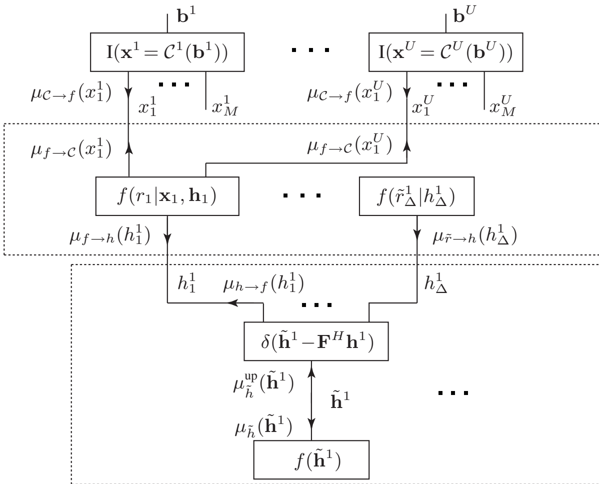

Fig. 2. Forney-style factor graph describing $f(\mathbf{X},\mathbf{H},\mathbf{r}|\mathbf{b})$ in (6) for the first symbol and one pilot symbol of the first user.

$$p(b_i^u|\mathbf{r}) \,=\, \sum_{\sim b_i^u} p(\mathbf{b}|\mathbf{r}) \,\propto \sum_{\sim b_i^u} f(\mathbf{r}|\mathbf{b})\,, \tag{4}$$

where $f(\mathbf{r}|\mathbf{b})$ is the conditional probability density function (pdf) of $\mathbf{r}$ given $\mathbf{b}$, $\sum_{\sim x}$ denotes summation over all unknown variables in the summand except x, and $\propto$ denotes equality up to factors irrelevant to the maximization (3). Based on the one-to-one correspondence between $\mathbf{b}$ and $\mathbf{X} \triangleq (\mathbf{x}^1 \cdots \mathbf{x}^U)$, we can factor $f(\mathbf{r}|\mathbf{b})$ as

$$f(\mathbf{r}|\mathbf{b}) \,=\, \sum_{\mathbf{X}} f(\mathbf{r}|\mathbf{X}) \prod_{u=1}^{U} \mathrm{I}\big(\mathbf{x}^u = \mathcal{C}^u(\mathbf{b}^u)\big)\,,$$

where the indicator function $\mathrm{I}(\cdot)$ is one if its argument is true and zero otherwise. We have $f(\mathbf{r}|\mathbf{X}) = \int f(\mathbf{r}|\mathbf{X},\mathbf{H})\, f(\mathbf{H})\, d\mathbf{H}$, where $\mathbf{H}$ is the $U \times N$ matrix of all h_n^u. Using (1), (2) and the definitions $\mathbf{x}_n \triangleq (x_n^1 \cdots x_n^U)^T$ and $\mathbf{h}_n \triangleq (h_n^1 \cdots h_n^U)^T$ yields

$$f(\mathbf{r}|\mathbf{X},\mathbf{H}) \,=\, \prod_{n\notin\mathcal{P}} f(r_n|\mathbf{x}_n,\mathbf{h}_n) \prod_{u=1}^{U} \prod_{n\in\mathcal{P}^u} f(\tilde{r}_n^u|h_n^u)\,.$$

Furthermore, using the independence of all channels and $\mathbf{h}^u = \mathbf{F}\tilde{\mathbf{h}}^u$, we have $f(\mathbf{H}) = \prod_{u=1}^{U} \int \delta(\tilde{\mathbf{h}}^u - \mathbf{F}^H\mathbf{h}^u)\, f(\tilde{\mathbf{h}}^u)\, d\tilde{\mathbf{h}}^u$. Combining these expressions and inserting them into (4), we obtain

$$p(b_i^u|\mathbf{r}) \,\propto \sum_{\sim b_i^u} \int f(\mathbf{X},\mathbf{H},\mathbf{r}|\mathbf{b})\, d\mathbf{H} \tag{5}$$

