

Dirichlet boundary-value problem for Chern-Simons modified gravityDaniel Grumiller,^{1,*} Robert Mann,^{2,3,+} and Robert McNees^{3,‡}¹*Center for Theoretical Physics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA*²*Department of Physics & Astronomy, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada*³*Perimeter Institute, 31 Caroline Street North, Waterloo, Ontario N2L 2Y5, Canada*

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Chern-Simons modified gravity comprises the Einstein-Hilbert action and a higher-derivative interaction containing the Chern-Pontryagin density. We derive the analog of the Gibbons-Hawking-York boundary term required to render the Dirichlet boundary-value problem well defined. It turns out to be a boundary Chern-Simons action for the extrinsic curvature. We address applications to black hole thermodynamics.

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In 1744–1746 Maupertuis formulated a general principle [1]: *When a change occurs in nature, the quantity of action necessary for that change is as small as possible.* In other words, the dynamics of a system follow from the condition that the first variation of the action vanishes. This includes all total derivative terms, which are addressed by imposing appropriate boundary conditions on the fields. Thus, a well-defined boundary-value problem is a prerequisite for any new theory formulated by means of an action principle.

Theories of gravity are notorious in this regard: The Einstein-Hilbert action of general relativity,

$$S_{\text{EH}} \sim \int d^4x \sqrt{-g} R \quad (1)$$

does not admit a well-defined Dirichlet boundary-value problem unless it is supplemented by the Gibbons-Hawking-York (GHY) boundary term [2,3]. This is because the Ricci scalar R contains second derivatives of the metric, so variation of the action produces boundary terms involving the variation of both the metric and its normal derivative. The GHY term cancels the latter contributions, and the remaining terms are addressed by placing Dirichlet boundary conditions on the metric.

String theory and other UV completions of Einstein gravity suggest that higher powers of curvature invariants appear in the low energy effective action, suppressed by powers of the Planck mass. These actions can also be considered as models in their own right, but in that case the problem described above reappears. Higher powers of the curvature introduce more derivatives of the metric, and the Dirichlet boundary-value problem is no longer well defined. A notable exception is Lovelock gravity [4], where it is possible to render the Dirichlet boundary-value

problem well defined by adding appropriate boundary terms that are nonlinear in extrinsic curvature [5].

Recently a CP -odd modification of general relativity dubbed “Chern-Simons modified gravity” was proposed. The action for this theory takes the form $S = S_{\text{EH}} + S_{\text{CS}}$ [6], where the new term is given by

$$S_{\text{CS}} \sim \int d^4x \sqrt{-g} \theta {}^*RR. \quad (2)$$

The action will be described in more detail momentarily. Here we just mention that *RR is the Chern-Pontryagin term and θ is a background scalar field, similar to an axion. The modification (2) has engendered a lot of interesting research—cf. e.g. [6–9] and references therein—but so far the boundary-value problem has not been addressed. Is there a boundary term analogous to the GHY term that can be added to the action to give a well-defined Dirichlet boundary-value problem?

We answer this question in the affirmative by constructing the boundary term required by the Chern-Simons (CS) modification (2). Schematically, our main result is that (2) has to be supplemented by a boundary term quadratic in the extrinsic curvature

$$S_{b\text{CS}} \sim \int d^3x \sqrt{h} \theta {}^*KDK \sim \int d^3x \sqrt{h} \theta \text{CS}(K). \quad (3)$$

This term may be interpreted as a boundary CS action for the (traceless part of) extrinsic curvature.

Our conventions are as follows. We use signature $- , + , + , +$. Indices from the beginning of the alphabet a, b, \dots range from 0 to 3 while indices from the middle of the alphabet i, j, \dots range from 0 to 2. Conventional factors and signs are chosen so that $A_{(ab)} := (A_{ab} + A_{ba})/2$, the Riemann tensor has sign $R^a{}_{bcd} := \partial_c \Gamma^a{}_{bd} - \dots$, and $R_{ab} := R^c{}_{acb}$. We denote the ϵ -tensor by ϵ^{abcd} and ϵ^{ijk} , and the ε -symbol by ε^{abcd} and ε^{ijk} . Covariant derivatives in four (three) dimensions are denoted by ∇_a (D_i), partial derivatives by ∂_a (∂_i).

