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Performance Assessment of MIMO-BICM Demodulators based on System Capacity: Further Results

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Abstract

This technical report serves as a supporting document for [1]. It provides a datasheet-like collection of complementary capacity results for various demodulation schemes for multiple-input multiple-output (MIMO) bit-interleaved coded modulation (BICM).

Contents

I. Introduction

This report presents a datasheet-like collection of ergodic capacity results and non-ergodic performance investigations (in terms of outage probability) for various hard-output and soft-ouput demodulation schemes for multiple-input multiple-output (MIMO) bit-interleaved coded modulation (BICM) [2]–[4] (note that our assessment focuses on non-iterative MIMO-BICM receivers). It complements the work in [1] by providing additional and more detailed numerical results under various system configurations (antenna configurations, symbol constellations, and bit labeling). We verify part of these results with bit error rate (BER) simulations using low-density parity-check (LDPC) codes [5]. Furthermore, we investigate the case when only imperfect channel state information (CSI) is available at the receiver. In particular, we consider training-based estimation of the channel matrix and the noise variance and analyze how the amount of training influences the performance of the demodulators.

The numerical results shown are based on the system capacity, i.e., mutual information of the equivalent modulation channel that comprises modulator, wireless channel, and demodulator, described in [1]. The advantage of this approach is that it allows for a code-independent assessment of the various MIMO-BICM demodulation schemes. For details on the computation of the performance measures, simulation parameter settings, and for a short review of the different demodulators studied in this report, we refer to [1]. We note that some of the figures in this report are also contained in [1] and discussed there in detail; this will be indicated wherever appropriate.

The remainder of this report is organized as follows. Section II presents ergodic system capacity results for fast Rayleigh fading and Section III shows corresponding BER simulations using regular LDPC codes. A performance comparison for the case of imperfect CSI is provided in Section IV. Finally, the rate-versus-outage trade-off of selected demodulators in quasi-static environments is given in Section V.

II. Ergodic Capacity Results

In the following we present numerical results for the system capacity for ergodic independent identically distributed (i.i.d.) fast Rayleigh fading channels for various antenna setups and labeling strategies based on the performance measures outlined in [1, Section III].

A block diagram of our MIMO-BICM model is shown in Fig. 1. A sequence of information bits b is encoded using an error-correcting code and then passed through a *bitwise* interleaver. The interleaved code bits are demultiplexed into M_T antenna streams ("layers") and mapped onto data symbols from a (complex) symbol alphabet. Each transmit vector $\mathbf{x}$ (comprising M_T data symbols) carries R_0 interleaved code bits c_l, $l=1,\ldots,R_0$, per channel use and satisfies the power constraint $\mathbb{E}\{\|\mathbf{x}[n]\|^2\} = E_s$ ($\mathbb{E}\{.\}$ denotes expectation and $\|\cdot\|$ is the ℓ^2 (Euclidean) norm). The M_R-length receive vector $\mathbf{y}$ (M_R denotes the number of receive antennas) is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v}, \tag{1}$$

where $\mathbf{H}$ is the $M_\mathrm{R} \times M_\mathrm{T}$ channel matrix with unit variance entries, and $\mathbf{v}$ is a M_R-length noise vector with i.i.d. circularly symmetric complex Gaussian elements with zero mean and variance σ_v^2. At the receiver, the optimum demodulator uses the received vector $\mathbf{y}$ and the channel matrix $\mathbf{H}$ to calculate log-likelihood ratios (LLRs) Λ_l for all code bits c_l, $l=1,\ldots,R_0$, carried by $\mathbf{x}$. In practice, the use of suboptimal demodulators or of a channel estimate $\hat{\mathbf{H}}$ will result in approximate LLRs $\tilde{\Lambda}_l$. The LLRs are deinterleaved and then passed on to the channel decoder that delivers the detected bits $\hat{b}$.

As shown in [1, Section III], we propose to measure the performance of sub-optimal MIMO-BICM demodulators via the system capacity of the associated equivalent "modulation" channel with *discrete* input c_l and *continuous* output $\tilde{\Lambda}_l$ (cf. Fig. 1). Thus, the system capacity is defined as the mutual information between c_l and $\tilde{\Lambda}_l$, which can be shown to equal

$$C \triangleq \sum_{l=1}^{R_0} I(c_l; \tilde{\Lambda}_l). \tag{2}$$

In the following, the capacity results were obtained by averaging over 10^5 fading realizations. The pdfs required for evaluating (2) are generally hard to obtain in closed form. Thus, we measured these pdfs using Monte-Carlo simulations and then evaluated all integrals numerically. All curves show maximum achievable rate in bits per channel use (bpcu) versus signal-to-noise ratio (SNR) $\rho \triangleq E_s/\sigma_v^2$. In some of the plots we show insets that provide zooms of the capacity curves around a target rate of $R_0/2$ bpcu in order to allow for a more detailed assessment of the demodulator performance.

We first discuss the simulation results for a selection of baseline MIMO-BICM demodulators reviewed in [1, Section IV.A]; max-log demodulation [3], hard maximum-likelihood (ML) demodulation [6], hard/soft zero-forcing (ZF) demodulation [7], and hard/soft minimum mean-square error (MMSE) demodulation [8]. In addition, we also include the capacity of coded modulation (CM) and MIMO-BICM (which equals the system capacity of BICM using optimum maximum a posteriori (MAP) demodulation) as well as the channel capacity with Gaussian inputs (labeled 'Gauss') (cf. [1, Section III.A and III.B]). The results have been assessed for two different labeling strategies, namely, Gray labeling and set partitioning labeling (as shown in Fig. 2 for a 16-QAM constellation). In addition, we provide numerical performance comparisons for other demodulation schemes like the list sphere decoder (LSD) [9], bit flipping demodulation [10], lattice-reduction (LR) aided demodulation [11], semidefinite relaxation (SDR) demodulation [12], ℓ^{∞}-norm sphere decoding [13], [14], as well as successive interference cancelation (SIC) [15] and soft interference cancelation (SoftIC) [16]. Additional observations can be found in [1, Section IV.C and Section V.A-E].

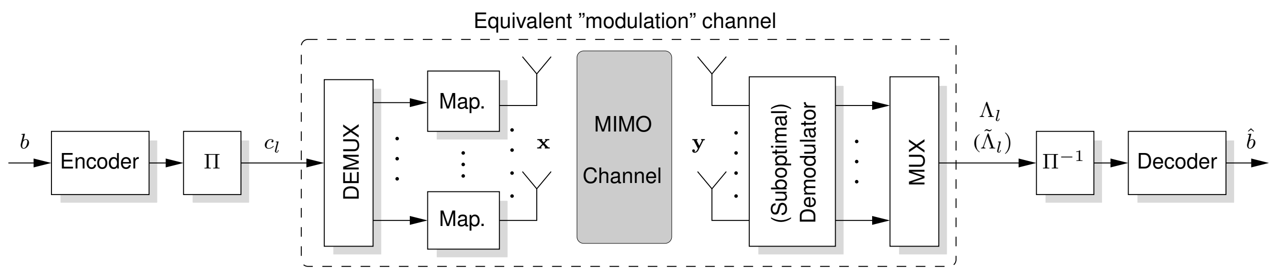


Fig. 1. Block diagram of a MIMO-BICM system.

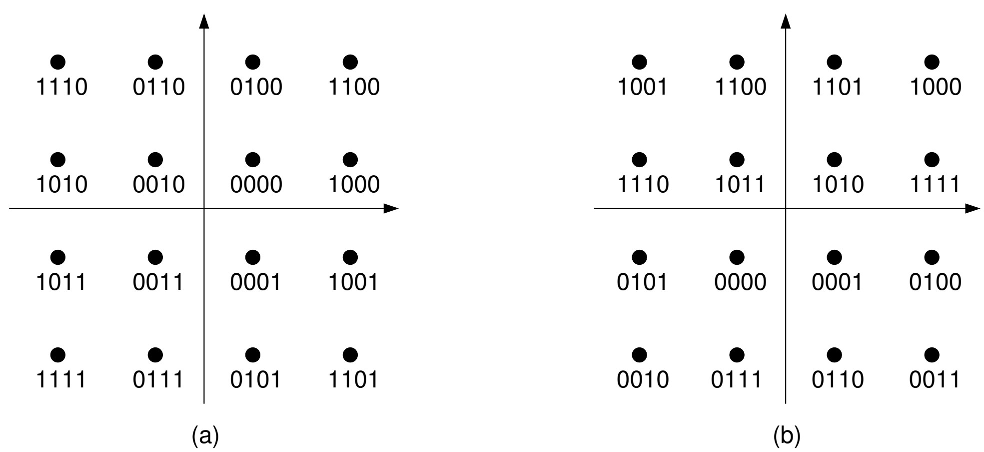
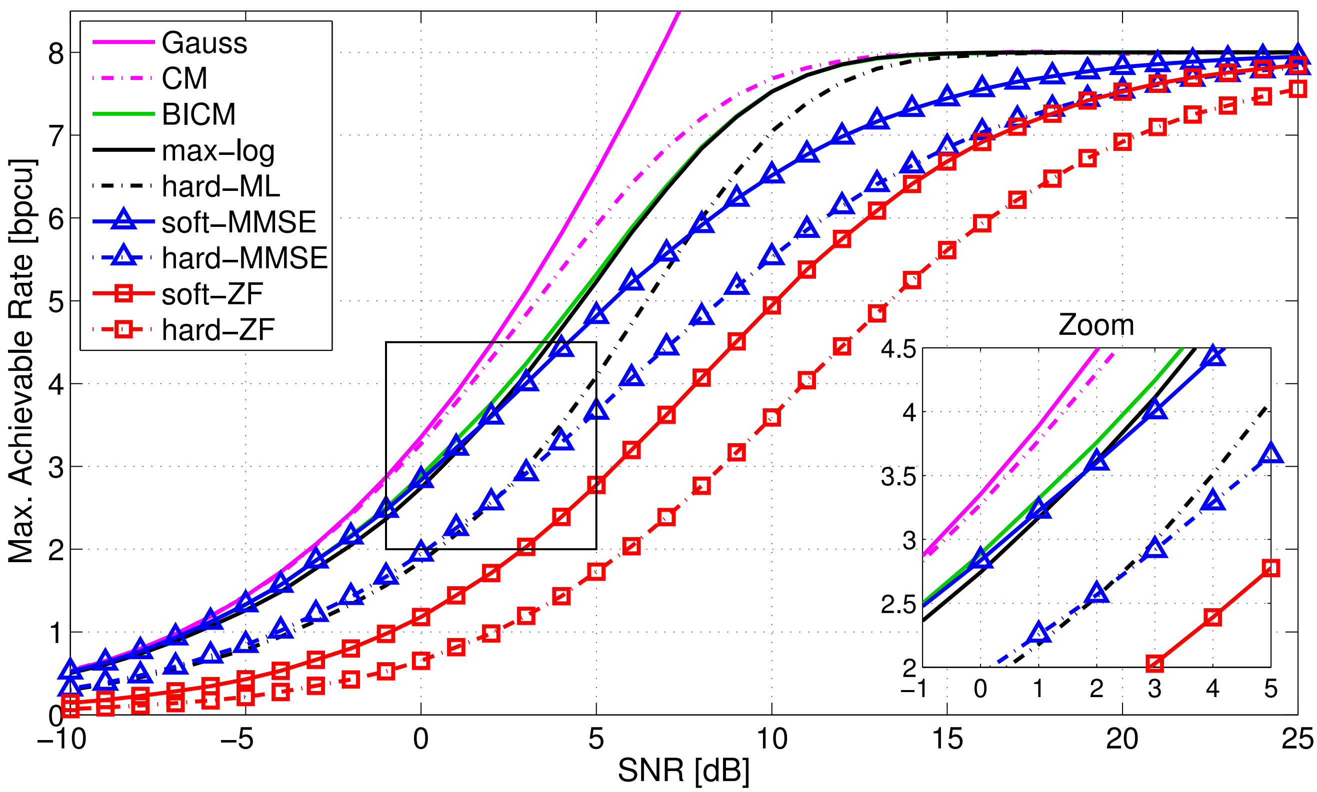


Fig. 2. 16-QAM constellation with (a) Gray labeling and (b) set partitioning labeling.

Baseline Demodulators

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

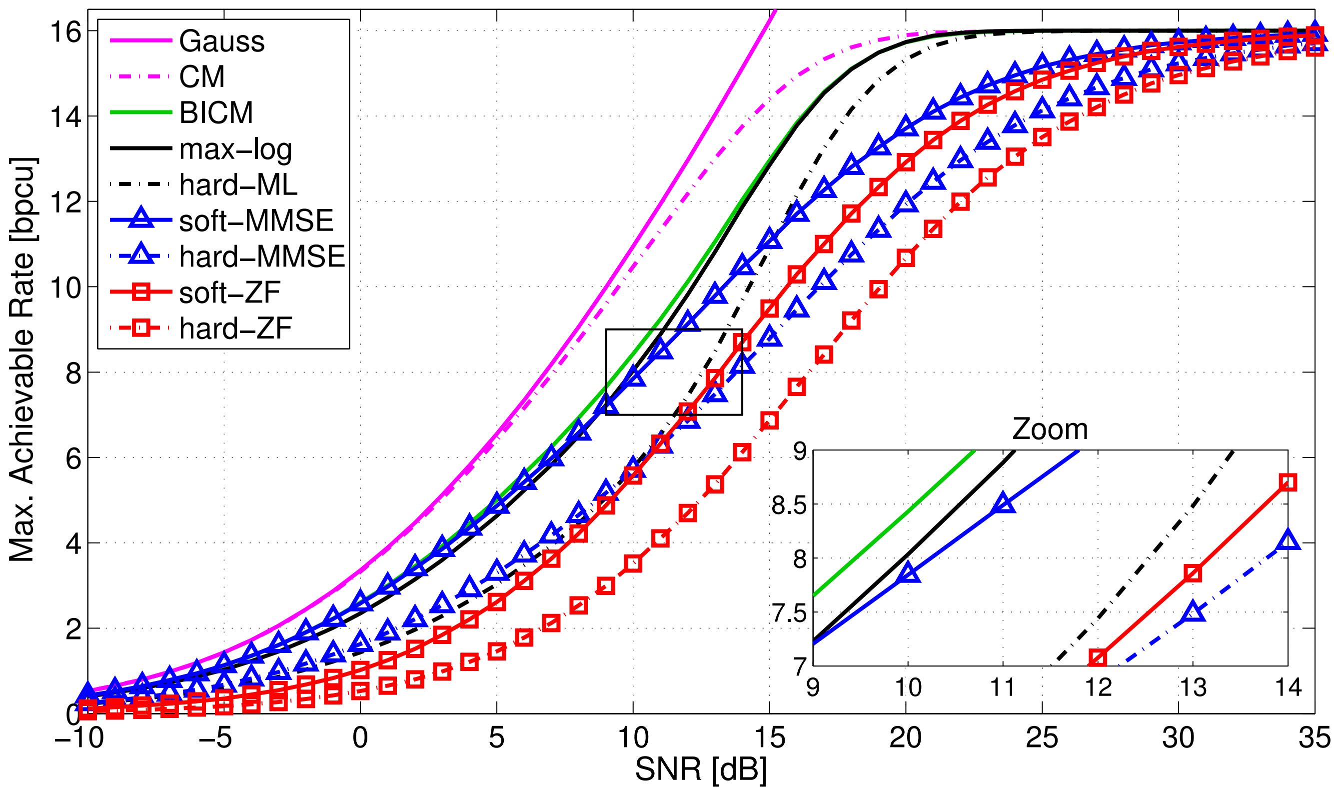


Observations: See [1, Section IV.C] for details. At a target rate of 4 bpcu, the SNR required for CM and Gaussian capacity is virtually the same, whereas that for BICM is larger by about 1.3 dB. The SNR penalty of using max-log demodulation instead of soft MAP is about 0.3 dB. Furthermore, hard ML demodulation requires a 2.1 dB higher SNR to achieve this rate than max-log demodulation; for soft and hard MMSE demodulation the SNR gaps to max-log are 0.2 dB and 3.1 dB, respectively, while for soft and hard ZF demodulation they respectively equal 5.1 dB and 8.1 dB. An interesting observation in this scenario is the fact that at low rates, soft and hard MMSE demodulation slightly outperform max-log and hard ML demodulation, respectively, whereas at high rates MMSE demodulation approaches ZF performance. Note that hard MMSE demodulation can perform better than hard ML demodulation, since the latter minimizes the vector symbol error but not the bit error probability. Surprisingly, at low rates soft MMSE essentially coincides with BICM capacity. Moreover, soft MMSE demodulation outperforms hard ML demodulation at low-to-medium rates whereas at high rates it is the other way around (the cross-over can be seen at about 5.8 bpcu). These observations reveal the somewhat unexpected fact that the demodulator performance ranking is not universal but depends on the target rate (or equivalently, the target SNR), even if the number of antennas, the symbol constellation, and the labeling are fixed.

Baseline Demodulators

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 16-QAM
- Labeling: Gray

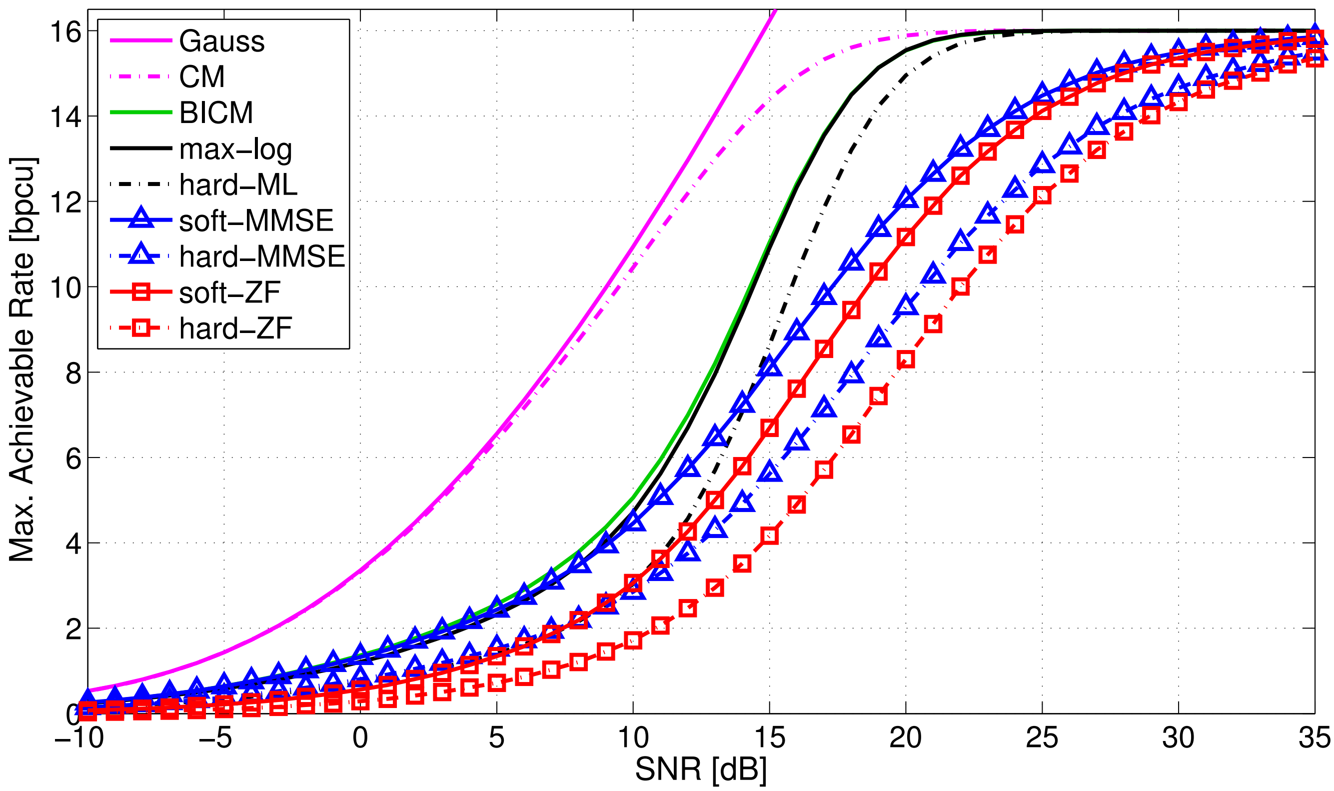


Observations: It can be seen that when using 16-QAM modulation instead of 4-QAM very much the same observations as for 4×4 MIMO with 4-QAM modulation apply. Apart from a general shift of all curves to higher SNRs, the larger constellation causes an increase of the gap between CM capacity and BICM capacity (cf. page 7). Note, however, that the gaps between hard ML, hard MMSE, and soft ZF demodulation are significantly reduced. In fact, soft ZF demodulation starts to outperform hard MMSE demodulation for rates larger than 6.2 bpcu and closely approaches hard ML demodulation for rates below 6 bpcu.

Baseline Demodulators

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 16-QAM
- Labeling: set partitioning

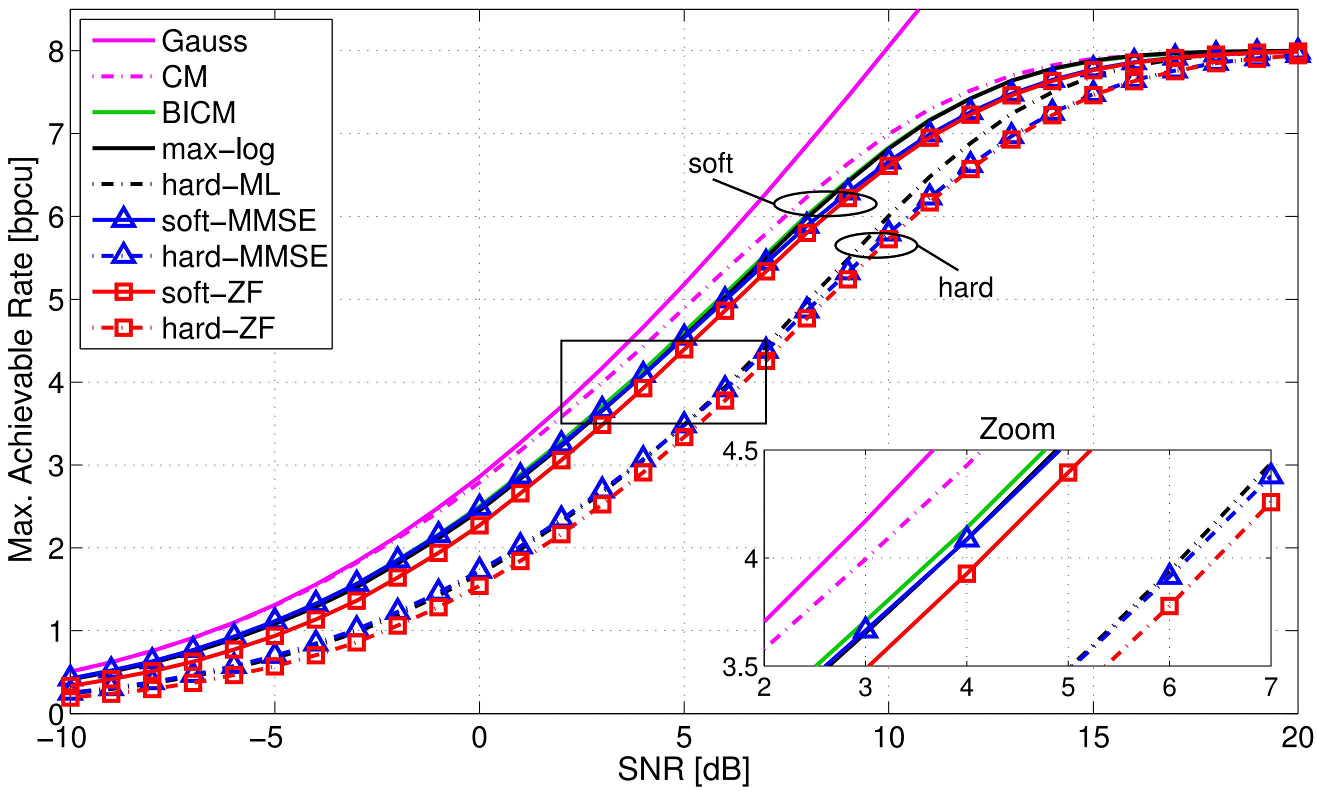


Observations: The gap between CM capacity and BICM capacity becomes even more pronounced when using set partitioning labeling instead of Gray labeling (cf. page 9). In fact, in this case the performance curves shift to even higher SNR values than for the case of Gray labeling. We hence conclude that Gray labeling is preferable over set partitioning labeling for MIMO-BICM systems with non-iterative receivers. Note that similar observations can be found in [2], [4].

Baseline Demodulators

System Setup:

- Number of transmit antennas: 2
- Number of receive antennas: 4
- Constellation: 16-QAM
- Labeling: Gray

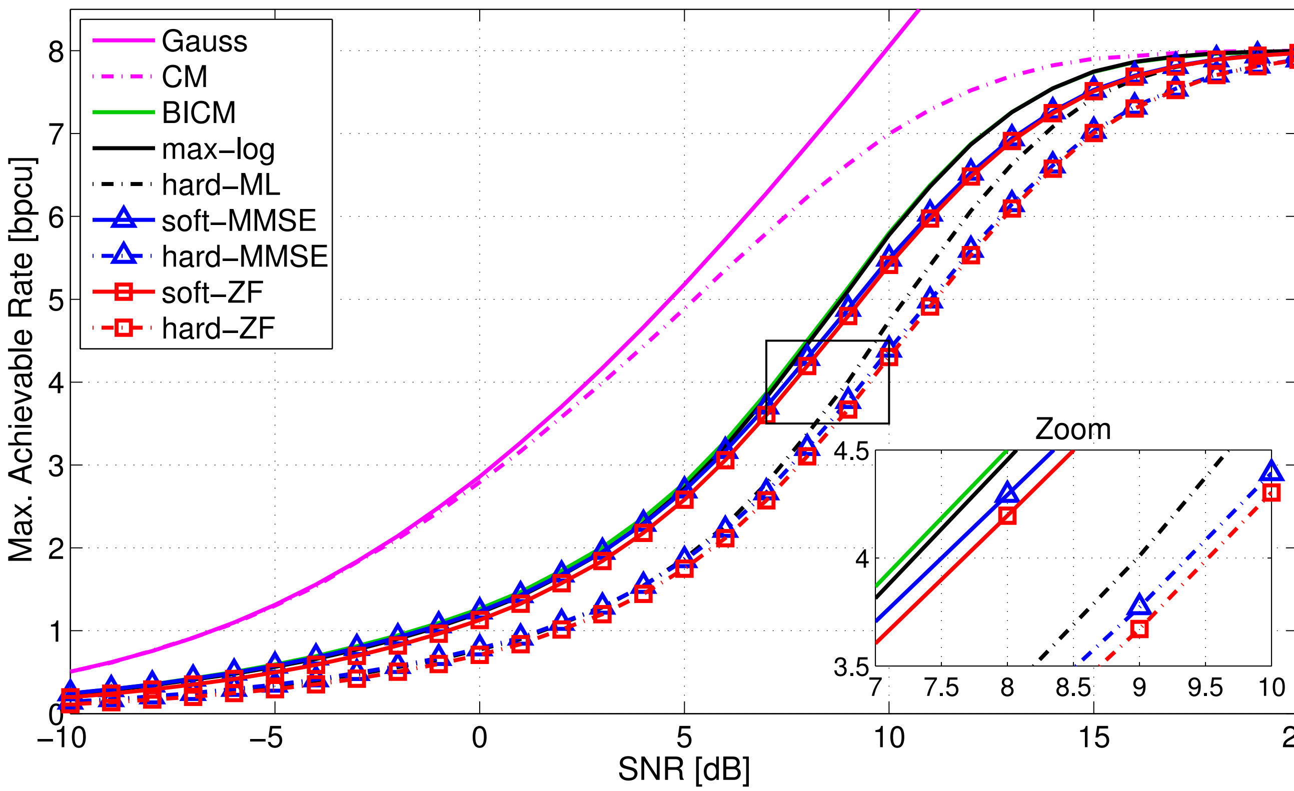


Observations: See [1, Section IV.C] for details. The increased SNR gap between CM and BICM capacity implied by the larger constellation (cf. 4×4 MIMO case with 16-QAM on page 9) is compensated by having more receive than transmit antennas (this agrees with observations in [3]). In addition, the performance differences between the individual demodulators are significantly reduced, revealing an essential distinction being between soft and hard demodulators. Having more receive than transmit antennas helps the linear demodulators approach their non-linear counterparts even at larger rates, i.e., soft ZF/MMSE perform close to max-log and hard ZF/MMSE perform close to hard ML, with an SNR gap of about 2.3 dB between hard and soft demodulators. Note that in this scenario soft MMSE and soft ZF both outperform hard ML demodulation at all rates.

