# Cooperative Space-Time Codes with Opportunistic Network Coding

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***Abstract*—Cooperative communications has emerged as one of the most efficient way of exploiting spatial diversity in wireless communications. Cooperative communications is a strategy where users, besides transmitting their own information, also relay re-encoded version of other users' information to a common destination. In this paper, we investigate a scheme called space-time (ST) coded cooperation with opportunistic network coding, which further enhances the cooperative communications benefits. In this scheme, two cooperating users using ST codes are taken. A third user, which is not demanding any service during the transmission of cooperating user, is used to send network coded message to the destination. We call it opportunistic because transmission of network coded data along with ST codes of cooperating users, depend on the availability of third user, acting as a relay in our case. The results show that the investigated scheme perform better than ST cooperation for a range of signal-to-noise ratio (SNR) from -10 to 10 dB. After this, the performance is degraded heavily in proportion to increase in SNR.**

## I. Introduction

In wireless communications, signal transmitted from transmitter when received at the destination receiver are very unreliable for decision making for the transmitted data. This unreliability is introduced because of reflection, refraction, diffraction, and scattering due to wireless nature of the channel [1].

Diversity is a way, where receiver is provided with different replicas of the same signal to average out the effects of wireless channel. Besides space, frequency, and time, cooperative communications is a recent idea to achieve not only diversity gain but capacity improvements as well [2]-[3]. Cooperative communications is a strategy where users, besides transmitting their own information, also relay re-encoded version of other user's information to a common destination.

The achievable gains are based on smart coding strategy of cooperative users to relay its partner's information to a common destination. Some of the many coding schemes are bilayer LDPC codes where relay generated parity bits are incorporated in specially designed bilayer graphical code structure [4]. Distributed turbo codes, where source and relay each have recursive encoder, the relay interleaves the data received from source and sends re-encoded version of it to destination [5]. ST codes [6] and network coding [7], [8].

We extend the last two strategies by combining both of them. Our study is based on the assumption that two users cooperation has already been established and both users are cooperating using ST codes. We argue that in a network, like in cellular networks, not all users in the network are demanding the service at the same time. In this situation, there is a third user, termed as idle user, can be used to further enhance the cooperative communications benefits. This idle user receives the information from cooperating users and sends network coded bits of the decoded information to the destination. We name this as opportunistic network coding on the basis of the fact that idle users' resources have been used opportunistically based on its availability during the cooperative transmission.

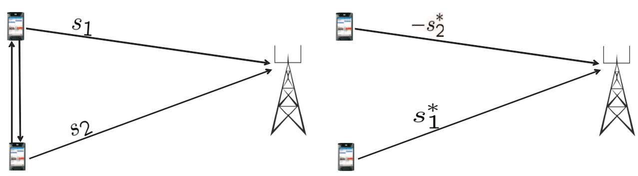
Our results show that, the use of opportunistic network coding on top of ST coded cooperation offers higher gain than the ST cooperation with Rayleigh fading channel between the cooperating users or even ideal inter-user channel between them.

The following structure of the paper is followed. Section II presents our system model. Section III is devoted to present the decoder used at the destination. Section IV shows our simulated results and section V is for conclusion of the paper.

*Notation:* Normal letters represent scalar quantities, boldface with lower case letters represent vectors and the boldface with capital letters represent matrices.

## II. System Model

Our system model contains two cooperating users $u_1$ and $u_2$ with random binary messages $(s_1, s_2 \in \{-1, 1\})$ transmitting to a common destination. We take cellular network as an example where there is third user which is idle in the sense that it is not demanding any service during the transmission of cooperating users.

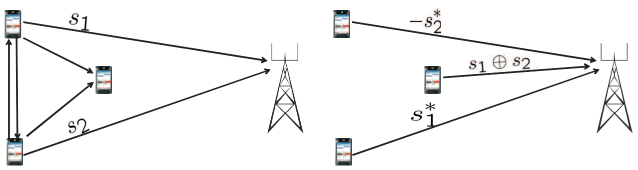



(a) First Phase (b) Second Phase

Fig. 1. ST Cooperative communications using Alamouti scheme

Fig. 1 shows two cooperating users using ST Alamouti codes, communication takes place in two transmissions or phases. During first phase user $u_1$ and $u_2$ send $s_1$ and $s_2$ as in Fig. 1(a). During second phase they send $-s_2^*$ and $s_1^*$ as shown in Fig. 1(b).

Fig. 2 shows our investigated ST cooperative communications setup with opportunistic network coding, which works as follows: During first phase of transmission users $u_1$ and $u_2$ broadcast $s_1$ and $s_2$ respectively as shown in Fig. 2(a). This message is received by their cooperating users $u_2$, $u_1$, relay, and destination. During second phase, after decoding the message, users $u_2$ and $u_1$, now the cooperating user for each other, send $s_1^*$ and $-s_2^*$ to destination. And the relay, which has received and decoded the message from both of the cooperating users, takes Ex-OR, $s_1 \oplus s_2$, of both the messages and sends it to the destination as shown in Fig. 2(b).