with

$$\begin{aligned} f(\mathbf{X},\mathbf{H},\mathbf{r}|\mathbf{b}) \,=\, & \prod_{n\notin\mathcal{P}} f(r_n|\mathbf{x}_n,\mathbf{h}_n) \prod_{u=1}^{U} \prod_{n\in\mathcal{P}^u} f(\tilde{r}_n^u|h_n^u) \\ & \times \prod_{u'=1}^{U} \int \delta(\tilde{\mathbf{h}}^{u'} - \mathbf{F}^H\mathbf{h}^{u'})\, f(\tilde{\mathbf{h}}^{u'})\, d\tilde{\mathbf{h}}^{u'} \\ & \times \prod_{u''=1}^{U} \mathrm{I}\big(\mathbf{x}^{u''} = \mathcal{C}^{u''}(\mathbf{b}^{u''})\big)\,. \end{aligned} \tag{6}$$

Due to (1) and (2), $f(r_n|\mathbf{x}_n,\mathbf{h}_n)$ and $f(\tilde{r}_n^u|h_n^u)$ are Gaussian with mean $\mathbf{h}_n^T\mathbf{x}_n$ and $h_n^u p_n^u$, respectively and variance σ_w^2. Fig. 2 depicts the Forney-style factor graph [6] corresponding to $f(\mathbf{X},\mathbf{H},\mathbf{r}|\mathbf{b})$ in (6). For simplicity, only the first symbol and one pilot symbol are shown.

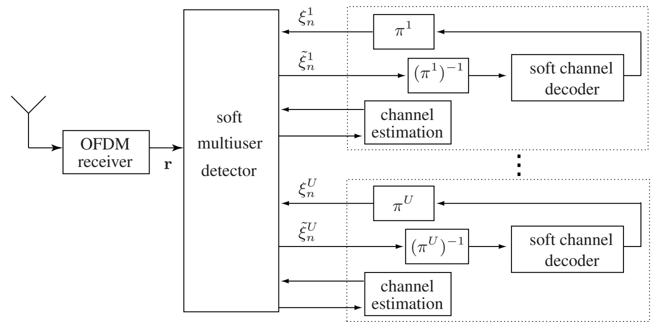

Fig. 3. Structure of the OFDM-IDMA receiver performing joint detection and channel estimation. (ξ_n^u and $\tilde{\xi}_n^u$ are defined in Section 4.)

Receiver structure. Applying the sum-product algorithm [5] to the factor graph in Fig. 2 yields an approximation to the marginal (5) for all b_i^u simultaneously. (This is only an approximation because the graph contains cycles.) Using parallel scheduling [8], we obtain the receiver structure shown in Fig. 3. The block termed “soft multiuser detector” (corresponding to the upper dotted box in Fig. 2) receives soft information from the channel decoders of the individual users [1] and updates it using the current channel estimate. The improved soft bits are sent back to the decoders and also passed to channel estimation units (corresponding to the lower dotted box in Fig. 2), which calculate refined estimates of the channel coefficients. The decoding units—each consisting of deinterleaver, soft channel decoder, and interleaver—correspond to the blocks $\mathrm{I}(\mathbf{x}^u{=}\mathcal{C}^u(\mathbf{b}^u))$ in Fig. 2. Upon termination of the sum-product algorithm, bit decisions are provided by the signs of the *a posteriori* soft information bits computed by the channel decoder.

4. MESSAGES

We now derive the messages that are propagated through the factor graph in Fig. 2.

Messages $\mu_{\mathcal{C}\to f}(x_n^u)$. Applying the sum-product algorithm to the code function nodes $\mathrm{I}(\mathbf{x}^u = \mathcal{C}^u(\mathbf{b}^u))$ yields the BCJR algorithm [5, 9] for soft-decoding the convolutional code and a summation of appropriate bit log-likelihood ratios (LLRs) for soft-decoding the repetition code. This soft decoder produces extrinsic LLRs $\xi_n^u \in \mathbb{R}$ for the BPSK symbols x_n^u. The corresponding messages are [5]

$$\mu_{\mathcal{C}\to f}(x_n^u) \,=\, \frac{\exp\big(\xi_n^u (x_n^u+1)/2\big)}{1+\exp(\xi_n^u)}\,, \quad x_n^u \in \{-1, 1\}\,.$$