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First we recall the basics of CS modified gravity. The bulk part of the action is given by [6,10]

$$S = S_{\text{EH}} + S_{\text{CS}} = \kappa \int d^4x \sqrt{-g} \left(R + \frac{1}{4} \theta {}^*RR \right), \quad (4)$$

where κ is the gravitational coupling constant and *RR is the Chern-Pontryagin term, defined as

$${}^*RR := {}^*R_b{}^c{}^d R^b{}_{acd} = \frac{1}{2} \epsilon^{cdef} R^a{}_{bef} R^b{}_{acd}. \quad (5)$$

Equation (5) can be expressed as the covariant divergence

$$\nabla_a J^a = \frac{1}{2} {}^*RR \quad (6)$$

of the CS topological current ($\Gamma^a{}_{bc}$ are the Christoffels),

$$J^a := \epsilon^{abcd} (\Gamma^n{}_{bm} \partial_c \Gamma^m{}_{dn} + \frac{2}{3} \Gamma^n{}_{bm} \Gamma^m{}_{cl} \Gamma^l{}_{dn}), \quad (7)$$

hence the name ‘‘Chern-Simons modified gravity.’’

The modified field equations are obtained by varying the action with respect to the metric. Using

$$\delta R^b{}_{acd} = \nabla_c \delta \Gamma^b{}_{ad} - \nabla_d \delta \Gamma^b{}_{ac} \quad (8)$$

and

$$\delta \Gamma^b{}_{ac} = \frac{1}{2} g^{bd} (\nabla_a \delta g_{dc} + \nabla_c \delta g_{ad} - \nabla_d \delta g_{ac}), \quad (9)$$

the variation of the action is ($G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R$)

$$\begin{aligned} \delta S = & -\kappa \int d^4x \sqrt{-g} (G^{ab} + C^{ab}) \delta g_{ab} \\ & + \text{boundary terms.} \end{aligned} \quad (10)$$

The symmetric, traceless tensor C^{ab} is given by

$$C^{ab} := (\nabla_c \theta) \epsilon^{cde(a} \nabla_e R^b)_{d} + (\nabla_c \nabla_d \theta) {}^*R^{d(ab)c}. \quad (11)$$

Surface terms that arise from repeated integration by parts are collected in the second line of Eq. (10). They will be studied in detail below.

Provided the boundary terms are dealt with appropriately, the modified field equations can be written as

$$R_{ab} + C_{ab} = 0. \quad (12)$$

If θ is not a constant then the contracted Bianchi identities applied to (12) imply the so-called Pontryagin constraint

$${}^*RR = 0. \quad (13)$$

The main qualitative difference to general relativity is the emergence of first derivatives of the Ricci tensor or, equivalently, of third derivatives of the metric in the equations of motion. The appearance of these higher derivatives has important repercussions on boundary issues.

Turning to the second line of Eq. (10), we are specifically interested in boundary terms involving normal derivatives of the metric variation. This is a small subset of the terms that appear when varying quantities like the action or extrinsic curvature, so we use a special notation to isolate them. Equivalence between two quantities up to

‘‘irrelevant terms’’ is denoted by \simeq . By definition ‘‘irrelevant terms’’ are bulk terms that are not total derivatives, or boundary terms that vanish when Dirichlet boundary conditions are imposed on the metric.

Before dealing with the modified action (4) we review the origin of the GHY boundary term. The variation of the Einstein-Hilbert action is

$$\begin{aligned} \delta S_{\text{EH}} &= \kappa \int d^4x \sqrt{-g} (g^{ab} \delta R_{ab} - G^{ab} \delta g_{ab}) \\ &\simeq \kappa \int d^4x \sqrt{-g} g^{ab} (\nabla_c \delta \Gamma^c{}_{ab} - \nabla_b \delta \Gamma^c{}_{ac}). \end{aligned} \quad (14)$$

