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First-forbidden continuum- and bound-state β^--decay rates of bare $^{205}Hg^{80+}$ and $^{207}Tl^{81+}$ ions

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We analyze the decay rates $\lambda_{\beta^-_c}$ and $\lambda_{\beta^-_b}$ of the continuum- and bound-state β^--decays for bare $^{205}Hg^{80+}$ and $^{207}Tl^{81+}$ ions. For the ratio of the decay rates $R_{b/c} = \lambda_{\beta^-_b}/\lambda_{\beta^-_c}$ we obtain the values $R_{b/c} = 0.161$ and $R_{b/c} = 0.190$ for bare $^{205}Hg^{80+}$ and $^{207}Tl^{81+}$ ions, respectively. The theoretical value of the ratio $R_{b/c} = 0.190$ for the decays of $^{207}Tl^{81+}$ agrees well with the experimental data $R^{\rm exp}_{b/c} = 0.188(18)$, obtained at GSI. The theoretical ratio $R_{b/c} = 0.161$ for $^{205}Hg^{80+}$ is about 20% smaller than the experimental value $R^{\rm exp}_{b/c} = 0.20(2)$, measured recently at GSI. We give arguments that the nuclear structure of the nuclei in the $^{205}Hg^{80+} \to {}^{205}Tl^{81+}$ transition cannot provide such an enhancement of the ratio. We estimate the values of the continuum- and bound-state β^--decay rates of the bare $^{205}Hg^{80+}$ ions.

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As has been shown in [1], the standard theory of weak interactions of heavy ions [2], allows to describe well the K-shell electron capture (EC) and β^+ decay rate ratios of the H-like and He-like ions $^{140}Pr^{58+}$ and $^{140}Pr^{57+}$, agreeing with the experimental data, obtained at GSI [3], with an accuracy better than 3%.

In this Rapid Communication we apply the standard theory of weak interactions of heavy ions [2] and the technique developed in [1] to the analysis of the decay rates $\lambda_{\beta^-_c}$ and $\lambda_{\beta^-_b}$ of the decays

$$\begin{aligned} {}^{205}\mathrm{Hg}^{80+} &\to {}^{205}\mathrm{Tl}^{81+} + e^- + \tilde{\nu}_e,\\ {}^{205}\mathrm{Hg}^{80+} &\to {}^{205}\mathrm{Tl}^{80+} + \tilde{\nu}_e,\\ {}^{207}\mathrm{Tl}^{81+} &\to {}^{207}\mathrm{Pb}^{82+} + e^- + \tilde{\nu}_e,\\ {}^{207}\mathrm{Tl}^{81+} &\to {}^{207}\mathrm{Pb}^{81+} + \tilde{\nu}_e, \end{aligned} \tag{1}$$

where $^{205}Hg^{80+}$, $^{207}Pb^{82+}$ and $^{205}Tl^{81+}$, $^{207}Tl^{81+}$ are two pairs of bare ions with quantum numbers $I^P = \frac{1}{2}^-$ and $I^P = \frac{1}{2}^+$, respectively, and $^{205}Tl^{80+}$ and $^{207}Pb^{81+}$ are H-like ions. The continuum- and bound-state β^--decays in Eq. (1) satisfy the selection rule $\Delta I^P = 0^-$, which corresponds to the selection rule of the first-forbidden β-decays [4,5]. The bound-state β^--decay to the K-shell is the time reversed orbital K-shell electron capture (EC) decay, which we analyzed in [1]. The main distinction of the bound state β^--decay from the EC-decay is that the bound electron can be not only on the K-shell in the $1s$ state but on any other shells in any excited ns state, the contribution of which is about 20%.

