/t . Then Eq. (4) can be resolved easily for $v_{0\pm}$, yielding

$$v_{0\pm} = \frac{x_2}{t} \left(1 \mp \frac{\Delta(t)}{2}\right) \quad (5)$$

with $\Delta(t) = (\ell/x_2)\varepsilon(x_2/t) + \int_L^{x_1} (dx/x_2)\varepsilon(x_2/t, x)$. The initial splitting of simultaneously arriving particles is thus described by $\Delta(t)$, which is even smaller in magnitude than $\varepsilon(x_2/t)$. After their passage through the energy splitting system their final velocities are simply $v_{\pm}(t) = x_2/t(1 \mp \Delta(t)/2)(1 \pm \varepsilon(x_2/t)/2)$. Ignoring second order terms, one obtains

$$v_{\pm}(t) = \frac{x_2}{t} \left(1 \pm \frac{\xi(t)}{2}\right) \quad (6)$$

with $\xi(t) \equiv \varepsilon(x_2/t) - \Delta(t) = (x_1/x_2)\varepsilon(x_2/t) - \int_L^{x_1} (dx/x_2)\varepsilon(x_2/t, x)$. The quantity $\xi(t)$ describes the final velocity splitting and can be expressed in terms of an ‘‘effective splitting field’’ $B_s^{\text{eff}}(t)$,

$$\xi(t) \equiv \frac{4\mu B_s^{\text{eff}}(t)}{m(x_2/t)^2}. \quad (7)$$

By definition, this effective field is equal to

$$B_s^{\text{eff}}(t) = \frac{1}{2} \left[\alpha_1 \hat{B}_s(t, \alpha_1) - \int_{\alpha_0}^{\alpha_1} d\alpha \hat{B}_s(t, \alpha) \right] \quad (8)$$

with the abbreviation $(\alpha_0 \equiv L/x_2, \alpha_1 \equiv x_1/x_2)$

$$\hat{B}_s(t, \alpha) = \int_{\alpha_0}^{\alpha} d\alpha' B_s(\alpha' t) \left| \frac{\partial b_s(\alpha' x_2)}{\partial \alpha'} \right|. \quad (9)$$

What did we achieve so far? We found that the final velocity splitting of simultaneously arriving neutrons is determined by the parameter $\xi(t)$ which is connected to an effective splitting field $B_s^{\text{eff}}(t)$. Similarly, in CMDNP the constant splitting was fixed by a parameter ε which is connected to NB_s . Obviously the ‘‘correspondence principle’’ is $NB_s \rightarrow B_s^{\text{eff}}(t)$ together with $v_0 \rightarrow x_2/t$. The spin exit angle is time-dependent, of course, but otherwise completely analogous to the CMDNP formula (1):

$$\vartheta_{\pm}(t) = \mp \frac{\pi}{2} + \int_{x_1}^{x_2} \frac{dx}{v_{\pm}(t)} |\gamma| B_p \left(t - \frac{x_2 - x}{v_{\pm}(t)} \right) b_p(x) \quad (10)$$

for $t \geq t_{\min} = (mx_2/h)\lambda_{\min}$. A change of the integration variable, $x \rightarrow \alpha = x/x_2$, plus a substitution of the velocity splitting $v_{\pm}(t)$, as well as a series expansion of the precession field amplitude centred at $B_p(\alpha t)$ and neglecting second order terms leads to

$$\vartheta_{\pm}(t) = \mp \frac{\pi}{2} + \int_{\alpha_1}^1 d\alpha |\gamma| t \left[B_p(\alpha t) \mp \frac{\xi(t)}{2} (B_p(\alpha t) - (1-\alpha)\dot{B}_p(\alpha t)t) \right] b_p(\alpha x_2). \quad (11)$$

Taking into account the spin alignment condition $\vartheta_+(t) - \vartheta_-(t) = 2n\pi$, then one obtains from the sum of

the two (for $n = 0$ identical) spin exit angles

$$\vartheta_+(t) = \vartheta_-(t) \equiv \vartheta(t) = |\gamma|t \int_{\alpha_1}^1 d\alpha B_p(\alpha t) b_p(\alpha x_2) \quad (12)$$

whereas their difference for $n = 0$ leads to

$$B_s^{\text{eff}}(t) \cdot B_p^{\text{eff}}(t) \ell \cdot \left(\frac{t}{x_2}\right)^3 = \frac{\pi m}{4\gamma\mu} \equiv \mathcal{A} \quad (13)$$

with the definition of an “effective precession field”

$$B_p^{\text{eff}}(t) \equiv \frac{1}{1 - \alpha_1} \int_{\alpha_1}^1 d\alpha (B_p(\alpha t) - (1 - \alpha) \dot{B}_p(\alpha t) b_p(\alpha x_2)). \quad (14)$$

Obviously the “generalised solution” (13) of the PPDNP setup problem is formally completely analogous to the CMDNP polarisation condition (2), provided that we add another correspondence rule: $B_p(b_p) \rightarrow B_p^{\text{eff}}(t)$.