We assume from now on that the boundary is a hypersurface with spacelike outward-pointing unit vector n^a , i.e., $n^a n_a = 1$ [11]. Then the induced metric on the boundary is given by

$$h_{ab} = g_{ab} - n_a n_b. \quad (15)$$

The extrinsic curvature is the Lie derivative of $\frac{1}{2} h_{ab}$ along n^a

$$K_{ab} = \frac{1}{2} \mathcal{L}_n h_{ab} = h_a^c h_b^d \nabla_c n_d, \quad (16)$$

with trace $K = K^a{}_a = \nabla_a n^a$. The term in the variation of the extrinsic curvature relevant to our calculations is given by

$$\delta K_{ab} \simeq \frac{1}{2} h_a^c h_b^d n^e \nabla_e \delta g_{cd}. \quad (17)$$

Applying Eqs. (9) and (15) to Eq. (14) and comparing with Eq. (17) yields

$$\delta S_{\text{EH}} \simeq -2\kappa \delta \int d^3x \sqrt{h} K. \quad (18)$$

This leads to the familiar result that the action

$$S_{\text{EH}} + S_{\text{GHY}} = \kappa \int d^4x \sqrt{-g} R + 2\kappa \int d^3x \sqrt{h} K \quad (19)$$

has a well-defined Dirichlet boundary problem [2,3].

We now perform a similar analysis for the second term in Eq. (4). Applying Eq. (8) to Eq. (4) yields

$$\delta S_{\text{CS}} \simeq -\kappa \int d^4x \sqrt{-g} \nabla_c (\theta {}^*R_a{}^{bcd} \delta \Gamma^a{}_{bd}). \quad (20)$$

To proceed we need a 3 + 1 decomposition of all tensors with respect to the induced metric and the normal vector. For simplicity we introduce an adapted coordinate system where the shift vector vanishes and the lapse function is unity, but the final result will be given in a manifestly covariant form. We denote tangential indices by i, j, \dots and indices contracted with the normal vector by \mathbf{n} . With respect to this decomposition we obtain ${}^*R_a{}^{bnd} \delta \Gamma^a{}_{bd} = {}^*R_j{}^{kni} \delta \Gamma^j{}_{ki} + 2 {}^*R_{\mathbf{n}}{}^{jni} \delta \Gamma^{\mathbf{n}}{}_{ji} \simeq 2 {}^*R_{\mathbf{n}}{}^{jni} \delta \Gamma^{\mathbf{n}}{}_{ji}$ and Eq. (20) reduces to

$$\delta S_{\text{CS}} \simeq -2\kappa \int d^4x \sqrt{-g} \nabla_{\mathbf{n}} (\theta {}^*R_{\mathbf{n}}{}^{inj} \delta \Gamma^{\mathbf{n}}{}_{ij}). \quad (21)$$

By virtue of Eq. (9) this further simplifies to

$$\delta S_{CS} \approx -\kappa \int d^3x \sqrt{h} \theta {}^*R^{innj} \nabla_n \delta g_{ij}. \quad (22)$$

The decomposition of the dual Riemann tensor ${}^*R^{innj} = \frac{1}{2} \epsilon^{nkl} R^{in}_{kl}$ requires us to calculate R^{in}_{kl} . Let D_i be the covariant derivative along the boundary whose connection γ^i_{jk} is torsionfree and compatible with the induced metric h_{ij} . Then the Codazzi equation $R_{nijk} = D_k K_{ij} - D_j K_{ik}$ together with the variation equation (17) can be used to rewrite Eq. (22) as

$$\delta S_{CS} \approx 2\kappa \int d^3x \sqrt{h} \theta \epsilon^{ijk} (D_j K_i^l) \delta K_{kl}, \quad (23)$$

where $\epsilon^{ijk} = \epsilon^{nijk}$. The expression (23) can be cancelled by adding the following boundary term to the action

$$S_{bCS} = \kappa \int d^3x \sqrt{h} \theta \epsilon^{ijk} K_i^l D_j K_{kl}, \quad (24)$$

i.e., $\delta S_{CS} \approx -\delta S_{bCS}$. With the abbreviation

$$CS(K) := {}^*K^{jkl} D_j K_{kl} = \frac{1}{2} \epsilon^{ijk} K_i^l D_j K_{kl} \quad (25)$$

we obtain the result announced in Eq. (3). The new boundary term depends only on the traceless part of the extrinsic curvature, and is therefore complementary to the GHY term in (19). The result (24) might have been anticipated on the grounds that it is required by the index theorem for manifolds with boundary, cf. e.g. Sec. 8 in Ref. [12].