Baseline Demodulators

System Setup:

- Number of transmit antennas: 2
- Number of receive antennas: 4
- Constellation: 16-QAM
- Labeling: set partitioning

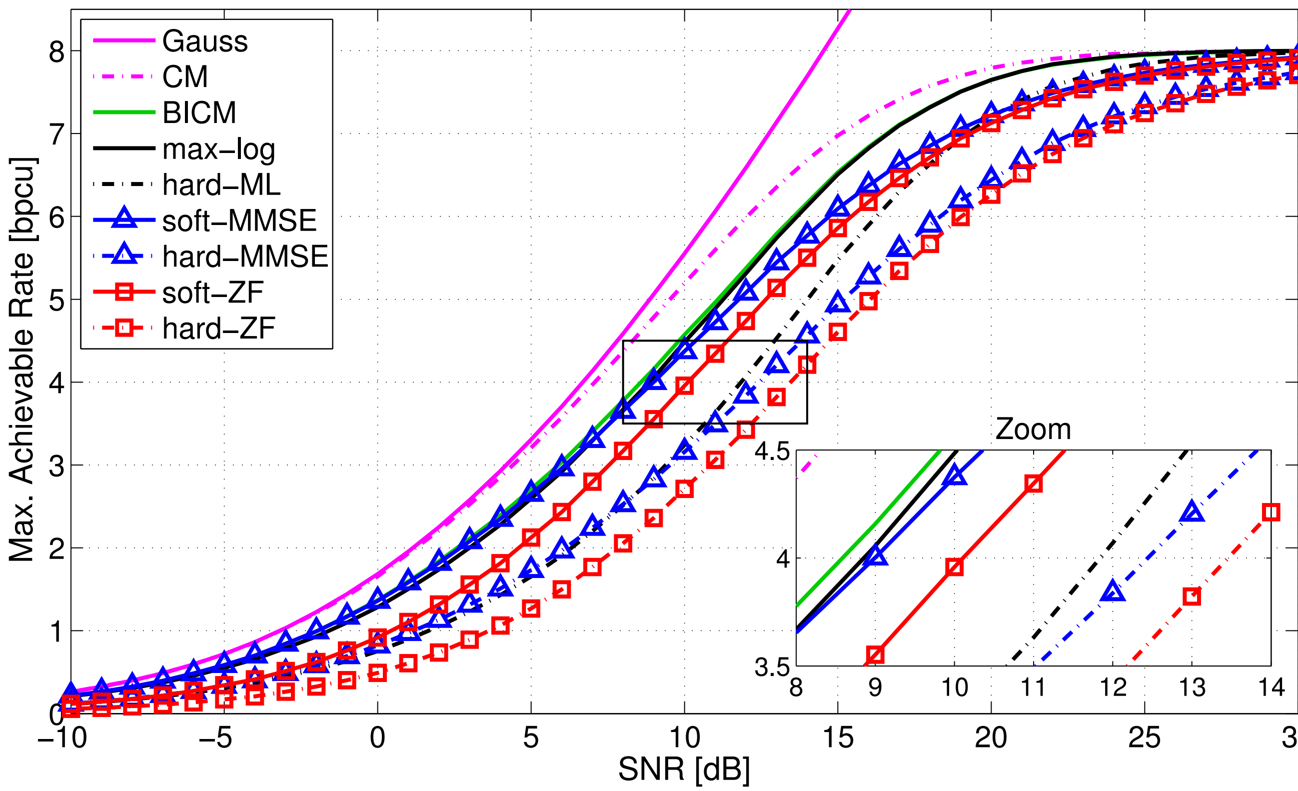


Observations: Again, set partitioning labeling induces a larger gap between CM and BICM capacity and results in a general shift to higher SNR values as compared to the case of Gray labeling (cf. page 11). However, apart from this fact very much the same observations apply.

Baseline Demodulators

System Setup:

- Number of transmit antennas: 2
- Number of receive antennas: 2
- Constellation: 16-QAM
- Labeling: Gray

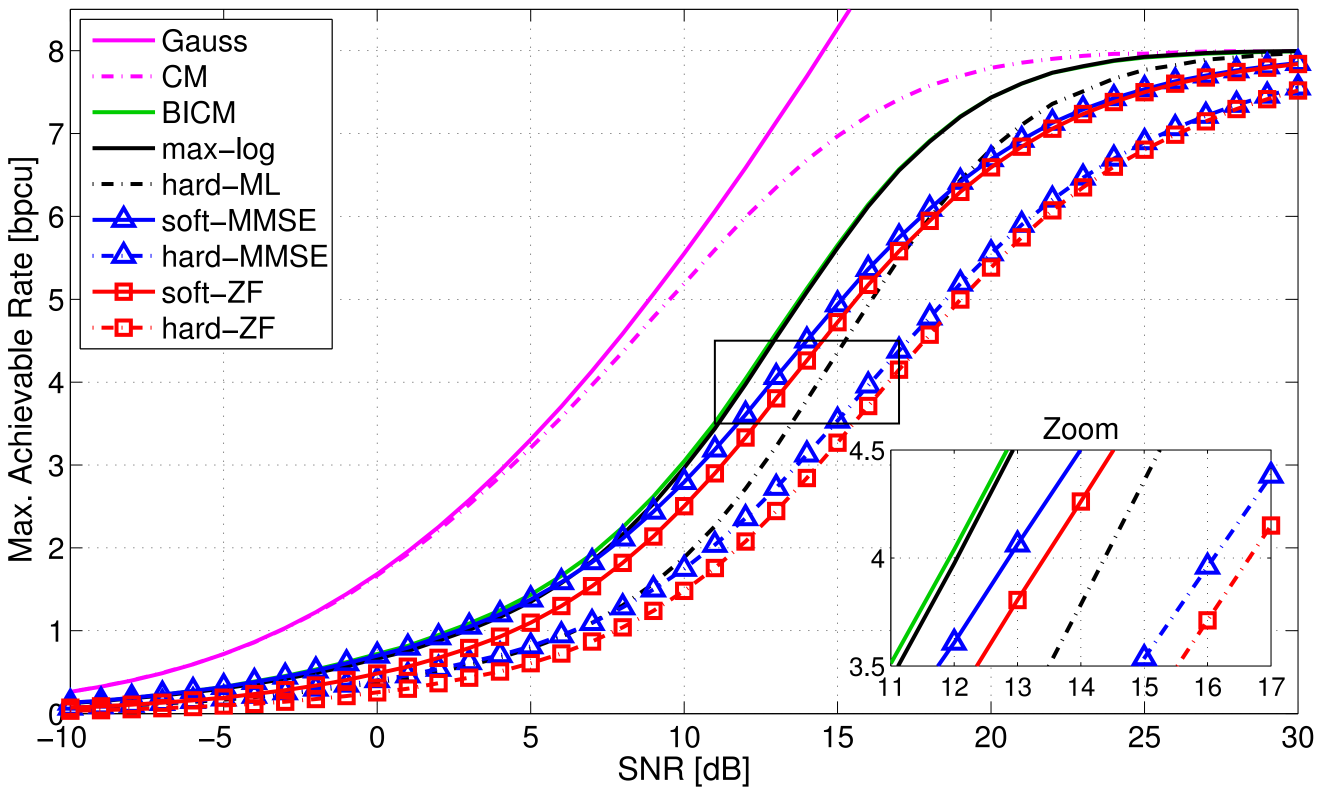


Observations: Surprisingly, with 2×2 MIMO and Gray labeling soft ZF outperforms hard ML demodulation for low-to-medium rates, e.g., by about $1.7\,$dB at $4\,$bpcu. In fact, the gap to BICM capacity is only within $1\,$dB in this SNR regime. Moreover, soft MMSE demodulation performs substantially better than hard ML demodulation up to $7\,$bpcu. The latter also holds true for soft ZF demodulation.

Baseline Demodulators

System Setup:

- Number of transmit antennas: 2
- Number of receive antennas: 2
- Constellation: 16-QAM
- Labeling: set partitioning

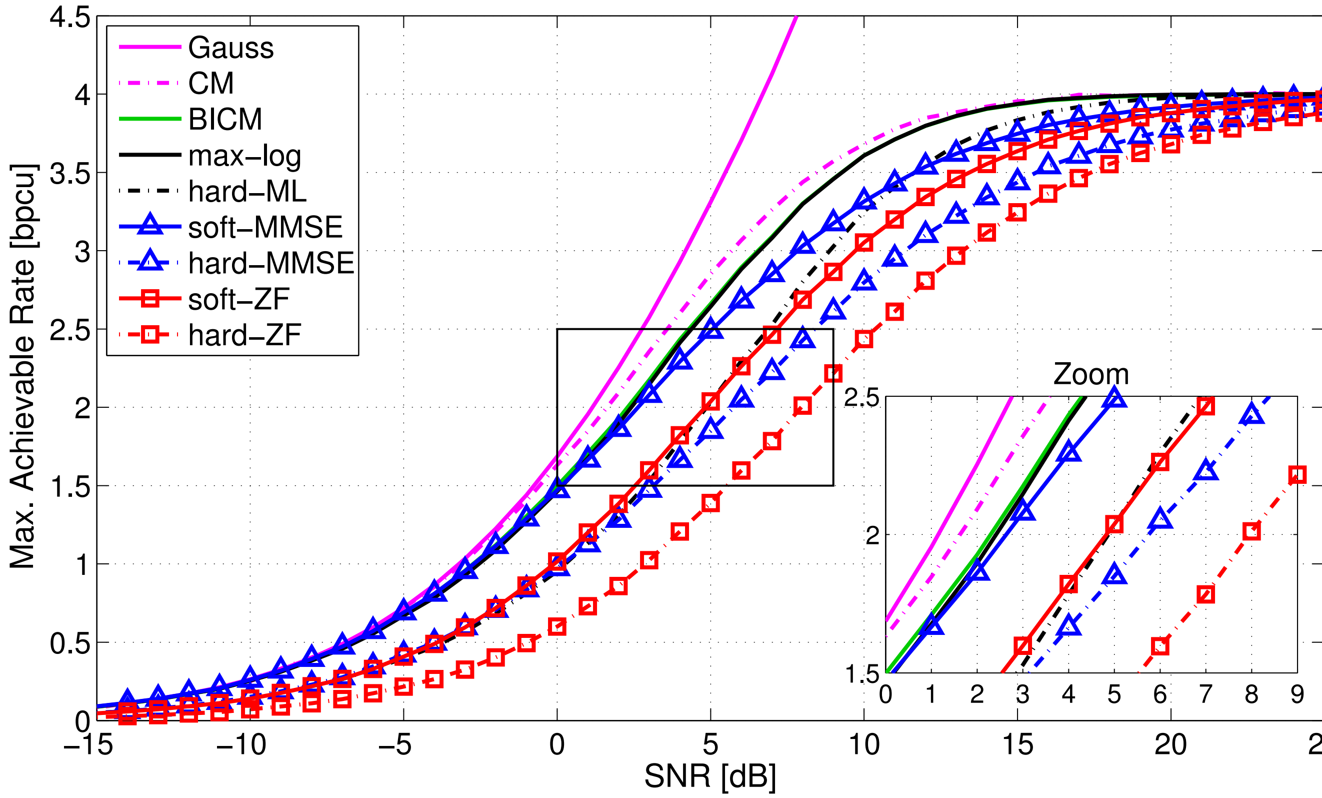


Observations: Similar observations as for the case of Gray labeling (cf. page 13) apply for set partitioning labeling apart from a shift to higher SNR values and a significantly larger gap between CM and BICM capacity.

Baseline Demodulators

System Setup:

- Number of transmit antennas: 2
- Number of receive antennas: 2
- Constellation: 4-QAM
- Labeling: Gray

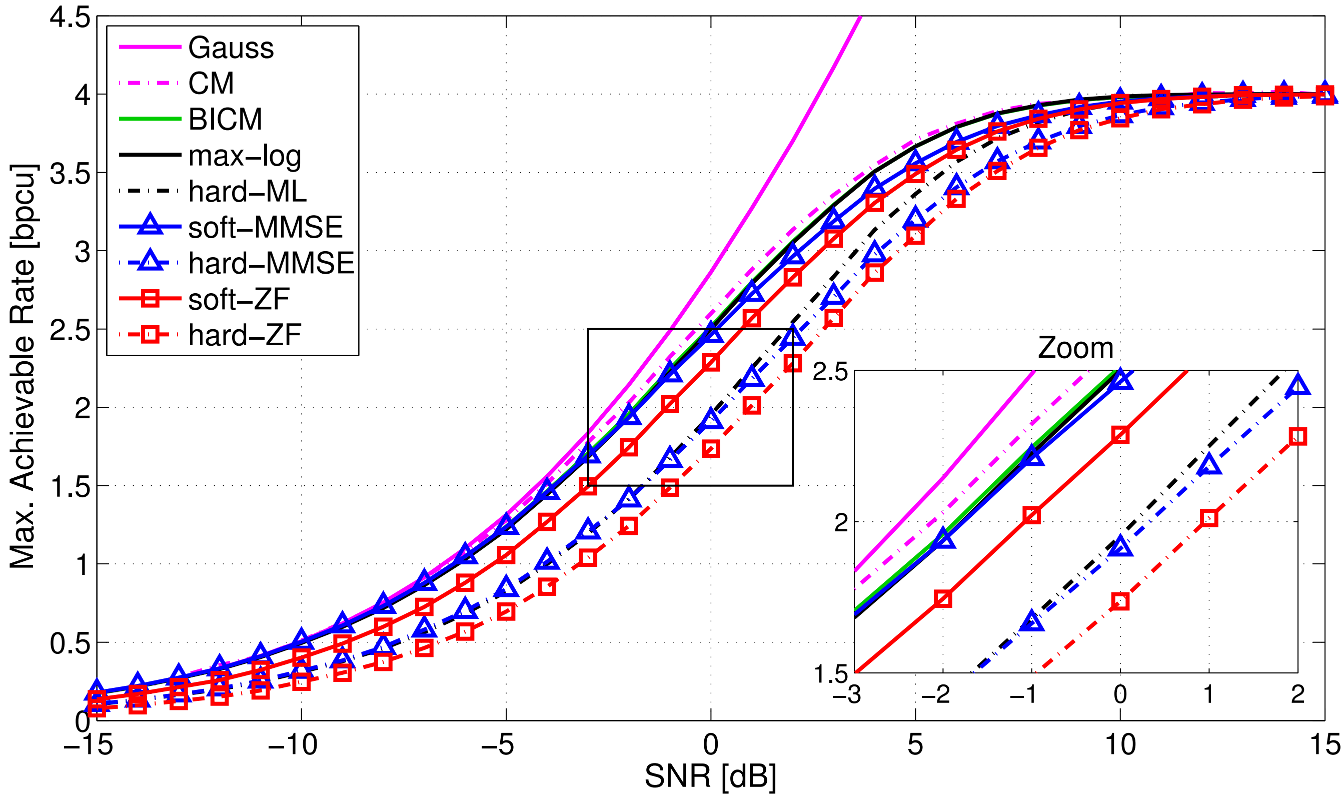


Observations: For the case of 2×2 with 4-QAM soft ZF demodulation outperforms hard ML for low rates whereas at high rates it is just the other way round. Note that here soft MMSE demodulation substantially performs better than hard ML demodulation for medium-to-low rates, i.e., an SNR gap of 2.8 db at 2 bpcu. Furthermore, hard MMSE demodulation approximates hard ML closely at low rates.

Baseline Demodulators

System Setup:

- Number of transmit antennas: 2
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

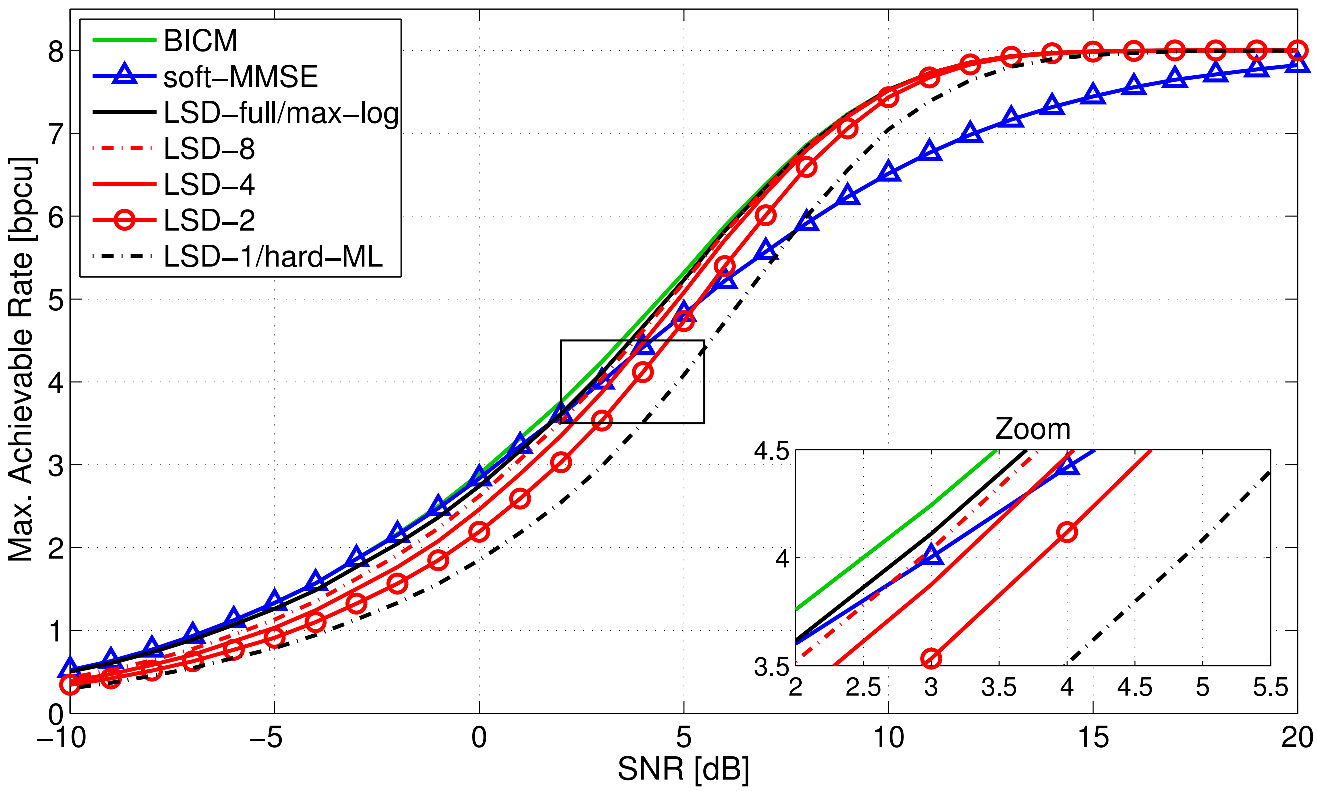


Observations: Similar to the 2×4 MIMO case with 16-QAM shown on page 11, also the performance results for the case of a 4-QAM constellation reveal an essential distinction between soft and hard demodulators, even though ZF-based schemes show a larger performance loss in this scenario.

List Sphere Decoder (LSD) [9]

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

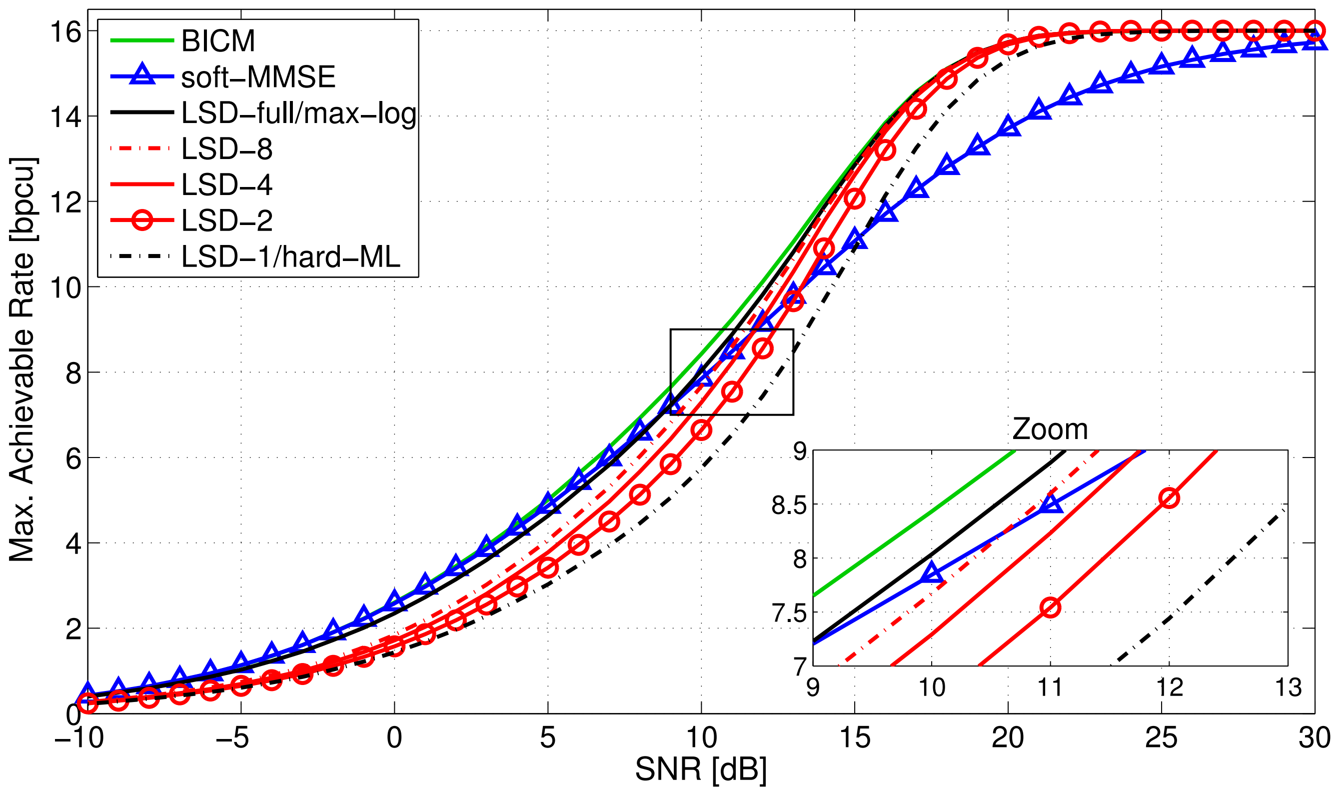


Observations: See [1, Section V.A-1] for more details. Here, the label 'LSD–X' denotes the list sphere decoder with list size $|\mathcal{L}|=\mathrm{X}$; in particular, 'LSD–full' refers to a list size containing all possible candidate vectors (for the current setup, the latter means that $|\mathcal{L}|=256$). Note that the LSD with $|\mathcal{L}|=256$ equals the max-log demodulator and with $|\mathcal{L}|=1$ equals hard ML demodulation. It is seen that with increasing list size the gap between LSD and max-log decreases rapidly, specifically at high rates. In particular, the LSD with list sizes of $|\mathcal{L}|\geq 8$ is already quite close to max-log performance. However, even with large list sizes LSD is outperformed by soft MMSE demodulation at low rates. Specifically, below 5.3 dB, 3.7 dB, and 2.8 dB the system capacity of soft MMSE demodulation is higher than that of LSD with list size 2, 4, and 8, respectively.

List Sphere Decoder (LSD) [9]

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 16-QAM
- Labeling: Gray

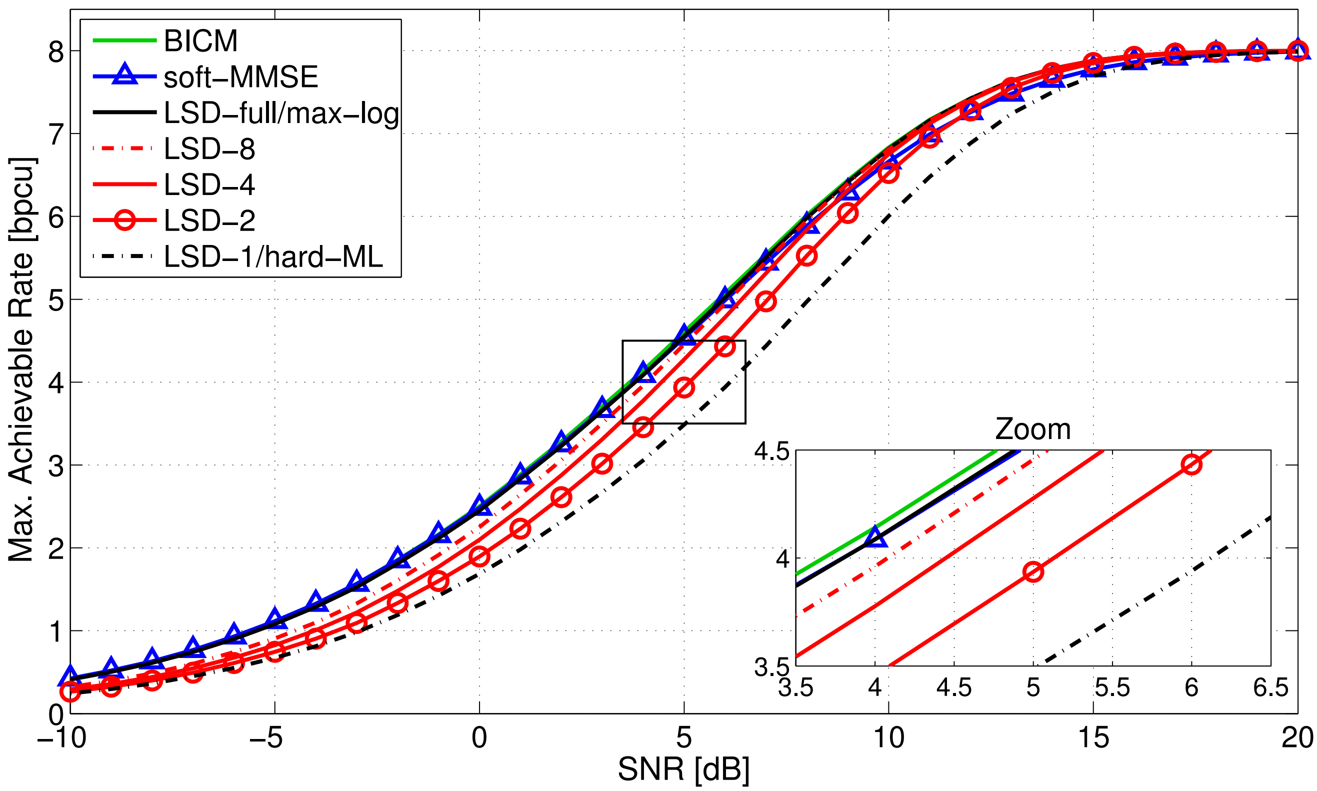


Observations: Here, the label 'LSD–X' denotes the list sphere decoder with list size $|\mathcal{L}| = \text{X}$; in particular, 'LSD–full' refers to a list size containing all possible candidate vectors. Here, very much the same observations as for the 4×4 MIMO case with 4-QAM apply (cf. page 17).

