A PRACTICAL FORWARDING SCHEME FOR WIRELESS RELAY CHANNELS BASED ON THE QUANTIZATION OF LOG-LIKELIHOOD RATIOS

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ABSTRACT

This paper considers a wireless relay network with one source, one relay, and one destination. We propose a low-complexity quantize-and-forward (QF) scheme in which the relay calculates log-likelihood ratios (LLRs) for the code bits transmitted by the source and passes quantized versions of these LLRs on to the destination. The destination combines the LLRs from the source-destination link and the quantized LLRs from the source-relay link to obtain improved information bit estimates. We verify the usefulness of the proposed scheme via numerical simulations in block and fast fading.

Index Terms— relay channel, quantize-and-forward, cooperative diversity, LLR quantization

1. INTRODUCTION

1.1. Background and Motivation

The use of relays in a wireless network promises power savings as well as an increase in diversity as compared to using only the direct link from source to destination. This enables reliable communication at higher data rates. Information theoretical results for the Gaussian relay channel have existed for a while, see e.g. [1], but the capacity of the general relay channel is still unknown. During the last few years, a lot of research efforts focused on the design of relaying schemes that can be implemented in practical hardware. Decode-and-forward (DF) is a scheme where the relay attempts to decode the source message and, in case of success, re-encodes it for the transmission to the destination (see [2] for a recent implementation using LDPC codes). In the amplify-and-forward (AF) scheme, the relay simply retransmits a scaled (amplified) version of the signal it has received. An advantage of AF is that the relay can also be active when the source-relay link is in outage, in which case DF fails. The downsides of AF are the additional noise from the relay and the difficulty of implementing the analog forwarding scheme into hardware. There are also schemes that try to combine the advantages of DF and AF like decode-amplify-and-forward (DAF) [3] and soft-DF [4]. Another possibility is quantize-and-forward (QF), also known as compress-and-forward or estimate-and-forward. Here, the relay compresses the information about the received signal and passes it on to the destination in digital form. Unlike DF, such schemes work well even when the source-relay link is in outage and, unlike AF, they do not require sophisticated analog processing at the relay.

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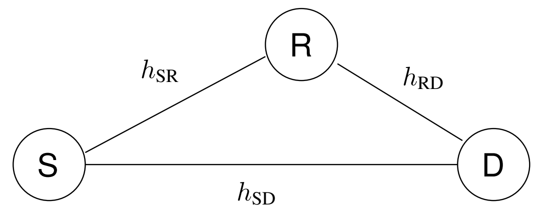


Fig. 1. *Schematic illustration of the relay channel.*

1.2. Contributions

In this paper we present a practical QF scheme in which the relay soft-demodulates the source symbols, performs a low-complexity quantization of the resulting log-likelihood ratios (LLRs), and forwards the quantized LLRs to the destination. The proposed relaying scheme has the following properties:

- performance similar to or better than AF or DF, unless the relay is close to the source;
- lower computational complexity than DF and DAF since channel decoding is not required;
- lower hardware complexity than AF thanks to digital operation.

Furthermore, the processing in the relay node is the same as the transmit and receive processing at the source and destination node, respectively. This is an advantage in larger cooperative networks, where each node can act in alternation as source, destination, and relay. Although we here consider only a three-node network to illustrate the basic principle, we think that the scheme is promising in setups with a larger number of nodes, specifically when combined with soft-information-based network coding in the spirit of [5].

The paper is structured as follows. Section 2 describes the relay channel system model. In Section 3, we present the proposed QF relaying scheme and in Section 4 we explain the underlying LLR quantizer. In Section 5, the performance of our scheme is illustrated with simulation results. Conclusions are provided in Section 6.

