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A Shared-Relay Cooperative Diversity Scheme Based on Joint Channel and Network Coding

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Abstract
In cooperative communications, the technique of algebraic code superposition (or *network coding*) may efficiently utilize limited communications resources to produce spatial diversity. In this paper, we propose a simple cooperative diversity scheme for the scenario of two sources sharing a single relay. No cooperative coding is required at the sources, and only very simple interleave-and-superpose operations are performed at the relay. We propose a novel computationally efficient message passing algorithm at the destination's decoder which utilizes spatial diversity gathered over multiple transmission frames. We show that despite the simplicity of the proposed scheme, diversity gains are efficiently leveraged by the simple combination of channel coding at the sources and network coding at the relay.

1. INTRODUCTION

While wireless channels suffer from fading, at the same time the broadcast nature of wireless channels provides the possibility of a third party other than the destination "overhearing" the information that the source transmits. Thus apart from the original transmission channel, the same information could be transmitted to the destination through another independently fading channel. This generated spatial diversity can effectively combat the deleterious effect of fading [1]. In recent years, there has been increasing interest in applying the idea of algebraic code superposition, also called "network coding" [2, 3, 4, 5, 6] to the cooperative communications scenario. The network coding approach provides an efficient way to generate spatial diversity under the constraint of limited resources. One challenge is the problem of decoder design which should be able to cope with the complicated decoding situation at the destination [2, 3, 4, 5, 6].

In [5, 6], the model of a typical network coding unit is considered in which the packets from the two sources are linearly combined at the relay. In our work we also focus on this cooperative transmission model for the situation where it is impractical for one mobile user to "capture" the other user's signal during its uplink transmission in the cellular network. Moreover, relay-based cooperative processing provides greater security than direct user cooperation in which user information must be shared.

In [4], a code superposition scheme employing low-density generator matrix (LDGM) codes is proposed to reduce the decoding complexity at the destination. But in order to do the graph-based decoding, the systematic bits must be retained without superposition which means that the potential superposition diversity is lower than that obtainable from fully superposed codewords. In [6], a combined LDPC code construction scheme including two channel code components and one network code component is produced by random parity-check matrix generation under certain constraints. The network codes are actually the parity checks for two channel codewords; this necessitates more complicated relay operations than simple superposition.

In this work we propose a cooperative coding scheme which, in contrast to previous work where TDMA-only or FDMA-only relaying is assumed, allows continuous transmission of superposed codewords by the relay, thus making efficient use of communication resources to leverage spatial diversity gains. The proposed scheme also arises from the observation that idle frequency channels do exist and could be exploited in current TDMA systems. We shall detail a novel efficient decoding algorithm based on message passing on the destination node's factor graph for the purpose of exploiting the spatial diversity contained in the algebraically superposed codewords. The algorithm attains a separation of the two soft-input soft-output (SISO) decoder modules corresponding to the codes employed by the two sources; for convolutional codes, this separation affords a complexity advantage over decoding of the "nested code" [2, 3]; for LDPC codes, it affords a more efficient Tanner graph schedule than fully parallel decoding [6].

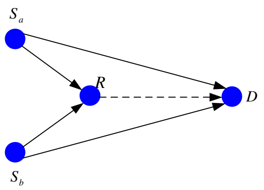

Figure 1: Four-node communications network. Sources S_a and S_b share a common relay R as well as having direct links to the destination D.

2. PROPOSED COOPERATIVE CODING SCHEME

We consider the four-node communications network depicted in Figure 1, with two sources S_a and S_b, one relay R, and one destination D common to the two sources. The communication period is divided into $L+1$ *time slots* $t = 0, 1, \cdots, L$; each time slot $t \in \{0, 1, \cdots, L\}$ is further subdivided into 2 *half slots* ($2t$, $2t+1$). Source S_a has L messages to transmit, which it encodes into L n-bit codewords $\{\mathbf{a}_t \,:\, t = 0, 1, \cdots, L-1\}$. The code used at source S_a is $\mathcal{C}_a$ and is defined by the $m_a \times n$ parity-check matrix $\mathbf{H}_a = (H_a(j,i))$. Similarly, source S_b has L messages to transmit, which it encodes into L n-bit codewords $\{\mathbf{b}_t \,:\, t = 0, 1, \cdots, L-1\}$. The code used at source S_b is $\mathcal{C}_b$ and is defined by the $m_b \times n$ parity-check matrix $\mathbf{H}_b = (H_b(j,i))$. Thus, the codes $\mathcal{C}_a$ and $\mathcal{C}_b$ have the same length but not necessarily the same rate. In general, the codes used at the two sources can be LDPC or convolutional; in this paper we concentrate on LDPC codes. S_a and S_b broadcast their modulated codewords to the relay and destination nodes using TDMA in frequency band f_1 and there is no cooperation between the two sources. For each $t \in \{0, 1, \cdots, 2L-1\}$, let $\mathbf{c}_t$ denote the codeword broadcast by the source in half slot t; thus $\mathbf{c}_{2t} = \mathbf{a}_t$ and $\mathbf{c}_{2t+1} = \mathbf{b}_t$ for $t \in \{0, 1, \cdots, L-1\}$.