List Sphere Decoder (LSD) [9]

System Setup:

- Number of transmit antennas: 2
- Number of receive antennas: 4
- Constellation: 16-QAM
- Labeling: Gray

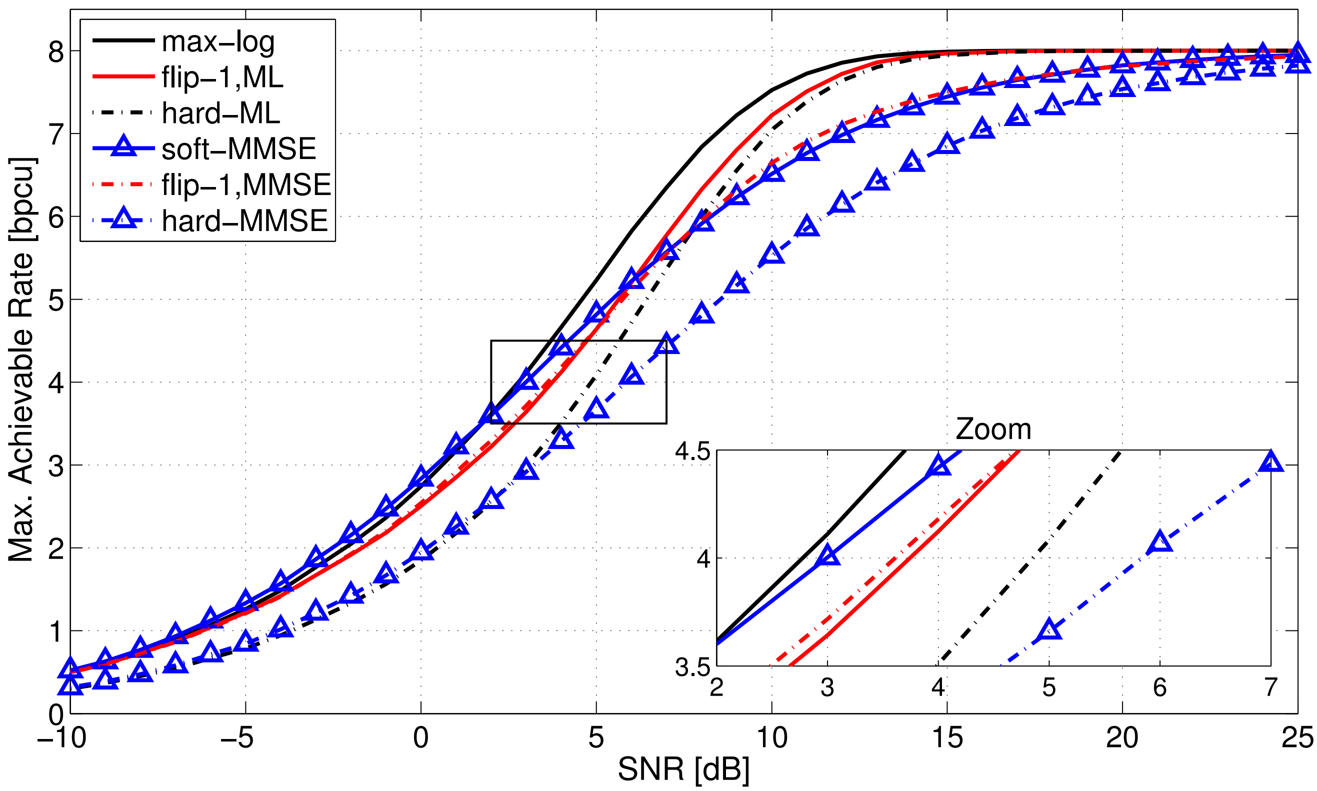


Observations: Here, the label 'LSD–X' denotes the list sphere decoder with list size $|\mathcal{L}| = \mathrm{X}$; in particular, 'LSD–full' refers to a list size containing all possible candidate vectors. Whereas an asymmetric MIMO constellation usually reveals an essential distinction between soft and hard demodulators (cf. page 11), the LSD shows a tradeoff between these two extremes depending on the candidate list size. However, for this setup it is almost always better to do soft MMSE demodulation which also closely approaches BICM capacity, but exhibits a much lower computational complexity.

Bit Flipping Demodulators (1-bit)

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

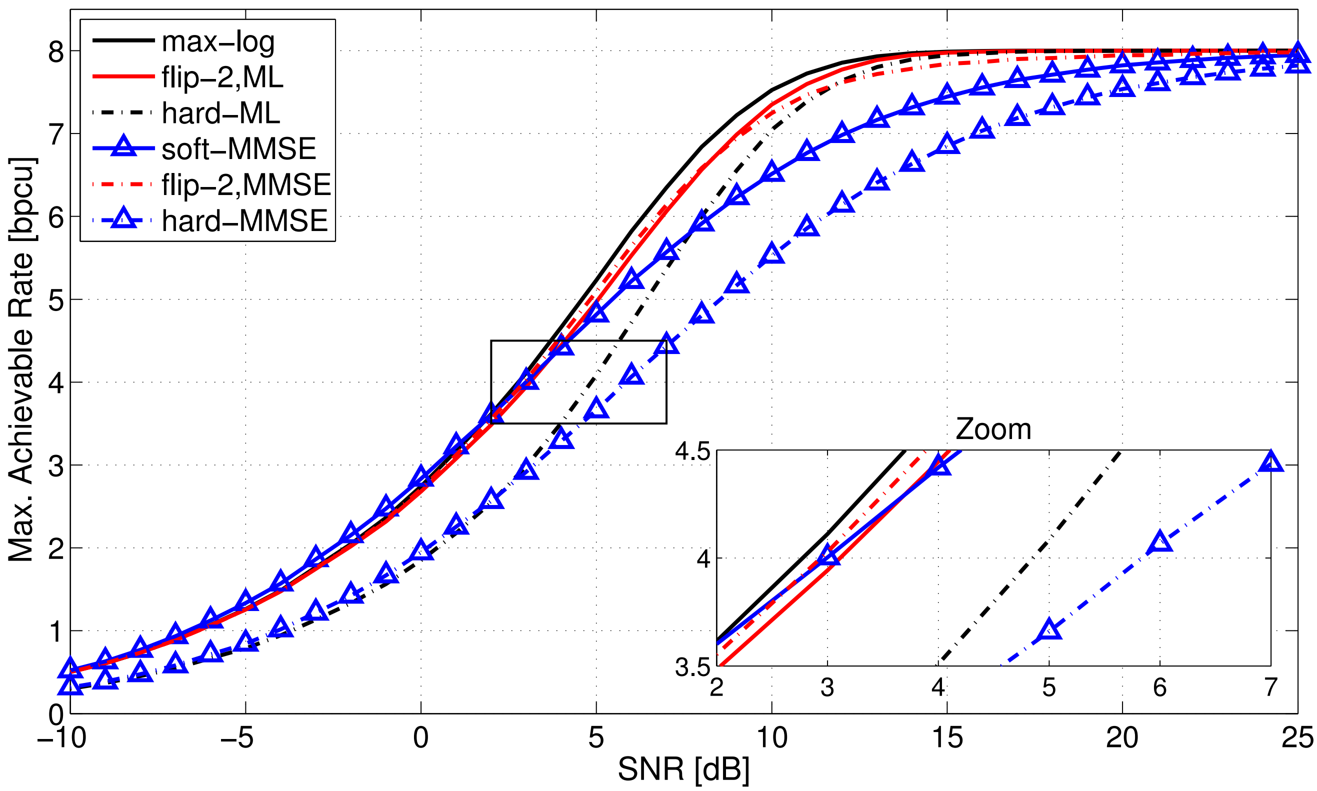


Observations: See [1, Section V.A-2] for more details. Here, we show the performance results for bit flipping demodulation based on an initial hard ML as well as a hard MMSE estimate. It can be seen that 1 bit flipping (labeled 'flip-1') with $|\mathcal{L}| = 9$ allows a significant performance improvement over the respective initial hard demodulator (about 2.1 dB at 2 bpcu). For rates below 5 bpcu, hard ML and hard MMSE initialization yield effectively identical results, showing a maximum gap of 0.9 dB to soft MMSE demodulation at 3.5 bpcu. At higher rates MMSE-based bit flipping even outperforms soft MMSE demodulation slightly.

Bit Flipping Demodulators (2-bit)

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

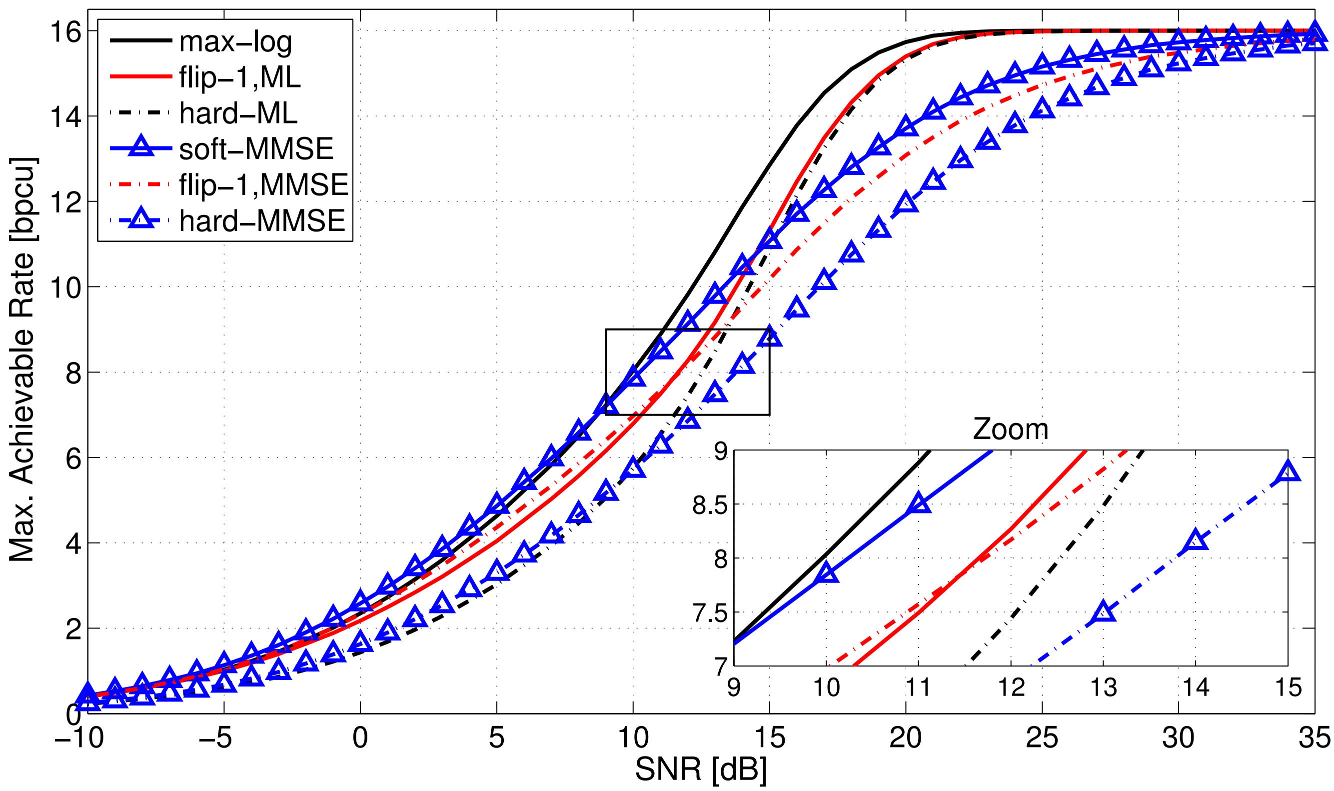


Observations: See [1, Section V.A-2] for more details. Compared with the results of the 1 bit flipping demodulators on page 20, a further performance improvement can be achieved by using 2 bits flipping yielding a list size of $|\mathcal{L}| = 37$. It can be seen that bit flipping demodulation performs close to max-log below 4 bpcu and that hard ML and hard MMSE initialization are very close to each other for almost all rates and SNRs; in fact, below 6.7 bpcu hard MMSE initialization performs slightly better than hard ML initialization while at higher rates ML initialization gives slightly better results.

Bit Flipping Demodulators (1-bit)

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 16-QAM
- Labeling: Gray

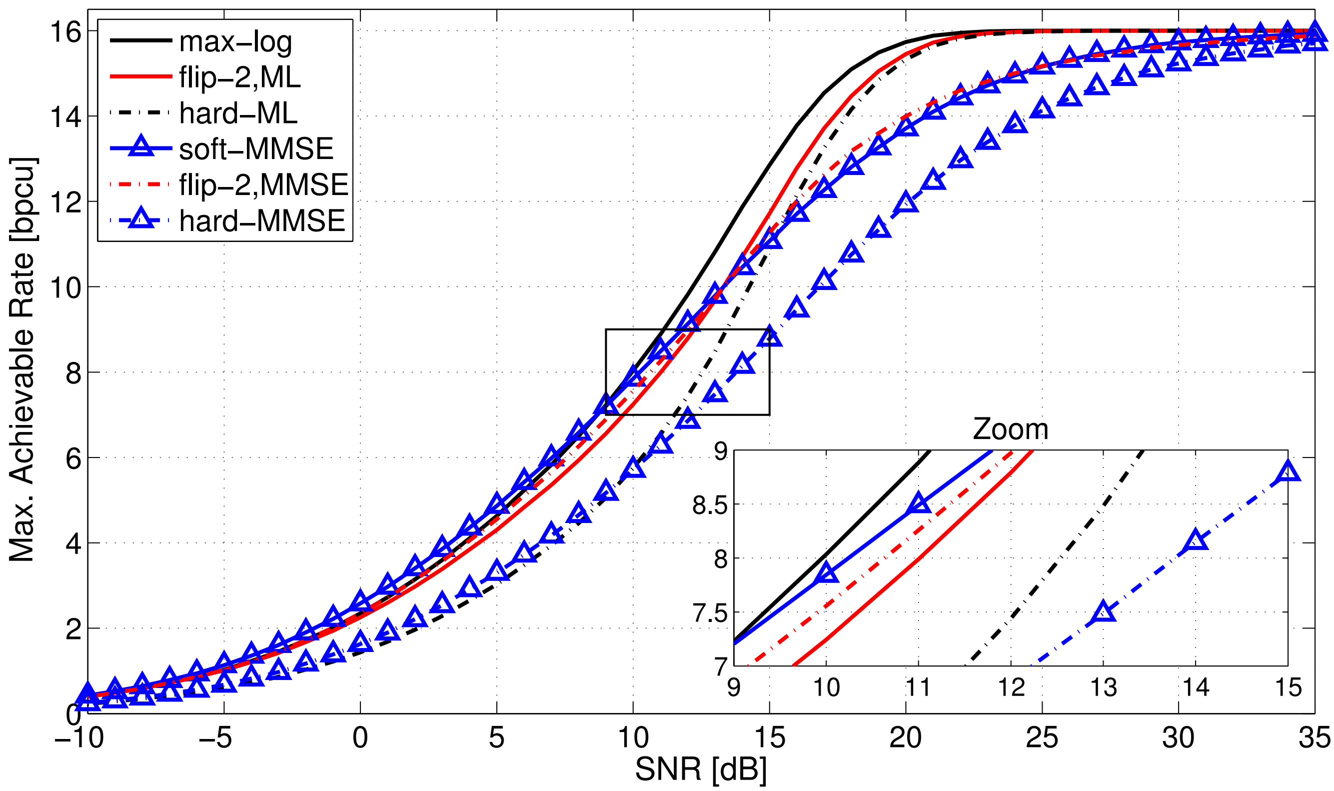


Observations: When increasing the constellation size from 4-QAM to 16-QAM for the $4{\times}4$ MIMO setup (cf. page 20 and page 21), it can be seen that the number of flipped bits (and thus the list size) has to scale with the systems size, i.e., with the number of code bits per channel use R_0, in order to maintain similar performance results. For example, MMSE-based 1 bit flipping outperforms soft MMSE demodulation for the case of 4×4 MIMO with 4-QAM at high rates (cf. page 20). However, this is not the case if the constellation size is increased to 16-QAM where MMSE-based 1 bit flipping performs poorer than soft MMSE demodulation. Nevertheless, the performance behavior can be retained when increasing the number of flipped bits to 2 (cf. page 23).

Bit Flipping Demodulators (2-bit)

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 16-QAM
- Labeling: Gray

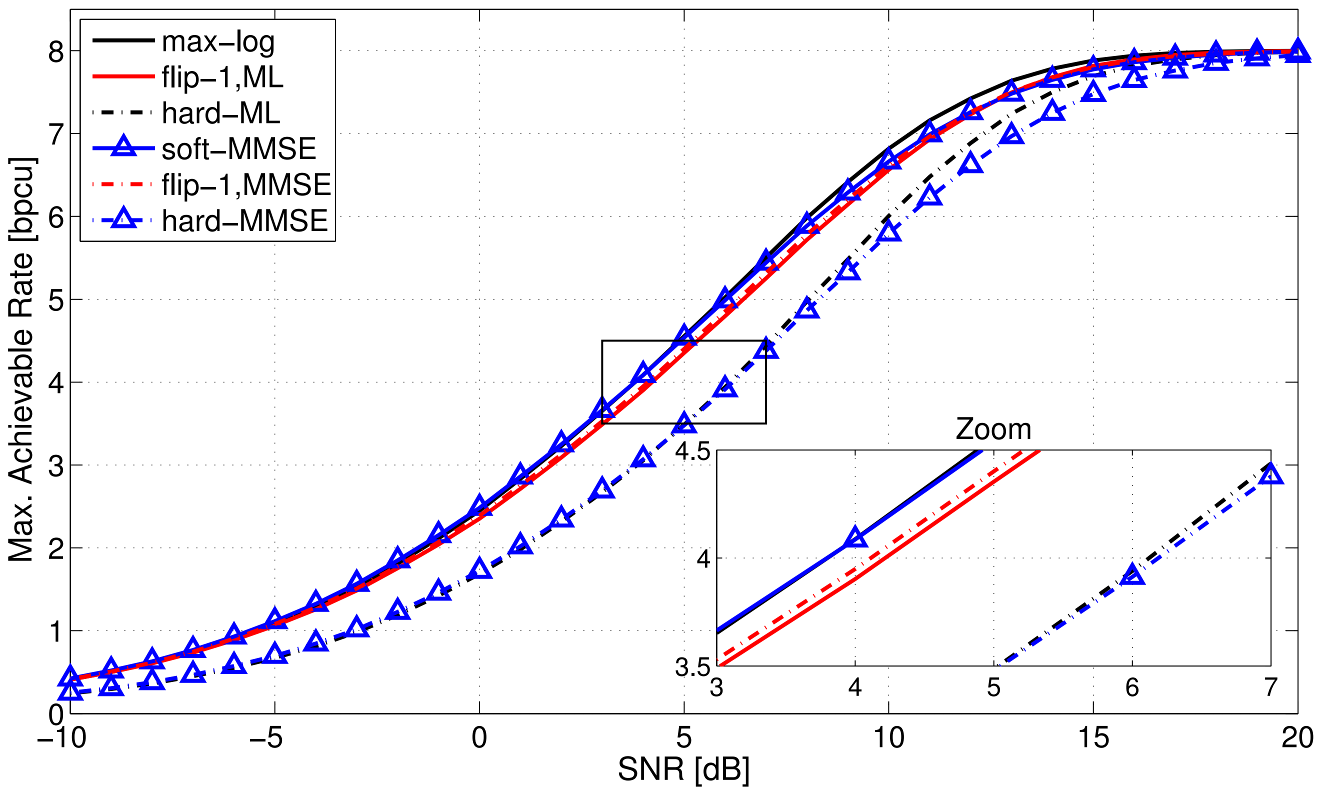


Observations: See observations from page 22 for more details. In general, it can be seen that soft MMSE demodulation can be outperformed at high rates if 2 bits flipping instead of 1 bit flipping is applied.

Bit Flipping Demodulators (1-bit)

System Setup:

- Number of transmit antennas: 2
- Number of receive antennas: 4
- Constellation: 16-QAM
- Labeling: Gray

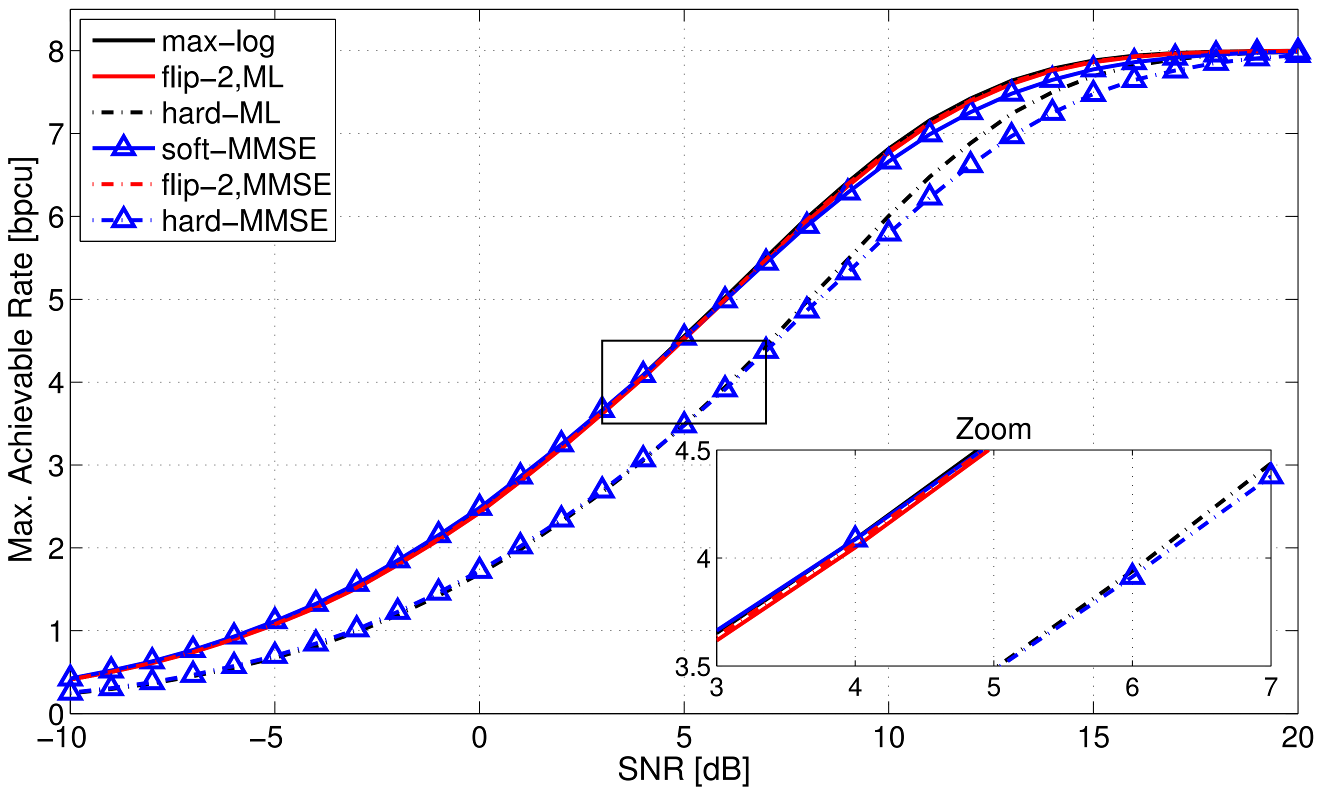


Observations: For the asymmetric case of 2×4 MIMO it can be seen that bit flipping demodulation achieves essentially the same performance results as the other soft demodulators (cf. page 11).

Bit Flipping Demodulators (2-bit)

System Setup:

- Number of transmit antennas: 2
- Number of receive antennas: 4
- Constellation: 16-QAM
- Labeling: Gray

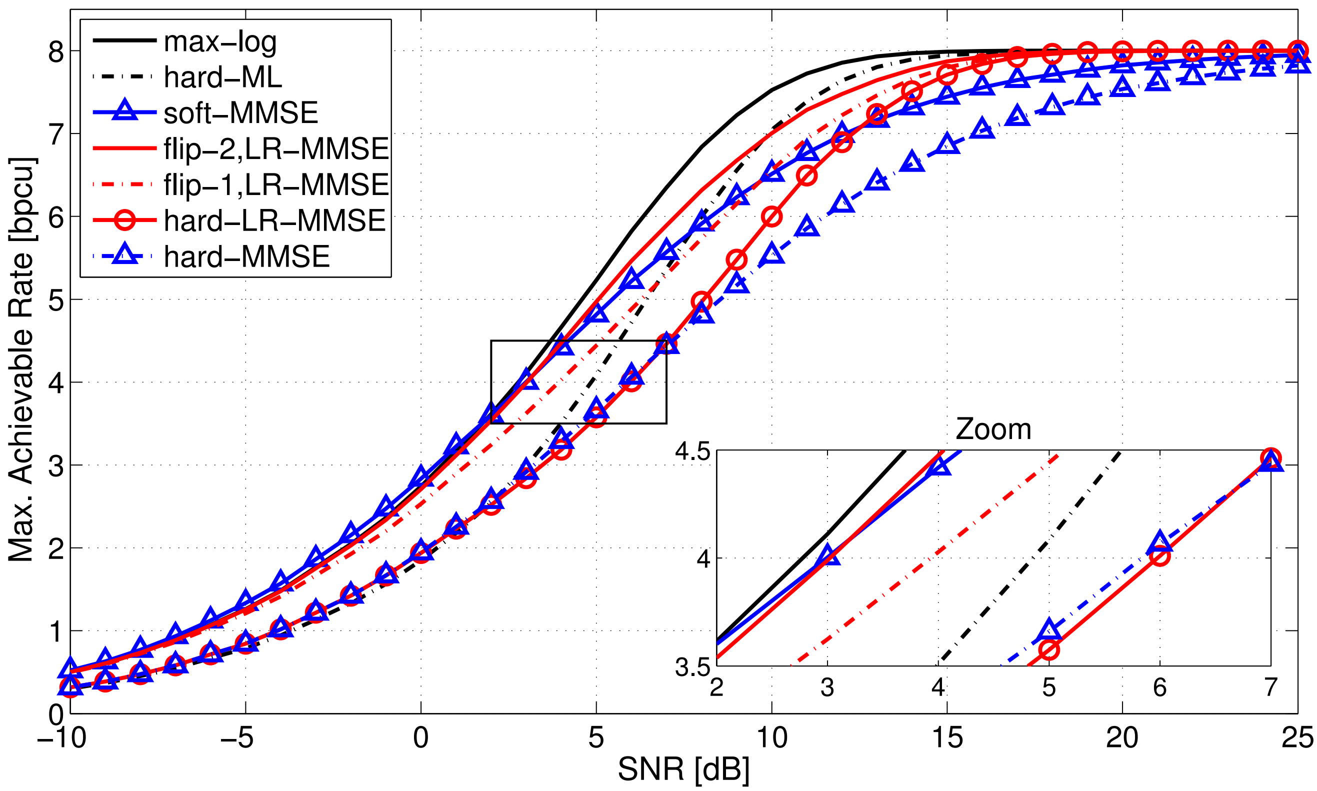


Observations: For the asymmetric case of 2×4 MIMO it can be seen that bit flipping demodulation aligns with the performance results of the other soft demodulators (cf. page 11 and page 24). In comparison to 1 bit flipping (cf. page 24), 2 bit flipping naturally performs slightly better in this case.

