Efficient RDO using sample recycling

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Abstract

Deterministic optimization does not consider uncertainties. This may lead to designs which are not robust or reliable. The use of safety factors is the common approach to cope with this problem. The main weaknesses of the achieved results are overdesign (too expensive) or underdesign (unreliable) because safety factors do not necessarily consider the special problem. Therefore robust design optimization uses stochastic values as constraint and/or objective to obtain a robust and reliable optimal design. In classical approaches the effort required for stochastic analysis multiplies with the complexity of the optimization algorithm. The suggested approach shows that it is possible to reduce this effort enormously by recycling previously obtained data. In a simple example, it will be shown that this is possible without loss of accuracy.

Keywords: Robust Design Optimization, Reliability Based Design Optimization, Response Surface, Robustness, Design for Six Sigma, Sample Recycling.

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1 Robust Design Optimization

The cost of a product includes more than the development, production and management costs. There are also costs that are related to the quality of the product e.g. scrap, re-work or recalling costs. The quality of a product is related to uncertainties in the production process, in component parts, etc. According to the PIMS-study (first introduced in Buzzell und Gale (1987)), providing the corresponding quality is clearly seen as one of the most essential competition advantage in international business rivalry.

A classical optimization produces a design that does not consider uncertainties. Without taking this into account (and without additional safety factors) nearly 50% of the products will fail. One point that causes this problem is that in many cases the optimal point lies near a constraint, i.e. at the boundary of the feasible domain. To avoid such high failure rates, classical approaches minimize internal strength, and (or) maximize external forces via safety factors. Because of the very general assumptions in these factors 4 things can happen.

1. Overdesign, (too safe, too expensive)
2. Underdesign, (not safe enough, more re-work and scrap)
3. Oversensitivity, (very sensitive with respect to small changes)
4. Cross independence, (interactions between inputs are neglected)

According to Boehm's *cost of change curve*, the earlier one starts with improvements in production process, the lower the costs of these improvements are. Drawing a conclusion avoiding errors early in development phase is much more cheaper than repairing them later. It is neccessary to spend every possible attention in the research and design phase. Otherwise, once made savings in early development phases may have to be payed 100 times in the production phase. For this reason companies start Robust Design Optimization to aim at "built-in quality".

One widely known approach is Design for Six Sigma. It has its roots in Crosby's approach on self paying quality Crosby (1979) "Zero Defects". At this point "Zero" should mean "very low". A proof for this assertion is Pareto's 80/20 law which postulates infinite costs for perfect systems. But only the wanted quality is paid for by a customer. Therefore more information is needed, and another definition for quality is needed. Six sigma quality should be the target that will not be reached at the first time practising the method. So it has to be used in a *Kaizen* (way to the better) - process .

1.1 Deterministic, random and total space

An RDO problem has variables that can be controlled, the deterministic design parameters

$$\mathbf{d} = [d_1, d_2, ...d_{n_d}] \tag{1}$$

They can be used for optimization. There also exist parameters that have inherent uncertainties, the random parameters

$$\mathbf{p} = [p_1, p_2, \ldots p_{n_r}] \tag{2}$$

Design parameters that also have uncertainties are called mixed variables. They can be found in $\mathbf{p}$ and $\mathbf{d}$. Now we can define a problem with two spaces. First there is the deterministic space which is defined by d(Fig. 1). And second, there is the random space which is defined by p (Fig. 2). Random and deterministic space are building the total space. Therein the responses

$$\mathbf{Y} = [y_1, y_2, \dots y_n] \tag{3}$$

are calculated as a function of all parameters.

$$y_i = f_i\left(\mathbf{p} \cup \mathbf{d}\right) \tag{4}$$

In real problems, this function can be a complex FE-model. So solving it can take minutes, hours or even days.

