Exact string black hole behind the hadronic Rindler horizon?

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The recently suggested interpretation P. Castorina, D. Kharzeev, and H. Satz, [Eur. Phys. J. C **52**, 187 (2007).] of the universal hadronic freeze-out temperature T_f ($\simeq 170$ MeV)—found for all high-energy scattering processes that produce hadrons e^+e^-, pp, $p\bar{p}$, πp, etc., and NN' (heavy-ion collisions)—as an Unruh temperature triggers here the search for the gravitational black hole (BH) that in its near-horizon approximation better *simulates* this hadronic phenomenon. To identify such a BH we begin our gravity-gauge theory phenomenologies matching by asking the question: which BH behind that Rindler horizon could reproduce the experimental behavior of $T_f(\sqrt{s})$ in NN', where $\sqrt{s}$ is the collision energy? Provided certain natural assumptions hold, we show that the exact string BH turns out to be the best candidate (as it fits the available data on $T_f(\sqrt{s})$) and that its limiting case, the Witten BH, is the unique candidate to explain the constant T_f for all elementary scattering processes at large energy. We also are able to propose an effective description of the screening of the hadronic string tension $\sigma(\mu_b)$ due to the baryon density effects on T_f.

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I. INTRODUCTION

Relativistic Heavy Ion Collider experimental data strongly suggest that, above the critical temperature T_c—and up to $2.5T_c$—QCD is a strongly interacting system of quarks and gluons (for a brief review see, e.g., [1])—a picture confirmed by lattice simulations [2,3]. Hence, standard perturbative techniques fail to describe such a system—similar to a liquid with small shear viscosity η [4–6]—and various attempts have been proposed. As part of that, the anti–de Sitter/conformal field theory (AdS/CFT) correspondence [7] recently came to the forefront as it predicts a universal bound on the ratio $\eta/\mathcal{S}$, with $\mathcal{S}$ the entropy density, given by [8–10] $\eta/\mathcal{S} \geq 1/4\pi$, close to the value obtained by fitting the relativistic heavy-ion collisions data by hydrodynamical models and in QCD lattice simulations [11]. Moreover, it has also been applied to evaluate the jet quenching parameter [12–15].

The previous "top-down" results are based on the AdS/CFT gravity-gauge theory duality between strings and supersymmetric SU(N) Yang-Mills theory in the limit of large t' Hooft coupling. The relation of these models with QCD at finite temperature is suggestive, but by no means obvious.

In this paper, we shall follow a "bottom-up" approach, instead. We ask the double-sided question: is QCD a good *analog system* of a black hole (BH)? Or, conversely, is there a specific BH whose thermodynamics *simulates* well QCD thermodynamics?

Our program starts by identifying the BH analog system and takes as initial inputs the proposals of Ref. [16]: i) that at high energy the universal hadronic freeze-out temperature $T_f \simeq 170$ MeV—obtained by statistical analysis of hadronic abundances in all collisions e^+e^-, pp, $p\bar{p}$, πp, etc, including nucleus-nucleus scattering [17–21]—is an Unruh temperature T_U

$$T_f|_{\text{large}\sqrt{s}} = T_U = \frac{a}{2\pi} = \sqrt{\frac{\sigma}{2\pi}} \simeq 170 \text{ MeV}, \qquad (1)$$

where $\sqrt{s} \geq 20$ GeV is the energy of the collision, a is the *deceleration* of quarks and antiquarks typical of the hadronic production mechanism, and $\sigma \simeq 0.18$ GeV^2 is the QCD string tension; ii) that the associated Rindler horizon can be identified with the "color-blind" horizon dynamically produced by the color-charge confinement during the $q\bar{q}$ pair productions.

In this picture the hadrons produced are formed by quarks and antiquarks, Rindler-Unruh quanta excited out of the QCD vacuum, that are "born in equilibrium" (in Hagedorn's words). This means that the hadron abundances in the final state follow a thermal distribution not because partons rescatter, but because of the random distribution of quarks and antiquarks entangled in such a vacuum. This mechanism of thermalization is encountered each time quantum fields are near a (event) horizon; hence, the vacuum is a condensate of entangled quanta living on the two (causally disconnected) sides (for a case simpler than QCD see, e.g., [22] and also [23,24]).