A measurement of the continuum- and bound-state β^- decays of bare $^{207}Tl^{81+}$ ion was reported by Ohtsubo *et al.* [6]. The experimental value of the ratio of the β^--decay rates $R^{\rm exp}_{b/c} = 0.188(18)$ agrees within one standard deviation with the theoretical value $R^{\rm th}_{b/c} = 0.171(1)$ [7], obtained from the theory employing spectra of allowed transitions [6]. Very recently an experimental value $R^{\rm exp}_{b/c} = 0.20(2)$ of the ratio of the bound- and continuum-state β^- decays of bare $^{205}Hg^{80+}$ ions has become known [8]. This has motivated us to carry out the calculation of the first-forbidden continuum- and bound-state β^- decays of bare $^{205}Hg^{80+}$ and $^{207}Tl^{81+}$ ions following the approach used in [1].

For the calculation of the rates of the β^--decays we use the Hamilton density operator [1,2]

$$\mathcal{H}_W(x) = \frac{G_F}{\sqrt{2}} V_{ud}\, [\bar{\psi}_p(x)\gamma^\mu(1 - g_A\gamma^5)\,\psi_n(x)] \times [\bar{\psi}_e(x)\gamma_\mu(1-\gamma^5)\psi_{\nu_e}(x)] + \text{h.c.} \tag{2}$$

with standard notations [1,2,9]. In our calculations the antineutrino is assumed to be a massless Dirac antiparticle as in [1].

The continuum-state β^--decay. In the rest frame of the mother ion the amplitudes of the continuum-state β^--decay are defined by

$$M_{II_z \to I'I'_z} = -{}_{I'I'_z}\langle \tilde{\nu}_e(\vec{k})e^-(\vec{p}_-)d(\vec{q}\,)|\mathcal{H}_W(0)|m(\vec{0})\rangle_{II_z}, \tag{3}$$

where $II_z = \frac{1}{2}, \pm\frac{1}{2}$ and $I'I'_z = \frac{1}{2}, \pm\frac{1}{2}$ determine the spinorial states of the mother m and daughter d ions. The decay rate $\lambda_{\beta^-_c}$ of the continuum-state β^--decay is defined by [1]

$$\lambda_{\beta^-_c} = \frac{1}{2M_m}\int \frac{d^3q}{(2\pi)^3 2E_d}\frac{d^3p_-}{(2\pi)^3 2E_-}\frac{d^3k}{(2\pi)^3 2E_{\tilde{\nu}_e}} \times (2\pi)^4\delta^{(4)}(k + p_- + q - k_m)\; F(Z+1, E_-) \times \frac{1}{2}\sum_{I_z, I'_z, \sigma_-} \left|M_{II_z\to I'I'_z}\right|^2, \tag{4}$$

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where $k = (E_{\bar{\nu}_e}, \vec{k})$, $p_- = (E_-, \vec{p}_-)$, $q = (E_d, \vec{q})$, and $k_m = (M_m, \vec{0})$ are four-momenta of the interacting particles, $F(Z+1, E_-)$ is the Fermi function [5,10]

$$F(Z+1, E_-) = \left(1 + \frac{1}{2}\gamma\right) \frac{4(2Rp_-)^{2\gamma}}{\Gamma^2(3+2\gamma)} e^{+\frac{\pi(Z+1)\alpha E_-}{p_-}} \times \left|\Gamma\left(1 + \gamma + i\,\frac{\alpha(Z+1)E_-}{p_-}\right)\right|^2, \quad (5)$$

with $\gamma = \sqrt{1-((Z+1)\alpha)^2} - 1$, $p_- = \sqrt{E_-^2 - m_e^2}$ and Z is the charge of the mother ion. The summation in Eq. (4) should be carried out over all polarizations of the interacting particles, where $\sigma_- = \pm\frac{1}{2}$ is a polarization of the electron. The antineutrino is polarized parallel to the momentum $\vec{k}$. Following [1], for the amplitudes of the continuum-state β^--decay we get the expressions