(a) First Phase (b) Second Phase

Fig. 2. ST Cooperative communications using Alamouti scheme with opportunistic network coding

All of the channels, from users to destination, inter-user, users to relay, and relay to destination are considered to be block-fading Rayleigh, which stay constant for two symbol periods. Additive white Gaussian noise (AWGN) with zero mean and unit variance is assumed at the receiver. All of the users are assigned orthogonal channels. This orthogonality can be achieved by using different time, frequency or code, as is done in today's cellular networks. Signals received at the respective receivers are as follows:

During first transmission signal received at $u_1$ and $u_2$ from each other is,

$$y_1 = s_1 h_{21} + n_{u_1} \quad (1)$$

$$y_2 = s_2 h_{12} + n_{u_2} \quad (2)$$

relay receives from user $u_1$ and $u_2$, written in matrix vector notation

$$\begin{bmatrix} y_{r_1} \\ y_{r_2} \end{bmatrix} = \begin{bmatrix} h_{r1} & 0 \\ 0 & h_{r2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_{r_1} \\ n_{r_2} \end{bmatrix} \quad (3)$$

and the destination, after two phases of transmission, again written in matrix vector notation, receives

$$\begin{bmatrix} y_d \\ y_d^c \end{bmatrix} = \begin{bmatrix} h_{d1} & h_{d2} & 0 \\ h_{d2}^* & -h_{d1}^* & h_{dr} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_1 \oplus s_2 \end{bmatrix} + \begin{bmatrix} n_{d_1} \\ n_{d_2} \end{bmatrix}$$

$$\boldsymbol{y} = \boldsymbol{H}\mathbf{s} + \boldsymbol{n} \quad (4)$$

$s_i$ shows the message transmitted by user $u_i$. $y_1$ and $y_2$ are the signal received at the cooperating users $u_1$ and $u_2$. $y_{R_1}$, $y_{R_2}$, $y_{d_1}$, and $y_{d_2}$ are the signals received at relay and destination from users $u_1$ and $u_2$. $h_{ij}$ shows the channel fading between receiver $i$ and transmitter $j$. For example, $h_{R1}$ is the channel gain between user $u_1$ and relay. Noise terms at destination are represented as $n_1$ and $n_2$, during first and second transmissions. For cooperating users and relay, noise terms are denoted as $n_{i_j}$, where $i$ is the particular receiver whose noise is under consideration and $j$ is the transmitting user. For example $n_{R_1}$ is the noise generated at the relay receiver during first user, $u_1$'s transmission.

We assume perfect channel information at the receivers. All transmitters are synchronized by some universal clock. All users satisfy an equal power constraint of $P_i$.

## III. Decoder

We use maximum likelihood (ML) decoder without taking into account the detection errors at the cooperating users and relay. ML decoder requires the computation of

$$\hat{\boldsymbol{s}} = \underset{\boldsymbol{s} \in \{-1,1\}}{\operatorname{argmin}} \operatorname{Re}\{(\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s})^H \boldsymbol{C} (\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s})\} \quad (5)$$

where $\boldsymbol{y}$, $\boldsymbol{H}$, and $\boldsymbol{s}$ are given in equation (4) and $\boldsymbol{C}$, is the inverse covariance matrix of the noise. Other possibility which also considers the detection errors at cooperating users and relay, by assuming the inter-user as well as user relay channel as binary symmetric channel (BSC), is our next target. One of its variants is discussed in [9]

## IV. Performance Evaluation

We present our simulation results for proposed opportunistic network coding on top of ST coding, based on the system model discussed in section II.

For the simulation purpose, we assume that the user $u_1$ and $u_2$ are at the same distance $d$ from destination. The inter-user distance is half of the user-destination distance, $\frac{d}{2}$. The idle user, the relay is located at the distance of $\frac{d}{4}$ from both of the users and is at $1 - \frac{d}{4}$ distance from destination. The distances are normalized to unity. We use binary phase shift keying (BPSK) uncoded transmission. Since both of the cooperating

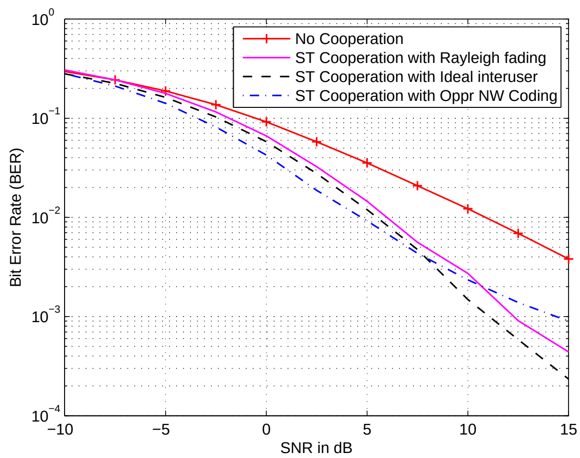



Fig. 3. Bit Error Rate for cooperative communications with different settings

users have symmetric channels to destination, we discuss the results only for one user and other users' performance is exactly same.

Fig. 3 shows our simulation results for the investigated ST cooperation with opportunistic network coding. For comparison purpose, we also have given the probability of error for a non cooperating user, a ST cooperating user with perfect inter-user and a ST cooperating user with Rayleigh fading channel between them.

As the figure shows, user cooperation in any case has considerable gain over non cooperative transmission. For example, two ST cooperating users have approximately 6 dB gain in terms of SNR at bit error rate (BER) of $10^{-2}$ over non cooperating users. There is a gap of 2 dB for two cooperating users using ST codes, when there is an ideal inter-user and when there is Rayleigh fading channel between them. Interestingly, the curve for ST cooperation with opportunistic network coding shows that it performs even better than the ST cooperation with ideal inter-user channel for SNR values from -10 to 8 dB. After this region, for high SNR, network coding on top of ST coding with discussed decoder actually worsen the situation.

## V. Conclusion

Cooperative communications benefits can be enhanced by exploiting idle user's resources. The investigated scenario of ST coded cooperation with opportunistic network coding gives us approximately 1.5 dB gain over ST cooperation for SNR range from -3 dB to 8 dB. Furthermore, it shows that for high SNR range, from 10 dB and onwards, network coding with ST cooperation degrades the performance compared to the only ST cooperation.

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