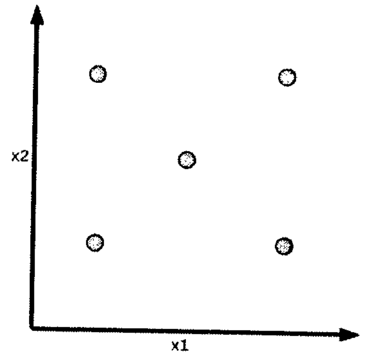

Figure 1: optimization space

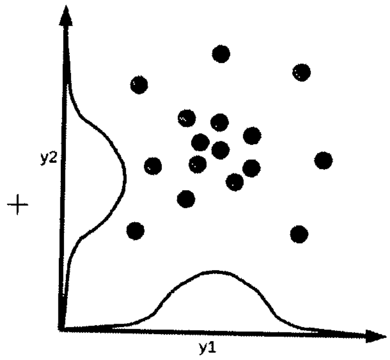

Figure 2: stochastic space

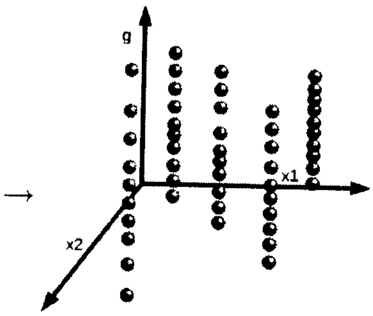

Figure 3: total space

Double loop approach Classical approaches solve the RDO-problem in a double loop method (fig.4). This means that the optimization takes place in deterministic space (outer loop) and for every deterministic design a value of robustness $\delta_q(d, p)$ is computed. Therefore robustness analysis (such as sampling, reliability) is executed in random space (inner loop). In fig. 3 it can be seen that in a projection of the results to the deterministic space a (hyper-)line appears for each optimization design. The number of evaluations for optimization N_{det} multiplies with that of stochastic analysis N_{sto}. So complexity is growing rapidly with the size of the problem. In most practical applications one evaluation(FE-solver run) can take a lot of time. For that reason methods are needed to reduce the effort.

A lot of approaches exist to reduce this effort some of them are shortly mentioned here. E.g.Egorov u. a. (2002) describes a method which adapts the methods while progressing. This approach uses mathematical models with varying accuracy (from the lowest to the highest) during the solution process. The usage of meta models is one possibility to work more efficiently. Improving by expanding the sampling strategy for stochastic dimensions according to the assumed probability level is suggested in Youn und Choi (2004). A comparison of different approaches for obtaining reliability measures and their convergence in deterministic space can be found in Youn u. a. (2003).

Decoupled loop approach Another option is to work iteratively. Optimization loop and stochastic loop are decoupled. Depending to the result of a stochastic analysis a shift in optimization space is accomplished. In Chen u. a. (2003) a method is presented which shifts the design itself with respect to the expected distance to a security level.