$$\begin{aligned}
&M_{\frac{1}{2},+\frac{1}{2}\to\frac{1}{2},+\frac{1}{2}} = \sqrt{2M_m 2E_d}\,\mathcal{M}_{m\to d} \\
&\times \left[\bar{u}_e(\vec{p}_-, \sigma_-)(g_A\gamma^0 - \gamma^3)(1-\gamma^5)v_{\bar{\nu}_e}\left(\vec{k}, +\frac{1}{2}\right)\right], \\
&\times M_{\frac{1}{2},+\frac{1}{2}\to\frac{1}{2},-\frac{1}{2}} = \sqrt{2M_m 2E_d}\,\mathcal{M}_{m\to d} \\
&\times \left[\bar{u}_e(\vec{p}_-, \sigma_-)(\gamma^1 + i\gamma^2)(1-\gamma^5)v_{\bar{\nu}_e}\left(\vec{k}, +\frac{1}{2}\right)\right], \\
&\times M_{\frac{1}{2},-\frac{1}{2}\to\frac{1}{2},+\frac{1}{2}} = \sqrt{2M_m 2E_d}\,\mathcal{M}_{m\to d} \\
&\times \left[\bar{u}_e(\vec{p}_-, \sigma_-)(\gamma^1 - i\gamma^2)(1-\gamma^5)v_{\bar{\nu}_e}\left(\vec{k}, +\frac{1}{2}\right)\right], \\
&\times M_{\frac{1}{2},-\frac{1}{2}\to\frac{1}{2},-\frac{1}{2}} = \sqrt{2M_m 2E_d}\,\mathcal{M}_{m\to d} \\
&\times \left[\bar{u}_e(\vec{p}_-, \sigma_-)(g_A\gamma^0 + \gamma^3)(1-\gamma^5)v_{\bar{\nu}_e}\left(\vec{k}, +\frac{1}{2}\right)\right],
\end{aligned} \quad (6)$$

where $\bar{u}_e$ and $v_{\bar{\nu}_e}$ are Dirac bispinors of the electron and the antineutrino, $\mathcal{M}_{m\to d}$ is the nuclear matrix element defined by

$$\mathcal{M}_{m\to d} = -\frac{G_F}{\sqrt{2}}\,V_{ud}\int d^3x\,\Psi_d^*(r)\,\Psi_m(r), \quad (7)$$

where $\Psi_d(r)$ and $\Psi_m(r)$ are the wave functions of the daughter and mother nuclei. For the numerical calculations we assume that the product $\Psi_d^*(r)\,\Psi_m(r)$ has the Woods-Saxon shape [1]. Substituting the amplitudes Eq. (6) into Eq. (4) and carrying out the summation over polarisations we get

$$\lambda_{\beta_c^-} = \left(3 + g_A^2\right)\frac{|\mathcal{M}_{m\to d}|^2}{\pi^3} f(Q_{\beta_c^-}, Z+1), \quad (8)$$

where the Fermi integral $f(Q_{\beta_c^-}, Z+1)$ is

$$f(Q_{\beta_c^-}, Z+1) = \int_{m_e}^{Q_{\beta_c^-}+m_e} (Q_{\beta_c^-} + m_e - E_-)^2 \times F(Z+1, E_-)\sqrt{E_-^2 - m_e^2}\,E_- dE_-. \quad (9)$$

The Q-values of the continuum-state β^--decays are equal to $Q_{\beta_c^-} = 1515.734$ keV and $Q_{\beta_c^-} = 1407.471$ keV for $^{205}Hg^{80+}$ and $^{207}Tl^{81+}$, respectively [10–12].

The bound-state β^--decay. In the bound-state β^--decay the electron in the final state can be in any bound ns-state. Due to hyperfine interactions the ns-state is split into two hyperfine states $(ns)_{F=0}$ and $(ns)_{F=1}$ with a total spin $F = 0$ and $F = 1$, respectively [13].