2. SYSTEM MODEL

We consider a wireless relay channel as shown in Fig. 1, consisting of one source (S), one relay (R) and one destination (D). The relay is assumed to have a half-duplex constraint, i.e., it cannot receive and transmit data at the same time; this is a realistic assumption in view of existing RF hardware. The overall transmit frame of length N is therefore partitioned into two time slots. In the first ("broadcast") slot, the source transmits a codeword $x_{\mathrm{S}}[n]$ of duration N_1 while relay and destination listen:

$$y_\mathrm{D}[n] = h_\mathrm{SD}[n]\, x_\mathrm{S}[n] + n_\mathrm{D}[n], \qquad n = 1,\ldots,N_1, \qquad (1)$$
$$y_\mathrm{R}[n] = h_\mathrm{SR}[n]\, x_\mathrm{S}[n] + n_\mathrm{R}[n], \qquad n = 1,\ldots,N_1.$$

Here, $y_\mathrm{D}[n]$ and $y_\mathrm{R}[n]$ denote the signals received at the destination and the relay, respectively, $h_\mathrm{SD}[n]$ is the the fading channel between source and destination, $h_\mathrm{SR}[n]$ is the source-relay channel, and $n_\mathrm{D}[n]$ and $n_\mathrm{R}[n]$ denote the white complex Gaussian noise at the destination and the relay, respectively. In the second time slot, the relay is transmitting a codeword $x_\mathrm{R}[n]$ of length $N_2 = N - N_1$ over the relay-destination channel $h_\mathrm{RD}[n]$ and the source listens again:

$$y_\mathrm{D}[n] = h_\mathrm{RD}[n]\, x_\mathrm{R}[n] + n_\mathrm{D}[n], \quad n = N_1+1,\ldots,N. \qquad (2)$$

The destination combines the signals received within the two time slots in order to recover the message transmitted by the source.

For simplicity we assume that source and relay transmit BPSK symbols $x_\mathrm{S}[n], x_\mathrm{R} \in \{-1,1\}$. Furthermore, the noise components $n_\mathrm{D}[n]$ and $n_\mathrm{R}[n]$ have zero mean and variance σ_D^2 and σ_R^2, respectively. The channel coefficients $h_\mathrm{SD}[n]$, $h_\mathrm{SR}[n]$, and $h_\mathrm{RD}[n]$ are assumed to be independent zero-mean complex Gaussian random variables, corresponding to a Rayleigh fading channel model. We consider both block fading (i.e., the channel coefficients remain constant during the time slot) and fast fading (i.e., the channel coefficients change with each channel use). The signal-to-noise ratio (SNR) for the source-destination link is defined as[1] $\mathrm{SNR}_\mathrm{SD} \triangleq \mathcal{E}\{|h_\mathrm{SD}[n]|^2\}/\sigma_\mathrm{D}^2$, and similar definitions apply for the SNRs of the source-relay link and the relay-destination link. With our proposed scheme, only receiver channel state information (CSI) is assumed at the relay and destination. For the AF scheme considered for comparison, the destination also needs to know the source-relay channel.

3. LLR-BASED QUANTIZE-AND-FORWARD

In the following, we describe the proposed QF scheme in detail.

Source. The source is active only in the first time slot. It encodes the information bits with an LDPC channel code $\mathcal{C}_1$ of rate R_1. The resulting code bits are mapped to BPSK transmit symbols $x_\mathrm{S}[n]$, $n = 1,\ldots,N_1$, which are broadcast to the relay and the destination.

Relay. During the first time slot, the relay demodulates the received signal $y_\mathrm{R}[n]$ using a soft-output demapper, resulting in LLRs $L_\mathrm{SR}[n]$ for the code bits transmitted by the source. For BPSK, the LLRs are obtained from (1) as (Re$\{\cdot\}$ denotes the real part)

$$L_\mathrm{SR}[n] = \frac{4\,\mathrm{Re}\{h^*_\mathrm{SR}[n]\, y_\mathrm{R}[n]\}}{\sigma_\mathrm{R}^2}. \qquad (3)$$

The relay then quantizes the LLRs using the scalar quantizer discussed in Section 4, which yields the quantization indices

$$i_L[n] = \mathcal{Q}(L_\mathrm{SR}[n]) \in \{0,1,\ldots,2^q-1\}.$$

Quantizing LLRs instead of the received signal has the advantage that the LLRs implicitly contain the channel state information, which otherwise would have to be transmitted separately to the destination. This advantage becomes even more important in scenarios with multiple relays. The binary representations of these indices $i_L[n]$, consisting of q bits per LLR value, are then optionally[2] encoded using an LDPC code $\mathcal{C}_2$ of rate R_2 and mapped to BPSK symbols $x_\mathrm{R}[n]$, $n = N_1+1,\ldots,N$. In the second time slot, these qN_1/R_2 BPSK symbols are transmitted to the destination. We note that increasing the number of quantization bits q per LLR is compensated for by a corresponding reduction of the effective SNR. However, the second time slot may be longer as in the AF and DF schemes. If the LLR quantizer is adapted to the current channel conditions (this is feasible e.g. for block fading), the relay furthermore has to transmit the mapping between the quantization indices $i_L[n]$ and the associated LLR reconstruction values $\hat{L}_\mathrm{SR}[n]$. In case of fast fading the LLR quantizer remains fixed and such overhead transmissions are not required.

