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Weak decays of H-like $^{140}Pr^{58+}$ and He-like $^{140}Pr^{57+}$ ions

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The nuclear K-shell electron-capture (EC) and positron (β^+) decay constants, $\lambda_{\rm EC}$ and λ_{β^+} of H-like $^{140}Pr^{58+}$ and He-like $^{140}Pr^{57+}$ ions, measured recently in the experimental storage ring (ESR) at GSI, were calculated using standard weak interaction theory. The calculated ratios $R = \lambda_{\rm EC}/\lambda_{\beta^+}$ of the decay constants agree with the experimental values within an accuracy better than 3%.

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I. INTRODUCTION

The neutral $^{140}Pr^{0+}$ atoms decay with a 99.4% branch to the ground state of $^{140}Ce^{0+}$ via a pure $1^+ \to 0^+$ Gamow-Teller (GT) transition [1]. The ratio of the electron-capture (EC) to positron (β^+) decay constants was measured as $R_{\rm EC/\beta^+} = 0.74(3)$ by Biryukov and Shimanskaya [2] and $R_{\rm EC/\beta^+} = 0.90(8)$ by Evans *et al.* [3]. Because of the discrepancy between these results, the experiment was repeated by Campbell *et al.*, who found $R^{\rm exp}_{\rm EC/\beta^+} = 0.73(3)$ [4]. The theoretical value $R^{\rm th}_{\rm EC/\beta^+} = 0.85(1)$ at a 68.3% confidence level [5–11], cited by Campbell *et al.* [4], agrees with the experimental value with an accuracy of about 15% [4]: $R^{\rm exp}_{\rm EC/\beta^+}/R^{\rm th}_{\rm EC/\beta^+} = 0.858(34)$.

The overview of the experimental data on the ratio $R_{\rm EC/\beta^+}$ cannot be complete without mentioning Refs. [12–14], the results of which agree well with recent experimental data at GSI [15]. Indeed, recently, Litvinov *et al.* [15] succeeded in measuring the β^+ and EC decay rates of fully ionized, H-like $^{140}Pr^{58+}$ and He-like $^{140}Pr^{57+}$ ions in the experimental storage ring (ESR) at GSI in Darmstadt [15]. The experimental data on the β^+ decay constants $\lambda^{\rm exp}_{\beta^+} = 0.00158(8)\,{\rm s}^{-1}$, $\lambda^{\rm exp}_{\beta^+} = 0.00161(10)\,{\rm s}^{-1}$, and $\lambda^{\rm exp}_{\beta^+} = 0.00154(11)\,{\rm s}^{-1}$, obtained for the nucleus $^{140}Pr^{59+}$, the H-like $^{140}Pr^{58+}$ ion, and the He-like $^{140}Pr^{57+}$ ion, respectively, agree well with the experimental data [12–14].

For the ratios of the EC and β^+ decay constants of the H-like $^{140}Pr^{58+}$ ion and the He-like $^{140}Pr^{57+}$ ion, Litvinov *et al.* [15] obtained the values $R^{\rm (H),exp}_{\rm EC/\beta^+} = 1.36(9)$ and $R^{\rm (He),exp}_{\rm EC/\beta^+} = 0.96(8)$; the latter ratio agrees well with the experimental data reported by Refs. [3,12,13].

In addition, Litvinov *et al.* [15] found that the H-like $^{140}Pr^{58+}$ ion with one electron in the K shell decays 1.49(8) times faster, i.e., $R^{\rm (H/He),exp}_{\rm EC/EC} = 1.49(8)$, than the He-like one with two electrons in the K shell.

In this work, we show that using standard weak interaction theory one can reproduce the experimental values of the ratios of the EC and β^+ decay constants of the H-like $^{140}Pr^{58+}$ and He-like $^{140}Pr^{57+}$ ions with an accuracy better than 3%.

