Herbert Steinrück
Editor

International Centre
for Mechanical Sciences

Asymptotic Methods in Fluid Mechanics: Survey and Recent Advances

CISM Courses and Lectures, vol. 523

INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES

COURSES AND LECTURES - No. 523

ASYMPTOTIC METHODS IN FLUID MECHANICS: SURVEY AND RECENT ADVANCES

EDITED BY

HERBERT STEINRÜCK
VIENNA UNIVERSITY OF TECHNOLOGY, AUSTRIA

SpringerWienNewYork

PREFACE

Rational asymptotic methods developed in the fifties and sixties of the last century have played an important role in theoretical physics, mechanics and in particular in fluid mechanics. Among the most powerful methods used in fluid mechanics are the method of matched asymptotic expansions and multiple scales methods. Matched asymptotic expansions are based on the idea of Prandtl's boundary-layer theory. In case of high Reynolds number flows the flow field can be approximated by an inviscid flow with the exception of a thin boundary-layer along the wall where the viscosity has to be taken into account. Both approximations have to match in an intermediate region. In some cases the inviscid flow and the viscous flow in a sub-layer have to be determined simultaneously. Thus one speaks of interacting boundary-layers. An introduction to triple deck problems and recent applications to internal flows, external sub- or supersonic flows, thermal flows and free surface flows will be presented.

Another fruitful application is the theory of separated laminar incompressible flows. Various examples of fluid flows involving separation will be considered, including self-induced separation of the boundary-layer in supersonic gas flows, and incompressible flow separation at the leading edge of an aerofoil. A characteristic feature of a multiple scales problem is that the solution exhibits almost periodic structures whose properties vary on a large scale. Recently, multiple scales methods have been applied to problems in meteorology. Thus well established ad hoc approximations have been verified by applying the method of multiple scales to the basic equations of fluid flow in the atmosphere. It will be demonstrated how a large collection of well-established models of theoretical meteorology can be recovered systematically, how new insight into scale interaction processes is gained, and how the asymptotic analyses provide hints for the construction of accurate and efficient numerical methods. The known limitations of the approach are also discussed.

Many problems in fluid mechanics involve asymptotic expansions in the form of power series. Such expansions necessarily fail to provide terms which are exponentially smaller than all terms in the series. Although small, these missing terms are often of physical importance. How to find such exponentially small terms, using as the

main tool matched asymptotic expansions in the complex plane and Borel summation will be discussed. The techniques will be developed in the context of model problems related to the theory of weakly nonlocal solitary waves which arise in the study of gravity-capillary waves and also for internal waves.

This volume comprises the lecture notes of a course with the title "Asymptotic Methods in Fluid Mechanics - Survey and Recent Advances" held at the Centre for Mechanical Sciences in Udine, September 21-25, 2009. Also included are contributed papers presented at a workshop embedded in the course.

The organizer of the course thanks all lectures and participants of the workshop for their valuable contributions and their cooperation. My personal thanks are to former rector of CISM Prof. Wilhelm Schneider who suggested this course and for his advice during the preparation. Thanks also to the staff of CISM for the perfect organization and the support in producing these lecture notes.

Herbert Steinrück

CONTENTS