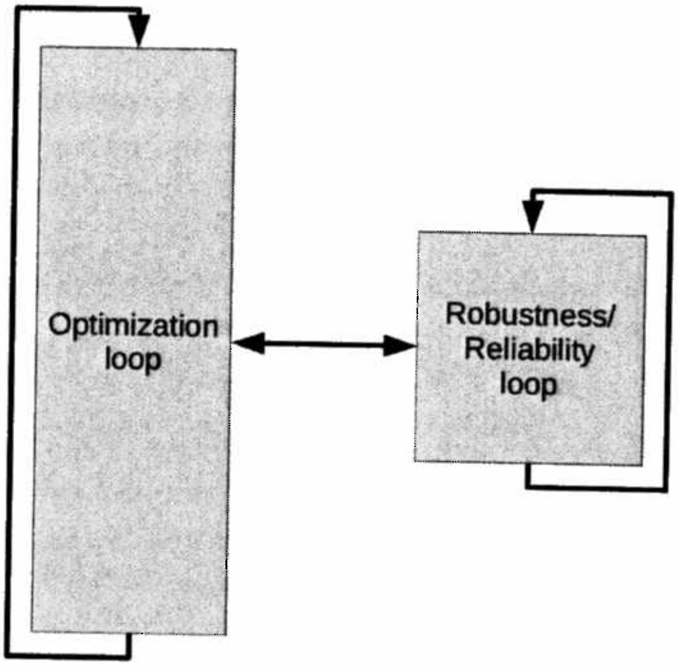

Figure 4: double loop

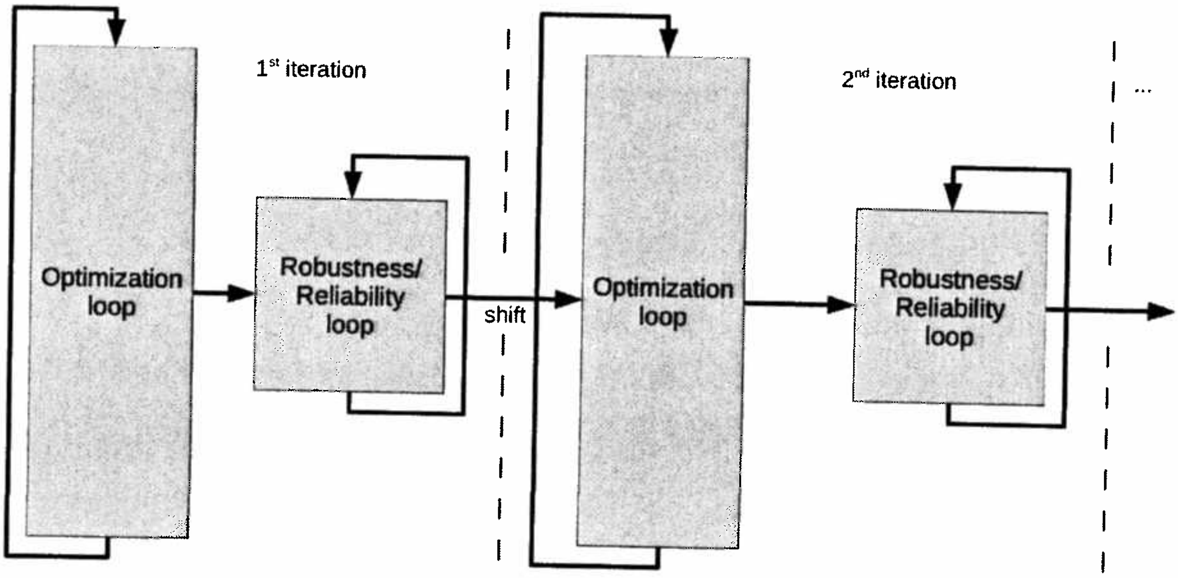

Figure 5: decoupled loop

Single loop approach Using the distance to the most probable point of failure (MPP) as part of the objective in optimization is presented in Kharmanda u. a. (2002). But this very efficient method can only be applied to small and smooth problems.

Orthogonal arrays and other DOE should be shortly mentioned here. They are widely used, but their use is in very controversial discussion. Reasons are the weaknesses of hard simulated samplings, neglecting interactions and so on. Advantages are a small effort of evaluations, an easy to implement method and a historically grown symphathy and understanding Otto und Antonsson (1993).

Critcism Most of the methods as mentioned are directly related to the kind of robustness value which is evaluated. Reliability methods often use FORM or other β/MPP based methods. Linearization and other (over-)simplifications inhibit their usage for large real problems with dynamic or nonlinear results. Additional a grouping in two fields of interest, reliability based design optimization (RBDO) and (variance based) robust design optimization (RDO), inhibits coupled analysis of failure modes (small probability) with a description of the robustness close to the mean design (higher probability).

2 Values of Robustness

The aim of RDO is to consider uncertainties during optimization. To do this it is neccessary to measure the robustness of an optimization design. These values are used in constraints or in objectives. Measuring robustness is a quantification of quality. Many, more or less suitable, definitions of quality were given in the past:

- "fitness for use"(Juran, Joseph M.)
- "loss to society" or "uniformity around a target value"(Taguchi G.)
- "conformance to requirements"(Crosby P.)
- "degree to which a set of inherent characteristic fulfills requirements." (ISO 9000)

It is possible to formulate 5 groups of problems:

Weighting the outputs with a loss function $f(x)$

$$\delta_q = \sum_N f(x_i)\frac{1}{N} \tag{5}$$

E.g. Taguchi Loss Functions Byrne und Taguchi (1987); Phadke (1989)
Target the Best ($\mu \ldots$ target mean value):

$$f(x) = k(x-\mu)^2 \tag{6}$$

The more one output differs from the target, the higher is its loss.
Maximum the Best:

$$f(x) = k(x)^2 \tag{7}$$

The higher all values are, the lower is its loss.
Minimum the Best:

$$f(x) = k\frac{1}{x^2} \tag{8}$$

It should be mentioned here, that Taguchi's approach is based on a Taylor series expansion. The three described functions above are well known, but only examples for a usage. They can be used if there is no additional data about the real loss function. Whenever it is possible, a better suiting loss function should be developed. Applications can be found in Fathi und Poonthanomsook (2007); Teeravaraprug (2002). A survey on cost of quality models is given in Schiffauerova und Thomson (2006).

Rating the probability of occurence $p(x)$ or density function
For discrete pdf's

$$\delta_q = \sum f(p(x_i)) \tag{9}$$

For continous pdf's

$$\delta_q = \int f(p(x_i)) \tag{10}$$

E.g. Shannon entropy Shannon (1948) (target: regular system/lowering standard deviation)

$$\delta_q = H = -K \sum_i p(x_i) log\left(p(x_i)\right) \tag{11}$$

Appraisal of a assumed function How good does the obtained distribution function fulfill the presumption? E.g. PRESS-value for response surfaces or chi-square test for distributions

Parameters of the distribution function E.g. mean, standard deviation

Exceeding or fall below limits or performance measure E.g. reliability index, process capability indices, probability of failure

The values of robustness are used in RDO as objective

$$\begin{aligned} minimize: &\quad \delta_q(d,p) \\ subject\ \ to: &\quad d_L \leq d \leq d_U \end{aligned} \tag{12}$$

or as constraint

$$\begin{aligned} minimize: &\quad Cost(d) \\ subject\ \ to: &\quad \delta_q(d,p) \geq C_{\delta_q} \\ &\quad d_L \leq d \leq d_U \end{aligned} \tag{13}$$

for the outer loop optimization. Methods for obtaining these values differ depending on the probability level and the desired accuracy. Mostly these values are only estimates. The outer loop has to cope with this fact. E.g. it is not useful to have a convergence criterion that is more stringent than the error in the estimation. The choice of the applied δ_q belongs to the field of application. In Thornton (2001) suggestions which description to use, according to the field of application are made. Combining all data in one cost of quality function may make it possible that no constraints are needed. The only objective for the optimization would then be the cost of quality for the design. Multiobjective problems can be reduced to an easier single objective function.

3 Sample Recycling

As it can be seen in fig.3 the more the optimization converges into one subspace the closer to each other the hyperlines are getting. The basic idea of this recycling approach is to decide whether it is neccessary to actually analyze a design or it is possible to use the previously analyzed designs that are nearby. Recycling of data is made in total space. The decision criterion whether recycling is possible (fig.6) or not is based (fig.7) on the quality of the meta model in the region of interest. There exist several methods to determine the quality of a response surface.