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II. EC AND β^+ DECAY CONSTANTS

For the calculation of the weak decay constants of H-like and He-like ions, we use the Hamiltonian of the weak interaction taken in the standard form [16]

$$\mathcal{H}_W(x) = \frac{G_F}{\sqrt{2}} V_{ud}\,[\bar{\psi}_n(x)\gamma^{\mu}(1 - g_A\gamma^5)\,\psi_p(x)] \times [\bar{\psi}_{\nu_e}(x)\gamma_{\mu}(1 - \gamma^5)\psi_e(x)] + {\rm h.c.}, \qquad (1)$$

where $G_F = 1.166 \times 10^{-11}\,{\rm MeV}^{-2}$ is the Fermi constant, $V_{ud} = 0.9738 \pm 0005$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, $g_A = 1.2695 \pm 0.0058$ is the axial coupling constant [17], and $\psi_n(x)$, $\psi_p(x)$, $\psi_{\nu_e}(x)$, and $\psi_e(x)$ are the operators of the neutron, proton, neutrino, and electron/positron fields, respectively.

The detailed calculations of the weak decay constants of the H-like $^{140}Pr^{58+}$ and He-like $^{140}Pr^{57+}$ ions are given in Ref. [18]. Since the EC decay of the H-like $^{140}Pr^{58+}$ ion from the hyperfine state $^{140}Pr^{58+}_{F=\frac{3}{2}}$ with $F = \frac{3}{2}$ is suppressed [15,18,19], we take into account that the H-like $^{140}Pr^{58+}$ ion decays from the hyperfine ground state $^{140}Pr^{58+}_{F=\frac{1}{2}}$ with $F = \frac{1}{2}$.

Using the Hamiltonian of the weak interaction (1) and following Ref. [11] for the EC decay constants of the H-like $^{140}Pr^{58+}_{F=\frac{1}{2}}$ and the He-like $^{140}Pr^{57+}_{I=1}$ ions in the ground states, we obtain the expressions [18]

$$\lambda^{\rm (H)}_{\rm EC} = \frac{1}{2F+1}\frac{3}{2}|\mathcal{M}_{\rm GT}|^2|\langle\psi^{(Z)}_{1s}\rangle|^2\frac{Q^2_{\rm H}}{\pi},$$
$$\lambda^{\rm (He)}_{\rm EC} = \frac{1}{2I+1}\frac{3}{2}|\mathcal{M}_{\rm GT}|^2|\langle\psi^{(Z-1)}_{1s}\psi^{(Z)}_{(1s)^2}\rangle|^2\frac{Q^2_{\rm He}}{\pi}, \qquad (2)$$

where $F = 1/2$ and $I = 1$ are the total angular momenta of the H-like $^{140}Pr^{58+}$ and the He-like $^{140}Pr^{57+}$ ions, respectively,[1] $Q_{\rm H} = (3348 \pm 6)\,{\rm keV}$ and $Q_{\rm He} = (3351 \pm 6)\,{\rm keV}$ are the Q values of the decays $^{140}Pr^{58+} \to {}^{140}Ce^{58+} + \nu_e$ and $^{140}Pr^{57+} \to {}^{140}Ce^{57+} + \nu_e$, $\mathcal{M}_{\rm GT}$ is the nuclear matrix element

[1]In the ratio $\frac{3}{2}$ of the EC decay constant $\lambda^{\rm (H)}_{\rm EC}$, the factors 3 and $\frac{1}{2}$ are caused by the hyperfine structure of the ground state of the H-like $^{140}Pr^{58+}_{F=\frac{1}{2}}$ ion and the phase volume of the final state of the decay, respectively [18].