The abbreviation ‘‘CS’’ emphasizes the fact that (25) resembles an Abelian Chern-Simons term. It should be contrasted with the gravitational CS term in three dimensions [13]

$$CS(\gamma) = \frac{1}{2} \epsilon^{ijk} (\gamma^l_{im} \partial_j \gamma^m_{kl} + \frac{2}{3} \gamma^l_{im} \gamma^m_{jp} \gamma^p_{kl}), \quad (26)$$

constructed from the intrinsic connection γ^i_{jk} . It is interesting to note that the terms comprising the CS modification are equivalent to

$$\begin{aligned} & \frac{1}{4} \int d^4x \sqrt{-g} \theta {}^*RR + 2 \int d^3x \sqrt{h} \theta CS(K) \\ &= -\frac{1}{2} \int d^4x \sqrt{-g} (\nabla_a \theta) J^a + \int d^3x \sqrt{h} \theta CS(\gamma), \end{aligned} \quad (27)$$

where J^a is the topological current defined in Eq. (7).

We conclude that the full action for CS modified gravity is given by

$$\begin{aligned} S &= S_{EH} + S_{CS} + S_{GHY} + S_{bCS} \\ &= \kappa \int d^4x \sqrt{-g} \left(R + \frac{1}{4} \theta {}^*RR \right) \\ &\quad + 2\kappa \int d^3x \sqrt{h} (K + \theta CS(K)). \end{aligned} \quad (28)$$

The action (28) has a well-defined Dirichlet boundary-value problem. This is our main result. We present now

Eq. (28) in a manifestly covariant form:

$$\begin{aligned} S &= \kappa \int d^4x \sqrt{-g} \left(R + \frac{1}{4} \theta {}^*RR \right) \\ &\quad + 2\kappa \int d^3x \sqrt{h} \left(K + \frac{1}{2} \theta n_a \epsilon^{abcd} K_b^e \nabla_c K_{de} \right) \\ &\quad + \kappa \int d^3x \sqrt{h} \mathcal{F}(h_{ab}, \theta). \end{aligned} \quad (29)$$

The last line is an additional term $S_{\mathcal{F}}$ that is intrinsic to the boundary. This ‘‘boundary counterterm’’ does not affect the Dirichlet boundary-value problem, and is necessary for a well-defined variational principle when the boundary is removed to spatial infinity [14–16].

The boundary terms in Eq. (29) are required for self-consistency of the theory and a prerequisite to a Hamiltonian formulation. They also contribute to the semi-classical approximation of the thermodynamical partition function Z in the Euclidean path integral approach [17,18]:

$$Z \approx \exp[-(S_{CS} + S_{GHY} + S_{bCS} + S_{\mathcal{F}})|_{\text{on-shell}}]. \quad (30)$$

A prime application where these boundary terms are important is black hole thermodynamics. To investigate this issue we make the ansatz

$$\mathcal{F} = \mathcal{F}_0(h_{ab}) + \Delta \mathcal{F}(h_{ab}, \theta), \quad (31)$$

where \mathcal{F}_0 is the boundary counterterm required by general relativity and $\Delta \mathcal{F}$ is a new contribution that must be linear in θ [19]. The contributions to the on-shell action that differ from general relativity are $\Delta S = S_{CS} + S_{bCS} + S_{\Delta \mathcal{F}}$:

$$\begin{aligned} \Delta S|_{\text{on-shell}} &= \frac{\kappa}{4} \int d^4x \sqrt{-g} \theta {}^*RR \\ &\quad + \kappa \int d^3x \sqrt{h} (2\theta CS(K) + \Delta \mathcal{F}(h_{ab}, \theta)). \end{aligned} \quad (32)$$

The bulk term in Eq. (32) vanishes due to the constraint (13). Therefore, all new contributions to the on-shell action are due to the boundary terms in the second line.