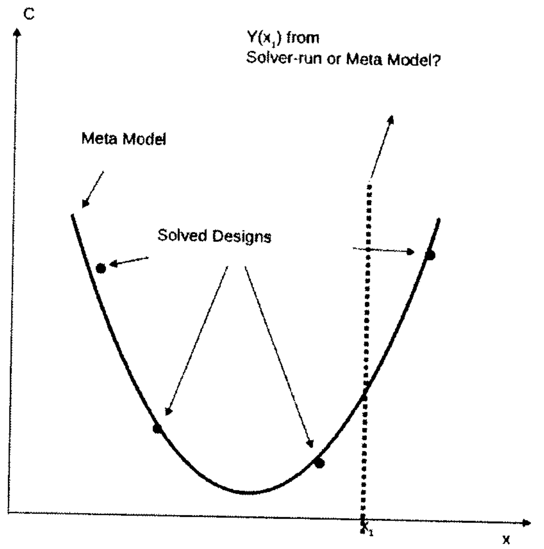

Figure 6: good approximation

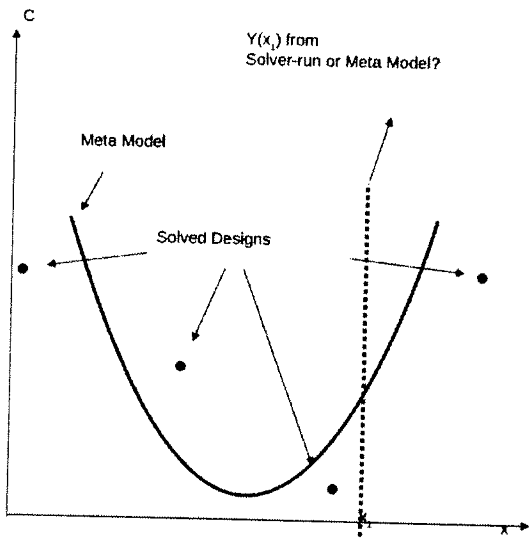

Figure 7: bad approximation

It can easily be defined e.g. by distances between the support points (offline quality). Offline quality means that a calculation of the responses is not neccessary. More complex methods use online quality therefore the responses are considered. Splitting the sample set in training and test data or cross-validation are examples for this kind Queipo u. a. (2005); Li (2007). With the help of these methods an error estimate $\hat{e}_P$ of the meta model for every point P in total space can be obtained.

In the proposed method the computed error is used to calculate a measure of quality for every point ever demanded in space. Depending on its value, it is decided whether the result can be taken from the meta model or a real calculation has to be done. The calculation of new support points will be only necessary in certain subregions, i.e. in those subregions where the quality is bad. In areas where the available data set can fully represent the calculation model no additional solver run is needed. Whenever a calculation is made its data is pushed back in a database and so it can be used as support point for the following steps. So the quality in this region grows. The entire functionality of the approach can be seen in fig.9.

Especially in RDO-problems this leads to massive reduction of the number of required solver runs. The main reason is that the outer looped optimization converges in a small area of the total space. In this region the meta model is refined until the needed quality is achieved. Subsequently, calculations are only neccessary if the optimizer finds new subregions of interest. Starting with low quality demands and increasing them with every iteration of the outer loop shows additional potential to reduce the number of solver runs.