The relay decodes and then re-encodes each codeword received from the source (the cooperative scheme is based on the scenario where the source is quite close to the relay). The relay also has a buffer in which it stores the codewords it obtained in the previous two half slots. At each half slot ($t = 2, 3, \cdots, 2L$), the relay interleaves the codeword received in half slot $t-1$ and superposes it (XOR operation) with the codeword received in half slot $t-2$; it then transmits the resulting codeword to the destination in frequency band f_2. Special cases arise at half slots 1 and $2L+1$ in which only a single codeword is stored at the relay and no XOR operation is performed. The allocation of different frequency bands to the transmission channel of the relay and the broadcast channels of the sources allows for simultaneous transmission and reception by the relay node, thus allowing efficient leveraging of communication resources for spatial diversity. Let $\mathbf{d}_t$ denote the codeword transmitted from the relay to the destination in half slot $t \in \{1, 2, \cdots, 2L+1\}$; thus $\mathbf{d}_t = \pi(\mathbf{c}_{t-1}) \oplus \mathbf{c}_{t-2}$, for $t = 2, 3, \cdots 2L$. For each $t = 0, 1, \cdots L-1$, source S_a's codeword $\mathbf{a}_t$ is decoded at the end of half slot $2t+2$ and source S_b's codeword $\mathbf{b}_t$ is decoded at the end of half slot $2t+3$. The transmission schedule for this cooperative coding scheme is illustrated in Figure 2. It may be seen from the Figure that spatial diversity for each message is contained in three codewords received at the destination.

Note that decoding and re-encoding of received codewords using a different code (as in the scheme of [2]) is not performed by the relay in this scheme; this is in order to keep the relay operation as simple as possible. The interleaver π actually provides the "interleaver gain" for decoding at the destination. The interleaver is not in general necessary in the case of LDPC coding; however it may be used to avoid a large multiplicity of 8-cycles in the Tanner graph for the case where $\mathbf{H}_a = \mathbf{H}_b$.

The transmission schedule for consecutive relaying (see Figure 3), in which the time and frequency allocations are the same as in the proposed cooperative scheme, except that the relay simply retransmits the previously received codeword rather than a codeword superposition. Also, the transmission schedules for simple TDMA and FDMA based relaying schemes are depicted in Figures 4 and 5 respectively. It is easily seen that in all three of these schemes, spatial diversity for each message is contained in two codewords received at the destination. A simulation-based comparison of all schemes described in this section under a transmit power constraint will be given in Section 5.