Messages $\mu_{f\to\mathcal{C}}(x_n^u)$. The channel function nodes pass the following messages back to the code function nodes:

$$\begin{aligned} \mu_{f\to\mathcal{C}}(x_n^u) \,=\, \sum_{\sim x_n^u} \int f(r_n|\mathbf{x}_n,\mathbf{h}_n) \prod_{u'=1}^{U} \mu_{h\to f}(h_n^{u'}) \\ \times \prod_{u''\neq u} \mu_{\mathcal{C}\to f}(x_n^{u''})\, d\mathbf{h}_n\,. \end{aligned} \tag{7}$$

The sum $\sum_{\sim x_n^u}$ is over 2^{U-1} terms. To achieve a complexity that is linear in U, we approximate the $\mu_{\mathcal{C}\to f}(x_n^u)$ by Gaussian messages,

$$\mu_{\mathcal{C}\to f}(x_n^u) \,=\, \exp\!\left(-\,\frac{(x_n^u - a_n^u)^2}{2\,b_n^u}\right), \tag{8}$$

with mean $a_n^u = \tanh(\xi_n^u)$ and variance $b_n^u = 1-(a_n^u)^2$ (cf. [1]). A Gaussian model is also used for the messages $\mu_{h\to f}(h_n^u)$, i.e.,

$$\mu_{h\to f}(h_n^u) \;=\; \exp\!\left(-\,\frac{|h_n^u - \alpha_n^u|^2}{\beta_n^u}\right), \tag{9}$$

where the mean α_n^u and variance β_n^u will be determined later.

Assuming that x_n^u and h_n^u are independent and distributed according to (8) and (9), respectively, the message $\mu_{f\to\mathcal{C}}(x_n^u)$ in (7) equals the conditional pdf $f(r_n|x_n^u)$. Since closed-form integration in (7) is impossible, we use the Gaussian approximation

$$\mu_{f\to\mathcal{C}}(x_n^u) \;=\; f_{\mathrm{G}}(r_n|x_n^u) \;\propto\; \exp\!\left(-\,\frac{|r_n - c_n^u|^2}{d_n^u}\right), \tag{10}$$

with mean $c_n^u = \mathrm{E}\{r_n|x_n^u\} = \alpha_n^u x_n^u + \sum_{u'\neq u} \alpha_n^{u'} a_n^{u'}$ and variance $d_n^u = \mathrm{E}\{|r_n - c_n^u|^2|x_n^u\} = |\alpha_n^u|^2 + \sigma_w^2 + \sum_{u'\neq u}\big(\beta_n^{u'} + |\alpha_n^{u'}|^2 b_n^{u'}\big)$. In the course of the iterations, $b_n^u \to 0$ and $\beta_n^u \to 0$, i.e., $\mu_{\mathcal{C}\to f}(x_n^u)$ in (8) and $\mu_{h\to f}(h_n^u)$ in (9) that enter as factors in (7) become increasingly narrow, and thus the Gaussian approximation becomes more accurate. Converting $\mu_{f\to\mathcal{C}}(x_n^u)$ into an LLR value yields

$$\tilde{\xi}_n^u \;=\; \log\frac{\mu_{f\to\mathcal{C}}(x_n^u\!=\!1)}{\mu_{f\to\mathcal{C}}(x_n^u\!=\!-1)} \;=\; \frac{\big(r_n - \sum_{u'\neq u}\alpha_n^{u'} a_n^{u'}\big)\,(\alpha_n^u)^*}{2\,d_n^u}\,,$$

which is passed to the associated channel decoder (see Fig. 3).

Messages $\boldsymbol{\mu_{f\to h}(h_n^u)}$. The messages from the channel function nodes to the channel variable nodes are

$$\mu_{f\to h}(h_n^u) \;=\! \sum_{\mathbf{x}_n} \int f(r_n|\mathbf{x}_n,\mathbf{h}_n) \prod_{u'=1}^{U} \mu_{\mathcal{C}\to f}(x_n^{u'}) \times\! \prod_{u''\neq u} \mu_{h\to f}(h_n^{u''})\, d\mathbf{h}_n^{\sim u},$$

where $\int d\mathbf{h}_n^{\sim u}$ denotes integration with respect to all entries of $\mathbf{h}_n$ except h_n^u. Based on arguments similar to those motivating (10), we use the approximation