The experimental value of the hyperfine energy level splitting for the $1s$-state $\Delta E_{1s}^{\exp} = E_{(1s)_{F=0}} - E_{(1s)_{F=1}} = -3.24409 \pm 0.00029$ eV [14] agrees well with the theoretical one $\Delta E_{1s}^{\rm th} = -3.275$ eV [13]. The hyperfine splitting $\Delta E_{ns}^{\rm th}$ of the energy level of the excited ns state is related to $\Delta E_{1s}^{\rm th}$ as [13]

$$\Delta E_{ns}^{\rm th} = \frac{\Delta E_{1s}^{\rm th}}{(3+2\gamma)[(n+\gamma)^2 - \gamma(2+\gamma)]^2} \times [2(n+\gamma) + \sqrt{(n+\gamma)^2 - \gamma(2+\gamma)}], \quad (10)$$

where $\gamma = \sqrt{1-((Z+1)\alpha)^2} - 1$ and Z is the charge of the mother ion. The decay rate $\lambda_{\beta_b^-}$ of the bound-state β^--decay into any hyperfine states $(ns)_{F=0}$ and $(ns)_{F=1}$ is defined by

$$\begin{aligned}
\lambda_{\beta_b^-} &= \sum_{n=1}^{\infty}\sum_{F=0,1}\lambda_{\beta_b^-}((ns)_F) = \frac{1}{2M_m} \\
&\times \int \frac{d^3q}{(2\pi)^3 2E_d}\frac{d^3k}{(2\pi)^3 2E_{\bar{\nu}_e}}(2\pi)^4\delta^{(4)}(q+k-k_m) \\
&\times \frac{1}{2}\sum_{n=1}^{\infty}\sum_{F=0,1}\sum_{I_z,M_F}\left|M^{(n)}_{II_z\to FM_F}\right|^2,
\end{aligned} \quad (11)$$

where $k = (E_{\bar{\nu}_e}, \vec{k})$, $q = (E_d, \vec{q})$ and $k_m = (M_m, \vec{0})$ are four-momenta of the interacting particles. In the rest frame of the mother ion the amplitudes of the bound-state β^--decay are determined by

$$M^{(n)}_{II_z\to FM_F} = -{}_{FM_F}\langle\bar{\nu}_e(\vec{k})d^{(n)}(\vec{q})|\mathcal{H}_W(0)|(\vec{0})\rangle_{II_z}. \quad (12)$$

Following [1] for the amplitudes of the transitions $II_z \to FM_F$ we get the expressions

$$\begin{aligned}
M^{(n)}_{\frac{1}{2},-\frac{1}{2}\to 0,0} &= \sqrt{2M_m 2E_d E_{\bar{\nu}_e}}(3-g_A)\sqrt{2}\,\mathcal{M}_{m\to d}\langle\psi_{ns}^{(Z+1)}\rangle, \\
M^{(n)}_{\frac{1}{2},+\frac{1}{2}\to 1,+1} &= \sqrt{2M_m 2E_d E_{\bar{\nu}_e}}(1+g_A)2\,\mathcal{M}_{m\to d}\langle\psi_{ns}^{(Z+1)}\rangle, \\
M^{(n)}_{\frac{1}{2},-\frac{1}{2}\to 1,0} &= \sqrt{2M_m 2E_d E_{\bar{\nu}_e}}(1+g_A)\sqrt{2}\,\mathcal{M}_{m\to d}\langle\psi_{ns}^{(Z+1)}\rangle,
\end{aligned} \quad (13)$$

where $\langle\psi_{ns}^{(Z+1)}\rangle$ is the wave function of the bound electron in the ns-state, averaged over the nuclear density [1]. The wave function $\psi_{ns}^{(Z+1)}$ is the solution of the Dirac equation [16,17]. Substituting the amplitudes Eq. (13) into Eq. (11) we get the decay rate

$$\begin{aligned}
\lambda_{\beta_b^-} &= (3-g_A)^2|\mathcal{M}_{m\to d}|^2\sum_{n=1}^{\infty}\left|\langle\psi_{ns}^{(Z+1)}\rangle\right|^2\frac{Q_{ns}^2}{2\pi} \\
&\quad + 3(1+g_A)^2|\mathcal{M}_{m\to d}|^2\sum_{n=1}^{\infty}\left|\langle\psi_{ns}^{(Z+1)}\rangle\right|^2\frac{Q_{ns}^2}{2\pi} \\
&= 2(3+g_A^2)\,|\mathcal{M}_{m\to d}|^2\sum_{n=1}^{\infty}\left|\langle\psi_{ns}^{(Z+1)}\rangle\right|^2\frac{Q_{ns}^2}{\pi},
\end{aligned} \quad (14)$$

where Q_{ns} is the Q-value of the β^--decays into the bound ns-state [15].