It is a non-trivial task, however, to find concrete solutions for the time dependencies of $B_s(t)$ and $B_p(t)$ in a PPDNP setup, although it is possible, indeed, to begin with an arbitrarily chosen time dependence of either of these two fields. But if, for instance, $B_s(t)$ is fixed and its effective field $B_s^{\text{eff}}(t)$ calculated from Eq. (8) then one obtains immediately an analytical form $B_p^{\text{eff}}(t)$ from Eq. (13). But to find $B_p(t)$ it will then be necessary to solve the defining Eq. (14), for which in general no unique solution exists. If, vice versa, $B_p(t)$ is fixed an analogous ambiguity occurs in the solution of Eq. (8).

Alternatively, the third important quantity, $\vartheta(t)$, which determines the motion of the final $\pi/2$ -turner’s field axis and follows directly from $B_p(t)$ by integration of Eq. (12), could be used also as the starting point in the setup development. But then one has to find an expression for $B_p(t)$ which fulfills Eq. (12)—again, a sophisticated task with no unique solution.

We have tackled this mathematical problems by means of a generic power series ansatz of $B_s^{\text{eff}}(t)$ and $B_p^{\text{eff}}(t)$ which is applicable for arbitrary spatial profiles $b_s(x)$ and $b_p(x)$. Introducing in Eq. (8) the ansatz $B_p(t) = \sum_{\rho} \beta_{\rho}^{(p)} \hat{t}^{\rho}$ ($\hat{t} \equiv t/t_n$) with a reference time t_n) yields for the effective field

$$B_p^{\text{eff}}(t) \equiv \sum_{\rho} g_{\rho} \hat{t}^{\rho} = \sum_{\rho} \beta_{\rho}^{(p)} \mathcal{F}_{\rho}^{(p)} \hat{t}^{\rho} \quad (15)$$

with

$$\mathcal{F}_{\rho}^{(p)} \equiv \frac{1}{1 - \alpha_1} \int_{\alpha_1}^1 d\alpha (\alpha^{\rho} - (1 - \alpha)\rho\alpha^{\rho-1}) b_p(\alpha x_2).$$

Hence one has a solution for $B_p(t)$ in terms of a series expansion with coefficients $\beta_{\rho}^{(p)} = f_{\rho}/\mathcal{F}_{\rho}^{(p)}$ if the sum is convergent. Of course, the effective field series coefficients g_{ρ} have to be determined beforehand. The sought-after amplitude coefficients are then $\beta_{\rho}^{(p)} = g_{\rho}/\mathcal{F}_{\rho}^{(p)}$.

By this ansatz one can easily reproduce the hitherto well-known “ k -mode” solutions [2], i.e. fields ramped with negative powers of t : If one starts with $B_s(t) = B_{s1} \hat{t}^{k-1}$

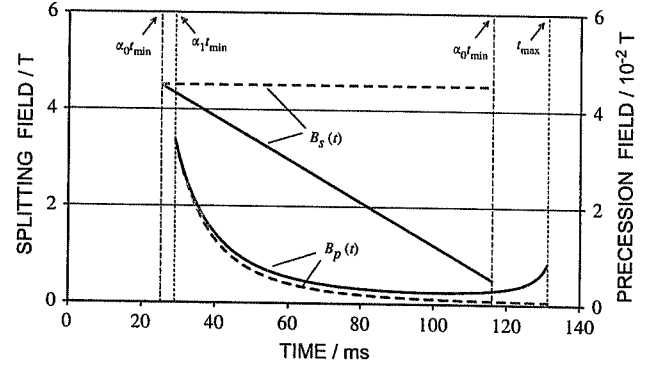


Fig. 2. “Non- k -mode” PPDNP solution for a linearly decreasing energy splitting field and “ k -mode” field time dependencies for $k = 0$. “Flat” field profiles with $L = 20$ m, $a' = 3$ m, $c = 0$, $\ell = 3$ m are assumed in both cases. The arrival time interval at the polarisers end position $t_{\min} \leq t \leq t_{\max}$ follows from the chosen wavelength interval $0.5 \text{ nm} < \lambda < 2 \text{ nm}$.