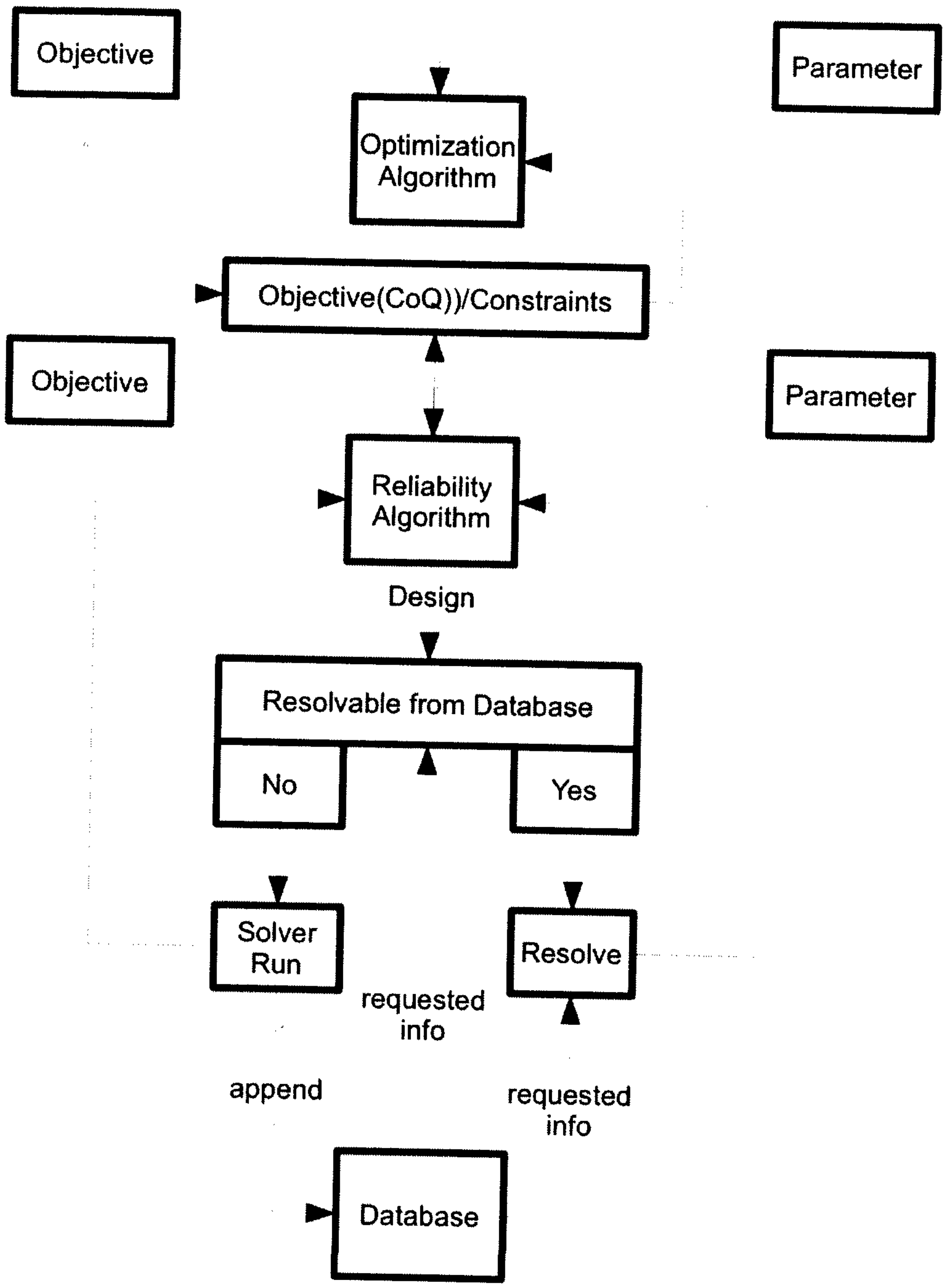

Figure 8: Algorithm for sample recycling

4 Example

As example a beam under dynamic load is choosen. The example was first introduced in Bucher (2005). The objective is to minimize the mass with a constraint on the probability of exceeding a maximum displacement. Since it was first introduced various methods were tested with it and a lot of improvement was achieved. Width d and height h are controllable (design) parameters. The parameters of the harmonic load $F_0 \quad (meanvalue = 20000)$, $\omega \quad (meanvalue = 60)$ are random variables with a coefficient of variation of 0.1. Constant values are young's modulus

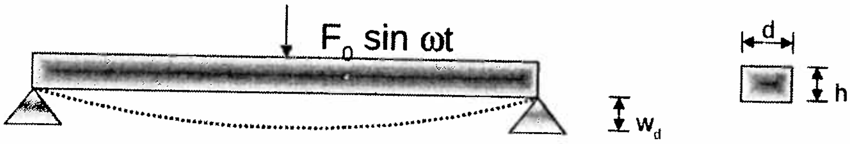

Figure 9: beam design principle

$E = 3E10$, poisson ratio $\nu = 0.2$, mass density $\rho = 2500$ and the length of the beam $L = 10$. The problem is defined as:

$$min(d \cdot h) \tag{14}$$

subject to

$$\begin{aligned} 0 \le d \le 1 \\ 0 \le h \le 1 \\ P[w_d < 0.005] \le 0.01 \end{aligned} \tag{15}$$

For the reason of a fair comparison with respect to Roos u. a. (2006) the problem is solved with ARSM for outer loop and ARSM for inner loop Roos und Bucher (2003). Starting at the deterministic optimum $d = 0.06$ / $h = 1.00$ it takes 1060 solver runs with the classical approach. The robust optimum lies at $d = 0.08/h = 1.00$. The estimated probability of failure is 0.01. Compared with the analytical solution this is the real optimal solution.