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According to that approach, the universal temperature T_f found in all scattering processes at large $\sqrt{s}$ is understood as a constant Unruh temperature. From now on we identify T_f with T_U. For heavy-ion collisions T_f could depend on other dynamical parameters of the produced system. For instance, experimental data show that T_f depends on the collision energy for $\sqrt{s} \leq 17$ GeV [17–21] and on the baryon chemical potential μ_b. These dependences are strongly correlated, since the limit of large energy corresponds to zero baryon chemical potential [25].

It is at this point that we venture into the analogy with a gravitational system (a BH) motivated by the well-known correspondences between acceleration, Rindler horizon, and BH horizon. Namely, we consider this hadronic Rindler spacetime as the near-horizon approximation of some BH spacetime and pose the question: which BH?

Of course, without further input there is no unique answer to this question, since many different BHs have the same near-horizon approximation. Thus, what we are doing here is to look for a BH that shares with the hadronization mechanism certain thermodynamical properties, possibly to the extent of enabling us to make predictions of certain behaviors, such as the dependence of T_f on the nucleus-nucleus collision energy $\sqrt{s}$ to begin with. In [16] the analogy with a Schwarzschild BH has been attempted, but the latter has unusual thermodynamical properties such as negative specific heat and does not exhibit a Hagedorn temperature. In the next section, we shall identify a BH with the same near-horizon approximation, but with more appropriate thermodynamical properties.

II. SEARCHING FOR THE RIGHT BLACK HOLE

Let us first clarify that, for the hadronization processes we are dealing with, the 2-dimensional (2D) study of the BH analog is more appropriate than the 4-dimensional (4D) case for the following two reasons:

(a) The dynamics of particle production is effectively 2D because it can be described in terms of the evolution in time of the hadronic strings (string breaking), that are one-dimensional objects.

(b) The near-horizon field dynamics is effectively 2D [26–28].

Let us now introduce the basic ingredients of 2D dilaton gravity. It is well known that the Einstein-Hilbert action in 2D does not generate equations of motion. Dilaton gravity is the most natural generalization, which leads to nontrivial dynamics.[1] Its action (dropping surface terms) is

$$I = -\frac{1}{16\pi G_2}\int d^2x\sqrt{-g}[XR - U(X)(\nabla X)^2 - \sigma_d V(X)]. \tag{2}$$

Here, G_2 is the 2D Newton constant, which we shall set to $1/(8\pi)$ henceforth, g is the metric, R the associated Ricci scalar, X is a scalar field (the "dilaton"), and σ_d is a coupling constant of dimension $1/\text{length}^2$. The two functions $U(X)$ and $V(X)$ are unconstrained *a priori* and define what kind of BH solutions (if any) we obtain. The (quasilocal) thermodynamics for generic models (2) has been extensively discussed in Ref. [30]. We recall here some of the main results, which we are going to need below.

First of all, we note that there is a dimensionfull coupling constant in the action σ_d, which controls the strength of the dilaton self-interactions. This feature is in contrast to Einstein gravity, which contains no such coupling constant besides the Newton constant. The classical solutions of the equations of motion descending from (2)

$$X = X(r), \qquad ds^2 = \xi(r)d\tau^2 - \frac{1}{\xi(r)}dr^2, \tag{3}$$

with

$$\partial_r X = e^{-Q(X)}, \qquad \xi(X) = w(X)e^{Q(X)}\left(1 - \frac{2M}{w(X)}\right) \tag{4}$$

are expressed in terms of two model-dependent functions,

$$\begin{aligned} Q(X) &:= Q_0 + \int^X d\tilde{X}U(\tilde{X}), \\ w(X) &:= w_0 - \sigma_d\int^X d\tilde{X}V(\tilde{X})e^{Q(\tilde{X})}. \end{aligned} \tag{5}$$

