



# Computer program to calculate resistivities and layer thicknesses from Schlumberger soundings at the surface, at lake bottom and with two electrodes down in the subsurface <sup>☆</sup>

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## ABSTRACT

A computer program is presented that calculates thicknesses and resistivities of a layered earth from three different variations of Schlumberger arrangements:

- (1) The usual Schlumberger method with 4 electrodes at the surface.
- (2) Lake bottom arrangement: 4 electrodes at the bottom of waters like lakes, rivers or the sea.
- (3) Two potential electrodes at the same depth in the ground and two electrodes at the surface.

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## 1. Introduction

One of the most popular methods for investigating the resistivity of the subsurface is DC sounding. The program presented here written to evaluate Schlumberger soundings and two further, similar arrangements (see Fig. 1). Schlumberger soundings have a very long tradition and are frequently used today in spite of the growing importance of two-dimensional measurements with multi-array arrangements. The reason is that the overall effort required to carry out multi-array measurements is much higher than that for Schlumberger measurements and in many cases the information obtained from Schlumberger measurements is sufficient.

The method can distinguish between layers with different resistivities, where sufficiently high resistivity contrast exists. In groundwater exploration a popular example is the differentiation between aquitards, consisting of low resistivity layers with high clay content, and aquifers, consisting mainly of gravel and sand with low clay content and high resistivity. There is no one-to-one relation between the resistivities obtained from the measurements and the materials found in the subsurface; however, in most cases a very confident model can be obtained.

A great variety of computer programs have been published, e.g.: Johansen (1977); Koefoed (1979); Zohdy (1989); Başokur, (1990); Vedanti et al. (2005) and are commercially available e.g. (1 × 1D from Interpex<sup>®</sup>, IPI2Win from Geoscan<sup>®</sup>, Res1d from Geotomo<sup>®</sup>). In most cases the source code is not available or the programs do not meet the requirements for easy usage. The computer program presented here offers an open source code for both calculation of resistivities and user interface. The program calculates layer thicknesses (or depth) and resistivities from surface measurements. The user must be aware of the fact that a unique solution is not possible. This is not a deficiency of the program, but a consequence of the theory of electrical soundings. Unique solutions may be obtained by combinations with other methods and by assuming layer continuity.

The program calculates layer thicknesses and resistivities from three different variations of Schlumberger arrangements (Fig. 1):

- (1) The usual Schlumberger method with 4 electrodes at the surface.
- (2) Lake bottom arrangement: 4 electrodes at the bottom of waters like lakes, rivers or the sea.
- (3) Two potential electrodes at the same depth in the ground and two electrodes on the surface.

The program gives a numerical error estimate for all parameters and allows for rapid judgment of the reliability of the results with

<sup>☆</sup> Code available from server at <http://www.iamg.org/CGEditor/index.htm>.

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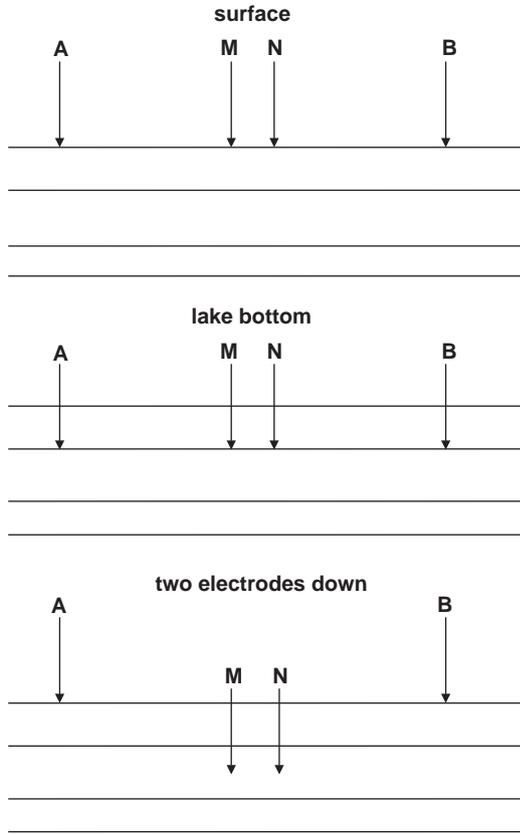


Fig. 1. Three electrode arrangements calculated by program. Currents are introduced as point sources at head of arrows.

different numbers of layers. Furthermore, data from several lines can be read and the calculated depth resistivity graphs can be compared in the final window. The final window may be copied to the clipboard and pasted into other programs.