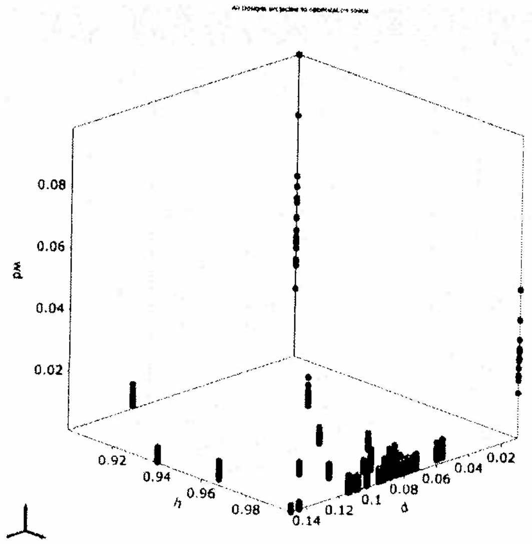

Figure 10: classical approach(1060samples)

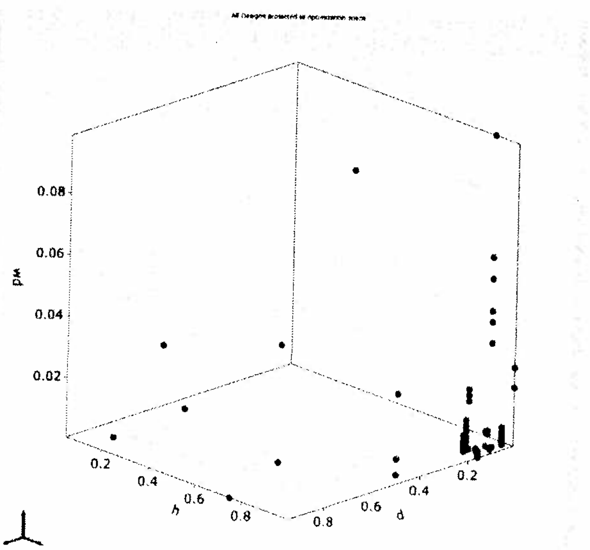

Figure 11: sample recycling(79samples)

The sample recycling is used for the same algorithm settings with ARSM on ARSM in 2 steps of quality criteria. Starting with a lower quality criteria the RDO needed 61 real calculations. This means that approximately 1000 designs could be calculated on the response surface. Using the calculated design with a higher quality level, it was necessary to perform 18 additional calculations to find the same optimal point as the classical approach did. The effort was reduced by more than 90% without any loss of accuracy. The differences of effort between classical and suggested approach can be seen in fig.10 and fig.11.

To show that the approach works also for small probabilities the same example as above is used but now with a much smaller demanded probability of failure.

$$min(d * h) \tag{16}$$

with respect to

$$\begin{aligned} 0 \leq d \leq 1 \\ 0 \leq h \leq 1 \\ P[wd < 0.005] \leq 3.4E - 6 \end{aligned} \tag{17}$$

Therefore the same 2 step evaluation as mentioned before was used. For the first step 63 solver runs were required. The calculated optimum was at $(0.20/1.0)$. In the second step 18 additional solver runs were needed. The resulting optimum was calculated at $(0.185/1.0)$ with the demanded probability of failure. That is the same point as the classical approach found with 4094 solver runs. This example shows that there is no loss of accuracy. Also, the number of neccessary solver runs is increased only by a small amount. Hence, the suggested approach is applicable for small probabilities of failure.

5 Conclusion

Providing the required quality is seen by far as the most essential advantage in international business competition. Because of the widely known cost of change curve "built-in" quality should be aimed at. One approach for doing that is using RDO. However, in classical approaches of RDO the effort of stochastic analysis multiplies with the complexity of the optimization algorithm. The suggested approach for sample recycling shows that it is possible to reduce the expenditure enormously by re-using previously obtained results. In a first example it has been shown that this is possible without loss of accuracy. More examples are needed to prove the applicability of this promising approach to practical problems.

