Imperfection Sensitivity or Insensitivity of Zero-stiffness Postbuckling … that is the Question

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Zero-stiffness postbuckling of a structure is characterized by a secondary load-displacement path along which the load remains constant. In sensitivity analysis of the (initial) postbuckling path it is usually considered as a borderline case between imperfection sensitivity and imperfection insensitivity. However, it is unclear whether zero-stiffness postbuckling as such is imperfection sensitive or insensitive. In this paper, Koiter's initial postbuckling analysis is used as a tool for sensitivity analysis. Distinction between two kinds of imperfections is made on the basis of the behavior of the equilibrium path of the imperfect structure. New definitions of imperfection *in*sensitivity of the postbuckling behavior are provided according to the classification of imperfections. A structure with two degrees of freedom with a zero-stiffness postbuckling path is studied, considering four different imperfections. The results from this example show that zero-stiffness postbuckling is a case of transition from imperfection sensitivity to imperfection insensitivity for imperfections of the first kind and that it is imperfection *in*sensitive for imperfections of the second kind.

1 Introduction

In the course of sensitivity analysis of the initial postbuckling behavior of a structure, a special case may occur that is referred to as *zero-stiffness postbuckling* [1]. In this paper the question will be answered whether zero-stiffness postbuckling is imperfection sensitive or imperfection insensitive.

2 Theory

2.1 Koiter's initial postbuckling analysis

Details of this analysis can be found in [2], [3].

2.2 Classification of imperfections

For perfect systems undergoing bifurcation buckling, the imperfections are classified in two categories [4] [5] depending on whether or not the imperfect system has a bifurcation point. With the help of the potential energy function referring to the imperfect structure $V^* = V^*(\boldsymbol{u}, \lambda, \varepsilon)$ where $\varepsilon \in \mathbb{R}$ denotes the imperfection parameter and * refers to variables or functions of the imperfect structure, the imperfection vector is defined as $\boldsymbol{E} = V^*_{,\boldsymbol{u}\varepsilon}\big|_{\boldsymbol{u}=\tilde{\boldsymbol{u}}}$. Classification of the imperfections yields: $\boldsymbol{E}^T \cdot \boldsymbol{v}_1 = 0$ for imperfections of first kind, ε_I; $\boldsymbol{E}^T \cdot \boldsymbol{v}_1 \neq 0$ for imperfections of second kind, ε_{II}.

2.3 Definitions of and criteria for imperfection insensitivity

Imperfections of first kind: **Definition I**: $\varepsilon_I \in [-\varsigma, \varsigma]$, where ς is an arbitrary small positive value. If all imperfect structures in this interval are still stable at the bifurcation point C^*, then the *initial* postbuckling path of the corresponding perfect structure is *imperfection insensitive* with respect to ε_I . **Criterion I**: If, in $\lambda(\eta) = \lambda_C + \lambda_1\eta + \lambda_2\eta^2 + \lambda_3\eta^3 + O(\eta^4)$, $\lambda_{m_{\min}} > 0 \wedge m_{\min}$ is even, where $m_{\min} := \min\{m \,|\, m \in \mathbb{N}\backslash\{0\}, \lambda_m \neq 0\}$, then the *initial* postbuckling path is *imperfection insensitive* with respect to ε_I .

Imperfections of second kind: **Definition II**: $\varepsilon_{II} \in [-\varsigma, 0) \cup (0, \varsigma]$, where ς is an arbitrary small positive value. If no imperfect structure in this interval has a load-displacement path with a snapthrough point $(\boldsymbol{u}_{D*}, \lambda_{D*})$ with $\lambda_{D*} < \lambda_C$, then the *initial* postbuckling path of the corresponding perfect structure is *imperfection insensitive* to ε_{II} . **Criterion II**: See **Definition II**.

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3 Example problem

A planar, static, conservative system with two degrees of freedom shown in Fig. 1 is studied to illustrate the special situation of zero-stiffness postbuckling. Four different imperfections are considered herein, including an imperfection of the stiffness of the top spring, an imperfection of the stiffness of the lateral spring, a shift of the load and a change of the initial angle between two rods. The first two imperfections belong to the first kind, and the last two to the second kind of imperfections. For the structures with imperfections of the first kind, there is no interval $\varepsilon_I \in [-\varsigma, \varsigma]$, in which the equilibrium of imperfect structure is either only stable or only unstable. Hence, for this type of imperfection, zero-stiffness postbuckling represents the case of transition from imperfection sensitivity to imperfection insensitivity. For the structures with imperfections of the second kind, there is $\varepsilon_{II} \in [-0.1, 0.1]$, in which the equilibrium paths of imperfect structures are monotonically increasing. Hence, zero-stiffness postbuckling is imperfection insensitive to this type of imperfections.

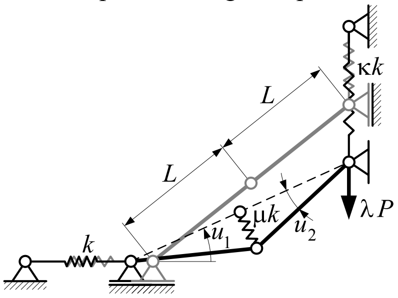

Fig. 1 Two-bar system

A surface $\bar{V}(\boldsymbol{u}) = \left(\boldsymbol{u}, V\left(\bar{\boldsymbol{u}}, \lambda(\bar{\boldsymbol{u}})\right)\right)\ \forall\ \boldsymbol{u} \in \mathbb{R}^2$ is calculated and plotted in Fig. 2. Its intersection with the horizontal plane $V_C = \left(\boldsymbol{u}, V(\boldsymbol{u}_C)\right)\ \forall\ \boldsymbol{u} \in \mathbb{R}^2$ is the closed curve $\gamma(\eta) = \left(\bar{\boldsymbol{u}}(\eta), V\left(\bar{\boldsymbol{u}}(\eta), \lambda\left(\bar{\boldsymbol{u}}(\eta)\right)\right)\right)\ \forall\ \eta \in \mathbb{R}$ which represents the potential energy along the zero-stiffness path. In an infinitesimal neighborhood of $\gamma(\eta)$, $\bar{V}(\boldsymbol{u})$ coincides (apart from terms that are of higher order small) with the potential-energy surface $V(\boldsymbol{u}, \lambda)$. In the infinitesimal neighborhood of an arbitrary point on $\gamma(\eta)$, $V_{,\boldsymbol{uu}} \geq 0$, where the equals sign holds for $\gamma(\eta)$. Consequently, the zero-stiffness postbuckling path is stable.

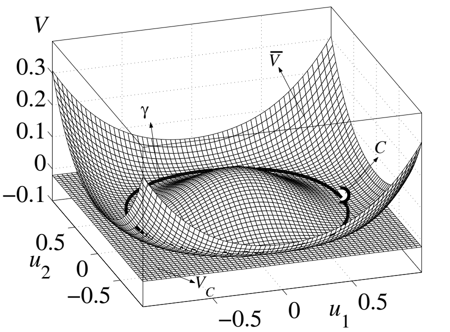

Fig. 2 Surface $\bar{V}(\boldsymbol{u})$ containing the curve $\gamma(\eta)$ which represents the zero-stiffness postbuckling mode

4 Conclusions

From the theoretical investigation and the results of the examples it follows that zero-stiffness postbuckling

- represents a case of transition from imperfection sensitivity to insensitivity for imperfections of first kind;
- is characterized by a stable postbuckling equilibrium path with constant potential energy and, hence, is imperfection insensitive to imperfections of second kind.

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