3. DECODING ALGORITHM AT DESTINATION NODE

Time Slot		0		1		…	t		t+1		…	L-1		L	
Half Slot		0	1	2	3	…	2t	2t+1	2t+2	2t+3	…	2L-2	2L-1	2L	2L+1
Relay Receives	From Source (f1)	$\mathbf{a}_0$	$\mathbf{b}_0$	$\mathbf{a}_1$	$\mathbf{b}_1$	…	$\mathbf{a}_t$	$\mathbf{b}_t$	$\mathbf{a}_{t+1}$	$\mathbf{b}_{t+1}$	…	$\mathbf{a}_{L-1}$	$\mathbf{b}_{L-1}$		
Destination Receives	From Source (f1)	$\mathbf{c}_0=\mathbf{a}_0$	$\mathbf{c}_1=\mathbf{b}_0$	$\mathbf{c}_2=\mathbf{a}_1$	$\mathbf{c}_3=\mathbf{b}_1$	…	$\mathbf{c}_{2t}=\mathbf{a}_t$	$\mathbf{c}_{2t+1}=\mathbf{b}_t$	$\mathbf{c}_{2t+2}=\mathbf{a}_{t+1}$	$\mathbf{c}_{2t+3}=\mathbf{b}_{t+1}$	…	$\mathbf{c}_{2L-2}=\mathbf{a}_{L-1}$	$\mathbf{c}_{2L-1}=\mathbf{b}_{L-1}$		
	From Relay (f2)		$\mathbf{d}_1=\pi(\mathbf{a}_0)$	$\mathbf{d}_2=\pi(\mathbf{b}_0)\oplus\mathbf{a}_0$	$\mathbf{d}_3=\pi(\mathbf{a}_1)\oplus\mathbf{b}_0$	…	$\mathbf{d}_{2t}=\pi(\mathbf{b}_{t-1})\oplus\mathbf{a}_{t-1}$	$\mathbf{d}_{2t+1}=\pi(\mathbf{a}_t)\oplus\mathbf{b}_{t-1}$	$\mathbf{d}_{2t+2}=\pi(\mathbf{b}_t)\oplus\mathbf{a}_t$	$\mathbf{d}_{2t+3}=\pi(\mathbf{a}_{t+1})\oplus\mathbf{b}_t$	…	$\mathbf{d}_{2L-2}=\pi(\mathbf{b}_{L-2})\oplus\mathbf{a}_{L-2}$	$\mathbf{d}_{2L-1}=\pi(\mathbf{a}_{L-1})\oplus\mathbf{b}_{L-2}$	$\mathbf{d}_{2L}=\pi(\mathbf{b}_{L-1})\oplus\mathbf{a}_{L-1}$	$\mathbf{d}_{2L+1}=\mathbf{b}_{L-1}$
Destination Decodes				$\mathbf{a}_0$	$\mathbf{b}_0$	…	$\mathbf{a}_{t-1}$	$\mathbf{b}_{t-1}$	$\mathbf{a}_t$	$\mathbf{b}_t$	…	$\mathbf{a}_{L-2}$	$\mathbf{b}_{L-2}$	$\mathbf{a}_{L-1}$	$\mathbf{b}_{L-1}$

Figure 2: Transmission schedule of proposed cooperative coding scheme.

Time Slot		0		1		…	t		t+1		…	L-1		
Half Slot		0	1	2	3	…	2t	2t+1	2t+2	2t+3	…	2L-2	2L-1	2L
Destination Receives	From Source (f1)	$\mathbf{a}_0$	$\mathbf{b}_0$	$\mathbf{a}_1$	$\mathbf{b}_1$	…	$\mathbf{a}_t$	$\mathbf{b}_t$	$\mathbf{a}_{t+1}$	$\mathbf{b}_{t+1}$	…	$\mathbf{a}_{L-1}$	$\mathbf{b}_{L-1}$	
	From Relay (f2)		$\mathbf{a}_0$	$\mathbf{b}_0$	$\mathbf{a}_1$	…	$\mathbf{b}_{t-1}$	$\mathbf{a}_t$	$\mathbf{b}_t$	$\mathbf{a}_{t+1}$	…	$\mathbf{b}_{L-2}$	$\mathbf{a}_{L-1}$	$\mathbf{b}_{L-1}$

Figure 3: Transmission schedule of consecutive relaying scheme.

Time Slot										
0	1	2	…	3t	3t+1	3t+2	…	3L-3	3L-2	3L-1
$\mathbf{a}_0$	$\mathbf{b}_0$	$\mathbf{a}_0\oplus\mathbf{b}_0$	…	$\mathbf{a}_t$	$\mathbf{b}_t$	$\mathbf{a}_t\oplus\mathbf{b}_t$	…	$\mathbf{a}_{L-1}$	$\mathbf{b}_{L-1}$	$\mathbf{a}_{L-1}\oplus\mathbf{b}_{L-1}$

Figure 4: Section of transmission schedule for the TDMA relaying scheme.

	0	1	…	t	t+1	…	L-1	
Frequency Band f1	$\mathbf{a}_0$	$\mathbf{a}_1$	…	$\mathbf{a}_t$	$\mathbf{a}_{t+1}$	…	$\mathbf{a}_{L-1}$	
Frequency Band f2	$\mathbf{b}_0$	$\mathbf{b}_1$	…	$\mathbf{b}_t$	$\mathbf{b}_{t+1}$	…	$\mathbf{b}_{L-1}$	
Frequency Band f3		$\mathbf{a}_0\oplus\mathbf{b}_0$	…	$\mathbf{a}_{t-1}\oplus\mathbf{b}_{t-1}$	$\mathbf{a}_t\oplus\mathbf{b}_t$	…	$\mathbf{a}_{L-2}\oplus\mathbf{b}_{L-2}$	$\mathbf{a}_{L-1}\oplus\mathbf{b}_{L-1}$

Figure 5: Section of transmission schedule for the FDMA relaying scheme.