of the Gamow-Teller transition,[2] $\psi_{1s}^{(Z)}$ and $\psi_{1s}^{(Z-1)}$ are the wave functions of the H-like ions $^{140}\mathrm{Pr}^{58+}$ and $^{140}\mathrm{Ce}^{57+}$, respectively, $Z = 59$ is the electric charge of the mother nucleus $^{140}\mathrm{Pr}^{59+}$, and $\psi_{(1s)^2}^{(Z)}$ is the wave function of the He-like ion $^{140}\mathrm{Pr}^{57+}$, with $\langle\psi_{1s}^{(Z)}\rangle$ and $\langle\psi_{1s}^{(Z-1)}\psi_{(1s)^2}^{(Z)}\rangle$ defined by [18]

$$\langle\psi_{1s}^{(Z)}\rangle = \frac{\int d^3x\, \psi_{1s}^{(Z)}(\vec{r}\,)\rho(r)}{\int d^3x\, \rho(r)},$$
$$\langle\psi_{1s}^{(Z-1)}\psi_{(1s)^2}^{(Z)}\rangle = \frac{\int d^3x_1\, d^3x_2\, \psi_{1s}^{(Z-1)}(\vec{r}_1)\psi_{(1s)^2}^{(Z)}(\vec{r}_1,\vec{r}_2)\rho(r_2)}{\int d^3x_2\, \rho(r_2)}, \quad (3)$$

where $\rho(r)$ is the nuclear density [20]. Using the Woods-Saxon shape for the nuclear density $\rho(r)$ and the Dirac wave functions for the H-like ion and the He-like ion [21],[3] one gets $\langle\psi_{1s}^{(Z)}\rangle = \langle\psi_{1s}^{(Z-1)}\psi_{(1s)^2}^{(Z)}\rangle = 1.66/\sqrt{\pi a_B^3}$, where $a_B = 1/Z\alpha m_e = 897\,\mathrm{fm}$ is the Bohr radius of the H-like ion $^{140}\mathrm{Pr}^{58+}$; $m_e = 0.511\,\mathrm{MeV}$ is the electron mass and $\alpha = 1/137.036$ is the fine-structure constant [17].

The β^+ decay constants of the H-like $^{140}\mathrm{Pr}^{58+}$ and the He-like $^{140}\mathrm{Pr}^{57+}$ ions are defined by [18]

$$\lambda_{\beta^+}^{(\mathrm{H})} = \frac{2}{2F+1}\frac{|\mathcal{M}_{\mathrm{GT}}|^2}{4\pi^3} f(Q_{\beta^+}^{\mathrm{H}}, Z-1),$$
$$\lambda_{\beta^+}^{(\mathrm{He})} = \frac{3}{2I+1}\frac{|\mathcal{M}_{\mathrm{GT}}|^2}{4\pi^3} f(Q_{\beta^+}^{\mathrm{He}}, Z-1). \quad (4)$$

Since the Q values of the decays $^{140}\mathrm{Pr}^{58+} \to {}^{140}\mathrm{Ce}^{58+} + e^+ + \nu_e$ and $^{140}\mathrm{Pr}^{57+} \to {}^{140}\mathrm{Ce}^{57+} + e^+ + \nu_e$ are equal, that is, $Q_{\beta^+}^{\mathrm{H}} = Q_{\beta^+}^{\mathrm{He}} = Q_{\beta^+} = (3396 \pm 6)\,\mathrm{keV}$, then the Fermi integral $f(Q_{\beta^+}, Z-1) = (2.21 \pm 0.03)\,\mathrm{MeV}^5$ is defined by the phase volume of the final states of the decays and the Fermi function, describing the Coulomb repulsion between the positron and the nucleus $^{140}\mathrm{Ce}^{58+}$ for $Z = 59$ [16].

The theoretical values of the weak decay constants are defined up to the unknown nuclear matrix element $\mathcal{M}_{\mathrm{GT}}$ of the Gamow-Teller transition, which cancels in the ratios. The theoretical ratios of the weak decay constants are given by