In addition to the source code, the executable program for Windows® operating system, together with examples, is also available.

## 2. Theoretical background

The calculation of layer resistivities and thicknesses from measurements is denoted as ‘inversion’ in the following context. The calculations start with an initial subsurface model consisting of horizontal layers of given thickness and resistivity (see Fig. 2 for notations). Currents  $I$  are induced at the current electrodes (A, B) and the potential difference  $\Delta V$  is measured at the potential electrodes (M, N). With the known positions of the current and potential electrodes, the resistances  $\Delta V/I$  can be calculated for all  $AB/2$  distances of electrode positions. Now the model parameters may be changed in order to minimize the sum of the squared differences between measured and calculated values.

The potentials of the model are calculated using the formulas derived by Schulz (1985), for electrodes placed arbitrarily within a layered model.

The potential at location  $\vec{r}$  caused by one current electrode at location  $\vec{r}_a$  may be expressed as

$$V(\vec{r}, \vec{r}_a) = \frac{I}{2\pi} \int_0^\infty f(z, z_a, \lambda) J_0(\lambda R_a) d\lambda \quad (1)$$

with

$$R_a = \sqrt{(x - x_a)^2 + (y - y_a)^2} \quad (2)$$

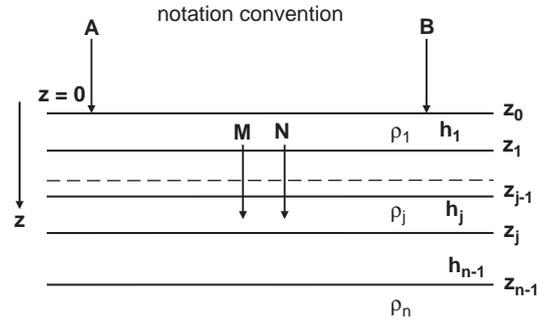


Fig. 2. Notations used in equations.

and  $J_0(\lambda R_a)$ , the Bessel function of order zero.  $(x, y, z)$  and  $(x_a, y_a, z_a)$  are the components of  $\vec{r}$  and  $\vec{r}_a$ , respectively, with  $z$  being positive downwards.

One recognizes that the horizontal components  $(x, y)$  of  $\vec{r}$  in Eq. (1) are only contained within the Bessel-function and that the vertical component  $z$  is contained solely within  $f(z, z_a, \lambda)$ . When the potential at  $\vec{r} = \vec{r}_1$  is known, the potential at  $\vec{r}_1 + \Delta\vec{r}$  with sufficiently small  $\Delta\vec{r}$  may be approximated as:

$$V(\vec{r}_1 + \Delta\vec{r}, \vec{r}_a) = V(\vec{r}_1, \vec{r}_a) + \left( \frac{\partial V(\vec{r}, \vec{r}_a)}{\partial x} \right)_{\vec{r}=\vec{r}_1} \Delta x + \left( \frac{\partial V(\vec{r}, \vec{r}_a)}{\partial y} \right)_{\vec{r}=\vec{r}_1} \Delta y + \left( \frac{\partial V(\vec{r}, \vec{r}_a)}{\partial z} \right)_{\vec{r}=\vec{r}_1} \Delta z$$

with  $(\Delta x, \Delta y, \Delta z)$  the components of  $\Delta\vec{r}$ .

The potential difference  $\Delta V_a$  between two adjacent points  $(\Delta x \ll R_a)$  located at the same  $z$ -level with  $y$  constant  $(\Delta y = 0, \Delta z = 0)$  is as follows:

$$\Delta V_a = V(\vec{r}, \vec{r}_a) - V(\vec{r} + \Delta\vec{r}, \vec{r}_a) = \frac{I \Delta x}{2\pi} \varepsilon \int_0^\infty f(z, z_a, \lambda) J_1(\lambda R_a) \lambda d\lambda$$

with  $J_1(\lambda \vec{r}_a)$  the 1st order Bessel function.  $R_a$  Eq. (2) simplifies to:

$$R_a = x - x_a \text{ and } \varepsilon = 1 \text{ case } x > x_a \\ R_a = x_a - x \text{ and } \varepsilon = -1 \text{ case } x < x_a$$