Here, the integrals are evaluated at X and Q_0, and w_0 are two constants. However, for physical solutions a single constant of integration $M \geq 0$ is enough (cf., e.g., [29]), and the Ricci scalar is given by

$$R = -\frac{\partial^2\xi}{\partial r^2} = -e^{-Q}[w'' + Uw' + U'(w - 2M)]. \tag{6}$$

For $e^Q w = 1$, $R \propto M$, and therefore the ground state solution $M = 0$ is Minkowski space. We call models with this property "Minkowskian ground-state models".

All classical solutions (3) exhibit a Killing vector ∂_τ, so we have a "generalized Birkhoff theorem." Therefore, each solution X_h of $\xi(X_h) = 0$ leads to a Killing horizon. The Hawking temperature is given by surface gravity or, equivalently, the inverse periodicity in Euclidean time

$$T_{\text{Haw}} = \frac{w'(X_h)}{4\pi}. \tag{7}$$

For instance, when the dilaton model is the one obtained by dimensional reduction of the 4D Schwarzschild BH—that is, we use spherical symmetry and consider the angular coordinates as spectators—one obtains $w(X) = \sqrt{2X/G_4}$, $X_h = 2M^2G_4$, where G_4 is the 4D Newton constant, hence,

[1]Other gravity actions, like nonlinear actions in the Ricci scalar or actions that introduce torsion and/or nonmetricity, can be reformulated as dilaton gravity actions, so our Ansatz is rather general. For a review, cf., e.g., [29].

$T_{\rm Haw} = w'(X_h)/4\pi = (8\pi G_4 M)^{-1}$, the well-known result of Hawking.

To study the thermodynamical properties (see Ref. [30]) one considers the 2D BH in a cavity with boundaries at $X = X_{\rm cav}$ in contact with a thermal reservoir at the Tolman temperature $T_{\rm cav} = T_{\rm Haw}/\sqrt{\xi(X_{\rm cav})}$ (i.e., the blue-/red-shifted temperature). In the limit $X_{\rm cav} \to \infty$, for Minkowskian ground-state models, the free energy is

$$F = M - T_{\rm Haw} S_{\rm BH}, \tag{8}$$

where

$$S_{\rm BH} = 2\pi X_h \tag{9}$$

is the Bekenstein-Hawking entropy [31–35], independent from the location of the cavity wall and just sensitive to local properties of the horizon, as it should be. To make contact again with well-known results we might consider once more the spherically symmetric 4D Schwarzschild BH reduced to 2D and use the previous result $X_h = 2M^2 G_4$, which gives $S_{\rm BH} = 2\pi X_h = 4\pi G_4 M^2$. Recalling that the Schwarzschild radius is $r_h = 2G_4 M$ and that the area of the event horizon is $A_{\rm BH} = 4\pi r_h^2$, we have $S_{\rm BH} = A_{\rm BH}/4G_4$, the well-known result of Bekenstein and Hawking.

We have now the necessary tools on the gravity side to focus on the search for our BH. Because we demand the Minkowski ground state property, the function Q is determined uniquely once the function w is known. Therefore, our BH is identified by constructing w with the phenomenological requirements from the Rindler hadronization process. Namely, we require that

(a) The BH mass is proportional to the energy of the collision

$$M = \gamma\sqrt{s}, \tag{10}$$

where γ is some numerical coefficient.
This requirement relies on the fact that since the Hawking temperature (7) depends on the BH mass M, also the near-horizon approximation (the Rindler description) and therefore the Unruh hadronization temperature must depend on M. We know that the hadronization temperature depends on energy $\sqrt{s}$, hence, it is natural to identify it with M.

(b) The coupling constant σ_d in (2) coincides with the string tension σ ($\sigma = \sigma_d$).
Indeed, the string tension σ is a fixed dimensionfull parameter, and there is only one such parameter available in (2), namely, σ_d.