An interesting subtlety occurs when $\theta = \theta_0$ is a constant. In that case the field equations reduce to the Einstein equations, and the constraint (13) no longer applies. However, the on-shell action still receives a contribution from the CS modification,

$$\Delta S|_{\text{on-shell}} = -4\pi^2 \kappa \theta_0 \tau(\mathcal{M}) + \kappa \theta_0 \int d^3x \sqrt{h} \Delta \mathcal{F}(h_{ab}), \quad (33)$$

where

$$\begin{aligned} \tau(M) &= \frac{1}{32\pi^2} \int_{\mathcal{M}} d^4x \epsilon^{cdef} R^a_{bef} R^b_{acd} \\ &\quad - \frac{1}{4\pi^2} \int_{\partial \mathcal{M}} d^3x \epsilon^{ijk} K_i^l D_j K_{kl} \end{aligned} \quad (34)$$

is the signature index for a manifold \mathcal{M} with boundary $\partial\mathcal{M}$ (for a list of examples with $\tau \neq 0$ see Table D.1 in Ref. [12]). Since $\tau(\mathcal{M})$ is just some (integer) number, it does not modify black hole thermodynamics.

The result (32) holds in general, but there may be additional simplifications for specific black hole solutions. For instance, the Schwarzschild solution persists in CS modified gravity [6] (at least for certain choices of the scalar field θ [8]), because the two tensors R_{ab} and C_{ab} vanish separately and thus the field equations (12) are fulfilled. We consider now the Schwarzschild black hole in CS modified gravity and assume that the boundary is a hypersurface of constant surface area. Because it is possible to simultaneously diagonalize the metric and the extrinsic curvature, the expression $\text{CS}(K)$ defined in Eq. (25) vanishes identically. Thus, the only remaining new contribution to the on-shell action comes from $\Delta\mathcal{F}$. However, since we have just shown that all terms linear in θ vanish identically we deduce that also the quantity $\Delta\mathcal{F}$ must vanish. We conclude that the thermodynamics of the Schwarzschild black hole in CS modified gravity is the same as the thermodynamics of the Schwarzschild black hole in general relativity. The same conclusion applies to other black hole solutions that solve the Einstein equations and have vanishing boundary term $\text{CS}(K) = 0$. In order to find some physical effect of our boundary term (24) it is suggestive to consider rotating black hole solutions. Unfortunately, little is known about such solutions, see Ref. [8] for a recent analysis. The only rotating black hole solution without pathologies found in that work is the Aichelburg-Sexl boosted Kerr solution with line element

$$ds^2 = -2dudv - h_0\delta(u)\ln\rho^2 + \rho^2d\phi^2 + d\rho^2 \quad (35)$$

and scalar field

$$\theta = \lambda v, \quad (36)$$

where h_0, λ are some nonvanishing constants. Evaluating the on-shell action (28) on the solution (35) and (36) we find that both bulk contributions vanish and also the new boundary contribution $\text{CS}(K)$ vanishes for a $\rho = \text{const}$

hypersurface. This is so, because the only nonvanishing components of extrinsic curvature are $K_{uu} = -h_0\delta(u)/\rho$ and $K_{\phi\phi} = \rho$. Clearly, the Aichelburg-Sexl boosted Kerr solution is too simple to yield a novel effect. Once a physically relevant rotating black hole solution is found our results above can be applied to discuss their thermodynamical properties and then we expect novel effects from our boundary term (24).

Another interesting application of our analysis is that it provides a shortcut to an effective field theory of single field inflation. Namely, after writing down all terms with four derivatives one obtains a correction to the action containing ten terms (see Eq. (3) in Ref. [9]). Requiring a well-defined Dirichlet boundary-value problem eliminates seven of these terms. The remaining three terms are identical to the ones contained in Eq. (6) of Ref. [9] and include the Gauss-Bonnet term studied in Ref. [5] and the gravitational Chern-Simons term studied in the present work.

In this paper we constructed the analog of the Gibbons-Hawking-York term for Chern-Simons modified gravity and showed that the action (29) has a well-defined Dirichlet boundary-value problem. However, for physically interesting boundary conditions this does not guarantee a well-defined variational problem. Even asymptotically flat boundary conditions require additional surface terms in the action [14–16]. It will be interesting to investigate the structure of these terms once nontrivial rotating black hole solutions of the theory are known.

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