Without loss of generality, we consider the decoding of codeword $\mathbf{a}_t$ at the end of half slot $2t+2$, for $t \in \{1, 2, \cdots, L-1\}$. We focus on the codeword $\mathbf{d}_{2t+2} = \pi(\mathbf{b}_t) \oplus \mathbf{a}_t$ for this decoding. We use $L_1(c_t^{(i)})$ and $L_1(d_t^{(i)})$ to denote the respective *a priori* LLRs derived from the corresponding received streams. Also, $L_2(\cdot)$ denotes the (updated) *a posteriori* LLRs which will be used as the *a priori* LLRs in the next decoding. We assume that *a priori* LLRs on $\mathbf{c}_{2t+1}$ and $\mathbf{c}_{2t}$, denoted $\{L_1(c_{2t+1}^{(i)})\}$ and $\{L_2(c_{2t}^{(i)})\}$ respectively, are available from the channel observation value and previous decoding respectively. In addition to the decoding of $\mathbf{a}_t$, the decoder will produce *a posteriori* LLRs $\{L_2(c_{2t+1}^{(i)})\}$ which will be used as *a priori* LLRs in the next decoding.

Next, we provide a concise description of the factor graph based decoding algorithm [7] at the destination decoder. The factor graph for the decoding is illustrated in Figure 6, where circles depict variable nodes and squares depict factor nodes. Extrinsic information is exchanged between the two soft-input soft-output (SISO) decoder modules for the constituent codes $\mathcal{C}_a$ and $\mathcal{C}_b$, via the factor nodes $\{F_i\}$ which correspond to the network coding operation at the relay. For simplicity, the graph is illustrated for the case $n = 3$, and where $\mathcal{C}_a$ and $\mathcal{C}_b$ are (trivial) LDPC codes. For convolutional constituent codes, the two SISO modules execute BCJR algorithms. Note that in the convolutional case, the separation of the two (e.g. M-state) decoder SISO modules gives a complexity advantage over schemes which use a larger (e.g. M^2-state) decoder to decode the "nested" code generated at the relay (see e.g. [2]). In the LDPC case this separation of SISO modules effects a more efficient message-passing schedule than does fully parallel decoding on the Tanner graph of the nested code (see e.g. [6]).

Next we introduce some notational conventions pertaining to the following algorithm description. In all cases, the letter λ is used to denote LLRs corresponding to messages passed on the factor graph. The interleaving "π" is interpreted as

$$\mathbf{x} = \pi(\mathbf{y}) \quad \Longleftrightarrow \quad x^{(i)} = y^{(\pi(i))}$$

Some index sets are defined as follows. $\mathcal{J}_a = \{1, 2, \cdots m_a\}$; $\mathcal{J}_b = \{1, 2, \cdots m_b\}$; $\mathcal{N}_a(i) = \{j \in \mathcal{J}_a \ : \ H_a(j,i) = 1\}$; $\mathcal{N}_b(i) = \{j \in \mathcal{J}_b \ : \ H_b(j,i) = 1\}$; $\mathcal{M}_a(j) = \{i \in \mathcal{I} \ : \ H_a(j,i) = 1\}$; $\mathcal{M}_b(j) = \{i \in \mathcal{I} \ : \ H_b(j,i) = 1\}$. Also, $\boxplus$ denotes the (commutative and associative) "box-plus" operation [8], i.e.

$$\boxplus_{s \in \mathcal{S}} \lambda_s = \log\left(\frac{1 + \prod_{s \in \mathcal{S}} \tanh(\lambda_s/2)}{1 - \prod_{s \in \mathcal{S}} \tanh(\lambda_s/2)}\right)$$

N denotes the maximum number of decoding iterations.