The potential difference  $\Delta V$  for a positive source at  $\vec{r}_a$  and a negative source of the same strength (as necessary for a current introduced into the earth) located at  $\vec{r}_b$  with

$$\vec{r}_a = \begin{pmatrix} x_a \\ y_a \\ z_a \end{pmatrix}, \vec{r}_b = \begin{pmatrix} -x_a \\ y_a \\ z_a \end{pmatrix}$$

follows as:

$$\Delta V = 2 \frac{I \Delta x}{2\pi} \int_0^\infty f(z, z_a, \lambda) J_1(\lambda R_a) \lambda d\lambda \quad (3)$$

This formula applies to all three electrode arrangements of the computer program described.

Placing the current electrode at the top layer ( $z_a = 0$ ), the expression for  $f(z, z_a, \lambda)$  from Schulz (1985) may be modified and expressed as (see Fig. 2 for notations and definitions of the layer index):

$$f(z, 0, \lambda) = \left[ \prod_{i=1}^{j-1} \left( \frac{\tilde{T}_i + \rho_i}{\tilde{T}_{i+1} + \rho_i} \right) \right] \frac{1}{2} (\tilde{T}_j + \rho_j) e^{-\lambda z} \times \left[ 1 + e^{-2\lambda(z_j - z)} \frac{\tilde{T}_{j+1} - \rho_j}{\tilde{T}_{j+1} + \rho_j} \right] \quad j \geq 2$$

$\bar{T}_i(\lambda)$  is defined by the following recursion formula:

$$\begin{aligned}\bar{T}_n &= \rho_n \\ \bar{T}_i &= \rho_i \frac{\bar{T}_{i+1} + \rho_i \alpha_i}{\rho_i + \bar{T}_{i+1} \alpha_i} \quad 1 \leq i < n \\ \alpha_i &= \tanh(\lambda h_i)\end{aligned}$$

with  $n$  being the number of the deepest layer and  $\tanh(x)$  the hyperbolic tangent.

This formula applies to the general case of two electrodes at the surface and two electrodes at the same level down.

For Schlumberger measurements we may use this formula with  $z = 0$ .

At lake bottom one obtains with  $z_i = h_i$ :

$$f(z_1, z_1, \lambda) = \begin{cases} \frac{\bar{T}_1 - \alpha_1 \rho_1}{1 - \alpha_1^2} & \alpha_1 \neq 1 \\ \frac{\rho_1 \rho_2}{\rho_1 + \rho_2} & \alpha_1 = 1 \end{cases}$$

The integral of Eq. (3) is carried out by convolution with a linear filter containing 141 coefficients. The filter method was first introduced to DC-geoelectrics by Ghosh (1971). The calculation of the model parameters from the measurements starts with Eq. (3). The measured quantity is  $\Delta V/I$ , which is multiplied by a weighting factor  $K$  to give the apparent resistivity  $\rho_a$ :

$$\rho_a = \frac{2\pi\Delta V}{I} K = 2\Delta x K \int_0^\infty f(z, z_a, \lambda) J_1(\lambda R_a) \lambda d\lambda \quad (4)$$

with  $K = (R_a^2 - a^2)/4a \approx R_a^2/4$ ,  $R_a$  the horizontal distance from the centre of the arrangement to the current electrodes and  $a = \Delta x/2$ .

$\rho_a$  approaches the true resistivity of a uniform half-space for Schlumberger measurements only. The designation for  $\rho_a$ , apparent resistivity, is not correct in the case of lake bottom measurements because  $\rho_a$  will not approach the true resistivity in that case.

Taking a set of measurements with different values of  $R_a = R_{a,m}$ . Eq. (4) with  $K$  substituted may be written as

$$\rho_{a,m} = R_a^2 \int_0^\infty f(z, z_a, \lambda) J_1(\lambda R_{a,m}) \lambda d\lambda \quad m = 1, M \quad (5)$$

This equation contains the known values  $\rho_{a,m}$ ,  $R_{a,m}$ ,  $z$ ,  $z_a$  and the  $(2N-1)$  unknown layer parameters  $(\rho_j, (j = 1, N); h_j, (j = 1, N-1))$  with  $N$  the number of layers and  $M$  the number of measurements. All unknowns are contained within  $f(z, z_a, \lambda)$ .