Destination. During the first time-slot, the destination uses a soft demapper to generate LLRs $L_\mathrm{SD}[n]$ based on the source-destination link (1), similar to (3). In the second time slot, the destination first calculates LLRs $L_\mathrm{RD}[n]$ from the relay-destination link (2). If an LDPC code was used at the relay, the destination uses these LLRs as input to an LDPC decoder for code $\mathcal{C}_2$, which produces estimates of the bit labels for the quantization index $i_L[n]$. If the packet from the relay is successfully decoded, it is used to retrieve the quantized source-relay LLRs $\hat{L}_\mathrm{SR}[n]$. Otherwise the packet is discarded. The rationale for this is the threshold behavior of LDPC codes: the packet will either be decoded with a vanishing number of wrong bits, or the error rate will be close to 1/2 if the relay-destination link is too weak. In the case of uncoded relay transmission over reliable relay-destination links, the sign of the LLRs $L_\mathrm{RD}[n]$ is used to make a hard decision about the quantization indices $i_L[n]$ and in turn about the quantized source-relay LLRs $\hat{L}_\mathrm{SR}[n]$. The destination finally combines the LLRs from the source-destination link and the quantized LLRs of the source-relay link that were forwarded by the relay according to

$$L_\mathrm{D}[n] = \hat{L}_\mathrm{SR}[n] + L_\mathrm{SD}[n].$$

This LLR combination is optimal since all channels were assumed to be independent. If the relay-destination link is in outage, the destination uses only the source-destination LLRs. The combined LLRs $L_\mathrm{D}[n]$ constitute the input to an LDPC decoder for the code $\mathcal{C}_1$ that finally delivers estimates of the source information bits.

4. QUANTIZER DESIGN

A scalar quantizer for quantize-and-forward relaying is described in [6] but has rather high design complexity. Since we strive for a low-complexity relay design, we base the design of the LLR quantizer on the results from [7]. We drop the time index n in this section.

We consider a scalar quantizer for real-valued LLRs with $K = 2^q$ bins $\mathcal{I}_k = [l_k, l_{k+1}]$, $k = 0,\ldots,K-1$ (we use the conventions $l_0 = -\infty$, $l_K = -\infty$) with associated quantization index i_L and reconstruction value $\hat{L}_\mathrm{SR}$, i.e., $i_L = k$ and $\hat{L}_\mathrm{SR} = \hat{l}_k$ iff $L_\mathrm{SR} \in \mathcal{I}_k$. The concatenation of source-relay channel, soft demodulator, and LLR quantizer can be seen as an equivalent discrete memoryless channel with binary input $x_\mathrm{S}[n] \in \{-1,1\}$ and K-ary output $\hat{L}_\mathrm{SR}$. The crossover probabilities $p_{bk} = \Pr\{\hat{L}_\mathrm{SR} = \hat{l}_k \,|\, x_\mathrm{S} = b\}$ of this channel are given by

$$p_{bk} = \Pr\{L_\mathrm{SR} \in \mathcal{I}_k \,|\, x_\mathrm{S} = b\} = \int_{\mathcal{I}_k} f_{L_\mathrm{SR}|x_\mathrm{S}}(\xi|b)\, d\xi, \qquad (4)$$

where $f_{L_\mathrm{SR}|x_\mathrm{S}}(\xi|b)$ is the conditional probability density function (pdf) of the LLR L_SR given $x_\mathrm{S} = b$ (cf. (3)).

The proposed quantizer design maximizes the mutual information $I(L_\mathrm{SR}; \hat{L}_\mathrm{SR})$ between the continuous-valued LLRs L_SR and the

[1] Here, $\mathcal{E}\{\cdot\}$ denotes mathematical expectation.