$$R_{\mathrm{EC}/\beta^+}^{(\mathrm{H}),\mathrm{th}} = \frac{3\pi^2 Q_{\mathrm{H}}^2 |\langle\psi_{1s}^{(Z)}\rangle|^2}{f(Q_{\beta^+}, Z-1)} = 1.40(4),$$
$$R_{\mathrm{EC}/\beta^+}^{(\mathrm{He}),\mathrm{th}} = \frac{2\pi^2 Q_{\mathrm{He}}^2 |\langle\psi_{1s}^{(Z-1)}\psi_{(1s)^2}^{(Z)}\rangle|^2}{f(Q_{\beta^+}, Z-1)} = 0.94(3), \quad (5)$$
$$R_{\mathrm{EC}/\mathrm{EC}}^{(\mathrm{H/He}),\mathrm{th}} = \frac{2I+1}{2F+1}\frac{|\langle\psi_{1s}^{(Z)}\rangle|^2}{|\langle\psi_{1s}^{(Z-1)}\psi_{(1s)^2}^{(Z)}\rangle|^2}\frac{Q_{\mathrm{H}}^2}{Q_{\mathrm{He}}^2} = 1.50(4),$$

calculated for $F = \frac{1}{2}$ and $I = 1$. The theoretical ratios agree well with the experimental data $R_{\mathrm{EC}/\beta^+}^{(\mathrm{H}),\mathrm{exp}} = 1.36(9)$, $R_{\mathrm{EC}/\beta^+}^{(\mathrm{He}),\mathrm{exp}} = 0.96(8)$, and $R_{\mathrm{EC}/\mathrm{EC}}^{(\mathrm{H/He}),\mathrm{exp}} = 1.49(8)$, obtained at GSI [15].

III. CONCLUDING DISCUSSION

We have calculated the EC and β^+ decay constants of the H-like and He-like ions $^{140}\mathrm{Pr}^{58+}$ and $^{140}\mathrm{Pr}^{57+}$, respectively. Following the standard theory of weak decays of heavy nuclei [11], we have expressed the decay constants in terms of the nuclear matrix element $\mathcal{M}_{\mathrm{GT}}$ of the Gamow-Teller transition. We have shown that the complete set of experimental data on the ratios of weak decay constants of the ions $^{140}\mathrm{Pr}^{58+}$ and $^{140}\mathrm{Pr}^{57+}$ can be explained within the standard theory of weak interactions of heavy ions.

Our theoretical values of the ratios of the decay constants for H-like $^{140}\mathrm{Pr}^{58+}$ and He-like $^{140}\mathrm{Pr}^{57+}$ ions agree with the experimental data within an accuracy better than 3%. This high precision of the theoretical analysis of the weak decays of the H-like $^{140}\mathrm{Pr}^{58+}$ and He-like $^{140}\mathrm{Pr}^{57+}$ ions is due to the small number of electrons, the behavior of which can be described by the solution of the Dirac equation. The dependence of the ratio R_{EC/β^+} on the electron structure of the decaying system is confirmed for both the experimental data and our theoretical analysis of the He-like ion $^{140}\mathrm{Pr}^{57+}$. Indeed, for the He-like $^{140}\mathrm{Pr}^{57+}$ ion, the ratio $R_{\mathrm{EC}/\beta^+}^{(\mathrm{He}),\mathrm{exp}} = 0.96(8)$ is smaller than that for the H-like $^{140}\mathrm{Pr}^{58+}$ ion, $R_{\mathrm{EC}/\beta^+}^{(\mathrm{H}),\mathrm{exp}} = 1.36(9)$. Of course, such a regularity should be confirmed experimentally by the measurements of EC and β^+ decays of the Li-like $^{140}\mathrm{Pr}^{56+}$ ions.

According to the hyperfine structure of the H-like ion $^{140}\mathrm{Pr}^{58+}$ [22], the bound electron can be in two states with a total angular momentum $F = \frac{1}{2}$ and $F = \frac{3}{2}$ with the energy splitting equal to

$$\Delta E = E_{1s_{F=\frac{1}{2}}} - E_{1s_{F=\frac{3}{2}}} = -2\alpha(\alpha Z)^3 \frac{\mu}{\mu_N}\frac{m_e^2}{m_p}\left\{\frac{(1-\delta)(1-\varepsilon)}{(1+\gamma)(1+2\gamma)} + x_{\mathrm{rad}}\right\}, \quad (6)$$