($k \in \mathfrak{R}$, $B_{s1} \equiv B_s(t_n)$), then $B_s^{\text{eff}}(t) = B_{s1} \mathcal{F}_{k-1}^{(s)} \hat{t}^{k-1}$. From the generalised PPDNP setup Eq. (13) then follows $B_p^{\text{eff}}(t) = \mathcal{A} x_2^3 \hat{t}^{-(k+2)} / (\ell t_n^3 B_{s1} \mathcal{F}_{k-1}^{(s)})$. Thus we obtain the solution for the time-dependent precession field

$$B_p(t) = B_{p1} \hat{t}^{-(k+2)} \quad \text{with} \quad B_{p1} = \frac{\mathcal{A} x_2^3}{\ell t_n^3 B_{s1} \mathcal{F}_{k-1}^{(s)} \mathcal{F}_{-(k+2)}^{(p)}}. \quad (16)$$

However, from solution (13) follows that there are many more possible field dependencies beyond k -modes which allow for more flexibility in DNP setup design. Specifically, we found that for any given $B_s(t)$ which is part of a properly working setup another possible time dependence for the splitting field amplitude is given by $B_s(t) + B_s^{\text{osc}}(\ln \hat{t})$, where B_s^{osc} is a general antiperiodic function of arbitrary amplitude [9]. It is important to notice that for “flat” field profiles, as e.g. $b_s(x) = \Theta_{L, L+a'}(x)$, $b_p(x) = \Theta_{x_1, x_2}(x)$, where the length of the splitting system is divided into a homogeneous field region (length a') and a field-free transition region (length c): $a = a' + c$, the analysis is much simpler and requires no regress to effective fields to calculate the required precession field time dependence for given $B_s(t)$. For instance, in the case of a splitting field decreasing linearly with time one obtains the “non- k -mode” solution of $B_p(t)$ that is shown in Fig. 2. For comparison the especially simple but in practice most important “ k -mode” solution for the case $k = 1$, i.e. for a static precession field $B_s(t) \equiv B_s$, is plotted as well in this figure. As mentioned previously, the required time dependence of the final spin turner’s field axis is governed by the spin exit angle $\vartheta(t)$. It can be determined once $B_p(t)$ has been found, and hence needs no further consideration.

3. Numerical simulations

Complementary to our analytical treatment we have also performed extensive numerical simulations of several possible PPDNP setup configurations under realistic

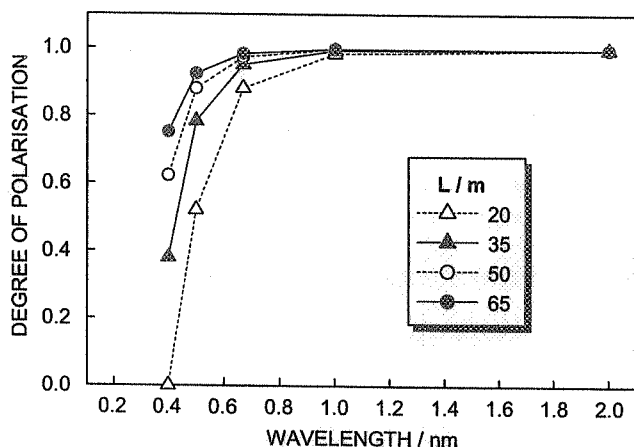


Fig. 3. Achievable degree of polarisation vs wavelength for various lengths of the TOF-path of the incident neutrons for a typical PPDNP setup ($NB_s = 8 \times 5$ T, source pulse duration: $\Delta t = 50 \mu\text{s}$, $\ell = 1$ m) with constant splitting field, i.e. $k = 1$ and $B_p(t) \propto t^{-3}$.

conditions. As in the theoretical considerations we treated the neutron motion again in a semi-classical way. In particular, the centre of mass moves in three dimensions, and the polarisation vector is a classical vector with length 1, subject to LARMOR rotation in the respective local magnetic field. Several technical details could be taken into account, as for instance, the source pulse length, the

length of the incident TOF path, the wavelength spectrum, the beam divergence, and the finite thickness of the final $\pi/2$ -spin turner. However, because of limited space in Fig. 3 we only present one typical example out of a manifold of results, namely the wavelength dependence of the achievable degree of polarisation for various lengths of time-of-flight path of the incident neutrons.

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