The inversion algorithm uses an iterative Levenberg–Marquardt algorithm with singular value decomposition described in Johansen (1977). The algorithm minimizes the sum of the squared differences between measured and calculated values  $\rho_{a,m}$  on a 20 points/decade grid of  $AB/2$  values on log scale. The measured data are interpolated to the 20 points/decade grid to fit this requirement.

### 3. Programming considerations

The program was written in Fortran95 using the Lahey<sup>®</sup> compiler with graphical interface to Windows by the Winter-acter<sup>®</sup> programming package.

The inversion algorithm was originally programmed by K. Arnason (personal communication, 1994). A 141 point filter was introduced by A. Başokur (personal communication 1997). The algorithm was further modified by the authors of this paper. The first-order partial derivatives for the parameters  $p$ , necessary for the Marquardt algorithm, are computed by calculating the function difference as  $\Delta\rho/\Delta p$  instead of  $\partial\rho/\partial p$ . A test showed that the accuracy of this approximation is sufficient. The computation

time is about the same as that for the analytical solution and the programming code is much simpler.

Eq. (5) allows to first calculate  $f(z, z_a, \lambda)$  with Bessel transform and then evaluate the unknown parameters from  $f(z, z_a, \lambda)$  (Başokur, 1990). This method is called the ‘direct method’. However, it turns out that the direct method is not stable in cases where the approximation of horizontal layering is not good enough. For that reason the direct method is not used in the presented computer program.

It is well known that – apart from one- and two-layer cases – clear cut calculations of layer-thicknesses and resistivities from measured soundings are not possible in most cases. A wide range of variations in layer thickness and resistivity will give the same sounding curve within the limits of data spread. This phenomenon is called the principle of equivalence in geoelectrics. Further information from other investigations must be included to resolve these uncertainties. The described program calculates the standard errors for layers and thicknesses as random errors from the variance matrix during inversion. Biased error distributions or systematic errors originating, for example, from non-horizontal layering are not considered. The error estimates are very important because they show the possible range of equivalent models.

The program allows for fixing layer thicknesses and/or resistivities. The number of layers can be given or can be selected automatically; likewise, a starting model for the iteration process can be selected manually or automatically. The comparison of the parameters with their error ranges allows for a quick estimate of realistic models.

One of the main advantages of the presented program is the possibility to adapt geological models to the measurements very easily. One can fix layers and/or resistivities during the approximation process and obtains error estimates for the variable layers and resistivities. High uncertainties for a specific layer thickness or resistivity indicate that other models are as likely as the current one.

#### 3.1. Program usage

After calling up the program, a main menu, a data dialog, a model dialog and a checkbox with the title “ready for calculation” appear (Fig. 3). From the main menu, the electrodes submenu should be selected first to start either surface, lake bottom or the ‘two electrodes down-two electrodes up’ electrode configuration. In case nothing is selected, a standard Schlumberger array is assumed.

Data can be input either directly by typing into the data dialog spreadsheet, or by copying the data via clipboard from another spreadsheet like Excel<sup>®</sup>, or by calling a data file from the file menu.

The next step is to create a starting model with the model dialog. The starting model can be typed into the spreadsheet either directly, or selected automatically. When ‘automatic’ is selected, the layers are created with exponentially increasing thicknesses on going downward. This selection is in accordance to the well-known fact that the resolution of DC soundings diminishes approx. exponentially (Johansen, 1977). The thicknesses are fixed in the following inversion process if the automatic switch is not turned off afterwards. When the number of layers is not given, following Zohdy (1989),  $M-1$  layers ( $M =$  number of measurements) are created. This is the most stable inversion and should be selected first when no further information on the subsurface is provided. For every layer, the thickness or the resistivity may be fixed or kept free during the inversion process.