Lattice-Reduction-Aided Detector

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

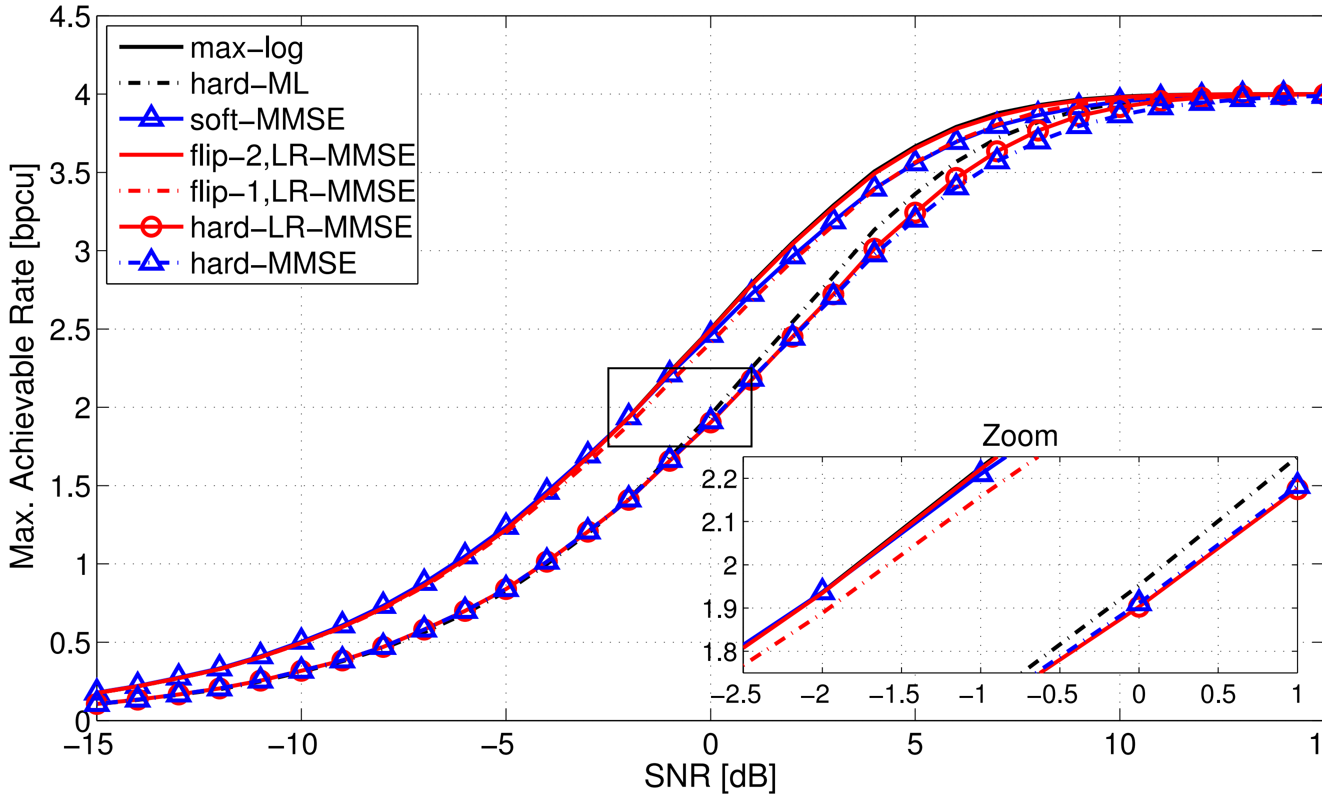


Observations: See [1, Section V.B] for more details. Here, soft LLR demodulation is obtained by extending the hard LR-MMSE demodulator in [11] via 1 bit and 2 bits flipping, respectively. It is seen that LR with hard MMSE demodulation shows a significant performance advantage over direct hard MMSE demodulation for SNRs above 7.2 dB (rates higher than 4.5 bpcu). At rates higher than about 7.1 bpcu, LR-aided hard demodulation even outperforms soft MMSE demodulation. Bit flipping is helpful particularly at low-to-medium rates. Thus, for SNRs below 6.8 dB (rates lower than 5.2 bpcu) LR-aided soft demodulation with $D = 1$ essentially performs better than hard ML. When flipping up to $D = 2$ bits, LR-aided soft demodulation closely approaches max-log performance and reveals a significant performance advantage over soft MMSE demodulation without LR in the high-rate regime.

Lattice-Reduction-Aided Detector

System Setup:

- Number of transmit antennas: 2
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

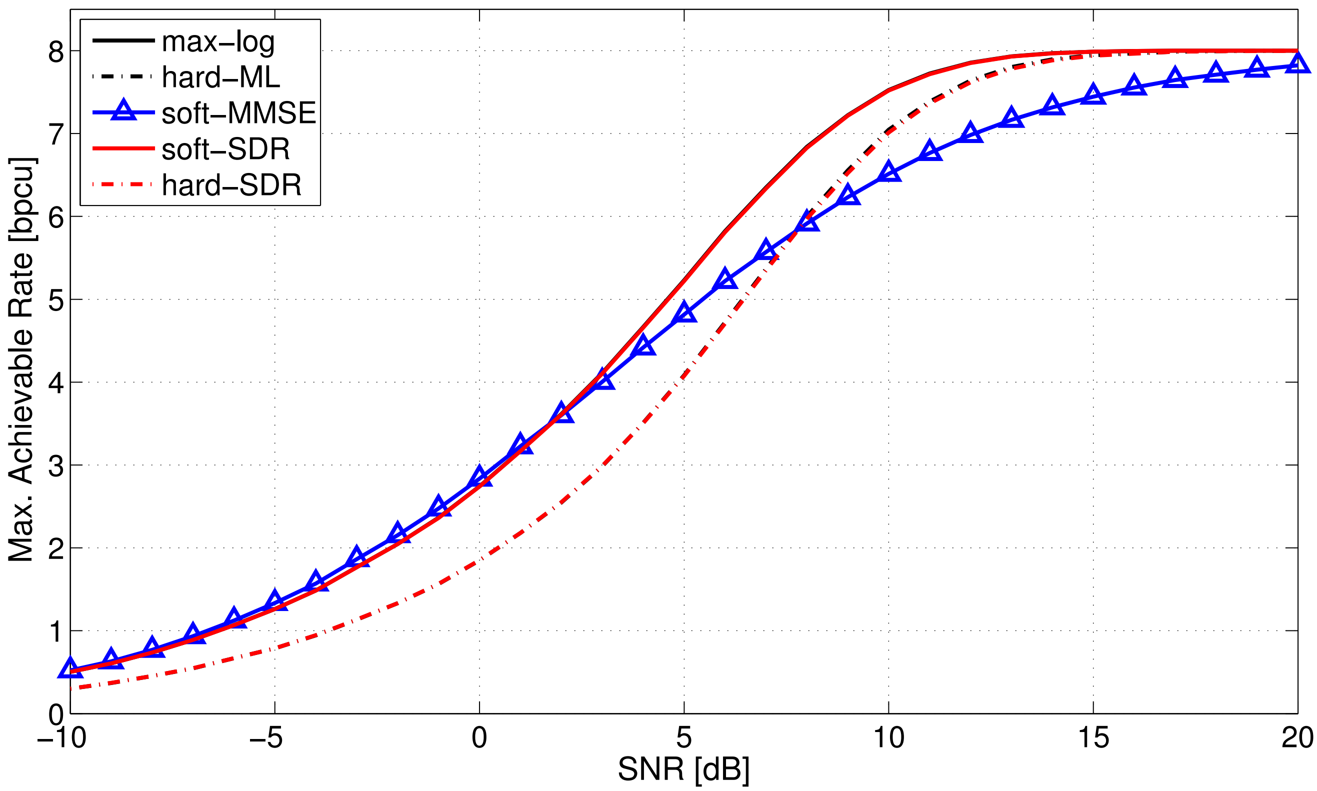


Observations: It can be seen that for the asymmetric case of 2×4 MIMO LR-based schemes confirm the essential distinction of hard and soft demodulator performance results observed on page 11.

Semidefinite Relaxation (SDR) Detector [12]

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

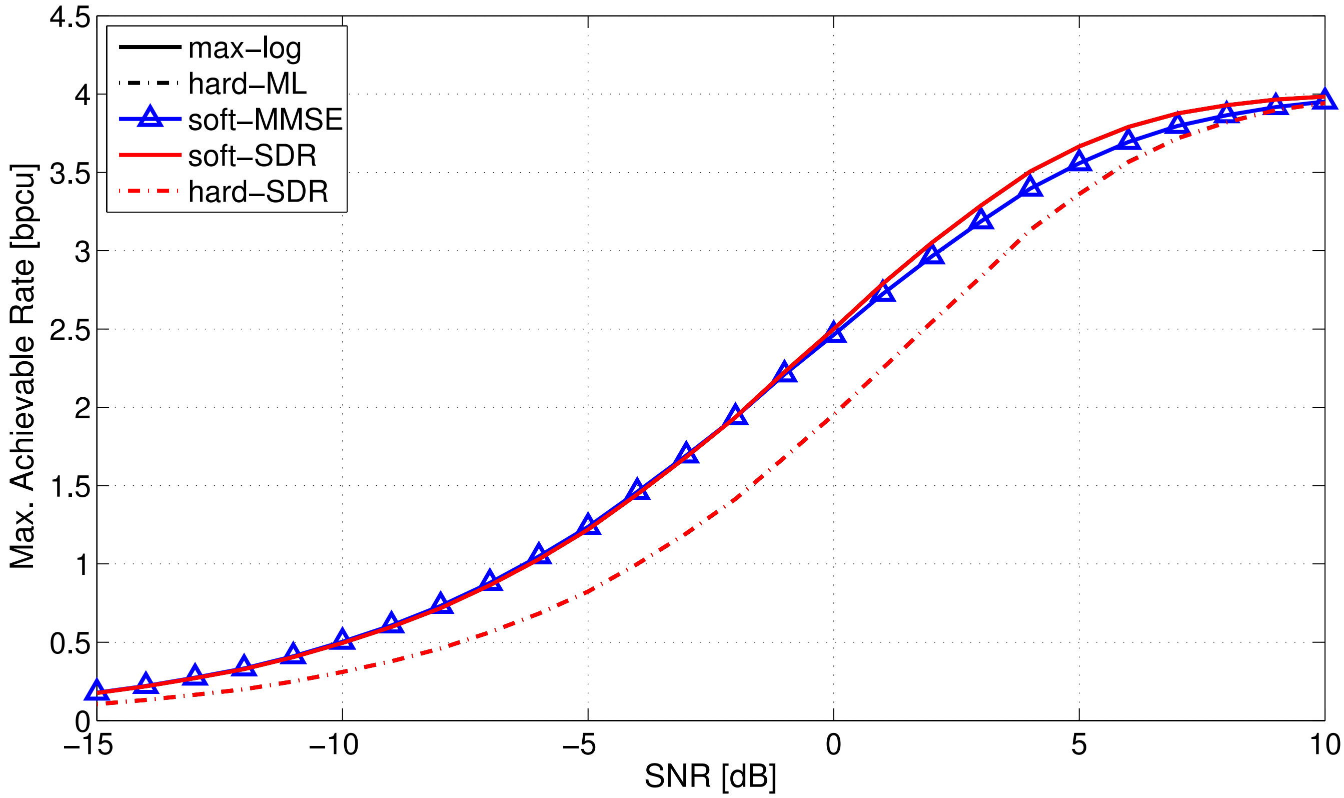


Observations: See [1, Section V.C] for more details. Here, we show the capacity results for hard and soft SDR demodulation (as described in [12] using randomization with 25 trials). Surprisingly, it can be seen that hard and soft SDR demodulation exactly match the performance of hard ML and max-log demodulation, respectively.

Semidefinite Relaxation (SDR) Detector [12]

System Setup:

- Number of transmit antennas: 2
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

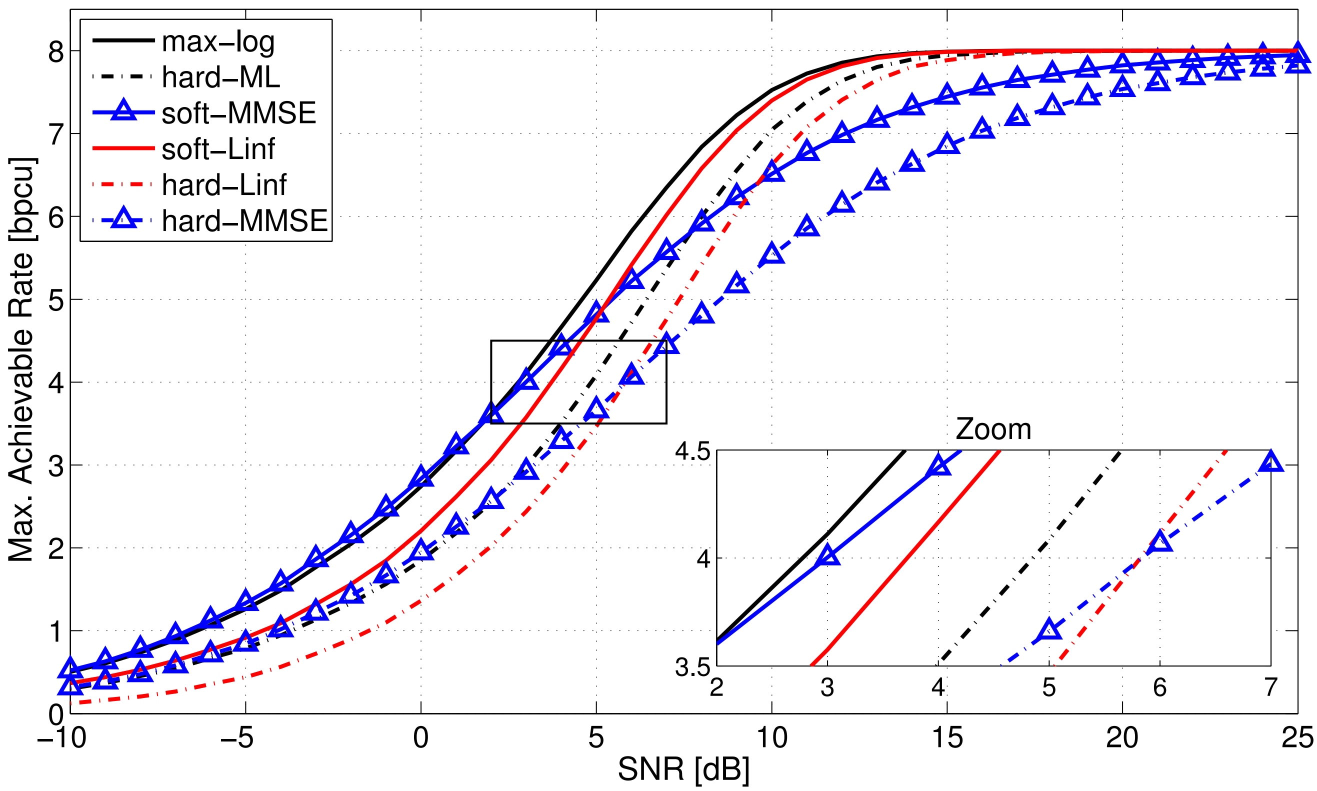


Observations: In the asymmetric antenna setup hard and soft SDR demodulators line up with the hard/soft performance behavior observed on page 11. In fact for this case, hard and soft SDR demodulation is again seen to exactly match hard ML and max-log demodulation, respectively (cf. page 28).

ℓ^∞-Norm Demodulator

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

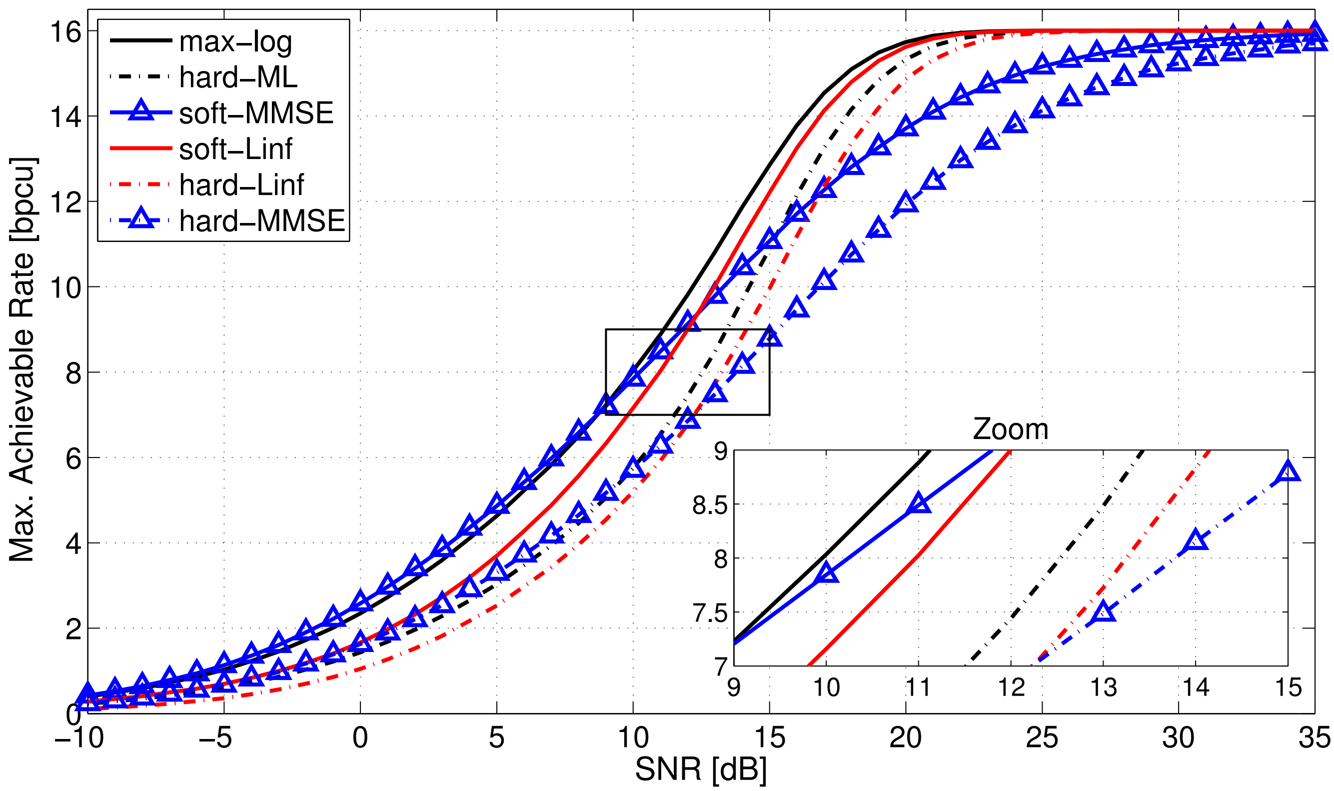


Observations: See [1, Section V.D] for more details. In comparison to hard ML demodulation, which makes use of the ℓ^2 norm, hard ℓ^∞-norm demodulation reveals an average SNR loss of about 1 dB. Note that for low rates (below 4 bpcu) hard MMSE outperforms hard ℓ^∞-norm demodulation. The same statements hold true when comparing soft ℓ^∞-norm demodulation with max-log performance. However, for high rates the gap to the max-log approximation tends to decrease, e.g., 0.6 dB at 6 bpcu.

ℓ^∞-Norm Demodulator

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 16-QAM
- Labeling: Gray

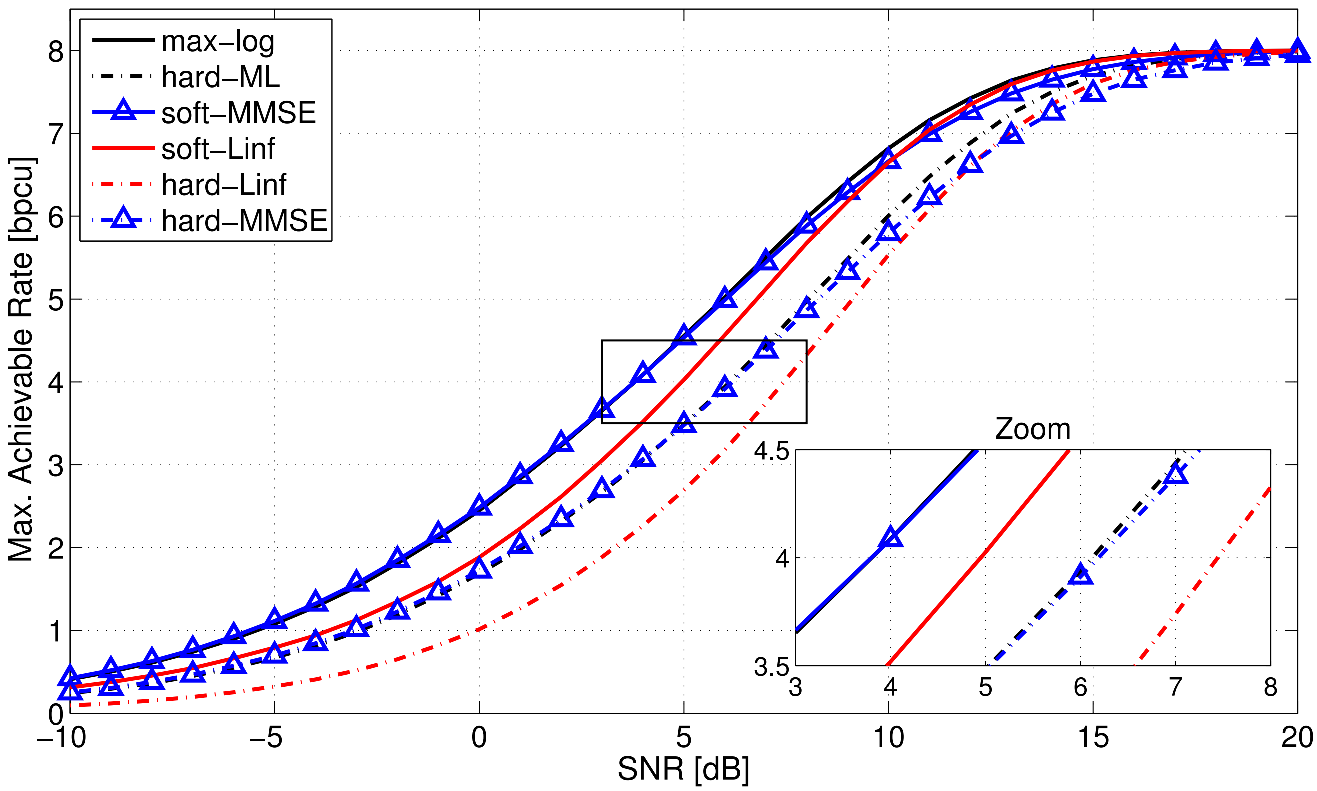


Observations: Very much the same observations as described on page 30 apply when the constellation size is increased from 4-QAM to 16-QAM .

ℓ^∞-Norm Demodulator

System Setup:

- Number of transmit antennas: 2
- Number of receive antennas: 4
- Constellation: 16-QAM
- Labeling: Gray

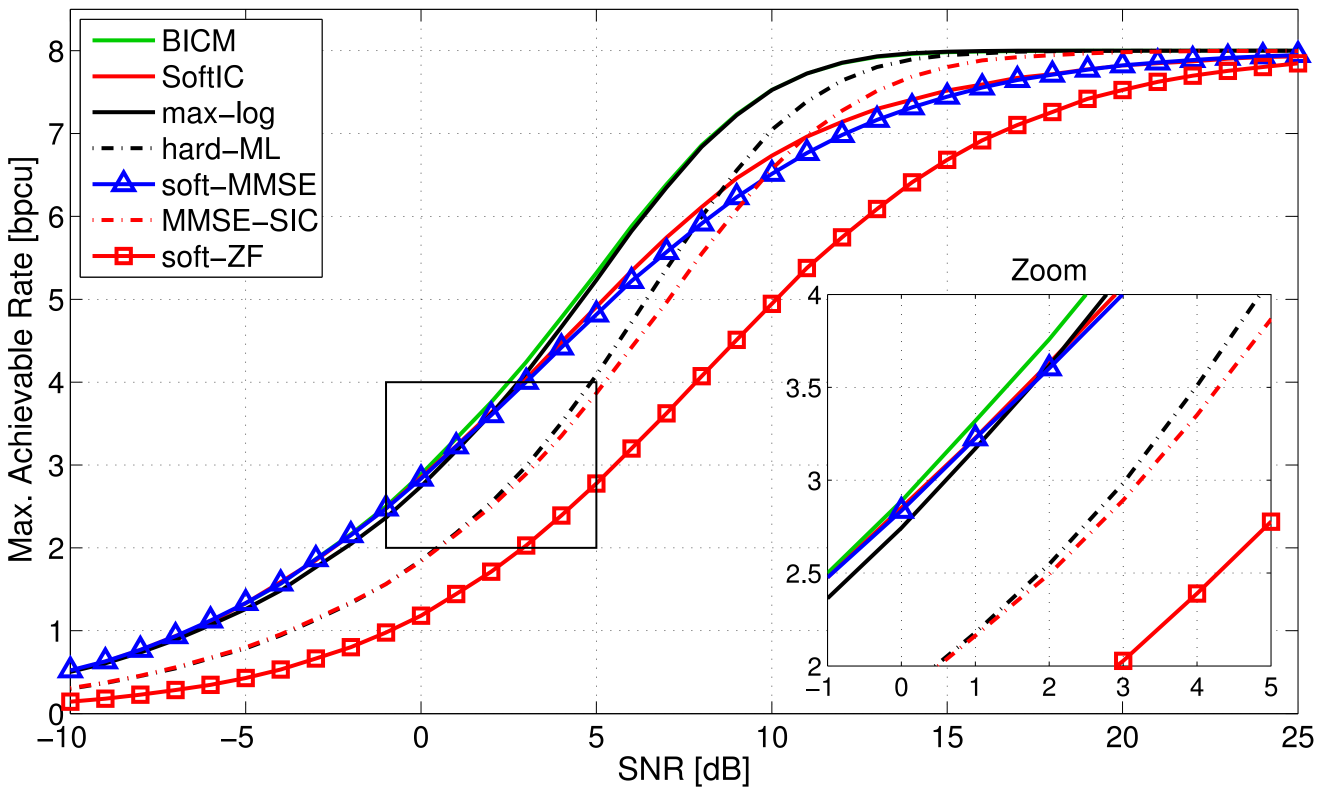


Observations: Note that ℓ^∞-norm demodulation is the only demodulation scheme that suffers from a significant performance loss when applied to an asymmetric antenna constellation (cf. page 11). In particular, hard and soft ℓ^∞-norm demodulation reveal a significant performance loss at low-to-medium rates. At 2 bpcu, soft ℓ^∞-norm demodulation requires 1.75 dB higher SNR than max-log and soft MMSE. Moreover, hard ℓ^∞-norm demodulation requires 2.3 dB higher SNR at this rate than hard ML/MMSE.

Successive Interference Canceler (SIC) [15] and Soft Interference Canceler (SoftIC) [16]

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

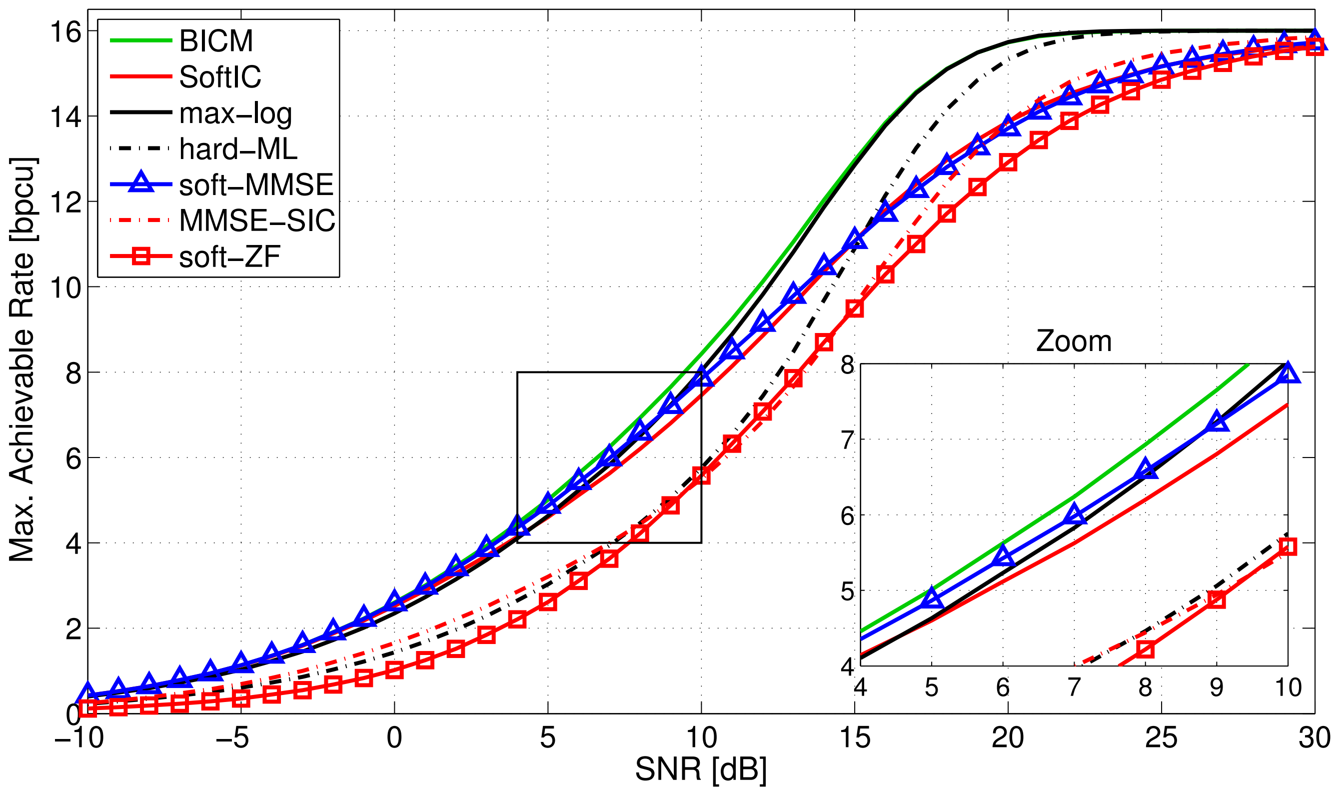


Observations: See [1, Section V.E] for more details. Hard MMSE-SIC demodulation is seen to perform similarly to hard ML demodulation at low rates and even outperforms it slightly at very low rates. While at high rates MMSE-SIC shows a noticeable gap to hard ML, it can outperform both soft MMSE and SoftIC (with 3 iterations and initialized using soft ZF demodulation) in this regime. SoftIC is superior to MMSE-SIC up to rates of 7 bpcu (and SNRs lower than 11 dB). At low rates, SoftIC even performs slightly better than max-log demodulation and essentially coincides with BICM capacity and soft MMSE. For the chosen system parameters, SoftIC closely matches soft MMSE at low rates and even outperforms it at high rates. This statement does not hold in general, however (cf. results for 4×4 MIMO with 16-QAM on page 34).