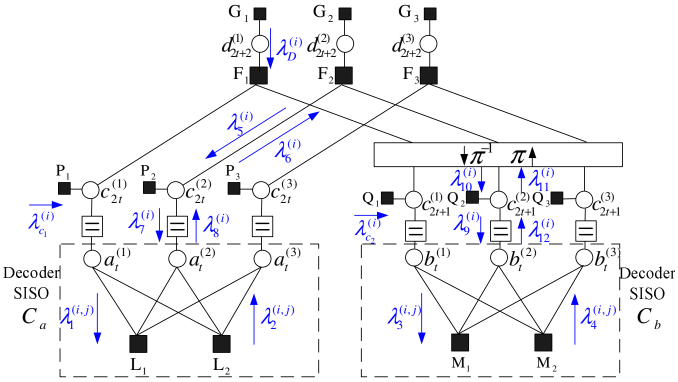

Figure 6: Factor graph corresponding to the destination's decoding of codeword $\mathbf{a}_t$. The message-passing schedule is such that extrinsic information is exchanged between the two decoder SISO modules for the constituent codes $\mathcal{C}_a$ and $\mathcal{C}_b$, via the factor nodes $\{F_i\}$ corresponding to the network coding operation at the relay. For ease of presentation, the factor graph is illustrated for the case $n = 3$ and trivial codes $\mathcal{C}_a$, $\mathcal{C}_b$.

Factor Graph Based Decoding Algorithm at Destination Node – Decoding of Codeword $\mathbf{a}_t$

Initialization:

- For $i \in \mathcal{I}$,

$$\lambda_{c_1}^{(i)} = L_2(c_{2t}^{(i)}) \quad (1)$$
$$\lambda_{c_2}^{(i)} = L_1(c_{2t+1}^{(i)}) \quad (2)$$
$$\lambda_8^{(i)} = 0 \quad (3)$$
$$\lambda_D^{(i)} = L_1(d_{2t+2}^{(i)}) \quad (4)$$

- For $i \in \mathcal{I}$, $j \in \mathcal{N}_a(i)$

$$\lambda_2^{(i,j)} = 0 \quad (5)$$

- For $i \in \mathcal{I}$, $j \in \mathcal{N}_b(i)$

$$\lambda_4^{(i,j)} = 0 \quad (6)$$

Main Loop: For $k = 1$ to N do

- For $i \in \mathcal{I}$,
$$\lambda_6^{(i)} = \lambda_{c_1}^{(i)} + \lambda_8^{(i)} \quad (7)$$

- Network coding constraints: for $i \in \mathcal{I}$,

$$\lambda_{10}^{(\pi(i))} = \lambda_6^{(i)} \boxplus \lambda_D^{(i)} \quad (8)$$

- For $i \in \mathcal{I}$,
$$\lambda_9^{(i)} = \lambda_{c_2}^{(i)} + \lambda_{10}^{(i)} \quad (9)$$

- SISO decoder $\mathcal{C}_b$: for $i \in \mathcal{I}$, $j \in \mathcal{N}_b(i)$

$$\lambda_3^{(i,j)} = \lambda_9^{(i)} + \sum_{l \in \mathcal{N}_b(i)\setminus\{j\}} \lambda_4^{(i,l)} \quad (10)$$
$$\lambda_4^{(i,j)} = \boxplus_{l \in \mathcal{M}_b(j)\setminus\{i\}} \lambda_3^{(l,j)} \quad (11)$$

For $i \in \mathcal{I}$,

$$\lambda_{12}^{(i)} = \sum_{j \in \mathcal{N}_b(i)} \lambda_4^{(i,j)} \quad (12)$$

- Obtain the *a posteriori* LLR (prior for decoding in next half slot)

$$L_2(c_{2t+1}^{(i)}) = \lambda_9^{(i)} + \lambda_{12}^{(i)} \quad (13)$$

- For $i \in \mathcal{I}$,
$$\lambda_{11}^{(i)} = \lambda_{c_2}^{(i)} + \lambda_{12}^{(i)} \quad (14)$$
$$\lambda_5^{(i)} = \lambda_D^{(i)} \boxplus \lambda_{11}^{(\pi(i))} \quad (15)$$

- For $i \in \mathcal{I}$,
$$\lambda_7^{(i)} = \lambda_{c_1}^{(i)} + \lambda_5^{(i)} \quad (16)$$

- SISO decoder $\mathcal{C}_a$: for $i \in \mathcal{I}$, $j \in \mathcal{N}_a(i)$

$$\lambda_1^{(i,j)} = \lambda_7^{(i)} + \sum_{l \in \mathcal{N}_a(i)\setminus\{j\}} \lambda_2^{(i,l)} \quad (17)$$
$$\lambda_2^{(i,j)} = \boxplus_{l \in \mathcal{M}_a(j)\setminus\{i\}} \lambda_1^{(l,j)} \quad (18)$$