[2] We observed that this encoding does not help when the relay is close to the destination.

quantizer output $\hat{L}_{\text{SR}}$. This quantity was shown in [7] to be maximized for a uniform distribution of the quantizer output $\hat{L}_{\text{SR}}$ which in turn leads to the optimum quantization thresholds

$$l_k = F_{L_{\text{SR}}}^{-1}\left(\frac{k}{K}\right), \qquad k = 1, \ldots, K-1. \tag{5}$$

Here, $F_{L_{\text{SR}}}(l) = \Pr\{L_{\text{SR}} \leq l\}$ is the unconditional cumulative distribution of the LLRs, i.e., $F_{L_{\text{SR}}}(l) = \frac{1}{2}\int_{-\infty}^{l}\left[f_{L_{\text{SR}}|x_{\text{S}}}(\xi|-1) + f_{L_{\text{SR}}|x_{\text{S}}}(\xi|1)\right] d\xi$.

For a fast Rayleigh fading channel, the LLR quantizer can be designed using the following analytic expression for the conditional pdf of the LLRs (see [8]):

$$f_{L_{\text{SR}}|x_{\text{S}}}(\xi|b) = \frac{\sigma_{\text{R}}^2}{4\sqrt{1+\sigma_{\text{R}}^2}} \exp\left(\frac{\xi\, b}{2} - \sqrt{1+\sigma_{\text{R}}^2}\,\frac{|\xi|}{2}\right). \tag{6}$$

For a block fading channel (i.e., fixed channel), the conditional LLR distribution is Gaussian and the LLR quantizer can be adapted for each channel realization by appropriate scaling. Alternatively, the quantizer designed for fast Rayleigh fading can be used for all blocks in a block fading scenario. Our simulations indicated that this results only in a slight performance degradation, which is payed off by the simplification of the LLR quantizer.

The optimum reconstruction values of the quantized LLRs are computed as the LLRs of the equivalent DMC described above [7,9], i.e.,

$$\hat{l}_k = \log\frac{\Pr\{i_L = k|x_{\text{S}} = 1\}}{\Pr\{i_L = k|x_{\text{S}} = 0\}} = \log\frac{p_{1k}}{p_{0k}}. \tag{7}$$

5. SIMULATION RESULTS

To illustrate the performance of the proposed QF scheme, we performed Monte Carlo simulations with the following parameters: the source used an LDPC code of rate $1/2$ and block length $N_1 = 64800$, optimized for an AWGN channel. The modulation format for all links was BPSK. Source, relay, and destination were assumed to be situated on a straight line, where source and destination have distance one and the relay is located between them, at distance d from the source. The mean-squared channel gain on the source-destination link is assumed to be $\gamma_{\text{SD}} = E\{|h_{\text{SD}}|^2\} = 1$, such that $\gamma_{\text{SR}} = E\{|h_{\text{SR}}|^2\} = d^{-\alpha}$ and $\gamma_{\text{RD}} = E\{|h_{\text{RD}}|^2\} = (1-d)^{-\alpha}$. The channel attenuation exponent was chosen $\alpha = 4$. With these conventions, it is sufficient to plot error rates versus the source-destination SNR SNR_{SD} with different distance parameters d. We compare the proposed QF scheme with both direct transmission and the following variants of AF and DF. For the direct transmission result we used a transmission power of $\mathcal{E}\{|x_{\text{S}}[n]|^2\} = 2$, i.e. the sum of the transmit powers of source and relay in the relaying schemes, for a fair comparison.

DF scheme. Here, the source encodes the information bits with an LDPC code $\mathcal{C}$ of rate $R = 1/2$ and transmits the resulting code bits during the first time slot. The relay tries to decode the received packet. If successful, it re-encodes the decoded source information bits with the same channel code $\mathcal{C}$ and transmits the result in the second time slot. The destination demaps the symbols received from the source and from the relay in the first and second time slot, respectively. The sum of the resulting LLRs is passed to the LDPC decoder for $\mathcal{C}$, which produces final estimates for the information bits. If decoding at the relay fails, the relay stays silent in the second time slot and the destination uses only the signal from the source.