(c) The Hawking temperature corresponds to the Unruh temperature, i.e., to the hadronization freeze-out temperature

$$T_{\rm Haw}(\sqrt{s}, \sigma) = T_f(\sqrt{s}, \sigma) \tag{11}$$

for all values of the energy $\sqrt{s}$ and for a given value of σ.
This requirement relies on the detailed analysis of Ref. [16] already mentioned.

(d) The BH partition function diverges at a given temperature, say T_c that, at $\mu_b = 0$, we identify in the following way:

$$T_c = \lim_{\mu_b \to 0} T_f = \lim_{\sqrt{s}\to\infty} T_f = \sqrt{\frac{\sigma}{2\pi}}, \tag{12}$$

where all limits are supposed to be sufficiently smooth.
This point is motivated by the fact that massless QCD at finite temperature and $\mu_b = 0$ has a deconfining first -order phase transition. Moreover, at zero baryon density, the critical temperature is associated with the QCD string breaking, i.e., with the Unruh hadronization mechanism. Another motivation for this requirement will be given later discussing the finite density effects.

These points are not sufficient to identify the BH. They constrain, though, the class of allowed models severely. Point d in the list implies that $T_{\rm Haw}$ must be bounded from above as a function of $\sqrt{s}$. Indeed, at that value of the temperature (T_c) the system undergoes a phase transition and the "hadronic Rindler horizon description" is no longer applicable, hence, our BH analog description also must break down there. This requirement excludes most of the well-known BHs, such as Schwarzschild or Reissner-Nordström in any dimension, which have no such a critical temperature. Furthermore, from the phenomenological analysis of the nucleus-nucleus scattering [25] the behavior of T_f at large but finite $\sqrt{s}$ turns out to be

$$T_{\rm Haw} = T_f \simeq T_c\left(1 - \frac{\sqrt{s_0}}{\sqrt{s}} + \mathcal{O}(1/s)\right). \tag{13}$$

This is consistent with the fourth requirement, but slightly stronger because it contains also information about the next-to-leading order term in a large s expansion.

Let us now consider first the leading order term T_c. Noticeably, this establishes a *unique* asymptotic BH model: since $T_{\rm Haw}$ to leading order must be given by the constant T_c, we can deduce from (7) that the function w must be linear in X in the limit of large M. The unique BH model that does the job is known as "Witten BH" [36]

$$w(X) = \sqrt{8\pi\sigma_d}X, \tag{14}$$

and arises as an approximate solution in 2D string theory to lowest order in α'. Its Hawking temperature is then $T_{\rm Haw} = \sqrt{\sigma_d/2\pi}$, and by the second and third phenomenological requirements we get

$$T_{\rm Haw} = T_f = \sqrt{\frac{\sigma}{2\pi}}. \tag{15}$$

Furthermore, the partition function of the Witten BH diverges,[2] hence, in particular, it is divergent at T_f. This we regard as an instance of the fulfillment of the fourth requirement. All this leads us to recognize the Witten BH as the unique BH reproducing the behavior of the freeze-out temperature for all the scattering processes at high energy considered in [16] except heavy-ion collisions (e^+e^-, pp, $p\bar{p}$, πp, etc.). That is to say that the Witten BH is the BH we wanted in all the high-energy scattering processes when the freeze-out temperature is constant and equal to the critical temperature.

For heavy-ion collisions we are therefore looking for a deformation of the Witten BH that, at finite values of s, is consistent with (13). Since the Witten BH emerged as the unique approximation to lowest order in α', the only natural candidate is the exact solution in 2D string theory to all orders in α', which is known as the "exact string BH" [38]. Its target space action was constructed in [39]. Like the Witten BH, it is a Minkowskian ground state model given by

$$w(X) = \sqrt{8\pi\sigma_d}\left(\sqrt{\rho^2+1}+1\right), \qquad e^{Q(X)}w(X) = 1, \tag{16}$$

where the canonical dilaton X is related to a new field ρ by

$$X = \rho + \operatorname{arcsinh}\rho. \tag{17}$$

Obviously, for $X \to \infty$ (16) with (17) asymptotes to (14).