For $i \in \mathcal{I}$,

$$\lambda_8^{(i)} = \sum_{j \in \mathcal{N}_a(i)} \lambda_2^{(i,j)} \quad (19)$$

- Calculate *a posteriori* LLRs for codeword $\mathbf{a}_t$:

$$L(a_t^{(i)}) = \lambda_7^{(i)} + \lambda_8^{(i)} \quad (20)$$

- Make decisions on the code bits

$$\hat{a}_t^{(i)} = \begin{cases} 0 & \text{if } L(a_t^{(i)}) \geq 0 \\ 1 & \text{if } L(a_t^{(i)}) < 0 \end{cases}$$

If $\hat{\mathbf{a}}_t \mathbf{H}_a^T = \mathbf{0}$ then *break;*

Endfor

For the special case of decoding the first codeword $\mathbf{a}_0$, we set $L_2(c_0^{(\pi(i))}) = L_1(c_0^{(\pi(i))}) + L_1(d_1^{(i)})$ for all $i \in \mathcal{I}$. Also, for the special case of decoding the final codeword $\mathbf{b}_{L-1}$, further modifications are made to the algorithm as follows:

- Equation (2), (6), (7)-(14), (15) are deleted
- Equation (16) is replaced by

$$\lambda_7^{(i)} = \lambda_{c_1}^{(i)} \quad (21)$$

In this case also, the decoding of $\mathbf{a}_t$ spans three transmission frames (half slots) $2t \rightarrow 2t{+}1 \rightarrow 2t{+}2$, resulting in the three-step decoding evolution $L_1(c_{2t}^{(i)}) \rightarrow L_2(c_{2t}^{(i)}) \rightarrow \hat{a}_t^{(i)}$.

4. A LOWER BOUND ON THE OUTAGE PROBABILITY

In this section a theoretical lower bound on the frame (codeword) error rate (FER) of the system is derived; the analysis follows the lines of [3].

In the case where an extra frequency band is used, let us consider the decoding of $\mathbf{a}_t$ and $\mathbf{b}_t$, transmitted in half slots $2t$ and $2t+1$ respectively. The codewords received by the destination from the sources in half slots $2t$ and $2t{+}1$ contain information only relating to $\mathbf{a}_t$ and $\mathbf{b}_t$. Due to latency constraints embodied in the relay operation, the message of $\mathbf{a}_t$ is contained in $\mathbf{d}_{2t+1}$ and $\mathbf{d}_{2t+2}$; in the same way, the message of $\mathbf{b}_t$ is also contained in $\mathbf{d}_{2t+2}$ and $\mathbf{d}_{2t+3}$. The received information which may be used in decoding $\mathbf{a}_t$ and $\mathbf{b}_t$ is contained in five received codewords at the destination – transmissions from sources to destination in half slots $2t$ and $2t + 1$, and transmissions from relay to destination in half slots $2t{+}1$, $2t{+}2$ and $2t{+}3$. Recall that the codeword length is n, and let $r = k/n$ denote the code rate of each encoder; also let $C(\gamma)$ denote the capacity of a binary-input point-to-point link with temporal SNR γ. Then the number of information bits at the destination which may be used for joint decoding of $\mathbf{a}_t$ and $\mathbf{b}_t$ is not greater than $n[C(\gamma_{AD}) + C(\gamma_{BD}) + C(\gamma_{RD1}) + C(\gamma_{RD2}) + C(\gamma_{RD3})]$, where γ_{RD1}, γ_{RD2} and γ_{RD3} (the relay-destination SNRs in consecutive half slots) are independent and identically distributed random variables, and γ_{AD} and γ_{BD} (the source-destination SNRs) are independent and identically distributed random variables if the two source-to-destination link average SNRs are equal. When this value is lower than $2k$, there is no possibility of jointly decoding both packets, and an outage event is inevitable. Thus, a lower bound for the FER is given by

$$\mathrm{FER} \geq \frac{1}{2} \cdot \Pr[C(\gamma_{AD}) + C(\gamma_{BD}) + C(\gamma_{RD1}) + C(\gamma_{RD2}) + C(\gamma_{RD3}) < 2r] \quad (22)$$

The Monte Carlo simulated lower bound is included in the simulation results of Section 5.