AF scheme. The operation of all nodes in the first time slot is the same as in the QF and DF case. In the second time slot the relay transmits a rescaled (amplified) version of the received signal $y_{\text{R}}[n]$, i.e. $x_{\text{R}}[n] = \alpha_{\text{R}}\, y_{\text{R}}[n]$, with $\alpha_{\text{R}} = \sqrt{\frac{N}{\sum_{n=1}^{N}|y_{\text{R}}[n]|^2}}$. The destination calculates LLRs for the Gaussian compound channel consisting of source-relay and relay-destination channel according to $L_{\text{SRD}}[n] = \frac{4\Re\{\alpha_{\text{R}} h_{\text{SR}}^*[n] h_{\text{RD}}^*[n] y_{\text{D}}[n]\}}{|\alpha_{\text{R}} h_{\text{RD}}[n]|^2\sigma_{\text{R}}^2 + \sigma_{\text{D}}^2}$. Note that this presupposes that the destination knows the channel coefficients and the noise variances for the source-relay link and the relay-destination link, which is unrealistic in fast fading scenarios. Finally, the destination feeds the sum of L_{SRD} and L_{SD} to the channel decoder.

We note that with the AF and the DF scheme, the relay transmits exactly as many symbols as the source. In our QF scheme, however, the number of symbols transmitted is $N_2 = qN_1/R_2$. In the simulations, this fact is accounted for in the power constraint to ensure a fair comparison.

5.1. Block fading

We first consider frequency-flat Rayleigh block fading channels. Here, the quantizer in the QF scheme was adapted to each block.

Fig. 2(a) depicts frame error rate (FER) versus source-destination SNR SNR_{SD} for our QF scheme (with $q = 1, 3$), for AF, and for DF, in a scenario where QF schemes are expected to work best, i.e., when the relay is close to the destination ($d = 0.9$), so that the relay-destination link is reliable. The use of a channel code on the relay-destination link is therefore not required, thus reducing complexity at the relay and the destination. The results show that our QF scheme indeed achieves diversity order two and, with $q = 3$, performs as well as AF. DF works worse than our scheme even for $q = 1$; this is due to the high outage probability of the source-destination link.

Fig. 2(b) shows the results for the case where the relay is located halfway between source and destination. For QF, only quantization with one bit is shown, because quantization with two bits already performs worse. This means that the gain in accuracy of the forwarded LLRs in the two bit vs. one bit case does not compensate the resulting loss in SNR on the relay-destination link. In this scenario, QF outperforms AF and looses about 1.5 dB against DF.

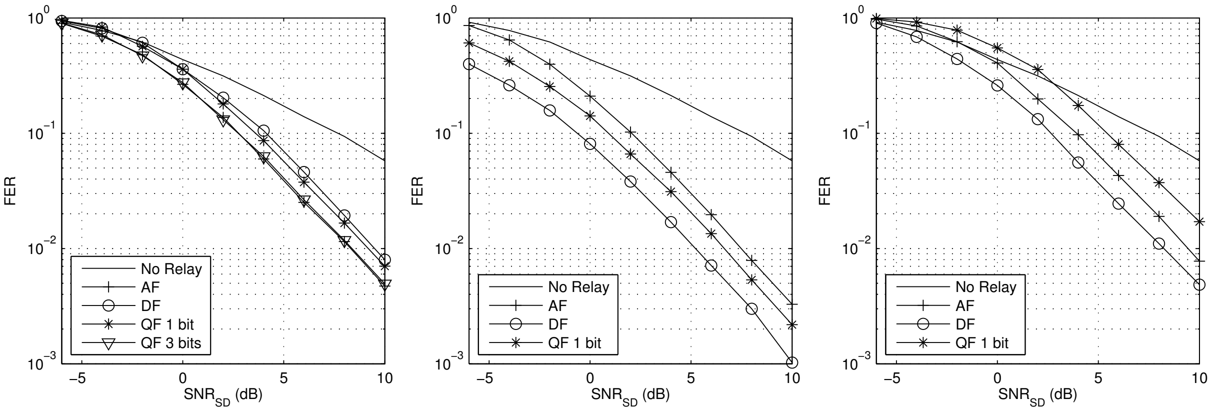
If the relay is close to the source ($d = 0.1$), QF performs worst as can be seen from Fig. 2(c). The reason is the low SNR on the relay-destination link. DF works best here, followed by AF. For low SNR, QF performs even worse than without relay. This can be attributed to the fact that we allocated equal power to the source and the relay. Apparently, for such low SNR all power should be allocated to the source. We note, however, that the diversity order of our QF scheme still equals two.