$$\mu_{f\to h}(h_n^u) \;=\; \exp\!\left(-\,\frac{|h_n^u - \nu_n^u|^2}{\gamma_n^u}\right), \tag{11}$$

with $\nu_n^u = \frac{1}{a_n^u}\big(r_n - \sum_{u'\neq u}\alpha_n^{u'} a_n^{u'}\big)$ and $\gamma_n^u = \frac{1}{|a_n^u|^2}\big[\sigma_w^2 + b_n^u|h_n^u|^2 + \sum_{u'\neq u}\big(\beta_n^{u'} + |\alpha_n^{u'}|^2 b_n^{u'}\big)\big]$. Because h_n^u appears in γ_n^u, $\mu_{f\to h}(h_n^u)$ in (11) is not Gaussian in h_n^u. We thus use the approximation $\gamma_n^u \approx \frac{1}{|a_n^u|^2}\big[\sigma_w^2 + \sum_{u'\neq u}\big(\beta_n^{u'} + |\alpha_n^{u'}|^2 b_n^{u'}\big)\big]$, which is justified because $b_n^u \to 0$ in the course of the iterations.

Messages $\boldsymbol{\mu_{\tilde{r}\to h}(h_n^u)}$. The messages from the pilot symbol function nodes are given by $\mu_{\tilde{r}\to h}(h_n^u) = \exp\big(-|\tilde{r}_n^u - h_n^u p_n^u|^2/\sigma_w^2\big)$. These messages can be rewritten as Gaussians in h_n^u:

$$\mu_{\tilde{r}\to h}(h_n^u) \;=\; \exp\!\left(-\,\frac{|h_n^u - \tilde{r}_n^u/p_n^u|^2}{\sigma_w^2/(p_n^u)^2}\right).$$

Messages $\boldsymbol{\mu_{\tilde{h}}(\tilde{\mathbf{h}}^u)}$. Let us combine the (Gaussian) messages $\mu_{\tilde{r}\to h}(h_n^u)$ and $\mu_{f\to h}(h_n^u)$ into a "vector message" $\mu_h(\mathbf{h}^u)$, which is Gaussian with mean $\mathbf{m}_h^u$ and diagonal covariance $\mathbf{C}_h^u$. In the first several iterations, we use only the pilot symbols for channel estimation. We thus have $\mu_h(\mathbf{h}^u) = \prod_{n\in\mathcal{P}^u}\mu_{\tilde{r}\to h}(h_n^u)$, from which it follows that $[\mathbf{m}_h^u]_n$ is $\tilde{r}_n^u/p_n^u$ if $n\in\mathcal{P}^u$ and zero otherwise, and $[\mathbf{C}_h^u]_{n,n}$ is $\sigma_w^2/(p_n^u)^2$ if $n\in\mathcal{P}^u$ and zero otherwise. In later iterations, we also use the messages $\mu_{f\to h}(h_n^u)$ for channel estimation, so $\mu_h(\mathbf{h}^u) = \prod_{n\in\mathcal{P}^u}\mu_{\tilde{r}\to h}(h_n^u)\prod_{n\notin\mathcal{P}}\mu_{f\to h}(h_n^u)$. Thus, $[\mathbf{m}_h^u]_n$ is $\tilde{r}_n^u/p_n^u$ if $n\in\mathcal{P}^u$, α_n^u if $n\notin\mathcal{P}$, and zero otherwise, and $[\mathbf{C}_h^u]_{n,n}$ is $\sigma_w^2/(p_n^u)^2$ if $n\in\mathcal{P}^u$, β_n^u if $n\notin\mathcal{P}$, and zero otherwise. Finally, because $\tilde{\mathbf{h}}^u = \mathbf{F}^H\mathbf{h}^u$, the desired message $\mu_{\tilde{h}}(\tilde{\mathbf{h}}^u)$ is Gaussian with mean $\mathbf{m}_{\tilde{h}}^u = \mathbf{F}^H\mathbf{m}_h^u$ and covariance $\mathbf{C}_{\tilde{h}}^u = \mathbf{F}^H\mathbf{C}_h^u\mathbf{F}$.