6 References

References

BUCHER, C.: Stochastic Analysis in Structural Optimization. In: *Weimarer Optimierungs- und Stochastik Tage* (2005)

BUZZELL, R. ; GALE, B.: The PIMS Principles: Linking Strategy to Performance. In: *Free Press, New York* (1987)

BYRNE, D.M. ; TAGUCHI, G.: The Taguchi Approach to Parameter Design. In: *40th Annual Quality Congress Transactions*. Milwaukee, Wisconsin : American Society for Quality Control, 1987, S. 19 – 26

CHEN, Wei ; ; LIU, Huibin ; SHENG, Jie ; GEA, Hae C.: Application of the sequential optimization and reliabilty assessment method to structural design problems. In: *Proceedings of DETC'03, ASME 2003 Design Engineering Technical Conferences and Computers and Information in Engineering Conference*. Chicao, Illinois USA, September 2 – 6 2003

CROSBY, P.B.: *Quality is free*. ISBN 0451625854 : McGrawHill, 1979

EGOROV, I.N. ; KRETININ, G.V. ; LESHCHENKO, I.A.: Robust Design Optimization Strategy of IOSO Technology. In: *Fifth World Congress on Computational Mechanics*. Vienna,Austria, July 7-12 2002

FATHI, Y. ; POONTHANOMSOOK, C.: A Quartic Quality Loss Function and its Properties. In: *Journal of Industrial and Systems Engineering* Vol. 1, No. 1 (2007), S. 8 – 22

KHARMANDA, G. ; MOHAMED, A. ; LEMAIRE, M.: Efficient reliability-based design optimization using a hybrid space withapplication to finite element analysis. In: *Structural and Multidisciplinary Optimization* 24 (2002), S. 233 – 245

LI, G.: *Online and Offline Approximations for Population based Multi-Objective Optimization*. Dissertation, Faculty of the graduate school of the university of Maryland, College Park, 2007

OTTO, K. ; ANTONSSON, E.: Extensions to the Taguchi Method of Product Design. In: *Journal of Mechanical Design* 115, (1993), S. 5–13

PHADKE, M.S.: *Quality Engineering using Robust Design*. New Jersey : Prentice Hall, Englewood Cliffs, 1989

QUEIPO, N.V. ; HAFTKA, R.T. ; SHYY, W. ; GOEL, T. ; VAIDYANATHAN, R. ; TUCKER, P.K.: Surrogate-based analysis and optimization. In: *Aerospace Sciences* 41 (2005), S. 1–28

ROOS, D. ; ADAM, U. ; BUCHER, C.: Robust Design Optimization. In: *Weimarer Optimierungs- und Stochastik Tage* (2006)

ROOS, D. ; BUCHER, C.: Adaptive Response Surfaces for Structural Reliability of Nonlinear Finite Element Structures. In: *Proceedings of the NAFEMS Seminar: Use of Stochastics in FEM Analyses*. Wiesbaden, Germany, May 7 – 8 2003

SCHIFFAUEROVA, A. ; THOMSON, V.: A review of research on cost of quality models and best practices. In: *International Journal of Quality an Reliability Management* 23 (2006), S. 647–669

SHANNON, C.E.: A Mathematical Theory of Communication. In: *The Bell System Technical Journal* 27 (1948), S. 623–656

TEERAVARAPRUG, J.: Integration of Loss Function in Two-Dimensional Deterministic Tolerance Synthesis. In: *Thammasat Int. J. Sc. Tech.* 7 (2002), Nr. 2

THORNTON, Anna C.: Optimism vs. Pessimism: Design Decisions in the Face of Process Capability Uncertainty. In: *Journal of Mechanical Design* 123 (2001), S. 313–321

YOUN, B.D. ; CHOI, K.K.: A new response surface methodology for reliability-based design optimization. In: *Computers and Structures* 82 (2004), S. 241 – 256

YOUN, B.D. ; CHOI, K.K. ; PARK, Y.H.: Hybrid Analysis Method for Reliability-Based Design Optimization. In: *Journal of Mechanical Design* 125 (2003), S. 221 – 232