where $\gamma = \sqrt{1-(\alpha Z)^2} - 1$, $\mu = +2.5\,\mu_N$ is the magnetic moment of the nucleus $^{140}\mathrm{Pr}^{59+}$ [15], $\mu_N = e/2m_p$ is the nuclear magneton, $m_p = 938.27\,\mathrm{MeV}/c^2$ is the proton mass, δ is the nuclear charge distribution correction, ε is the nuclear magnetization distribution correction (the Bohr-Weisskopf correction [23]), and x_{rad} denotes the radiative correction, calculated to lowest order in α and αZ [24]. Numerical estimates, carried our for different ions by Shabaev *et al.* [22], show that for the calculation of ΔE with an accuracy better than 1% one can drop the contributions of the corrections δ, ε, and x_{rad} and get $\Delta E = -1, 12\,\mathrm{eV}$. The lifetime $\tau_{F=\frac{3}{2}}$ of the hyperfine state of the H-like $^{140}\mathrm{Pr}^{58+}_{F=\frac{3}{2}}$ ion is defined by the radiative transition $^{140}\mathrm{Pr}^{58+}_{F=\frac{3}{2}} \to {}^{140}\mathrm{Pr}^{58+}_{F=\frac{1}{2}} + \gamma$ only. It is equal to $\tau_{F=\frac{3}{2}} = 8.5 \times 10^{-3}\,\mathrm{s}$ [15]. Since the lifetime $\tau_{F=\frac{3}{2}} = 8.5 \times 10^{-3}\,\mathrm{s}$ is much shorter than the cooling time of

[2]The nuclear matrix element of the GT transition is defined by $\mathcal{M}_{\mathrm{GT}} = -2g_A G_F V_{ud}\int d^3x \Psi_d^*(r)\Psi_m(r)$, where $\Psi_d^*(r)$ and $\Psi_m(r)$ are the wave functions of the daughter and mother nuclei, respectively. We set $\Psi_d^*(r)\Psi_m(r) \sim \rho(r)$, where $\rho(r)$ has the Woods-Saxon shape with $R = 1.1A^{1/3}\,\mathrm{fm}$ and slope parameter $a = 0.50\,\mathrm{fm}$ taken from Ref. [20].

[3]Note that $\epsilon_{(1s)^2}^{(\mathrm{He})}(Z) = -94.723\,\mathrm{keV}$ and $\epsilon_{(1s)^2}^{(\mathrm{He})}(Z-1) = -91.539\,\mathrm{keV}$ are binding energies of the He-like ions $^{140}\mathrm{Pr}^{57+}$ and $^{140}\mathrm{Ce}^{56+}$ with $Z = 59$.

about 2 s [15], all H-like $^{140}Pr^{58+}_{F=\frac{3}{2}}$ ions decay into the ground hyperfine states $^{140}Pr^{58+}_{F=\frac{1}{2}}$.

In our calculation of the EC decay of the H-like $^{140}Pr^{58+}$ ion from the ground hyperfine state $^{140}Pr^{58+}_{F=\frac{1}{2}}$, the hyperfine structure is taken into account in the spinorial wave function of the bound electron [18].

The influence of the hyperfine structure on the probabilities of weak decays of heavy ions at finite temperature has been investigated by Folan and Tsifrinovich [19]. As they mentioned [19], the EC decay of the H-like ion $^{140}Pr^{58+}_{F=\frac{3}{2}}$ from the hyperfine state with $F = \frac{3}{2}$ is forbidden by a conservation of angular momentum. This agrees with our analysis [18].