Successive Interference Canceler (SIC) [15] and Soft Interference Canceler (SoftIC) [16]

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 16-QAM
- Labeling: Gray

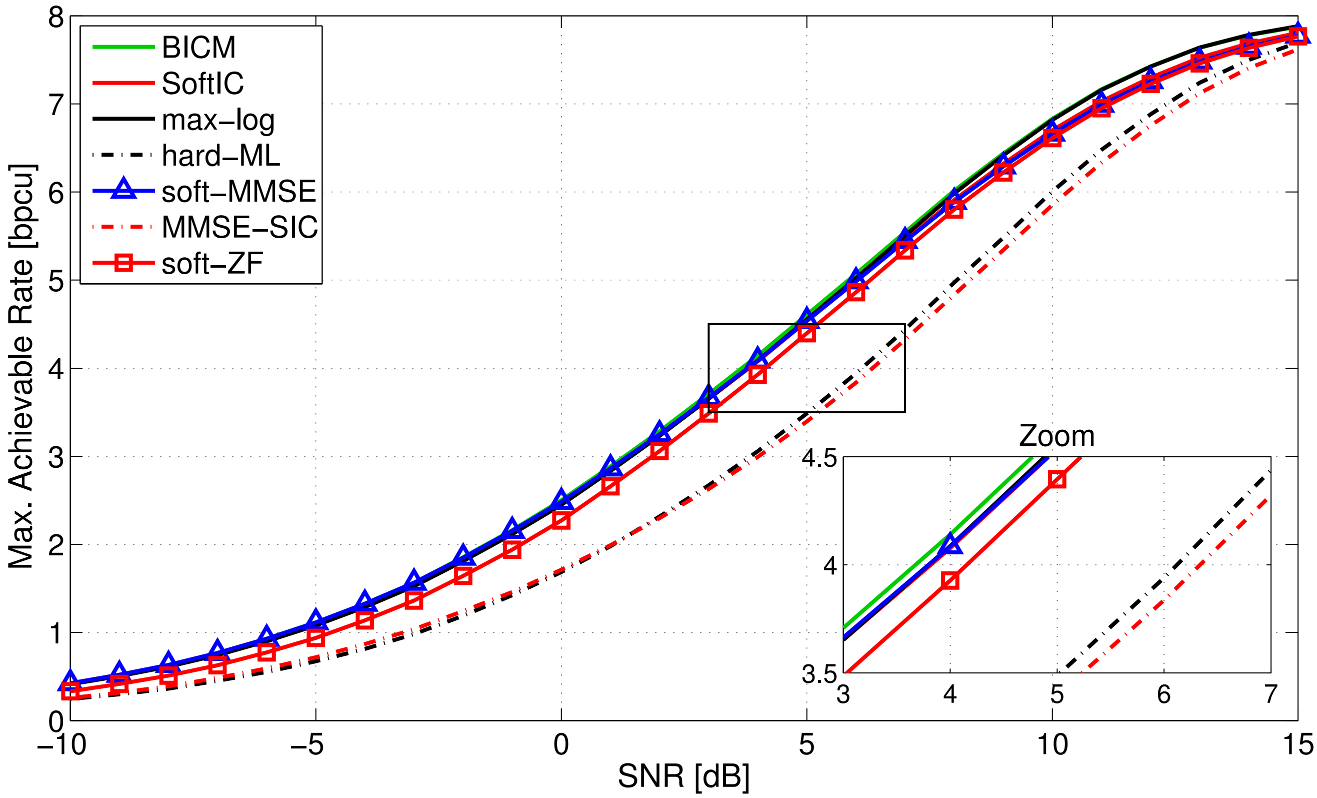


Observations: Whereas SoftIC matches soft MMSE at low-to-medium rates and even slightly outperforms soft MMSE at high rates for $4{\times}4$ MIMO with 4-QAM (cf. page 33), this is not the case when increasing the constellation size to 16-QAM. In this setup the performance of SoftIC drops below soft MMSE performance at medium rates. MMSE-SIC tends to slightly outperform hard ML demodulation at very low rates. However, compared to 4-QAM case (cf. page 33), MMSE-SIC shows a larger SNR gap to hard ML at high rates.

Successive Interference Canceler (SIC) [15] and Soft Interference Canceler (SoftIC) [16]

System Setup:

- Number of transmit antennas: 2
- Number of receive antennas: 4
- Constellation: 16-QAM
- Labeling: Gray

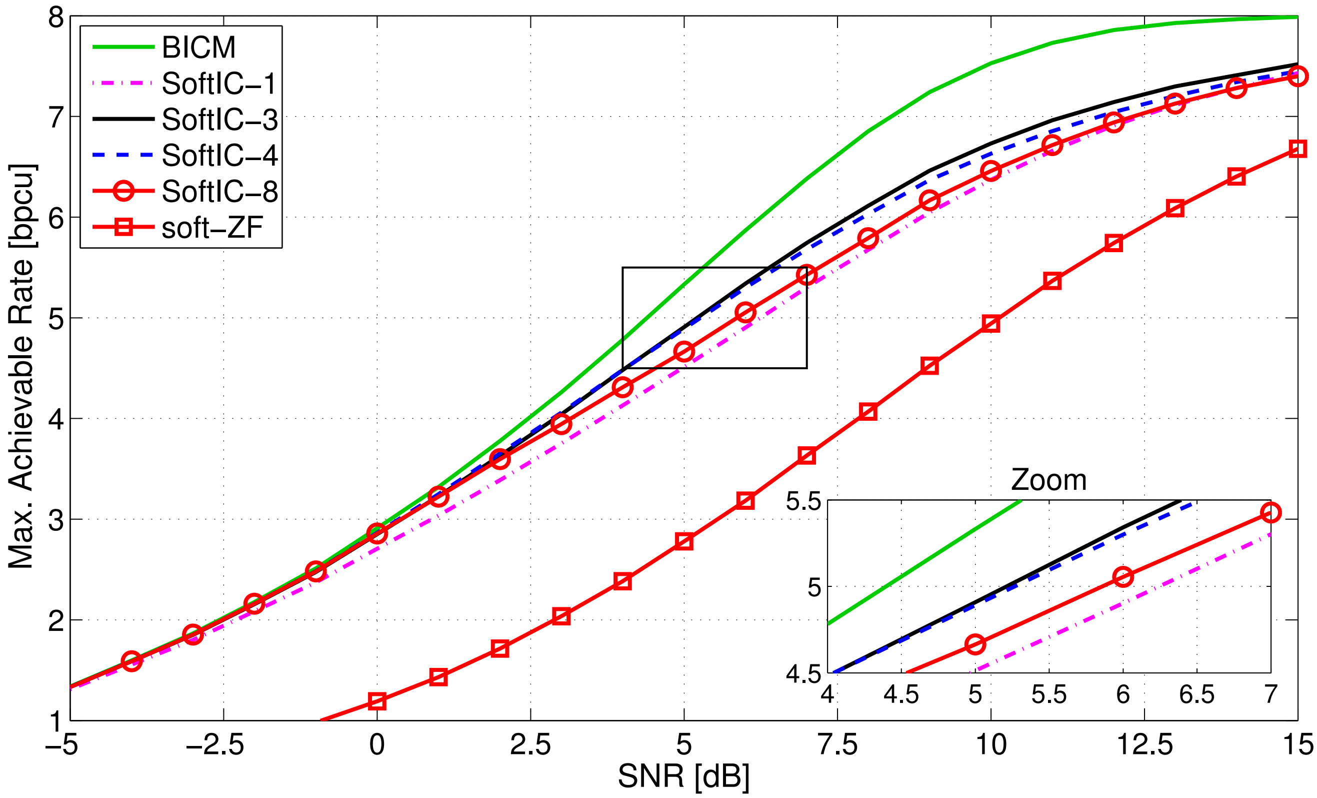


Observations: It can be seen that for the asymmetric antenna setup SoftIC demodulation aligns with the other soft demodulators and MMSE-SIC aligns with the hard demodulation schemes (cf. page 11).

Soft Interference Canceler (SoftIC) versus iterations

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray



Observations: We observed that the performance of the SoftIC algorithm depends strongly on the number of iterations (here, SoftIC is initialized using soft ZF demodulation and plotted for 1, 3, 4, and 8 iterations). Specifically at high SNRs, SoftIC suffers from a severe performance loss if is iterated too long. The above simulation results show that the SoftIC algorithm performs best when terminated after $2-3$ iterations. It can be seen that a large performance gain can be already achieved with one iteration, e.g., an SNR improvement of 4.2 dB at 4 bpcu in comparison to soft ZF demodulation. However, after 3 iterations the performance starts to deteriorate for medium-to-high SNRs. For example, comparing SoftIC with 3 and with 8 iterations at 6 bpcu reveals an SNR loss of 0.9 dB.

III. BER Performance

In the following we verify the foregoing observations by showing BER results for a 4×4 MIMO-BICM system with 4-QAM in conjunction with irregular LDPC codes of block length 64000, designed[1] for code rates $1/4$ and $3/4$ (corresponding to 2 and 6 bpcu, respectively). Here, we focus on the results of the baseline demodulators (reviewed in [1, Section IV.A]). For the case of soft demodulation, we used LDPC codes designed for additive white Gaussian noise channels whereas with hard detectors LDPC codes designed for a binary symmetric channel (BSC) were employed. At the receiver, message passing LDPC decoding [5] was performed. In the case of hard demodulation, the message-passing decoder was provided with the LLRs

$$\tilde{\Lambda}_l = (2\hat{c}_l - 1) \, \log \frac{1 - p_0}{p_0}, \tag{3}$$

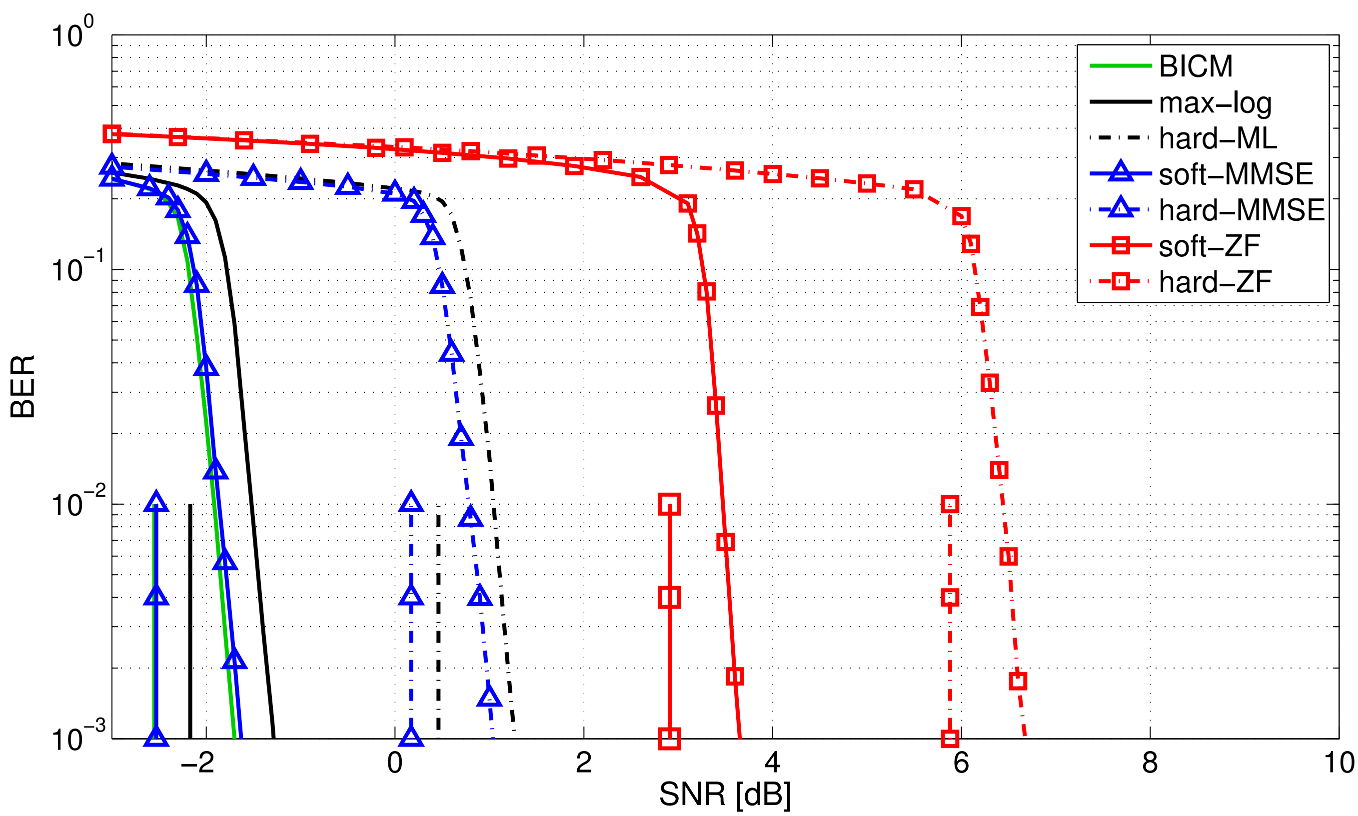
where $\{\hat{c}_1, \ldots, \hat{c}_{R_0}\}$ are the corresponding detected code bits obtained by the hard-output demodulator and $p_0 = \mathrm{P}\{\hat{c}_l \neq c_l\}$ is the cross-over probability of the equivalent BSC, which was determined via Monte-Carlo simulations. In addition, we used an LLR correction unit (implemented via a lookup table, similarly as in [17]) to correct the approximate LLRs delivered by the suboptimal soft demodulators before being fed to the channel decoder. Using LLR correction for approximate soft demodulators as well as (3) in the case of hard demodulators is critical in order to provide the channel decoder (here, the LDPC decoder) with appropriate reliability information [17]–[21]. Additional observations and information about parameter settings can be found in [1, Section IV.D].

[1] The LDPC code design was performed using the EPFL web-tool at `http://lthcwww.epfl.ch/research/ldpcopt/`.

Baseline Demodulators

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray
- Code design: rate-$1/4$ regular LDPC code

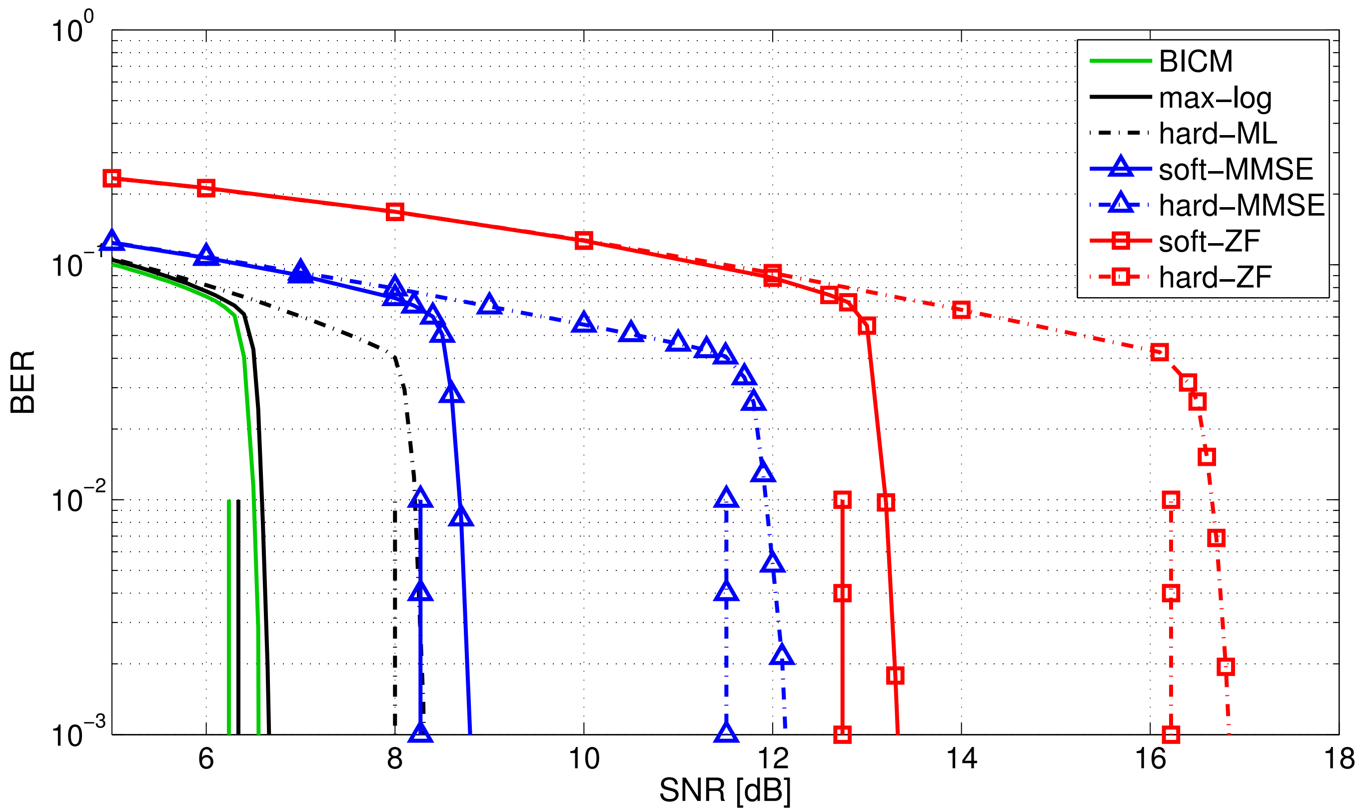


Observations: See [1, Section IV.D] for more results. Here, also the capacity limits in terms of the minimal SNRs which are required to achieve a code rate of $1/4$ are indicated by corresponding horizontal, gray lines (cf. capacity results on page 7). In general, no universal statements about the BER performance ranking of the demodulators can be made. At low rates soft MMSE demodulation performs best and hard ML demodulation performs worst whereas at high rates the same holds true for max-log and hard MMSE demodulation, respectively. Specifically, at rate $1/4$ soft MMSE outperforms max-log and hard ML demodulation by 0.3 dB and 2.9 dB, respectively; however, at rate $3/4$ soft MMSE performs 0.5 dB poorer than hard ML and 2.1 dB poorer than max-log (see results on page 39). These results confirm the capacity-based observation on page 7 that there is no universal (i.e., rate- and SNR-independent) demodulator performance ranking.

Baseline Demodulators

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray
- Code design: rate-$3/4$ regular LDPC code



Observations: See observations on page 38 and [1, Section IV.D] for more details. It is shown that whereas hard ML demodulation performs poorer than soft MMSE demodulation at rate $1/4$ (cf. page 38), for rate $3/4$ its just the other way round (by about $0.5\,$dB). This result agrees with the observations on page 7.

IV. Imperfect Channel State Information and Noise Variance

The following section investigates the ergodic system capacity for the case of imperfect channel state information (CSI) at the receiver. In particular, we consider training-based estimation of the channel matrix $\mathbf{H}$ and the noise variance σ_v^2. The transmitter sends $N_\mathrm{p} > M_\mathrm{T}$ training vectors[2] which are arranged into a full rank $M_\mathrm{T} \times N_\mathrm{p}$ training matrix $\mathbf{X}_\mathrm{p}$. We assume that the transmit power per channel use for training and actual data is the same such that the Frobenius norm [22] of $\mathbf{X}_\mathrm{p}$ equals $\|\mathbf{X}_\mathrm{p}\|_\mathrm{F}^2 = N_\mathrm{p} E_s$. Assuming that the channel stays constant for the duration of one block (which contains training and actual data), the $M_\mathrm{R} \times N_\mathrm{p}$ receive matrix $\mathbf{Y}_\mathrm{p}$ induced by the training is given by (cf. (1))

$$\mathbf{Y}_\mathrm{p} = \mathbf{H}\,\mathbf{X}_\mathrm{p} + \mathbf{V}\,. \tag{4}$$

Here, the $M_\mathrm{R} \times N_\mathrm{p}$ matrix $\mathbf{V}$ contains the noise received during the training period.

Using (4), the least-squares channel estimate (identical to the ML estimate under a Gaussian i.i.d. assumption for the noise) is computed as [23]

$$\hat{\mathbf{H}} = \mathbf{Y}_\mathrm{p}\mathbf{X}_\mathrm{p}^\mathrm{H}(\mathbf{X}_\mathrm{p}\mathbf{X}_\mathrm{p}^\mathrm{H})^{-1}\,. \tag{5}$$

The estimated channel matrix $\hat{\mathbf{H}}$ is then used to obtain the noise variance estimate

$$\hat{\sigma}_v^2 = \frac{1}{M_\mathrm{R}(N_\mathrm{p} - M_\mathrm{T})}\|\mathbf{Y}_\mathrm{p} - \hat{\mathbf{H}}\mathbf{X}_\mathrm{p}\|_\mathrm{F}^2\,, \tag{6}$$

which essentially amounts to measuring the mean power of $\mathbf{Y}_\mathrm{p}$ in the $(N_\mathrm{p}$–$M_\mathrm{T})$-dimensional orthogonal complement of the range space of $\mathbf{X}_\mathrm{p}^\mathrm{H}$.

We consider the suboptimum generation of LLRs based on a mismatched demodulator metric (derived under the perfect CSI assumption), i.e., we evaluate the demodulator outputs by replacing the perfect channel matrix and noise variance in the corresponding metrics with its estimates.

In the following, we provide numerical results for the ergodic system capacity where we compare the case of perfect and imperfect CSI. The latter assumes that both the channel and the noise variance are estimated using a training-based ML estimator (cf. (5) and (6)) with the minimum number of $N_\mathrm{p} = 5$ pilot vectors. Throughout, a 4×4 MIMO system with 4-QAM and Gray labeling is considered ($R_0 = 8$). For the baseline demodulators we restrict to max-log, hard ML, and soft MMSE demodulation.

Furthermore, we analyze how the quality of the estimates influences the performance of the demodulators. Therefore, we present a performance comparison in terms of the required minimum SNR to achieve a target code rate of 2 bpcu and 6 bpcu, respectively, over the amount of training vectors N_p. For the baseline demodulators we consider different variants which assume imperfect channel matrix (using (5)) and imperfect noise variance (using (6)), imperfect channel matrix (using (5)) and perfect noise variance, and perfect channel matrix and imperfect noise variance. For the latter case, the noise variance is estimated according to

$$\hat{\sigma}_v^2 = \frac{1}{M_\mathrm{R} N_\mathrm{p}}\|\mathbf{Y}_\mathrm{p} - \mathbf{H}\mathbf{X}_\mathrm{p}\|_\mathrm{F}^2\,,$$

[2] While $N_\mathrm{p} = M_\mathrm{T}$ is sufficient to estimate $\mathbf{H}$, extra training is required for estimation of σ_v^2.

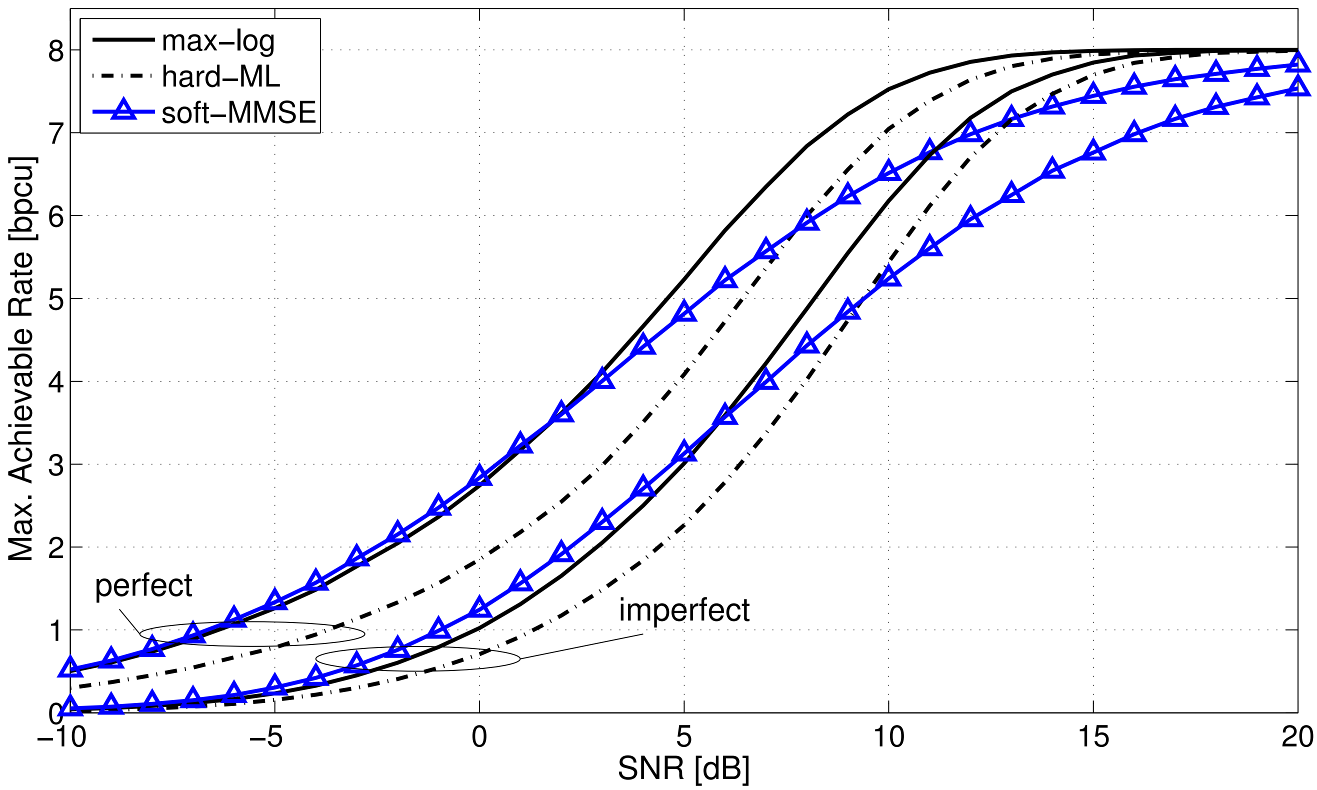
(note that this equals the unbiased estimator for this case). As a reference, the results for perfect channel matrix and perfect noise variance are indicated by corresponding horizontal, gray lines. For the remaining demodulators we focus only on the case of imperfect channel and imperfect noise variance.