The ratio of the bound- and continuum-state β^--decays. For the ratio of the bound- and continuum-state β^--decay rates we obtain the following expression:

$$R_{b/c} = \sum_{n=1}^{\infty} \frac{2\pi^2 Q^2_{ns}\big|\langle\psi^{(Z+1)}_{ns}\rangle\big|^2}{f(Q_{\beta^-_c}, Z+1)}, \qquad (15)$$

independent of the nuclear matrix element $\mathcal{M}_{m\to d}$. The numerical values of the ratio of the β^-–decays of bare $^{205}Hg^{80+}$ and $^{207}Tl^{81+}$ ions are

$$^{205}Hg^{80+}: \; R^{\rm th}_{b/c} = 0.161,$$
$$^{207}Tl^{81+}: \; R^{\rm th}_{b/c} = 0.190. \qquad (16)$$

These results are obtained for $R = 1.1A^{1/3}$ fm and the Woods-Saxon shape of the nuclear density with the diffuseness parameter $a = 0.50$ fm [1]. One can show that the variations of the nuclear radius $R = r_0\, A^{1/3}$ and the diffuseness parameter a in the limits $1.1\,\text{fm} \leqslant r_0 \leqslant 1.5\,\text{fm}$ and $0.45\,\text{fm} \leqslant a \leqslant 0.55\,\text{fm}$, respectively, lead to the 0.5% deviations of the ratios $R^{\rm th}_{b/c}$ from the values, given by Eq. (16).

Concluding discussion. Using the standard theory of weak interactions of heavy ions we have calculated the ratios of the decay rates of the bound- and continuum-state β^--decays of bare $^{205}Hg^{80+}$ and $^{207}Tl^{81+}$ ions. These are first-forbidden β-decays [4,5]. Our result for the ratio of the β^--decay rates of the bare $^{207}Tl^{81+}$ ions $R^{\rm th}_{b/c} = 0.190$ agrees well with the mean value of the experimental data $R^{\rm exp}_{b/c} = 0.188(18)$ obtained at GSI [6].

In turn, the ratio of the β^--decay rates of bare $^{205}Hg^{80+}$ ions deviates from the mean value of the experimental data $R^{\rm exp}_{b/c} = 0.20(2)$ by about 20%. Such a discrepancy cannot be attributed to the dependence of the ratio $R_{b/c}$ on the electric charge. Indeed, the replacement of $Z = 80$ by $Z = 81$ changes the ratio of the β^--decay rates $R_{b/c}$ of bare $^{205}Hg^{80+}$ ions to the value $R_{b/c} = 0.167$.

A possible influence of a nuclear structure of interacting nuclei on the ratios of the β^--decay rates of bare $^{207}Tl^{81+}$ and $^{205}Hg^{80+}$ ions should not be tangible. Really, our approach to the calculation of the nuclear matrix elements of the transitions $^{207}Tl^{81+} \to {}^{207}Pb^{82+}$ and $^{205}Hg^{80+} \to {}^{205}Tl^{81+}$ corresponds to the single-particle approximation [18]. Since the numbers of neutrons and protons in the interacting nuclei are very close to the magic numbers $N = 128$ and $Z = 82$, the single-particle approximation should work well [18,19].