Messages $\boldsymbol{\mu_{\tilde{h}}^{\mathrm{up}}(\tilde{\mathbf{h}}^u)}$. The message $\mu_{\tilde{h}}(\tilde{\mathbf{h}}^u)$ is multiplied by the *a priori* message $\mu_{f_{\tilde{h}}}(\tilde{\mathbf{h}}^u)$ (corresponding to $f(\tilde{\mathbf{h}}^u)$), which is Gaussian with mean $\mathbf{m}_{\tilde{h}_p}$ and covariance $\mathbf{C}_{\tilde{h}_p}$. The product $\mu_{\tilde{h}}^{\mathrm{up}}(\tilde{\mathbf{h}}^u)$ is again Gaussian [5], with mean $\mathbf{m}_{\tilde{h}}^{u,\mathrm{up}}$ and covariance $\mathbf{C}_{\tilde{h}}^{u,\mathrm{up}}$. Suppose that the receiver uses a fixed channel length $\tilde{L}_c$ (in Section 5, we propose a method for estimating the channel length). Then, only the first $\tilde{L}_c$ entries of $\mathbf{m}_{\tilde{h}}^{u,\mathrm{up}}$ (denoted by $\mathbf{n}_{\tilde{h}}^{u,\mathrm{up}}$) and the top-left $\tilde{L}_c\times\tilde{L}_c$ submatrix of $\mathbf{C}_{\tilde{h}}^{u,\mathrm{up}}$ (denoted $\mathbf{D}_{\tilde{h}}^{u,\mathrm{up}}$) are nonzero. Let $\mathbf{n}_{\tilde{h}}^u$ and $\mathbf{n}_{\tilde{h}_p}$ denote the first $\tilde{L}_c$ entries of $\mathbf{m}_{\tilde{h}}^u$ and $\mathbf{m}_{\tilde{h}_p}$, respectively, and let $\mathbf{D}_{\tilde{h}}^u$ and $\mathbf{D}_{\tilde{h}_p}$ denote the top-left $\tilde{L}_c\times\tilde{L}_c$ submatrix of $\mathbf{C}_{\tilde{h}}^u$ and $\mathbf{C}_{\tilde{h}_p}$, respectively. Then $\mathbf{D}_{\tilde{h}}^{u,\mathrm{up}} = \big((\mathbf{D}_{\tilde{h}}^u)^{-1} + \mathbf{D}_{\tilde{h}_p}^{-1}\big)^{-1}$ and $\mathbf{n}_{\tilde{h}}^{u,\mathrm{up}} = \mathbf{D}_{\tilde{h}}^{u,\mathrm{up}}\big((\mathbf{D}_{\tilde{h}}^u)^{-1}\mathbf{n}_{\tilde{h}}^u + \mathbf{D}_{\tilde{h}_p}^{-1}\mathbf{n}_{\tilde{h}_p}\big)$ [5].

Messages $\boldsymbol{\mu_{h\to f}(\mathbf{h}^u)}$. Because $\mathbf{h}^u = \mathbf{F}\tilde{\mathbf{h}}^u$, the message $\mu_{h\to f}(\mathbf{h}^u)$ (the "vector message" combining the messages $\mu_{h\to f}(\tilde{h}_n^u)$) is Gaussian with mean $\mathbf{m}_h^{u,\mathrm{up}} = \mathbf{F}\mathbf{m}_{\tilde{h}}^{u,\mathrm{up}}$ and covariance $\mathbf{C}_h^{u,\mathrm{up}} = \mathbf{F}\mathbf{C}_{\tilde{h}}^{u,\mathrm{up}}\mathbf{F}^H$. The mean α_n^u and variance β_n^u in (9) equal the nth element of $\mathbf{m}_h^{u,\mathrm{up}}$ and the nth diagonal element of $\mathbf{C}_h^{u,\mathrm{up}}$, respectively.

Scheduling. The messages $\mu_{f\to\mathcal{C}}(x_n^u)$ for all users at the input of the multiuser detector are simultaneously updated by the channel decoders and used by the multiuser detector to calculate the messages for all users at its output (parallel message scheduling [8]). We propose not to update the messages $\mu_{h\to f}(h_n^u)$ (using the messages $\mu_{f\to h}(h_n^u)$) during the first three iterations. Initially, a maximum value of $\tilde{L}_c$ is used; an estimate of L_c is calculated (see Section 5) in the fifth iteration and then used for $\tilde{L}_c$ in all subsequent iterations.