Additional observations and information about the system settings can be found in [1, Section VI].

Baseline Demodulators – Perfect vs. Imperfect CSI

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray
- Number of pilot vectors for estimation: 5

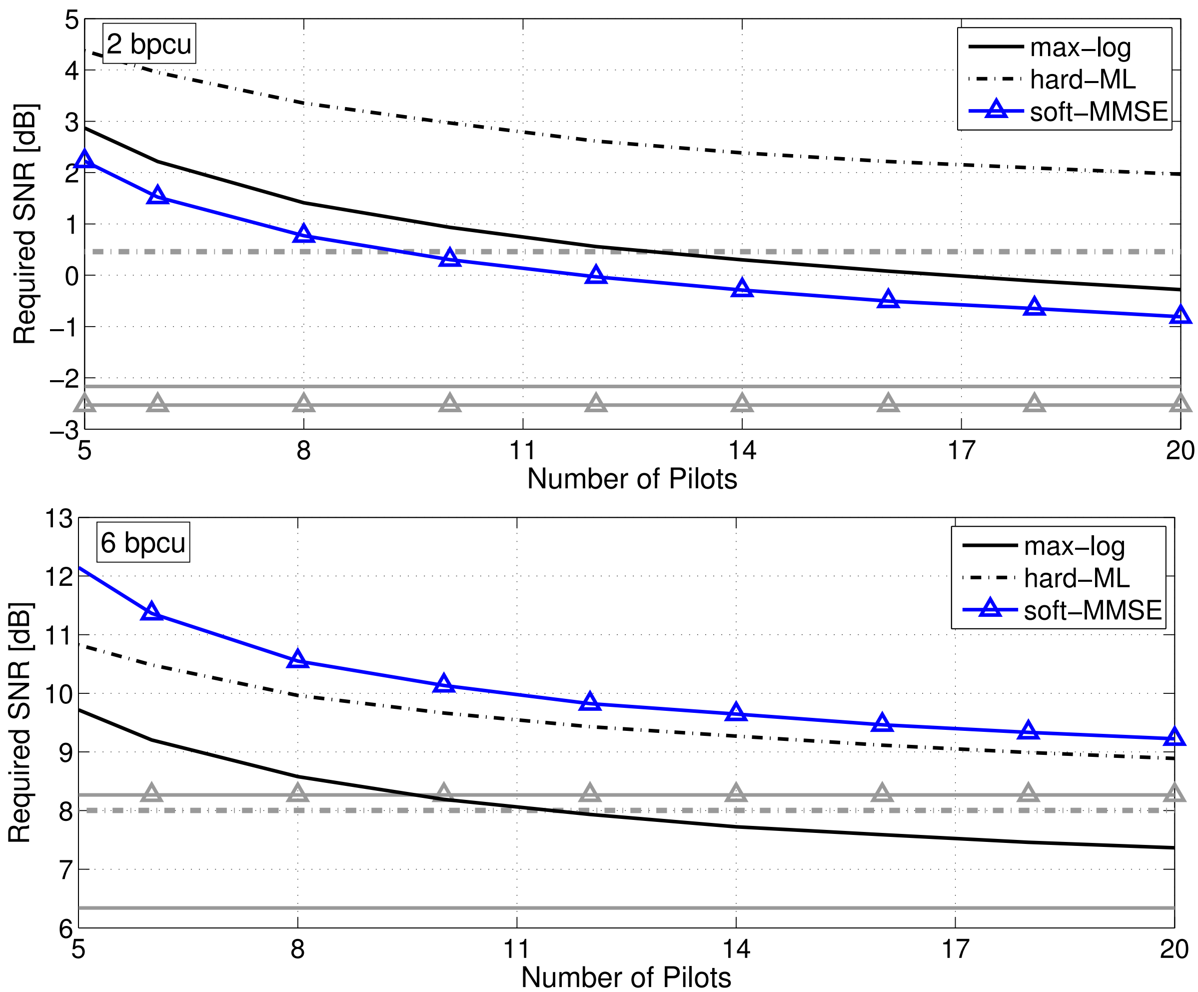


Observations: See results on page 44 and [1, Section VI] for more details. It is seen that for all three detectors imperfect CSI results in a significant performance loss, e.g., at 4 bpcu the SNR loss for max-log, hard ML, and soft MMSE is 3.9 dB, 3.2 dB, and 4 dB, respectively. In the considered worst case imperfect CSI setup the performance advantage of soft MMSE demodulation over hard ML demodulation at low rates is a little bit less pronounced; note that the cross-over between hard ML and soft MMSE performance shifts from 5.8 bpcu (at an SNR of about 7.7 dB) for perfect CSI to 5 bpcu (at 9.4 dB) for the case of imperfect CSI. However, the gap between soft MMSE and max-log is slightly larger at low rates, e.g., 0.7 dB at 2 bpcu. The performance losses for all demodulators tend to be smaller at high rates, which may be partly attributed to the fact that the CSI (in particular, the noise variance estimate) becomes more accurate with increasing SNR. In general it can be observed that the performance loss of hard ML is the smallest while soft MMSE and max-log performance deteriorates stronger; note that hard ML does not use the noise variance and hence is more robust to estimation errors in σ_v^2. Here, hard ML comes within less than $1.3\,\mathrm{dB}$ of max-log performance. In contrast, soft MMSE uses the imperfect channel and the noise variance estimate in the MMSE equalization stage and in the LLR calculation and is thus most strongly affected.

Baseline Demodulators – Imperfect Channel/Imperfect Noise Variance

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

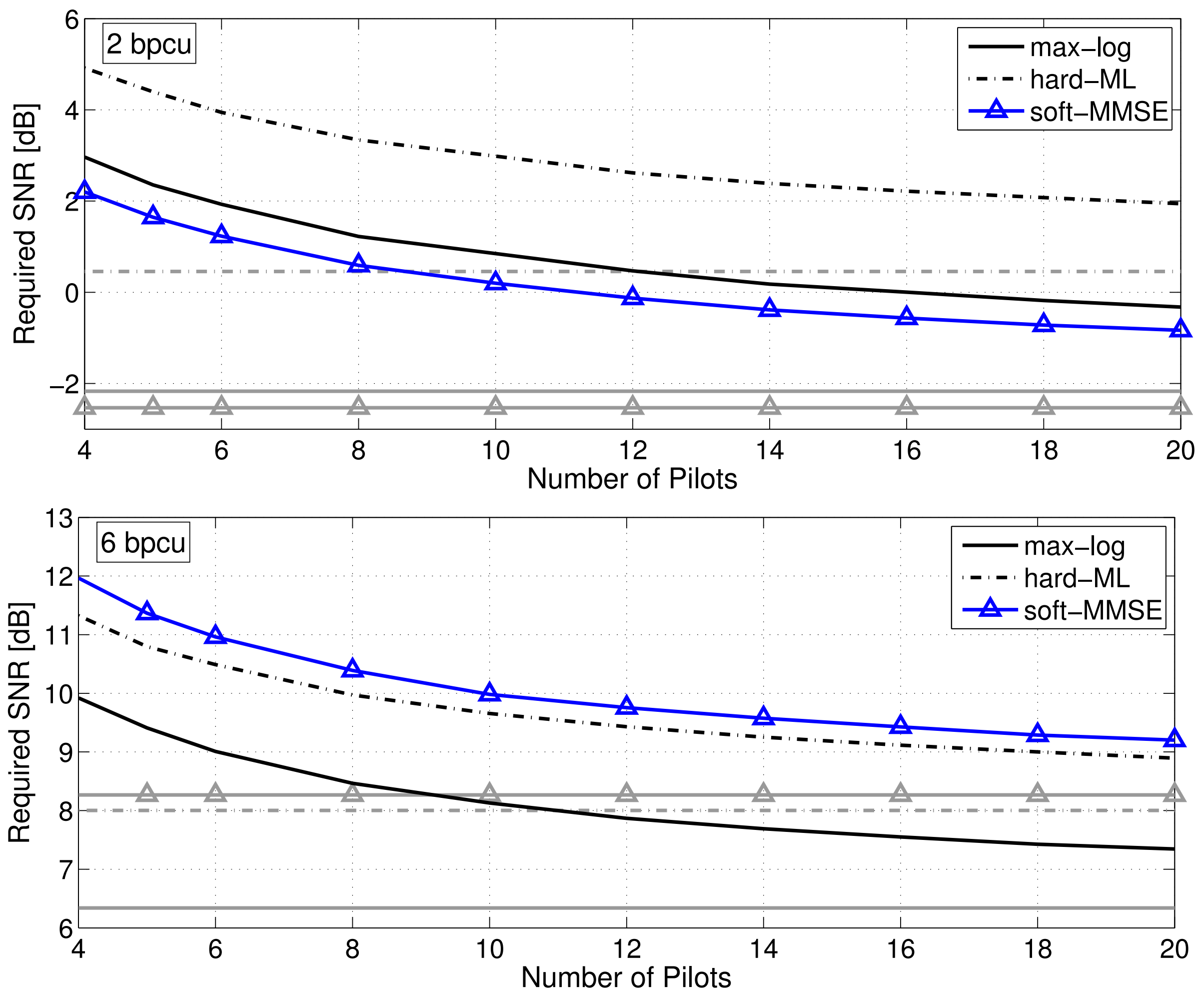


Observations: See [1, Section VI] for more details. It is seen that for all demodulators, the required SNR decreases rapidly with increasing amount of training. Yet, even for $N_\mathrm{p} = 20$ there is a significant gap of 1 to 2 dB to perfect CSI performance. Here, soft MMSE consistently performs better than max-log and hard ML at 2 bpcu. Moreover, it increases its SNR gap to hard ML by about 1 dB with a larger amount of training. In contrast, at 6 bpcu hard ML outperforms soft MMSE, especially for very small training durations.

Baseline Demodulators – Imperfect Channel/Perfect Noise Variance

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

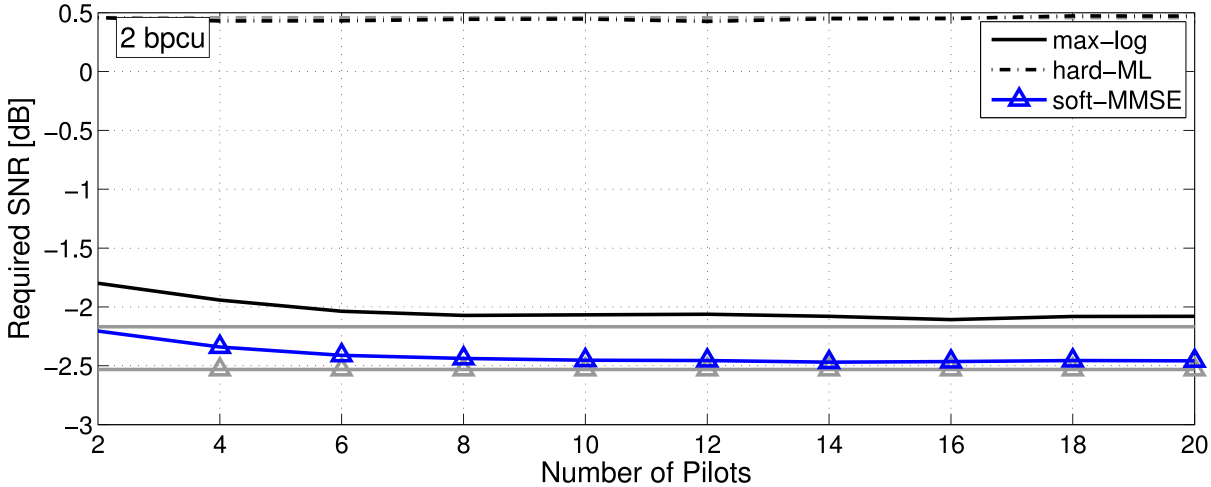


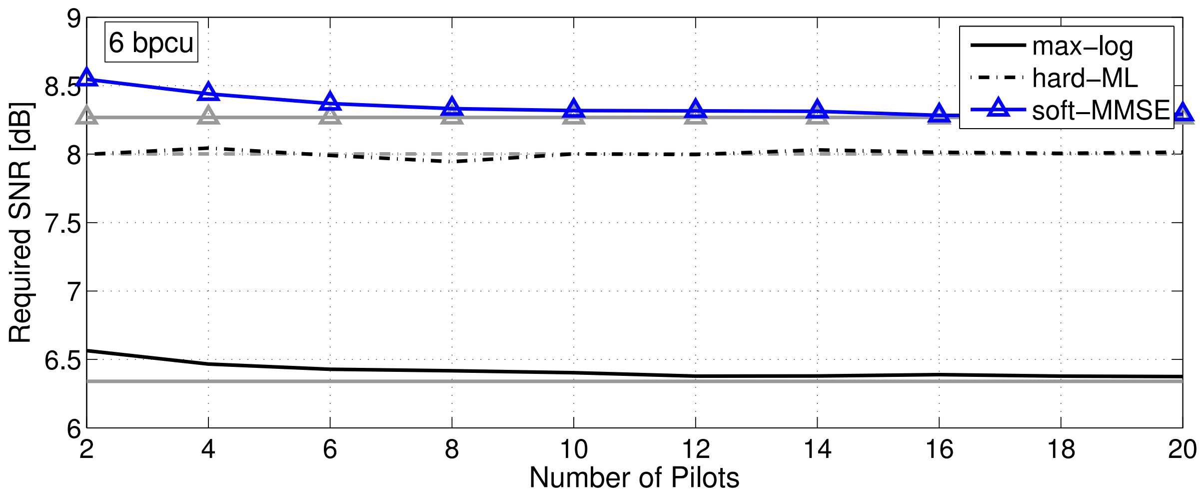
Observations: Here, we assume that only the channel matrix is affected by imperfections due to channel estimation; the noise variance is considered to be perfectly known at the receiver. The performance differences between the demodulators are not as pronounced as in our worst case scenario (cf. page 44). The latter observations can be explained by the fact that all demodulation techniques, that explicitly make use of the noise variance, may suffer from further performance degradations if provided with very poor quality estimates of the noise variance. In particular, this may affect the performance of all soft demodulation schemes, such as soft MMSE and max-log demodulation, which require an accurate noise variance estimate for the LLR computation. However, the impact of the imperfect noise variance on the demodulator performance is only very small (cf. also results on page 47). The differences lie within $0.5\,\mathrm{dB}$ for the worst case of minimum training, i.e., $N_{\mathrm{p}} = 5$ pilots. Note that the performance of hard ML demodulation essentially remains the same, since it does not require knowledge of the noise variance.

Baseline Demodulators – Perfect Channel/Imperfect Noise Variance

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray



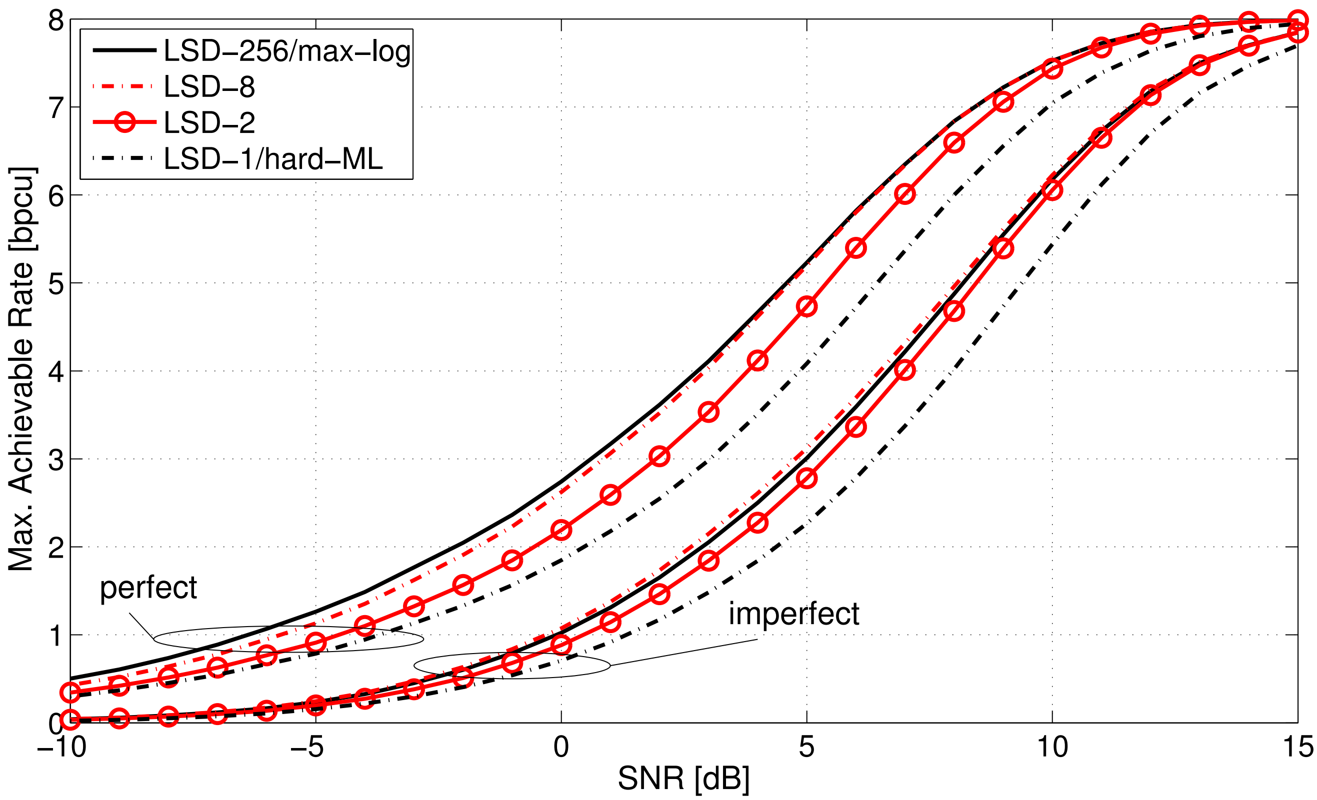


Observations: It can be seen that the demodulator performance degrades if only a small amount of training pilots is available to estimate the noise variance. However, the impact of the imperfect noise variance on the performance is not very critical and lies within 0.5 dB even for very small amounts of training. Note that hard ML does not use the noise variance and hence is more robust to estimation errors in σ_v^2. Using a sufficient amount of pilots, i.e., $N_\mathrm{p} \geq 8$, already closely approaches the performance obtained with perfect noise variance.

List Sphere Decoder (LSD) – Perfect vs. Imperfect CSI

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray
- Number of pilot vectors for estimation: 5

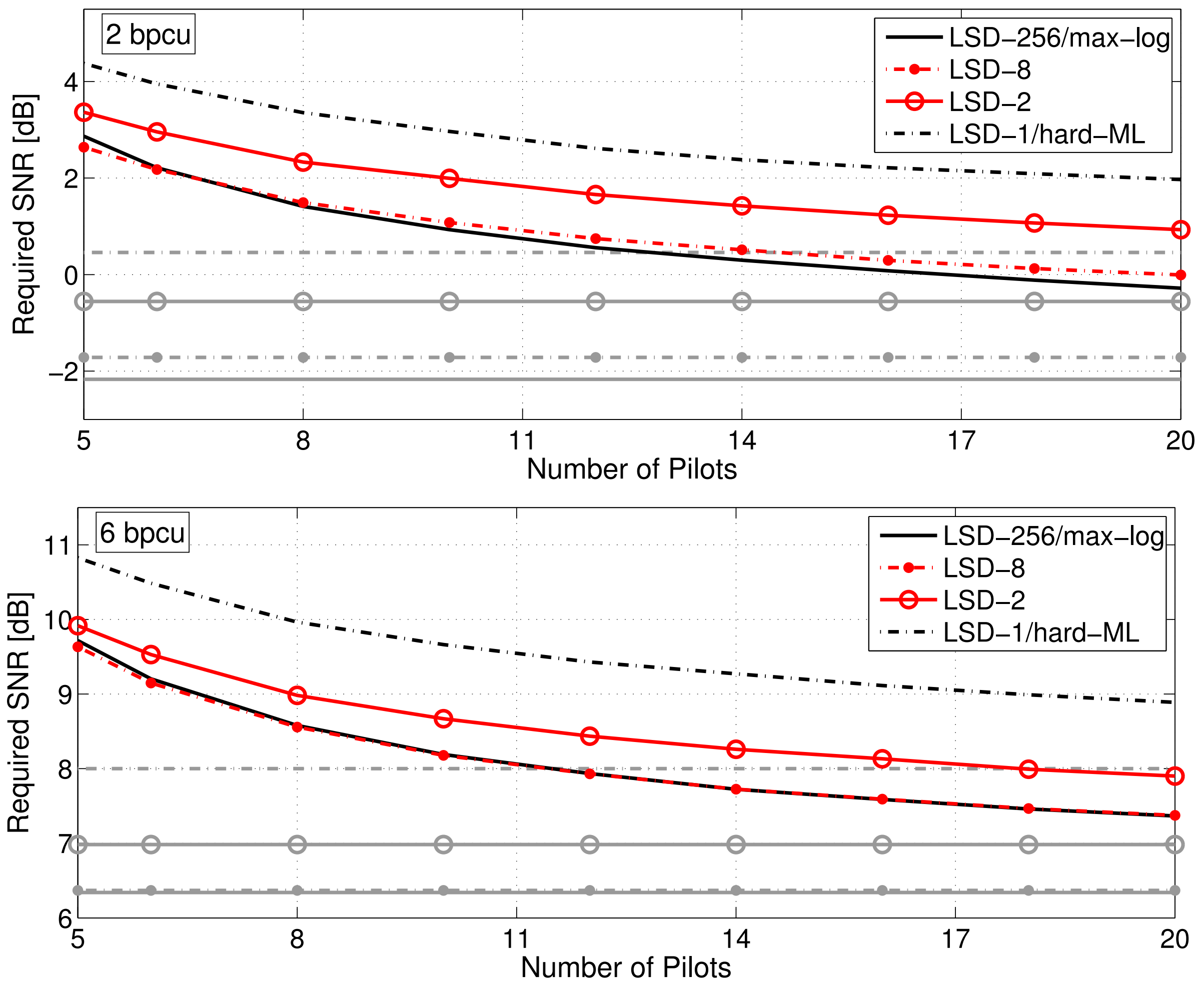


Observations: It can be seen that in this worst case scenario of imperfect channel and imperfect noise variance ($N_{\mathrm{p}} = 5$) changing the list size of the LSD no longer allows to go from hard ML to max-log performance (cf. page 17). In fact, the LSD with list size $|\mathcal{L}| = 8$ slightly outperforms max-log demodulation; here, the latter equals the LSD with a full candidate list ($|\mathcal{L}| = 256$). We note that the usual performance tradeoff of the LSD can however be experienced if more pilots for estimation are available at the receiver (cf. page 49).

List Sphere Decoder (LSD) – Imperfect Channel/Imperfect Noise Variance

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

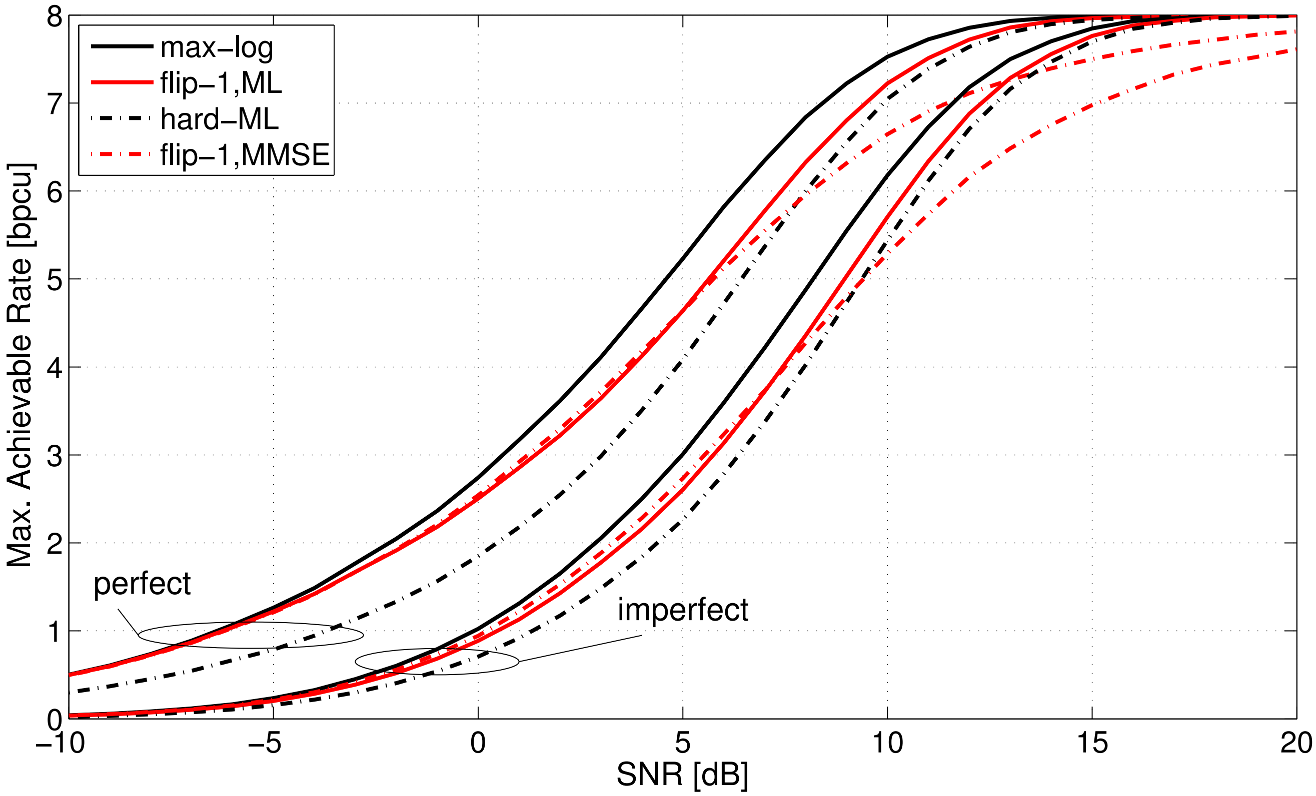


Observations: If only a small number of pilots is available for channel and noise variance estimation ($N_\mathrm{p} = 5$), it can be seen that the LSD with a list size of $|\mathcal{L}| = 8$ outperforms the LSD with full list ($|\mathcal{L}| = 256$). Although this is not the case for larger training durations, LSD with $|\mathcal{L}| = 8$ performs mostly very close to max-log for both rates.