5. SIMULATION RESULTS

In this section, we provide a comparison of the proposed cooperative coding scheme with two reference cooperative schemes. The first is the consecutive relaying scheme of Figure 3. Here a single LDPC decoding per codeword is sufficient for reception, where the LLRs for decoder initialization are found by adding the LLRs corresponding to the broadcast and relayed versions of the pertinent codeword. The second reference scheme is the simple TDMA/ FDMA code superposition relaying scheme of Figures 4 and 5. For this scheme, joint decoding of $\mathbf{a}_t$ and $\mathbf{b}_t$ is performed using a decoding algorithm similar to that of Section 3. The decoder also uses message passing between the two constituent decoders via network coding constraints; full details are omitted due to space limitations. The fair comparison of the three cooperative schemes is based on the constraint that in simulations, each scheme uses the same codes $\mathcal{C}_a$ and $\mathcal{C}_b$, and the same total energy E for transmission of the $2L$ source messages.

The codes used for simulations are randomly generated rate 1/2 regular LDPC codes of block length $n = 1200$ with column weight 3 and no 4-cycles in the Tanner graph. In simulations we choose $\mathbf{H}_a = \mathbf{H}_b$, and assume BPSK modulation for all systems. A random interleaver π is used to avoid 8-cycle multiplicity in the Tanner graph. We consider a quasi-static Rayleigh fading channel, for which the fading coefficients are constant within each half slot (one codeword) and change independently from one half slot to the next. We assume equal average signal-to-noise ratio (SNR) on the two source-destination links and the relay-destination link, and we assume that the destination has perfect knowledge of the channel fading coefficients and noise variances. As for the two source-relay links, which play a key role in the performance of the system since poor link quality may lead to catastrophic error propagation at the destination decoder, the simulation setup is that the source-relay links are both ideally error-free.

Simulated performance results in terms of BER and FER are shown in Figures 7 and 8 respectively. The curve corresponding to the proposed cooperative coding scheme again exhibits an increased diversity gain with respect to the reference schemes in the SNR region of interest. The proposed scheme attains approximately an order of magnitude decrease in both BER and FER over the other schemes at an E_b/N_0 of 8 dB.

6. CONCLUSION

In this paper, we have proposed a simple but effective cooperative coding scheme for the shared-relay scenario. The scheme requires only channel coding at

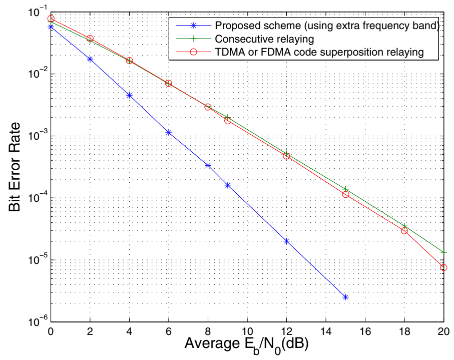

Figure 7: Comparative BER performance for the proposed cooperative scheme. The performance is shown with respect to two reference schemes: simple TDMA/FDMA code superposition relaying; and consecutive relaying using the extra frequency band.

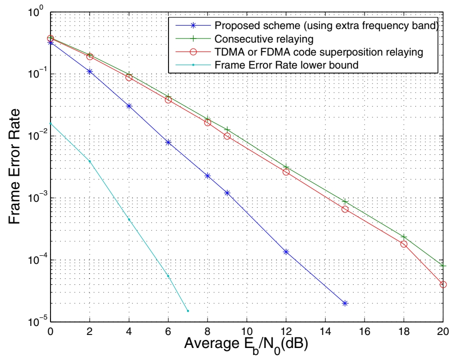

Figure 8: Comparative FER performance for the proposed cooperative scheme. The performance is shown with respect to two reference schemes: simple TDMA/FDMA code superposition relaying; and consecutive relaying using the extra frequency band. Also plotted is the theoretical lower bound on the FER given by (22).

the sources and interleaving and network coding at the relay, thus relegating complexity to the base station for uplink transmission. The decoding algorithm at the destination node is based on message passing on a factor graph corresponding to multiple received frames at the destination, and extracts spatial diversity gains in a computationally efficient manner. Simulation results demonstrate that the proposed scheme outperforms competitive schemes based on consecutive relaying and TDMA/FDMA based code superposition relaying.

7. ACKNOWLEDGEMENT

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