5. CHANNEL LENGTH ESTIMATION

Estimating the channel length L_c can improve receiver performance, because fewer channel coefficients have to be estimated. We consider the maximum likelihood estimator

$$\hat{L}_c = \arg\max_{L_c\in\mathbb{N}} f(\mathbf{r}|L_c)\,, \tag{12}$$

where $f(\mathbf{r}|L_c)$ can be obtained by the following marginalization:

$$f(\mathbf{r}|L_c) \;\propto\; \sum_{\mathbf{X},\mathbf{b}} \int f(\mathbf{X},\mathbf{H},\mathbf{r}|\mathbf{b},L_c)\, d\mathbf{H}\,. \tag{13}$$

We have $f(\mathbf{X},\mathbf{H},\mathbf{r}|\mathbf{b},L_c) = f(\mathbf{r}|\mathbf{X},\mathbf{H},\mathbf{b},L_c) f(\mathbf{X},\mathbf{H}|\mathbf{b},L_c) = f(\mathbf{r}|\mathbf{X},\mathbf{H})\, f(\mathbf{X}|\mathbf{b})\, f(\mathbf{H}|L_c)$, with

$$f(\mathbf{H}|L_c) \;=\; \prod_{u=1}^{U} \int \delta(\tilde{\mathbf{h}}^u - \mathbf{F}^H\mathbf{h}^u)\, f(\tilde{\mathbf{h}}^u|L_c)\, d\tilde{\mathbf{h}}^u.$$

The marginalization (13) equals (5), except that the summation is over all entries of $\mathbf{b}$. Using the sum-product algorithm, we pass messages analogous to those used for approximating (5), except that now they depend on L_c. For a given L_c, the messages $\mu_{\tilde{h}}(\tilde{\mathbf{h}}^u)$ are multiplied by the *a priori* messages $\mu_{f_{\tilde{h}}}(\tilde{\mathbf{h}}^u|L_c)$ (formerly $\mu_{f_{\tilde{h}}}(\tilde{\mathbf{h}}^u)$), yielding $\mu_{\tilde{h}}^{\mathrm{up}}(\tilde{\mathbf{h}}^u|L_c)$. From these, the messages $\mu_{h\to f}(h_n^u|L_c)$ are obtained as in Section 4, and (13) can be expressed as

$$f(\mathbf{r}|L_c) \;\propto\; \prod_{n\notin\mathcal{P}} \int f(r_n|\mathbf{x}_n,\mathbf{h}_n) \prod_{u=1}^{U} \mu_{\mathcal{C}\to f}(x_n^u) \times \mu_{h\to f}(h_n^u|L_c)\, dx_n^u\, dh_n^u\,.$$

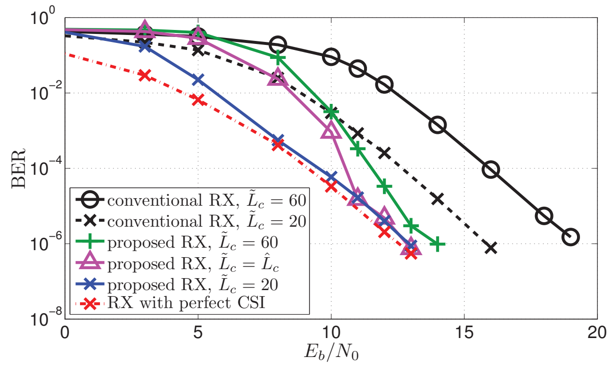

Fig. 4. Average BER versus E_b/N_0 obtained with different OFDM-IDMA receivers for $U = 4$ users and channel length $L_c = 20$.