Bit Flipping Demodulators (1-bit) – Perfect vs. Imperfect CSI

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray
- Number of pilot vectors for estimation: 5

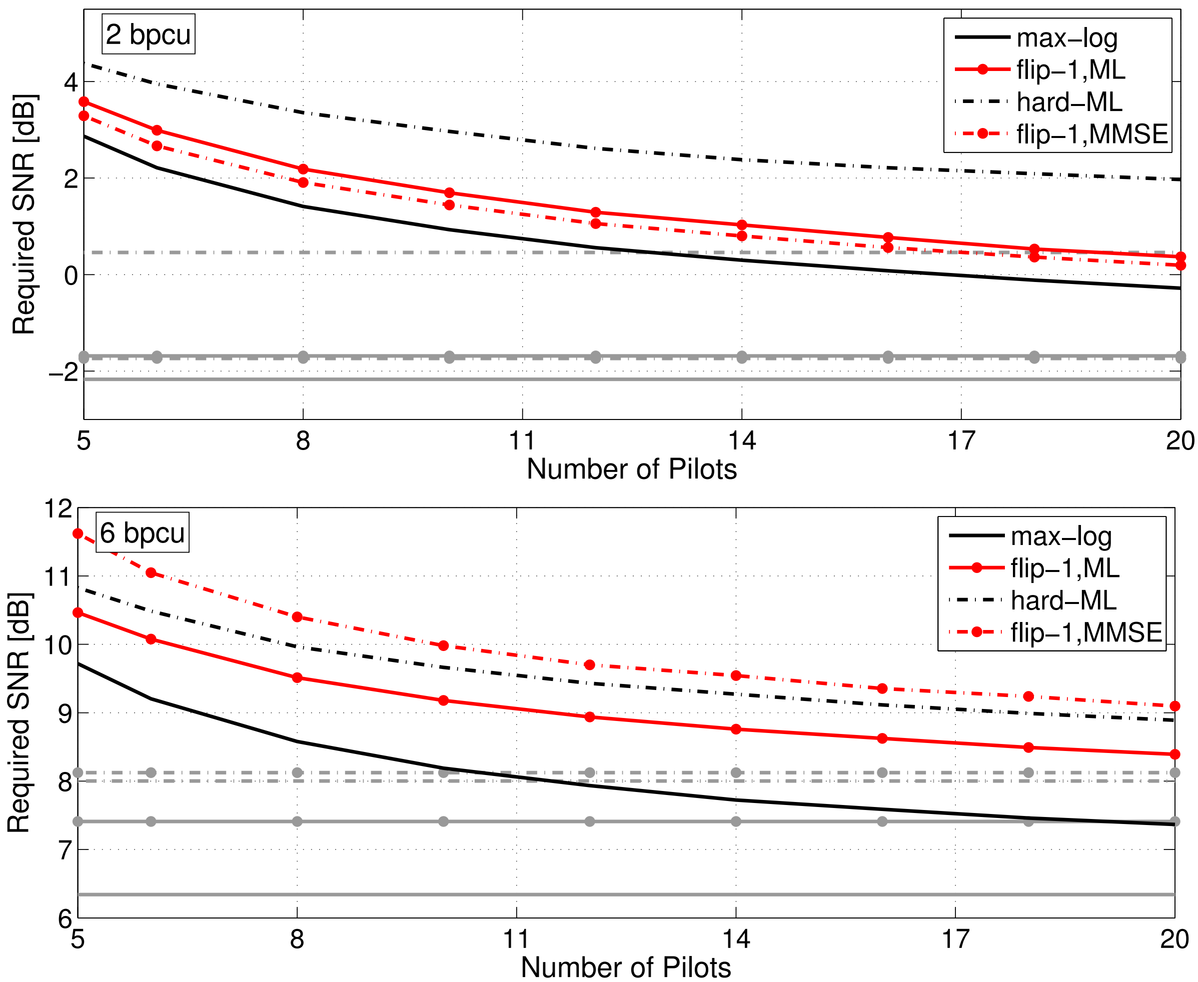


Observations: It can be seen that the two bit flipping variants suffer from a severe performance degradation in the case of imperfect CSI ($N_{\mathrm{p}} = 5$), i.e., a SNR shift of about $5\,$dB. The performance advantage of ML-based bit flipping over pure hard ML demodulation is much smaller than in the case of perfect CSI; in fact, the SNR gap lies only within $0.8\,$dB. Moreover, the performance cross-over between MMSE-based bit flipping demodulation and hard ML demodulation shifts to lower rates, i.e., from $5.9\,$bpcu (at $7.8\,$dB) for perfect CSI to $4.9\,$bpcu (at $9.3\,$dB) for imperfect CSI.

Bit Flipping Demodulators (1-bit) – Imperfect Channel/Imperfect Noise Variance

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

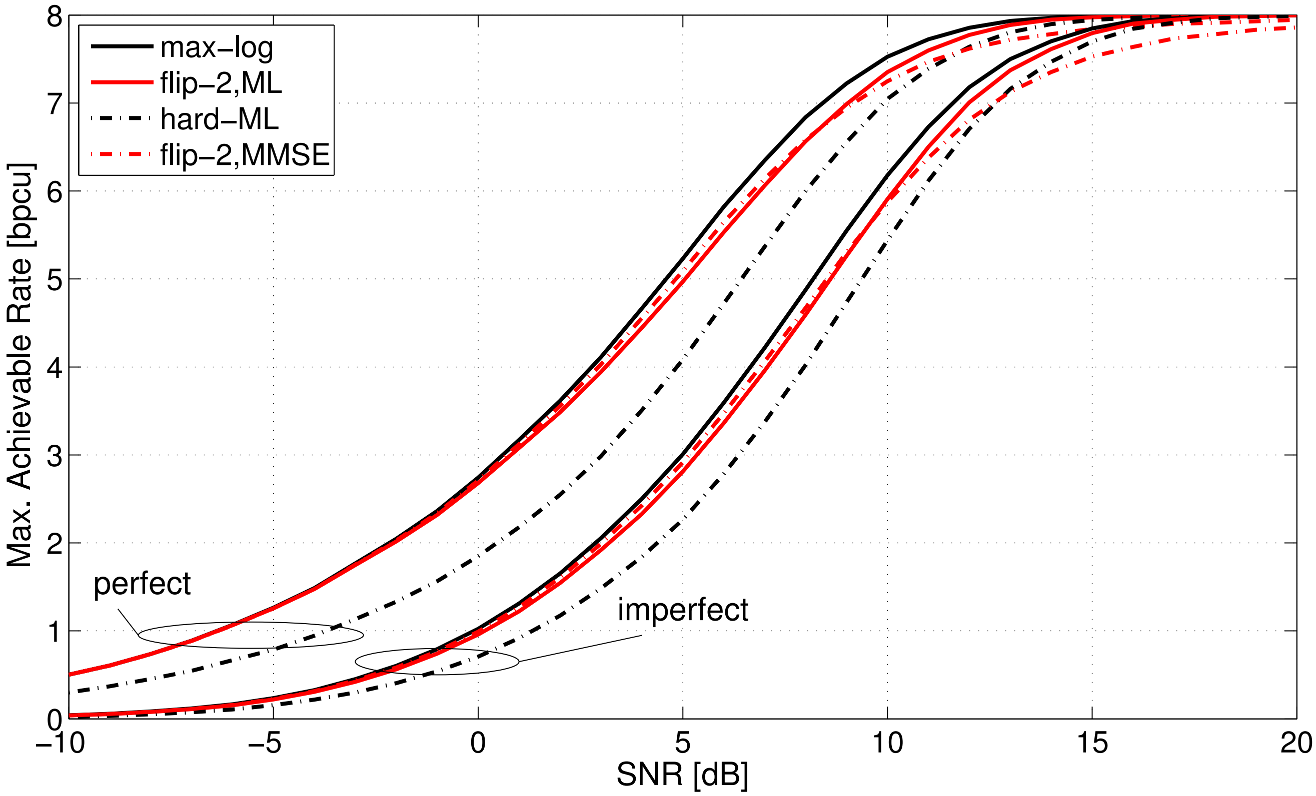


Observations: At 2 bpcu, MMSE-based flipping slightly outperforms ML-based bit flipping by about 0.25 dB, independent of the amount of training. At high rates and under imperfect CSI, the performance of MMSE-based bit flipping is affected most of all. For example, at 6 bpcu and for $N_{\mathrm{p}} = 5$ the SNR gap to perfect CSI is 3.5 dB for MMSE-based bit flipping whereas ML-based flipping reveals a gap of 3.1 dB.

Bit Flipping Demodulators (2-bit) – Perfect vs. Imperfect CSI

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray
- Number of pilot vectors for estimation: 5

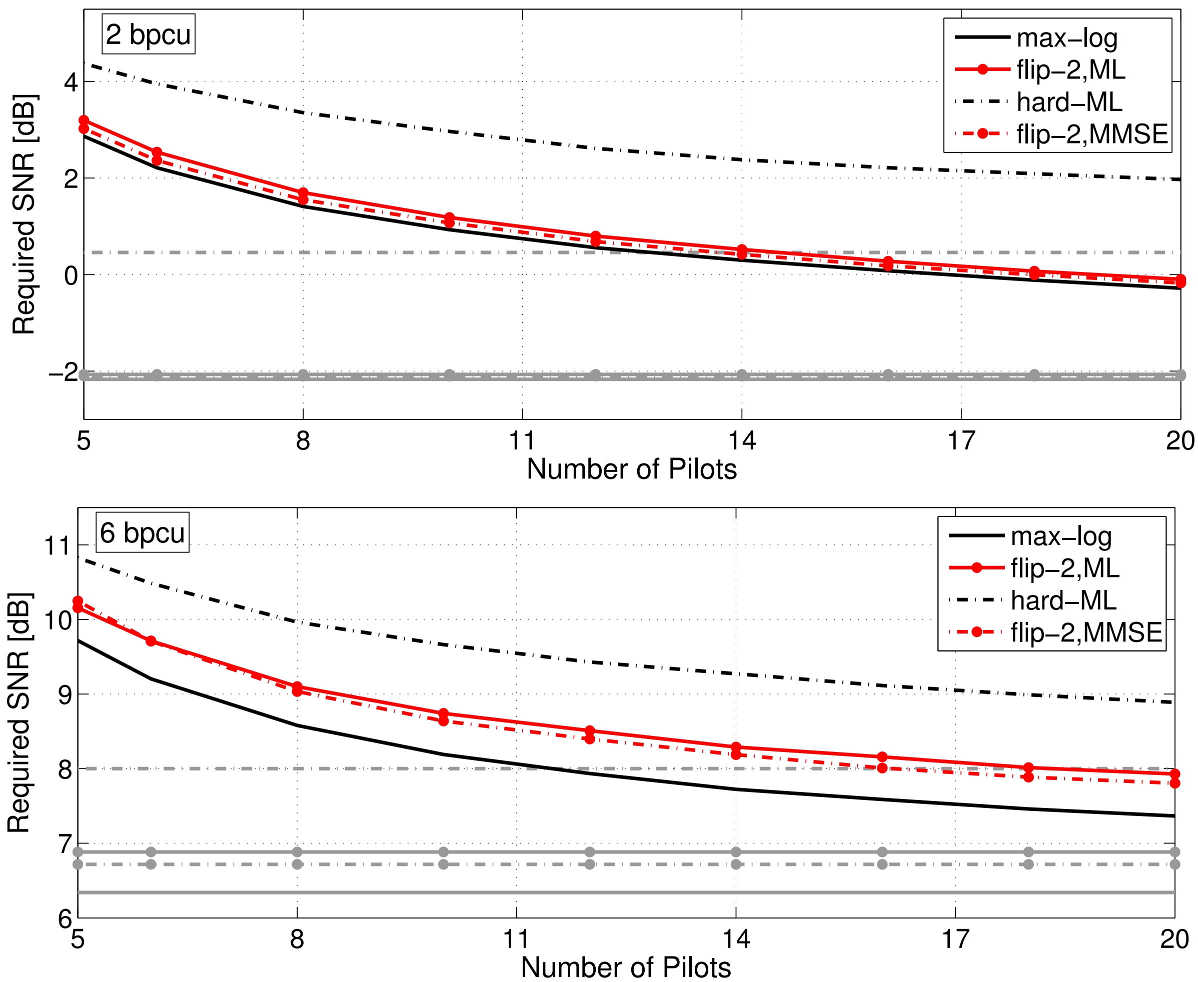


Observations: Under imperfect CSI the performance of the MMSE-based bit flipping demodulator can be greatly enhanced if the flipping is based on 2 bits instead of 1 bit. Except for an SNR shift, MMSE- and ML-based flipping show the same performance behavior as for perfect CSI. MMSE-based bit flipping outperforms hard ML demodulation and performs close to ML-based bit flipping and max-log demodulation at low-to-medium rates. Only at very high rates it starts to perform poorer than hard ML demodulation.

Bit Flipping Demodulators (2-bit) – Imperfect Channel/Imperfect Noise Variance

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

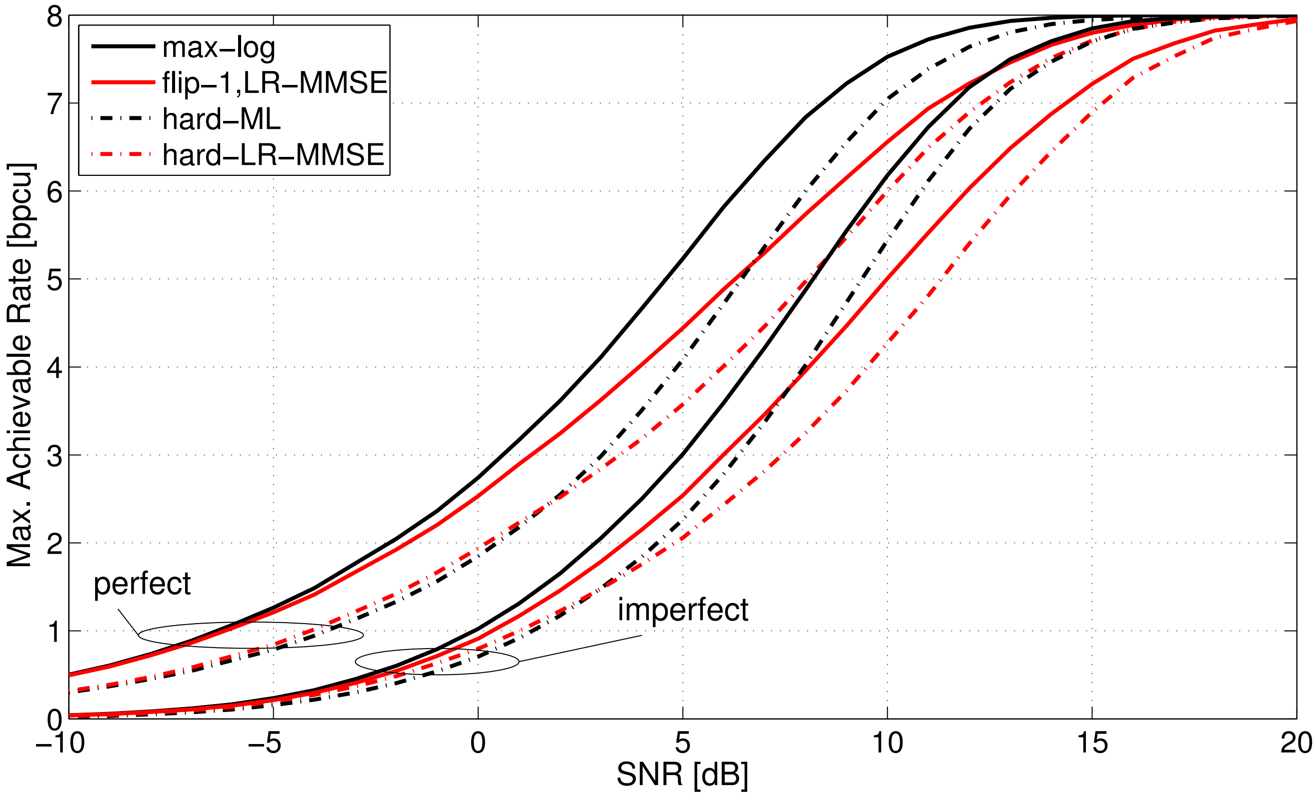


Observations: At 2 bpcu MMSE-based and ML-based bit flipping as well as max-log virtually show the same performance, independent of the amount of training. At 6 bpcu the two flipping variants also show almost the same performance for small training durations; however, when increasing the amount of training MMSE-based flipping starts to slightly outperform ML-based flipping.

Lattice-Reduction-Aided Detector – Perfect vs. Imperfect CSI

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray
- Number of pilot vectors for estimation: 5

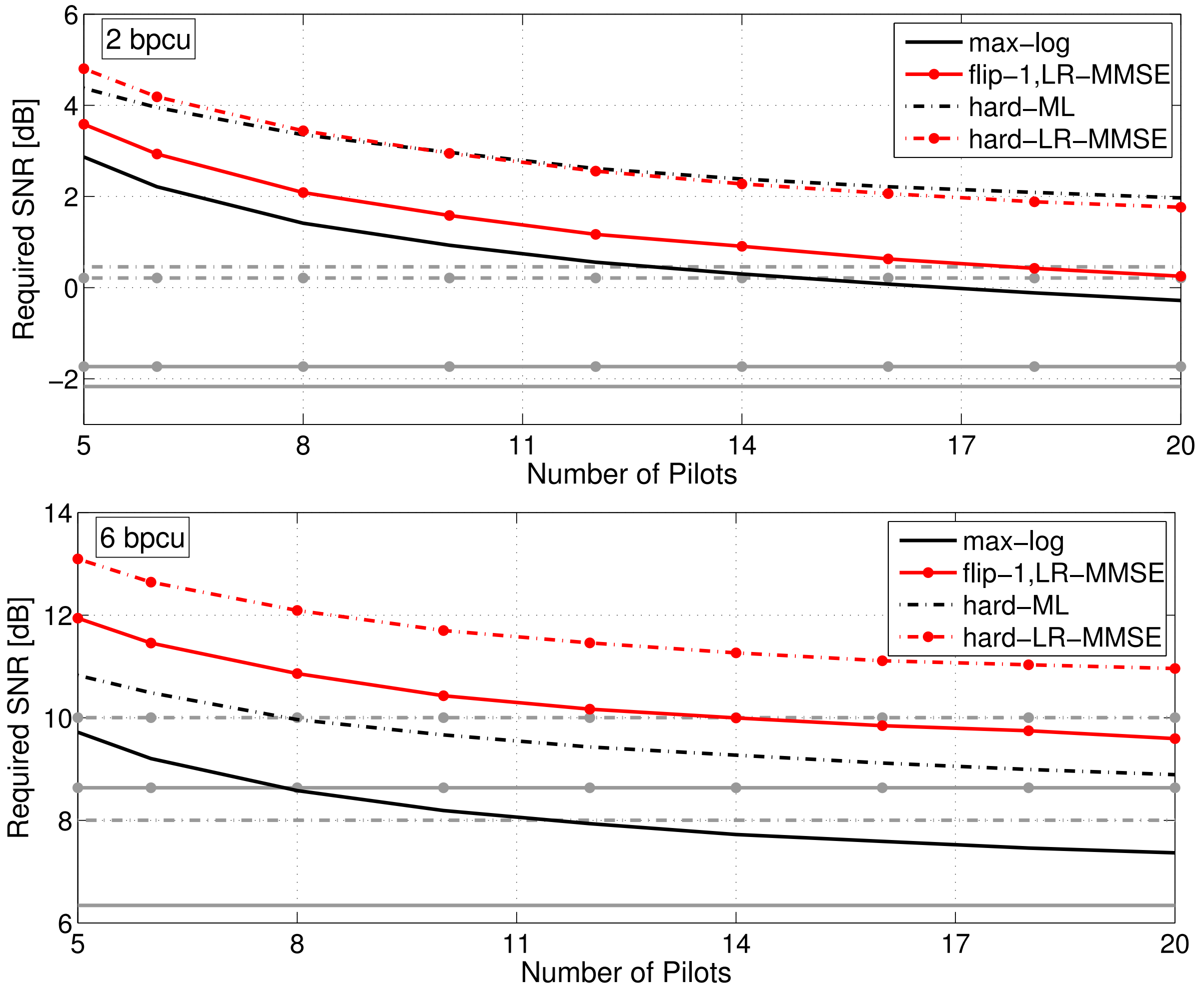


Observations: In comparison to the case of perfect CSI, hard and soft LR-based demodulation experience a significant shift to higher SNRs under imperfect CSI estimation with minimum training length ($N_{\mathrm{p}} = 5$). It can be seen that at low rates the performance gaps between the hard and soft demodulators are reduced. The performance cross-over between soft LR demodulation and hard ML demodulation shifts from 5.2 bpcu (at an SNR of about 6.7 dB) for the case of perfect CSI to 3.7 bpcu (at 7.6 dB) for imperfect CSI.

Lattice-Reduction-Aided – Imperfect Channel/Imperfect Noise Variance

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

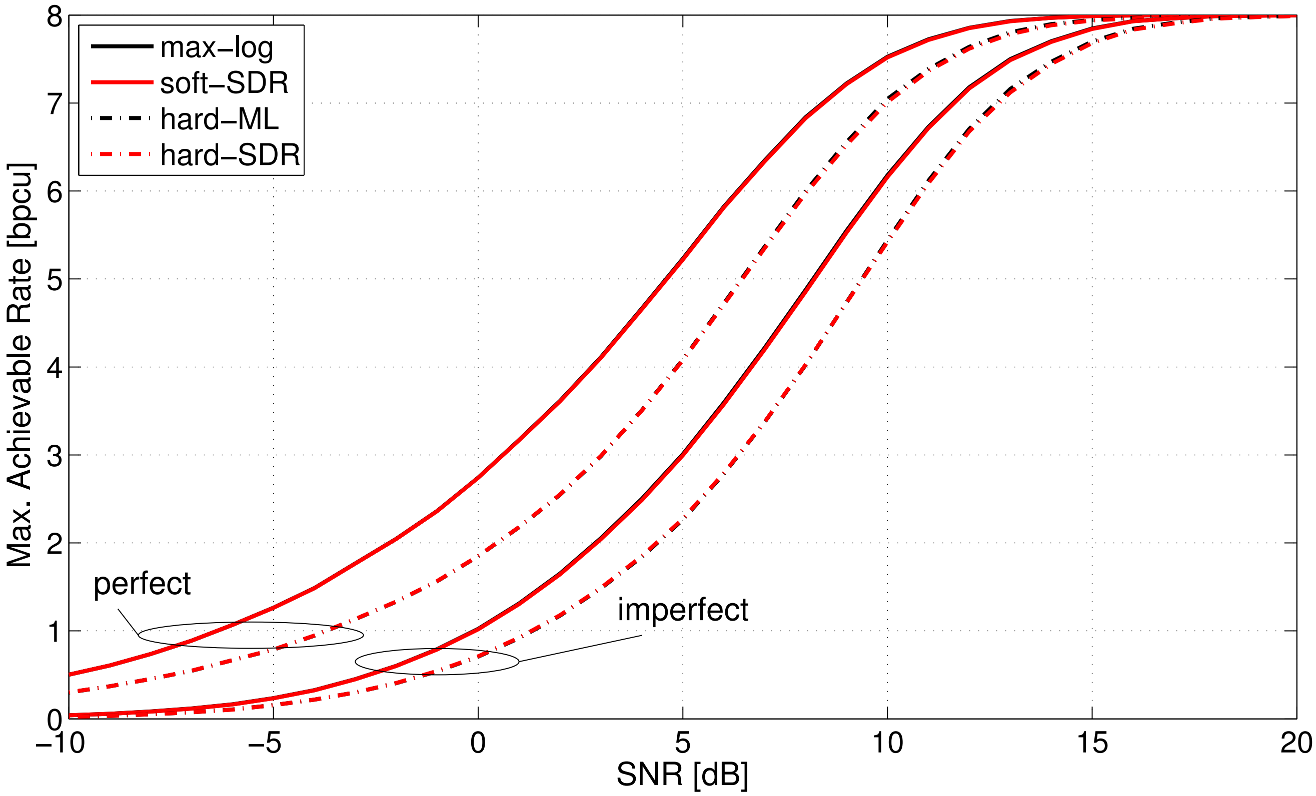


Observations: At 2 bpcu it can be seen that hard ML slightly outperforms hard LR demodulation for small training durations; however, for $N_{\mathrm{p}} > 10$ it is the other way around. At 6 bpcu all the demodulators show the same SNR gap of about 3 dB which decreases consistently with increasing amount of training.

Semidefinite Relaxation (SDR) Detector – Perfect vs. Imperfect CSI

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray
- Number of pilot vectors for estimation: 5

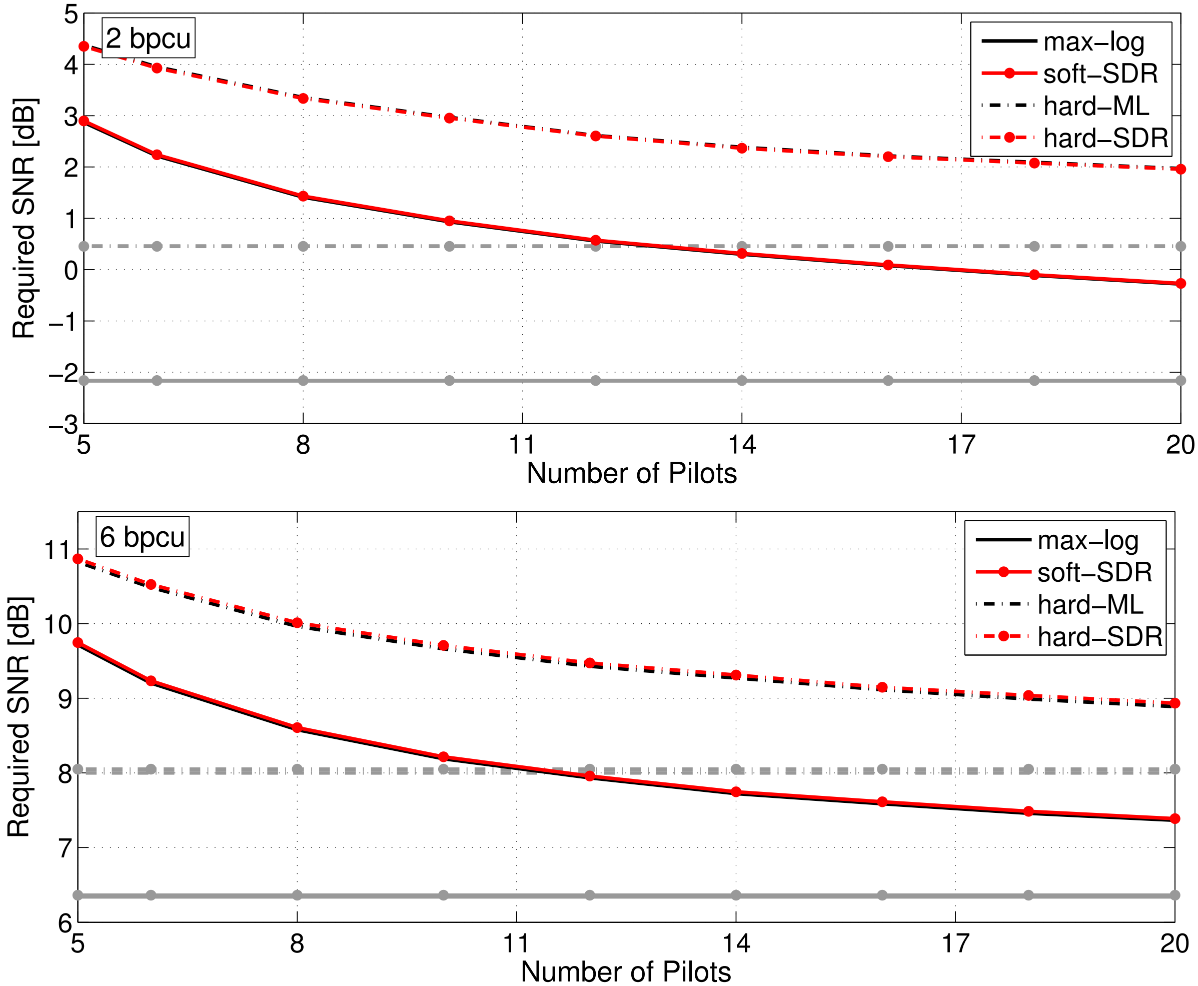


Observations: Similarly as for the perfect CSI case, hard and soft SDR demodulation also exactly coincide with hard ML and max-log performance in the case of imperfect CSI. This behavior is independent of the amount of training, as shown on page 57. However, the gap between hard and soft demodulation reduces by about $1\,\text{dB}$ for the worst-case scenario of minimum training length ($N_\text{p} \leq 5$).