5.2. Fast fading

We next consider fast Rayleigh fading channels with a fixed LLR quantizer designed based on (6). Fig. 3(a) shows the bit error rate obtained with the various relaying schemes versus SNR for the case where the relay was close to the destination. Again, our QF scheme with 3 bits per LLR performs as well as AF. QF with $q = 2$ still performs better than DF.

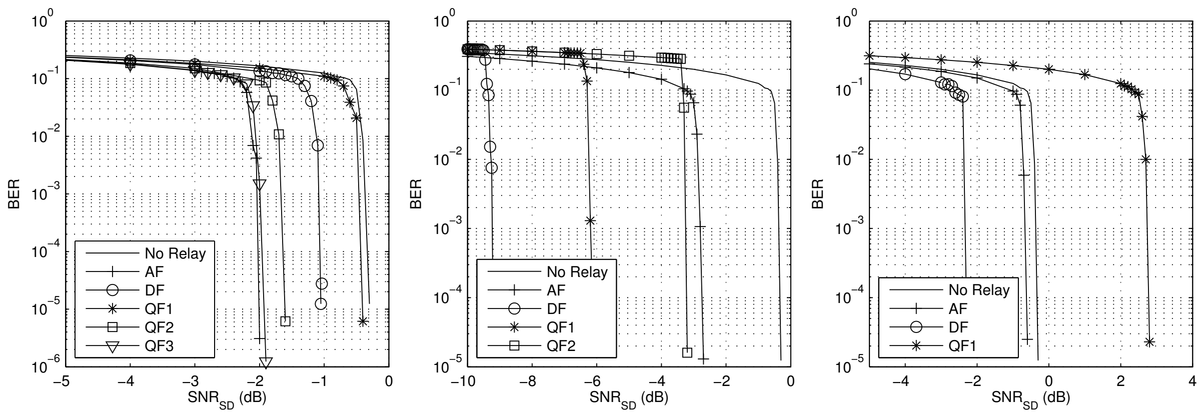
With the results shown in Fig. 3(b), the relay was located at $d = 0.5$. Here, our QF scheme performs about 3.5 dB worse than DF but 3.5 dB better than AF.

Finally, Fig. 3(c) depicts the case with the relay located near the source ($d = 0.1$). Here, the QF scheme fails to improve the BER over the no-relay case. This is due to the fact that the relay-destination link is in outage most of time. We therefore have effectively direct transmission only, but with half the source transmit power of the “no relay” reference case.



(a) (b) (c)

Fig. 2. *Frame error rate versus source-destination SNR in block fading: (a) relay near destination ($d = 0.9$), (b) relay halfway between source and destination ($d = 0.5$), (c) relay near source ($d = 0.1$).*



(a) (b) (c)

Fig. 3. *Bit error rate versus source-destination SNR in fast fading: (a) relay near destination ($d = 0.9$), (b) relay halfway between source and destination ($d = 0.5$), (c) relay near source ($d = 0.1$).*

6. CONCLUSIONS

We presented a transmission scheme for a simple relay channel where the relay i) performs soft-demodulation to obtain log-likelihood ratios (LLRs) for the source code bits and ii) quantizes and forwards these LLRs to the destination. The LLR quantizer was designed to maximize a mutual information criterion. In spite of its reduced computational and hardware complexity, simulations showed that our scheme performs comparably to decode-and-forward (DF) and amplify-and-forward (AF) schemes, except for scenarios where the relay is very close to the source. Another advantage is that (contrary to AF) the relay uses conventional transceiver processing, allowing a node to easily switch between source/destination/relay mode. Possible future research directions include an analysis of the outage behavior and extensions in which unequal power allocations are used or only a subset of LLRs is forwarded. Furthermore, while here we have considered a simple three-terminal network, we believe that our approach offers further gains in larger networks since it nicely integrates with network coding.

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