Using the approximation $\mu_{h\to f}(h_n^u|L_c) = \delta(h_n^u - \alpha_n^u(L_c))$ (cf. (9)), the h_n^u integral collapses. This yields

$$f(\mathbf{r}|L_c) \propto \prod_{n\notin\mathcal{P}} \int f(r_n|\mathbf{x}_n, \mathbf{h}_n = \boldsymbol{\alpha}_n(L_c)) \prod_{u=1}^{U} \mu_{\mathcal{C}\to f}(x_n^u)\, dx_n^u\,,$$

with $\boldsymbol{\alpha}_n(L_c) \triangleq \big(\alpha_n^1(L_c) \cdots \alpha_n^U(L_c)\big)^T$. The $\mu_{\mathcal{C}\to f}(x_n^u)$ are Gaussian (see (8)), so the integration can be carried out and we obtain

$$f(\mathbf{r}|L_c) \propto -\sum_{n\notin\mathcal{P}} \bigg| r_n - \sum_{u=1}^{U} a_n^u \alpha_n^u(L_c)\bigg|^2.$$

The maximization (12) is then done by exhaustive search.

6. SIMULATION RESULTS

We consider an OFDM-IDMA system with $U = 4$ users, each transmitting $K = 256$ information bits. An overall code rate of $1/8$ is achieved by serially concatenating a terminated rate-$1/2$ convolutional code (code polynomial $[2\ 3]_8$) and a rate-$1/4$ repetition code (hence, the sum-rate is $1/2$). $N = 2360$ subcarriers are used, with a pilot spacing of $\Delta = 30$. The *a priori* variances of the nonzero channel coefficients were set equal to 1.

For a channel of length $L_c = 20$, Fig. 4 shows the average bit error rate (BER) versus the signal-to-noise ratio (SNR) E_b/N_0 for the following receivers: (i) a conventional receiver that performs pilot-based MMSE channel estimation assuming $\tilde{L}_c = 60$ and uses the resulting channel estimates for iterative data detection; (ii) the same receiver but assuming the correct channel length, i.e., $\tilde{L}_c = L_c = 20$; (iii) the proposed receiver without channel length estimation, assuming $\tilde{L}_c = 60$; (iv) the proposed receiver with channel length estimation; (v) the proposed receiver without channel length estimation but assuming $\tilde{L}_c = L_c = 20$; and (vi) an iterative receiver with perfect channel state information (CSI). All receivers performed 10 iterations. The conventional receiver with $\tilde{L}_c = 60$ is seen to perform about 3 dB worse than the conventional receiver assuming the correct L_c. At a BER of 10^{-6}, the proposed receiver using channel length estimation and the proposed receiver using the true L_c perform almost as well as the receiver with perfect CSI, while the proposed receiver using $\tilde{L}_c = 60$ performs about 1 dB worse. These results demonstrate the advantages of joint data detection and channel estimation, as well as of channel length estimation, in the OFDM-IDMA context considered.

To illustrate the performance of the proposed channel length estimation method, Fig. 5 shows histograms of $\hat{L}_c$ for two channels with $L_c = 20$ and $L_c = 40$, at $E_b/N_0 = 11$ dB. It is seen that the estimates

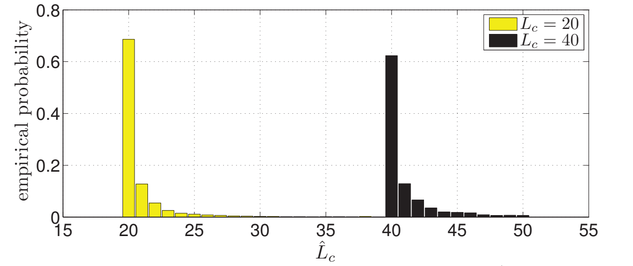

Fig. 5. Histogram of the estimated channel length $\hat{L}_c$ for channel lengths $L_c = 20$ and $L_c = 40$, at $E_b/N_0 = 11$ dB.

are correct in the majority of cases. There is a tendency for overestimation, but estimation errors larger than 2 are quite unlikely. We note that overestimation of L_c causes a much smaller BER increase than underestimation.

7. CONCLUSION

Using a factor graph approach, we developed an iterative OFDM-IDMA receiver whose complexity scales linearly with the number of users. Performance improvements relative to conventional receivers were achieved by joint multiuser data detection and channel estimation and by explicit estimation of the length of the frequency-selective channels. Extensions to MIMO-OFDM-IDMA systems with spatial multiplexing (cf. [10]) and to time-varying channels, as well as possible combination with reduced-rank approximations [11] are interesting directions for future research.

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