Its Hawking temperature is given by [39]

$$T_{\rm Haw} = \sqrt{\frac{\sigma_d}{2\pi}}\sqrt{1 - \frac{2\sqrt{2\pi\sigma_d}}{M}}, \tag{18}$$

where M is the Arnowitt-Deser-Misner mass of the BH.[3]

[2]As discussed in [34]we consider the Witten BH in a cavity whose wall is located at some fixed value of the dilaton $X = X_{\rm cav}$. The cavity is in contact with a thermal reservoir at $T = T_{\rm cav}$. Allowing for all paths where the metric is continuous (but not necessarily differentiable) gives a Euclidean partition function Z, which is an infinite sum over instantons. Most of them exhibit a conical defect [37]. For the Witten BH the resulting integral can be exactly solved, giving $Z \propto X_{\rm cav}$ for very large $X_{\rm cav}$. Eventually, we move the cavity wall to infinity—because that is where the asymptotic observer sits, measuring the Hawking temperature—and this means that $Z \to +\infty$. Physically, the reason for this divergence is the singular specific heat of the Witten BH, i.e., the divergence of fluctuations. Another way to put it is to observe that the Witten BH is marginally unstable against decay into conical defects [37].

[3]The mass M is also related to the level k of the current algebra underlying the CFT description of the exact string BH in terms of an $SL(2,\mathbb{R})/U(1)$ gauged Wess-Zumino-Witten model (for a review, cf., e.g., [40]). That k can be seen as a running parameter allowed by the CFT is discussed in [41].

Thus, the exact string BH is a 2D BH fulfilling all the phenomenological requirements: (1) The first condition is satisfied by identifying $\gamma\sqrt{s} = M$; (2) The second requirement is simply $\sigma = \sigma_d$; (3) The third postulate will allow us to make predictions about $T_f(\sqrt{s}, \sigma)$, which we shall discuss in Sec. III; 4. The fourth postulate is met, because (18) obviously is bounded from above by (12).

Having demonstrated that the exact string BH is phenomenologically viable, we address now the issue of uniqueness. We have shown above that asymptotically (for large s) the Witten BH emerges as the *unique* BH model consistent with all requirements. While there is a whole family of models that asymptotes to the Witten BH, we have also noted that from a CFT point of view there is a *unique* BH model that deforms the Witten BH for finite values of s, namely, the exact string BH. In that sense our results are unique.

III. MATCHING THE PHENOMENOLOGICAL RESULTS FOR $T_f(\sqrt{s})$

From the above discussion, Eq. (18) leads to the following prediction for the energy dependence of the freeze-out temperature in heavy-ion collisions

$$T_f(\sqrt{s}) = \sqrt{\frac{\sigma}{2\pi}}\sqrt{1 - \sqrt{\frac{s_0}{s}}}, \tag{19}$$

where $\sqrt{s_0} = 2\sqrt{2\pi\sigma}/\gamma$ is a free parameter. In Fig. 1, we compare Eq. (19) with the phenomenological results of Ref. [25] for the dependence of the freeze-out temperature on the collision energy, for different nuclei, for $\sqrt{s_0} = 2.4$ GeV. At large energy the universal value $T_f \simeq 170$ MeV is obtained.

As previously discussed, the $\sqrt{s}$ dependence of T_f is strongly correlated with its dependence on the baryon chemical potential μ_b [25]. We stress that the μ_b dependence of T_f is different from the μ_b dependence of T_c,