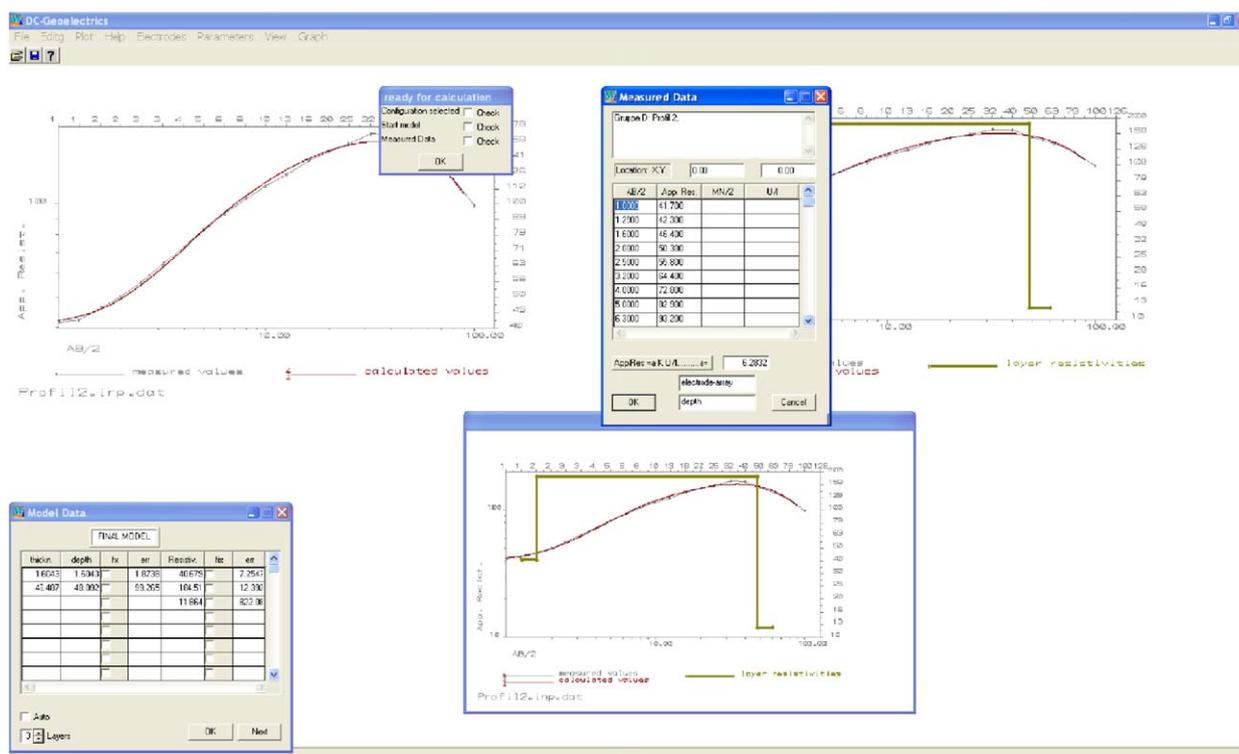


Fig. 3. Screen with dialogs and graphics. Highlighted graphic may be exported or plotted with different formats.

The checkbox shows if all three steps have been selected. Thereafter, the inversion starts. The first frame shows the sounding curve of the measured apparent resistivities and the starting model. After finishing the inversion process, the final depth model together with the sounding curve is shown in another frame. Layer thicknesses and resistivities are drawn into the model dialog and written onto an output file. The final frame may be copied to the clipboard and inserted into other reports, or plotted onto a file using the plot option on the main menu.

Two output files are created: '\*.mod' and '\*.res'. The asterisk represents the name of the data input file without extension. These two files will rarely be used. '\*.mod' contains the calculated resistivities and thicknesses and may be used as an input file for the starting model in further calculations. '\*.res' contains detailed information on the calculations: The eigenvalues, the eigenvectors, the correlation matrix, and the calculated resistivities. The user may read the paper by Inman (1975), in addition to the paper by Johansen (1977), for a deeper understanding of the singular value decomposition and the ridge regression method used within the program. An asterisk to the right of a parameter marks that the parameter has been kept constant during the inversion process.

Two further files, 'testf' and 'TEMP1' are created as scratch files and deleted when the program is closed. Programmers may change the output to be permanent. 'testf' contains programming information in order to find errors or to control the program flow. This file is of interest for programmers only. The content of this program will be changed to the needs of the programmer. 'TEMP1' contains output information for the final graph.

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## Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.cageo.2009.03.001.

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