Our result for the ratio $R^{(H/He),th}_{EC/EC} = \frac{3}{2}$ agrees quantitatively well with that obtained by Patyk *et al.* [25], who also accounted for the hyperfine structure of the H-like $^{140}Pr^{58+}$ ion, and the estimate carried out by Litvinov *et al.* [15]. According to Patyk *et al.* [25] and Litvinov *et al.* [15], the ratio $R^{(H/He),th}_{EC/EC} = \frac{3}{2}$ is fully caused by statistical factors, that is,

$$R^{(H/He)}_{EC/EC} = \frac{2I+1}{2F+1} = \frac{3}{2}, \tag{7}$$

calculated for $I = 1$ and $F = \frac{1}{2}$. Unfortunately, generally speaking this assertion is not completely correct. The result $R^{(H/He),th}_{EC/EC} = \frac{3}{2}$ appears only at the neglect of the electron screening of the electric charge of the nucleus Z in the He-like $^{140}Pr^{57+}$ ion. Having neglected the electron screening of the electric charge of the nucleus Z, one can represent the wave function of the bound $(1s)^2$ state of the He-like ion in the form of the product of the one-electron Dirac wave functions. In this case $\langle\psi^{(Z)}_{1s}\rangle = \langle\psi^{(Z-1)}_{1s}\psi^{(Z)}_{(1s)^2}\rangle$, and the ratio $R^{(H/He),th}_{EC/EC}$, given by Eq. (5), reduces to Eq. (7). Unlike our analysis of the weak decays of the H-like and He-like ions, the work of Patyk *et al.* [25] and Litvinov *et al.* did not analyze the contributions of the Coulomb wave functions of the bound electrons averaged over the nuclear density at all. Nevertheless, such contributions are important for the correct description of both the ratio $R^{(H/He)}_{EC/EC}$ for ions with small electric charges Z and the ratios $R^{(H)}_{EC/\beta^+}$ and $R^{(He)}_{EC/\beta^+}$ [see Eq. (5)]. Indeed, since the β^+ decays of the H-like and He-like ions are caused by the weak interactions of protons in the content of the nucleus, they do not depend on the electron structure of ions. As a result, the β^+ decay constants of the H-like and He-like ions are equal [see Eq. (4) for $F = \frac{1}{2}$ and $I = 1$] and equal to the β^+ decay constant of the nucleus $^{140}Pr^{59+}$ that agrees well with the experimental data [15].

Because of the independence of the β^+ decay constant of the electron structure of ions, the factors $|\langle\psi^{(Z)}_{1s}\rangle|^2$ and $|\langle\psi^{(Z-1)}_{1s}\psi^{(Z)}_{(1s)^2}\rangle|^2$ in the EC decay constants of the H-like and He-like ions play an important role in the correct description of the ratios $R^{(H),th}_{EC/\beta^+}$ and $R^{(He),th}_{EC/\beta^+}$. It is also important that the wave functions of the bound electrons be solutions of the Dirac equation. In the nonrelativistic limit $|\langle\psi^{(Z)}_{1s}\rangle|^2 \simeq |\langle\psi^{(Z-1)}_{1s}\psi^{(Z)}_{(1s)^2}\rangle|^2 \simeq 1/\pi a^3_B$, the ratios $R^{(H),th}_{EC/\beta^+}$ and $R^{(He),th}_{EC/\beta^+}$ are equal to $R^{(H),th}_{EC/\beta^+} \simeq 0.51$ and $R^{(He),th}_{EC/\beta^+} \simeq 0.34$, respectively, and disagree with the experimental data [15].

In the present paper, we have calculated the EC and β^+ decay rates of the H-like $^{140}Pr^{58+}$ ions using standard weak interaction theory without massive neutrinos and neutrino mixing. Recently, Litvinov *et al.* [26] measured the EC decay rate of the H-like $^{140}Pr^{58+}$ ions and found a nonexponential behavior of the rate of the number of daughter ions $^{140}Ce^{58+}$, varying with a period $T_d = 7.06(8)$ s and leading to a periodic time dependence of the EC decay rate $\lambda^{(H)}_{EC}(t)$. Such a periodic dependence on time of the EC decay rate has been explained as a consequence of mass-differences of massive neutrino mass-eigenstates [27–29]. The averaged value of the time-dependent EC decay rate is equal to the EC decay constant given by Eq. (4) [27–29].

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