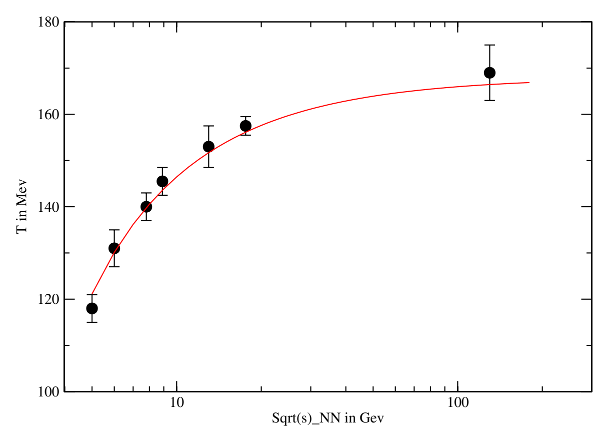

FIG. 1 (color online). Freeze-out temperature versus $\sqrt{s_{NN}}$ from Ref. [25] compared with Eq. (19) for $\sqrt{s_0} = 2.4$ GeV and $T_c = 169$ MeV.

where the corresponding BH partition function diverges [42]. Indeed, at $\mu_b = 0$, the deconfinement temperature is related, in the Hagedorn model [43] or in the dual resonance model [44,45], to the resonance formation and decay and therefore string-formation and -breaking is the relevant dynamical mechanism. This means that at $\mu_b = 0$, it is reasonable to consider the freeze-out temperature essentially equal to the temperature at the point of deconfinement. This is another motivation for our assumption 4 in the previous list. At finite μ_b, the interaction does not lead to the formation of resonances but the screening effects and Fermi statistic (at large μ_b) play the most important role. Hence there is no *a priori* reason for $T_f \simeq T_c$. Accordingly, in the language of the 2D BH thermodynamics the μ_b dependence of T_c should be studied by introducing in the dynamics a new conserved U(1) charge, corresponding to the baryon number, and considering the critical line in the $T - \mu_b$ plane.

We now come back to the μ_b dependence of T_f. At a purely phenomenological level it can be described by the empirical relation

$$\mu_b \simeq \frac{c}{\sqrt{s}}, \tag{20}$$

where c is a constant. The approximate relation (20) comes from the statistical analysis of the species abundances in heavy-ions collisions [25]. Thus, by inserting (20) into (19) we obtain

$$T_f(\mu_b) \simeq \sqrt{\frac{\sigma}{2\pi}}\sqrt{1 - \frac{\mu_b}{\mu_b^0}}, \tag{21}$$

where μ_b^0 is a free parameter. In Fig. 2, we compare the prediction of Eq. (21), for $\mu_b^0 = 1.2$ GeV, with the phenomenological analysis. Our curve merely gives a rough estimate of the μ_b dependence of T_f and could be regarded as a theoretical prediction that is only indirectly based on the BH analogy, i.e., via Eq. (20). Keeping these limitations in mind, we nonetheless notice that Eq. (21), if taken at face value, predicts a linear screening of the string tension due to finite density effects in heavy-ions collisions

$$\sigma(\mu_b) \simeq \sigma(1 - \mu_b/\mu_b^0). \tag{22}$$

This behavior of σ, being $\mu_b^0 = 1.2$ GeV, gives as critical quark chemical potential $\mu_q^0 \simeq 400$ MeV.

Finally, we address what happens if we relax the first assumption (10) and allow for a more general relation between BH mass and collision energy,

$$M = \Gamma(\sqrt{\sigma/s})\sqrt{s}, \tag{23}$$

where Γ is a free function with the only constraint that it asymptotes to a constant γ for large $\sqrt{s}$. On dimensional grounds, this is the most general Ansatz possible. The asymptotic condition ensures that in the limit of vanishing string tension the BH mass scales with the appropriate power of $\sqrt{s}$.

With the more general assumption (23) the conclusions of Sec. 1 still hold without any essential change: the Witten BH is the unique asymptotic BH model, and the exact string BH its natural deformation at finite values of $\sqrt{s}$. In particular, Eq. (18) for the Hawking temperature still applies.

However, Eq. (19) is replaced by

$$T_f(\sqrt{s}) = \sqrt{\frac{\sigma}{2\pi}}\sqrt{1 - \frac{2\sqrt{2\pi\sigma}}{\Gamma(\sqrt{\sigma/s})}\frac{1}{\sqrt{s}}}, \tag{24}$$

which only for large $\sqrt{s}$ coincides with Eq. (19). The phenomenological implications of Eq. (24) are more problematic to handle in the case of NN' scattering.