Thus, the obtained value $R_{b/c} = 0.161$ of the ratio of the β^--decay rates of the bare $^{205}Hg^{80+}$ ions one can justify as follows. The Q-values $Q_{\beta^-_c} = 1515.734$ keV and $Q_{\beta^-_c} = 1407.471$ keV of the continuum-state β^--decays result in the Fermi integrals $f(Q_{\beta^-_c}, Z+1) = 22.119\,\text{MeV}^5$ and $f(Q_{\beta^-_c}, Z+1) = 17.747\,\text{MeV}^5$ for $^{205}Hg^{80+}$ and $^{207}Tl^{81+}$, respectively. On the other hand the bound-state β^--decay rates scale with Q^2_{ns}, where $Q_{1s} = 1614.557$ keV and $Q_{1s} = 1509.053$ keV for $^{205}Hg^{80+}$ and $^{207}Tl^{81+}$, respectively. This implies that the ratio of the bound- and continuum-state β^--decay rates for bare $^{205}Hg^{80+}$ ions should be a minimum 1.1 times smaller than the ratio of the bound- and continuum-state β^--decay rates for bare $^{207}Tl^{81+}$ ions. This gives $R_{b/c} \sim 0.171$ for the ratio of the β^--decay rates of the bare $^{205}Hg^{80+}$ ions. A reduction of the value $R_{b/c} \sim 0.171$ to $R_{b/c} = 0.161$ is caused by the effective densities of electrons in the ns-states for different $(Z+1)$ values of electric charges of the ions $^{205}Tl^{80+}$ and $^{207}Pb^{81+}$ in the final state of the bound-state β^--decays.

The theoretical decay rates of the continuum- and bound-state β^--decays are expressed in terms of the nuclear matrix element $\int d^3x\, \Psi^*_d(r)\Psi_m(r)$. From the experimental data $\lambda^{\rm exp}_{\beta^-_c} = 2.29(12) \times 10^{-3}\,\text{s}^{-1}$ and $\lambda^{\rm exp}_{\beta^-_b} = 4.29(29) \times 10^{-4}\,\text{s}^{-1}$ [6] of the β^--decay rates of the bare $^{207}Tl^{81+}$ ions, we can estimate the value of the nuclear matrix element $\int d^3x\, \Psi^*_d(r)\Psi_m(r)|_{\rm Tl} = 0.094$, obtained for $R = 1.1\,A^{1/3}$ and $a = 0.50$ fm. The deviations from the value $\int d^3x\, \Psi^*_d(r)\Psi_m(r)|_{\rm Tl} = 0.094$, caused by the variations of the nuclear radius $R = r_0\, A^{1/3}$ and the diffuseness parameter a in the limits $1.1\,\text{fm} \leqslant r_0 \leqslant 1.5\,\text{fm}$ and $0.45\,\text{fm} \leqslant a \leqslant 0.55\,\text{fm}$, make up of about 6.5%.

The value $\int d^3x\, \Psi^*_d(r)\Psi_m(r)|_{\rm Tl} = 0.094$ agrees well with $\int d^3x\, \Psi^*_d(r)\Psi_m(r)|_{\rm Tl} = 0.095$, obtained for $R = 1.1\,A^{1/3}$ from the experimental data $\lambda^{\rm exp}_{\beta^-} = 2.42(1) \times 10^{-3}\,\text{s}^{-1}$ of the β^--decay rate of the neutral atom $^{207}Tl^{0+}$ [12].

Using the experimental data $\lambda^{\rm exp}_{\beta^-} = 2.22(1) \times 10^{-3}\,\text{s}^{-1}$ on the β^--decay rate of the neutral atom $^{205}Hg^{0+}$ [12], we deduce for $R = 1.1\,A^{1/3}$ the value $\int d^3x\, \Psi^*_d(r)\Psi_m(r)|_{\rm Hg} = 0.083$ of the nuclear matrix element of the $^{205}Hg^{80+} \to {}^{205}Tl^{81+}$ transition, which is commensurable with the value $\int d^3x\, \Psi^*_d(r)\Psi_m(r)|_{\rm Tl} = 0.095$ of the nuclear matrix element of the $^{207}Tl^{81+} \to {}^{207}Pb^{82+}$ transition. For $\int d^3x\, \Psi^*_d(r)\Psi_m(r)|_{\rm Hg} = 0.083$ we predict the values of the continuum- and bound-state β^--decay rates of the bare $^{205}Hg^{80+}$ ions $\lambda_{\beta^-_c} = 2.34 \times 10^{-3}\,\text{s}^{-1}$ and $\lambda_{\beta^-_b} = 3.77 \times 10^{-4}\,\text{s}^{-1}$.

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