Semidefinite Relaxation (SDR) Detector – Imperfect Channel/Imperfect Noise Variance

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

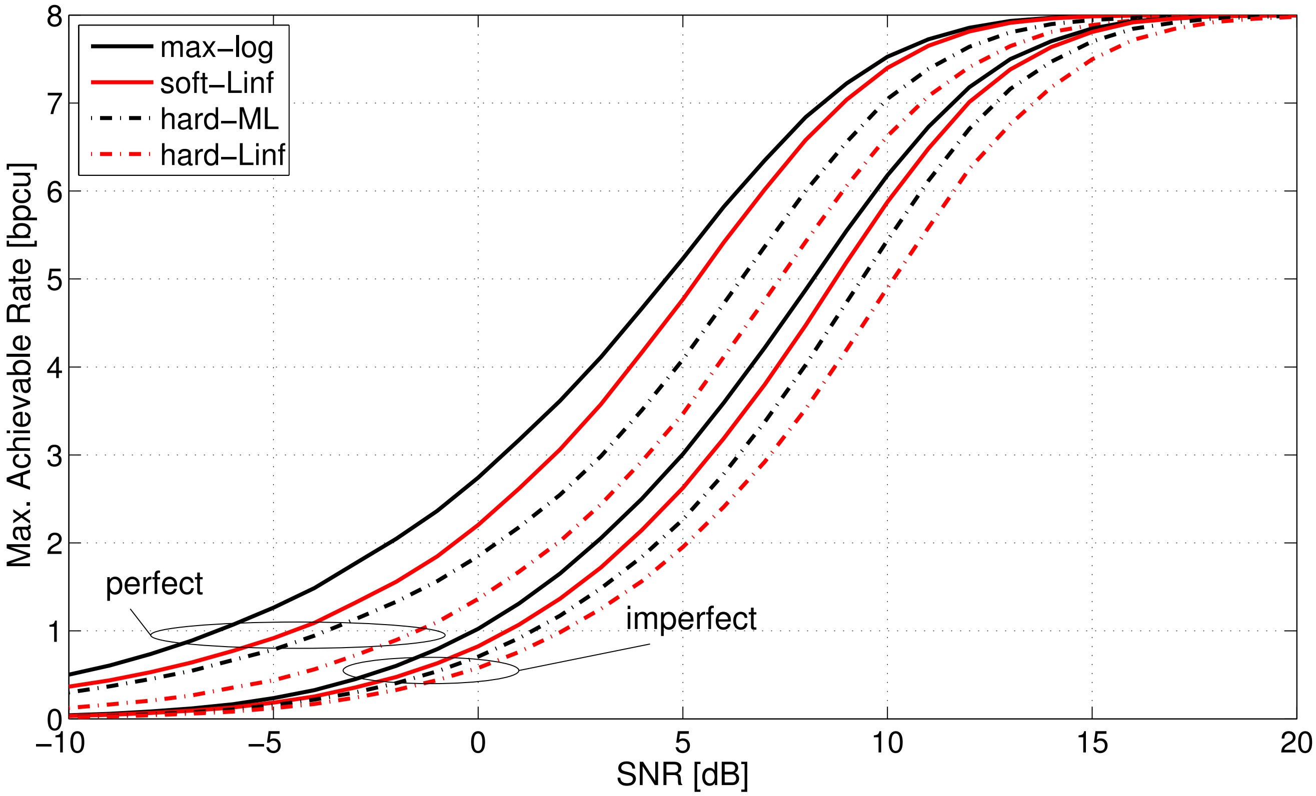


Observations: Even in the case of imperfect CSI hard and soft SDR demodulation match the performance of hard ML and max-log demodulation. This behavior is independent of the amount of training. The gap between hard and soft demodulation increases by about $1\,\mathrm{dB}$ when increasing the amount of training.

ℓ^∞-Norm Demodulator – Perfect vs. Imperfect CSI

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray
- Number of pilot vectors for estimation: 5

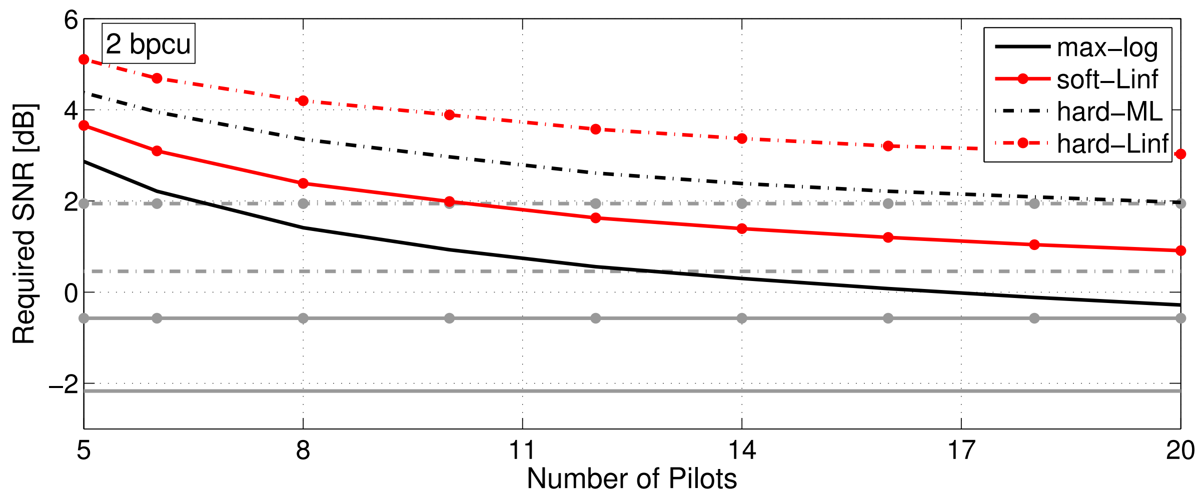


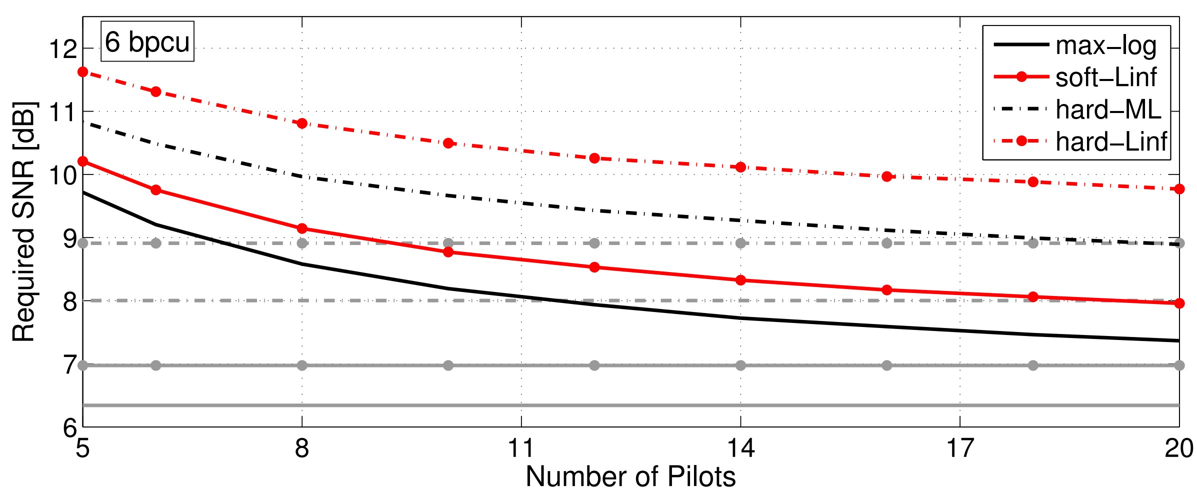
Observations: The main observation in this worst-case scenario ($N_{\mathrm{p}} = 5$) is that the performance gaps between max-log, soft ℓ^∞-norm, hard ML, and hard ℓ^∞-norm demodulation are significantly reduced. In general, soft ℓ^∞-norm demodulation appears to be more sensitive to poor quality estimates of the channel and the noise variance than hard ℓ^∞-norm demodulation (cf. page 59).

ℓ^∞-Norm Demodulator – Imperfect Channel/Imperfect Noise Variance

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray



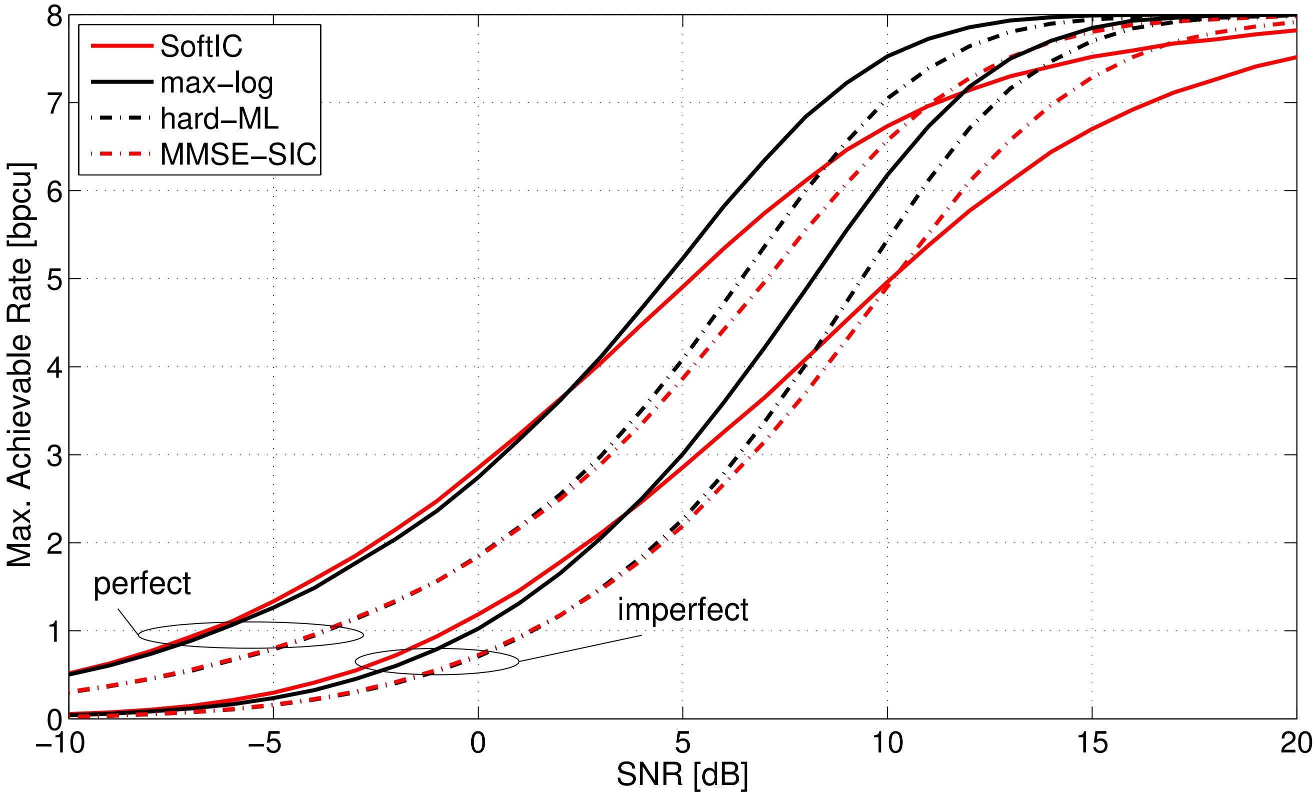


Observations: In general, soft ℓ^∞-norm demodulation appears to be more sensitive to poor quality estimates of the channel and the noise variance than hard ℓ^∞-norm demodulation. For example, at 2 bpcu and for $N_\mathrm{p} = 5$ the performance gap to perfect CSI equals 4.23 dB. In contrast, hard ℓ^∞-norm demodulation is more robust, showing only a gap of 3.2 dB in this case.

Successive Interference Canceler (SIC) and Soft Interference Canceler (SoftIC) – Perfect vs. Imperfect CSI

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray
- Number of pilot vectors for estimation: 5

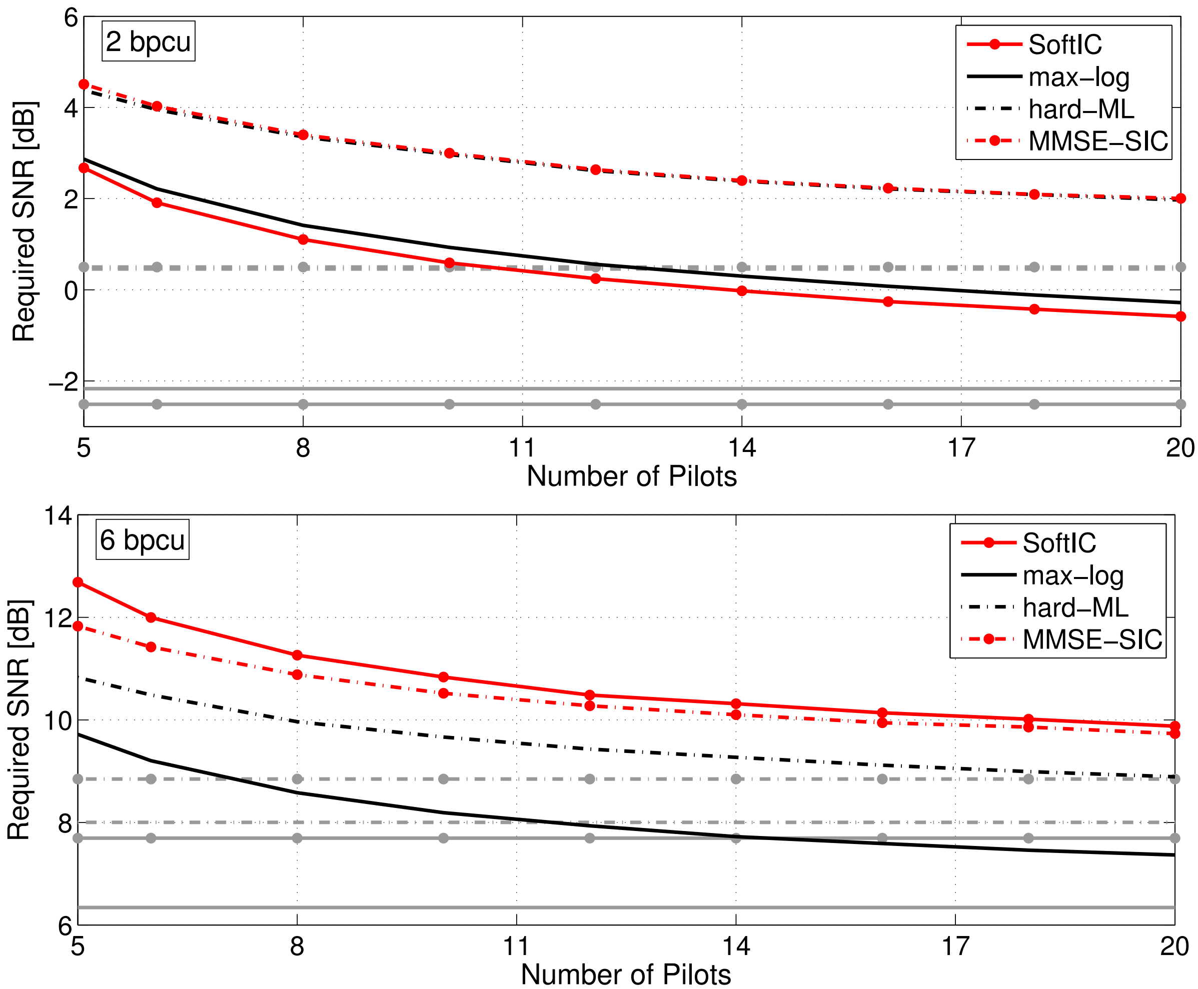


Observations: With imperfect CSI, the performance curves of SoftIC demodulation (with 3 iterations and initialized with soft ZF) experience a significant shift to lower SNRs. In comparison to hard ML demodulation, the performance of SoftIC demodulation suffers from a stronger degradation; the cross-over between SoftIC and hard ML demodulation shifts from 6.3 bpcu to 4.3 bpcu at an SNR of about 8.5 dB. The performance gap between MMSE-SIC and max-log demodulation reduces for the case of imperfect CSI by about 1 dB. Although, in general, MMSE-SIC demodulation has inferior performance than SoftIC at low rates, it remains more robust for the case of imperfect CSI.

Successive Interference Canceler (SIC) and Soft Interference Canceler (SoftIC) – Imperfect Channel/Imperfect Noise Variance

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray



Observations: Although, in general, MMSE-SIC demodulation has inferior performance than SoftIC at low rates, it remains more robust for the case of imperfect CSI; for example, at 2 bpcu MMSE-SIC shows a performance gap to perfect estimation of 4 dB for $N_{\mathrm{p}} = 5$ whereas SoftIC demodulation has a gap of 5.2 dB in this case. The inferior robustness of SoftIC over MMSE-SIC can also be seen at high rates (6 bpcu) where the performance degradation of SoftIC is more pronounced for the case of small training durations.

V. Quasi-static Fading

In this section we provide a demodulator performance comparison for the i.i.d. quasi-static fading MIMO channel. In this case the channel $\mathbf{H}$ is random but constant over time, i.e., each codeword can extend over only one channel realization [24]. Since in this regime, an ergodic system capacity of the equivalent modulation channel is no longer meaningful [24], [25], we consider the *outage probability*

$$P_{\text{out}}(R) \triangleq \mathrm{P}\{R_{\mathbf{H}} < R\}, \tag{7}$$

where $R_{\mathbf{H}}$ is a random variable defined as

$$R_{\mathbf{H}} \triangleq \sum_{l=1}^{R_0} I_{\mathbf{H}}(c_l; \tilde{\Lambda}_l).$$

Here, $I_{\mathbf{H}}(c_l; \tilde{\Lambda}_l)$ denotes the conditional mutual information given a fixed channel realization (cf. (2)). Note that the ergodic system capacity C in (2) equals $C = \mathbb{E}_{\mathbf{H}}\{R_{\mathbf{H}}\}$. The outage probability $P_{\text{out}}(R)$ can be interpreted as the smallest probability of error achievable at rate R [25]. A closely related concept is given by the *ϵ-capacity* of the equivalent modulation channel, defined as [25]

$$C_\epsilon \triangleq \sup \left\{ R \mid \mathrm{P}\{R_{\mathbf{H}} < R\} < \epsilon \right\}. \tag{8}$$

The ϵ-capacity may be interpreted as the maximum rate for which a probability of error less than ϵ can be achieved. We refer to [1, Section III.D] for more details on these measures.

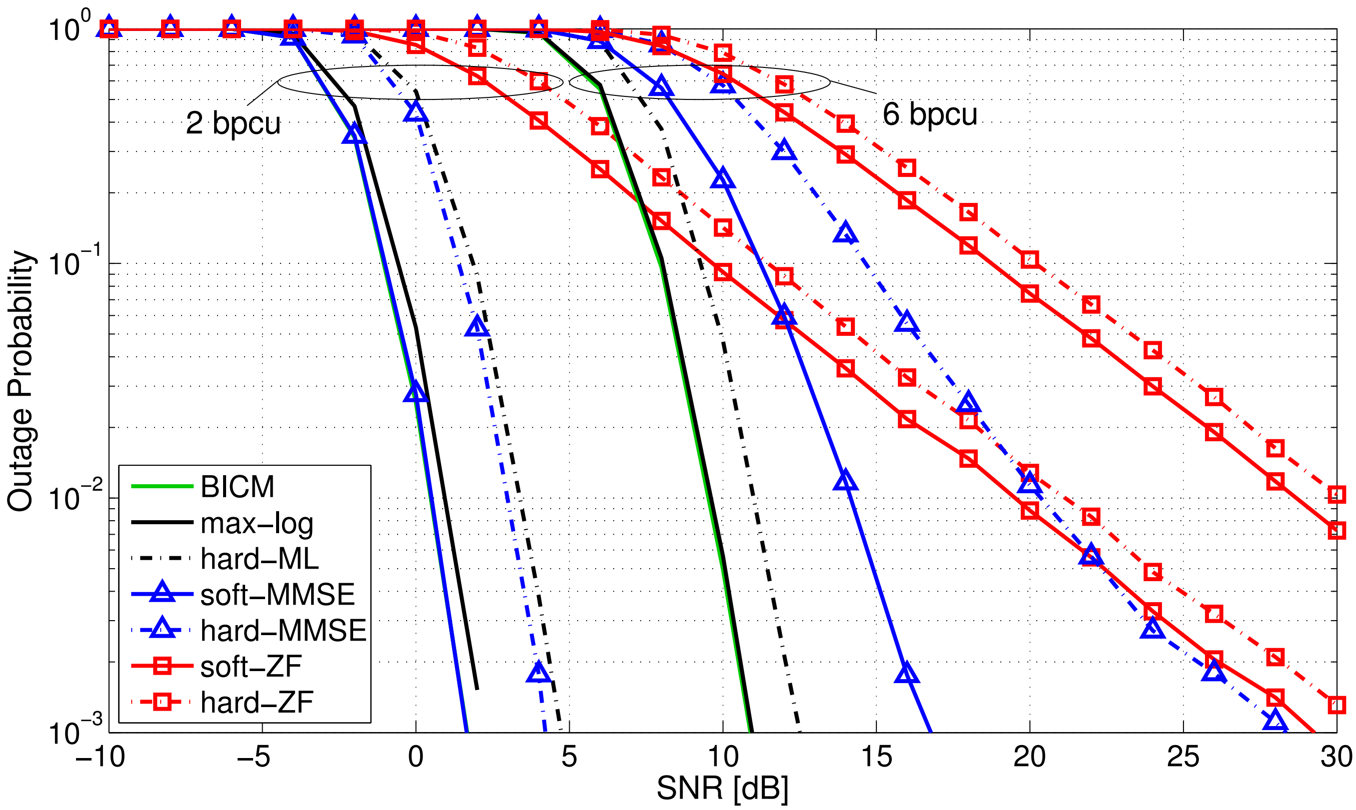
In our discussion we focus on the baseline demodulators (reviewed in [1, Section IV.A]) and investigate a 4×4 MIMO system with 4-QAM employing Gray labeling. We first show the outage probability in (7) versus SNR for a target rates of $R = 2\,\text{bpcu}$ and $R = 6\,\text{bpcu}$. The outage probability $P_{\text{out}}(R)$ was measured using 10^5 blocks (affected by independent fading realizations), each consisting of 10^5 symbol vectors. Furthermore, we plot the ϵ-capacity in (8) over SNR for a maximum outage probability of $\epsilon = 10^{-2}$ and note that these results are qualitatively very similar to the results obtained for the ergodic capacity on page 7.

Additional observations can be also found in [1, Section VII].

Baseline Demodulators

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray

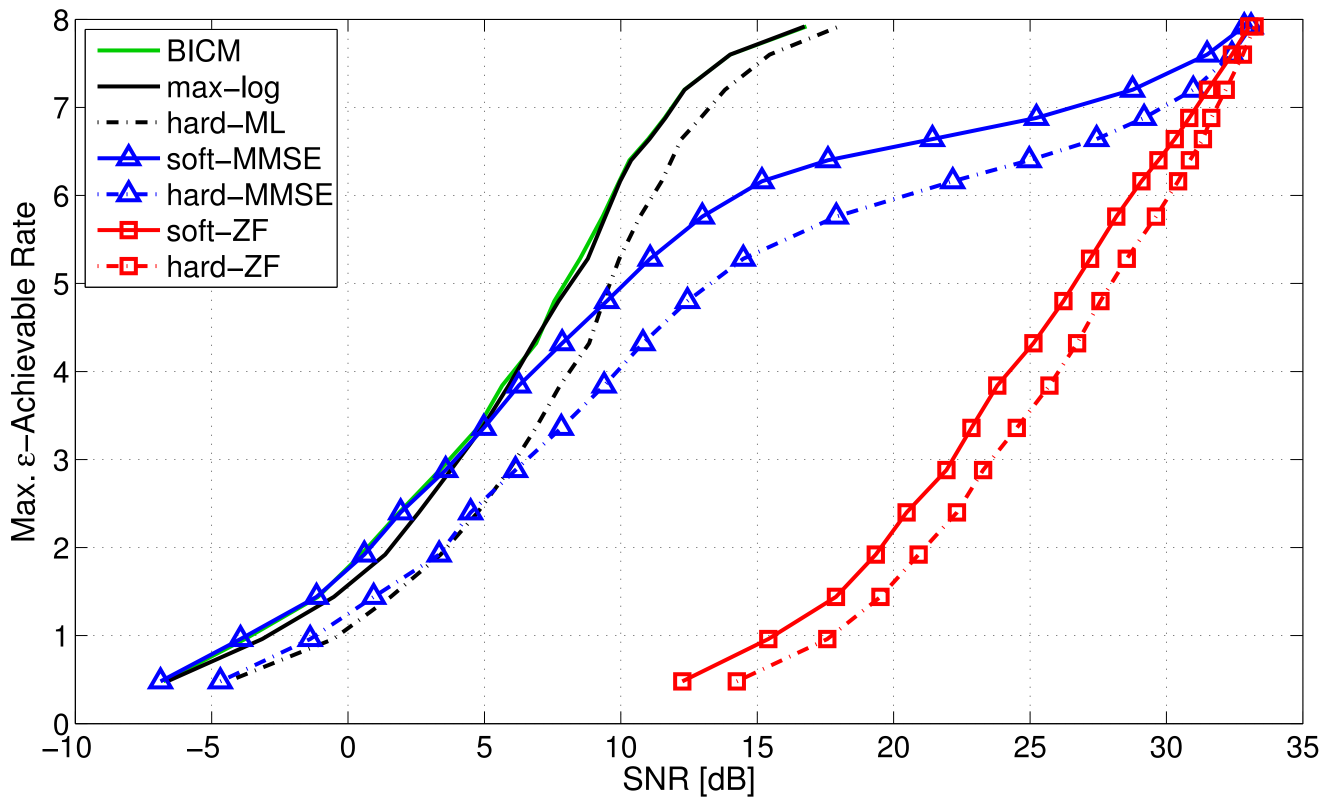


Observations: See [1, Section VII] for more details. For $R=2$ bpcu, it is seen that optimum soft MAP demodulation (labeled 'BICM' for consistency with previous sections) and soft MMSE demodulation exactly coincide and show a gap to max-log demodulation of about 0.5 dB. In such a low-rate regime, max-log performs about 2.5 dB better than hard ML. While max-log, hard ML, and soft MMSE demodulation all achieve full diversity (cf. the slope of the corresponding outage probability curves), soft and hard ZF only have diversity order one, resulting in a huge performance loss (almost 19 dB and 20.5 dB at $P_{\text{out}}(R) = 10^{-2}$, respectively). At $R=6$ bpcu the situation is quite different: here, max-log coincides with soft MAP and hard ML looses only 1.4 dB (again, those three demodulators achieve full diversity). Surprisingly, hard and soft MMSE deteriorate at this rate, starting to loose all diversity. At $P_{\text{out}}(R) = 10^{-2}$, the SNR loss of soft MMSE and soft ZF relative to max-log equals about 4.4 dB and 19 dB, respectively.

Baseline Demodulators

System Setup:

- Number of transmit antennas: 4
- Number of receive antennas: 4
- Constellation: 4-QAM
- Labeling: Gray
- Fixed maximum outage probability: $\epsilon = 10^{-2}$



Observations: See [1, Section VII] for more details. Some of the behavior of the ϵ-capacity curves is qualitatively very similar to the results obtained for the ergodic capacity on page 7: at low rates soft MMSE demodulation outperforms hard ML demodulation (by up to $2.8\,$dB for rates less than $4.7\,$bpcu) while at high rates it is the other way round. Furthermore, for low rates soft MMSE demodulation essentially coincides with optimum soft MAP demodulation whereas at high rates it approaches soft ZF performance. We note that a similar rate-dependent diversity behavior of MMSE demodulation has been observed in [26]. In particular, the authors proved that in case of joint spatial encoding the upper bound on the outage probability of hard MMSE receivers decays with diversity order of $M_{\mathrm{T}}M_{\mathrm{R}}$ at low rates and decays with order $M_{\mathrm{R}} - M_{\mathrm{T}} + 1$ at high rates; in contrast, hard ZF always has diversity no better than $M_{\mathrm{R}} - M_{\mathrm{T}} + 1$. This very well fits with our observations where the MMSE demodulator is seen to loose all its diversity for rates larger than $5\,$bpcu.

A comparison of these results with the ergodic results on page 7 even suggests that there is a connection between the diversity of the demodulators in the quasi-static scenario and their SNR gap to optimum demodulation in the ergodic scenario. For all rates (and all SNRs), the max-log and hard ML demodulator both achieve constant (full) diversity in the quasi-static regime and maintain a roughly constant gap to soft MAP in the ergodic scenario. On the other hand, the diversity of soft and hard MMSE demodulation in the quasi-static case and their SNR gaps to soft MAP in the ergodic scenario both deteriorate with increasing rate/SNR. Note that also here hard and soft ZF demodulation perform worst, with a diversity equal to 1 for all rates.

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