This is so because, while we are focusing here on high-energy scattering processes ($\sqrt{s}$ beyond 1 GeV) we deal with two distinct energy regimes: (i) what we might call the asymptote (beyond 20 GeV) and (ii) below that asymptote. For all scattering processes but NN', there is one single T_f for both regimes (i.e., T_f is $\sqrt{s}$ independent) and the Hawking temperature for the Witten BH has the same behavior. Thus, the Witten BH is phenomenologically viable also with assumption (23). In the case of NN', the interesting energy regime is the second one (above 1 GeV—below 20 GeV), and there we are not at the asymptote, but below. That is why, although $\Gamma \to \gamma$ in the limiting case, we should expect some $\sqrt{s}$ dependent contribution of Γ to M below the asymptote and, being the fit we obtain in Fig. 1 impressively good even a small change of the $M(\sqrt{s})$ behavior (as, for instance, a more modest increase with energy, see, e.g., [46]) would have a big impact on that.

Hence, the simplest Ansatz $\Gamma = \gamma$ employed in Eq. (10)—i.e., that the BH mass is sensitive to the collision energy only and does not depend on the string tension—appears to be the most phenomenologically viable.

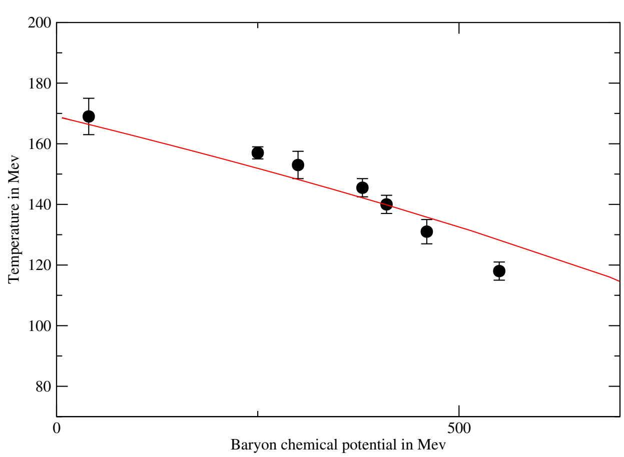

FIG. 2 (color online). Freeze-out temperature versus baryon chemical potential compared with Eq. (21) for $\mu_b^0 = 1.2$ GeV.

IV. CONCLUSIONS

In this work we have further investigated the recent proposal that for all high-energy hadron productions the universal hadronic freeze-out temperature $T_f \simeq 170$ MeV can be understood as an Unruh temperature. Here, we identified the exact string BH (for the heavy-ion collisions) and its limiting case, the Witten BH (for all the other processes), as the unique BHs whose thermodynamical properties well simulate some thermodynamical properties of hadronization. In particular, exploiting the behavior of Hawking temperature for the exact string BH we provided an analytical expression for the energy dependence of the freeze-out temperature in nucleus-nucleus scattering, which gives a very good fit of the experimental data obtained via the statistical hadronization model. We also proposed a linear screening of the hadronic string tension as a function of the baryon chemical potential based on an empirical relation.

In view of taking this work as a first step of a bottom-up program of finding a BH whose thermodynamics could simulate finite temperature QCD, it is perhaps suggestive to recall here some of the thermodynamical properties of the exact string BH: The partition function diverges at T_c; The specific heat is positive; The third law of thermodynamics holds, i.e., the specific heat vanishes linearly with temperature as the latter approaches zero.

Finally, let us merely report here the following coincidence. Besides the exact string BH that we advocate here there is another BH that has been applied to QCD, namely, the well-known BH in AdS_5. We can look at the near singularity behavior of the AdS_5 BH and compare it with the near singularity behavior of the T dual of the exact string BH. It turns out [39], that they have the same behavior.

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