Turbulent bluff-body separation: recent advances of a self-consistent flow description for large Reynolds numbers

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Seminar lecture

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The research presented has been carried out in collaboration with

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Institute of Fluid Mechanics and Heat Transfer

Prof Frank T. Smith, FRS
Department of Mathematics

Their contributions are greatly acknowledged.
“Turbulent boundary-layer separation is normally listed as one of the most important unsolved problems in fluid mechanics...”
Overview

1. Turbulent wall-bounded flows
   Numerically-based methods
   Analytically-based methods

2. Classical theory of turbulent small-defect BLs
   General framework
   Routes to separation

3. Problem formulation – motivation
   Global flow picture
   Separation – experimental findings

4. Asymptotic structure of the flow
   ‘Ideal-fluid limit’ – Kirchhoff-type potential flow
   Boundary layer

5. Incident boundary layer – numerical results

6. Preliminary conclusions – outlook
Wall-bounded high-Reynolds-number flows: \( \text{Re} := \tilde{U}\tilde{L}/\tilde{v} \gg 1 \)

Navier–Stokes eqs \((\rho = \text{const})\)

\[
\nabla \cdot \mathbf{u} = 0, \quad \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u}, \quad \mathbf{u}|_{y=0} = 0
\]

Reynolds-averaging, ergod hypothesis (nominally steady 2D flow)

\[
Q(x, t; \text{Re}) = \langle Q \rangle(x, y; \text{Re}) + Q'(x, t; \text{Re})
\]

\[
\langle Q \rangle := \lim_{t_{av} \to \infty} \frac{1}{t_{av}} \int_{-t_{av}/2}^{t_{av}/2} Q(x, t + t'; \text{Re}) \, dt'
\]
Wall-bounded high-Reynolds-number flows: \( \text{Re} := \frac{\tilde{U} \tilde{L}}{\tilde{\nu}} \gg 1 \)

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Numerically-based methods

- **DNS**: resolution of all scales $\Rightarrow$ no modelling needed but at present restricted to moderately large values of $Re$
- **LES**: modelling of short scales $\Rightarrow$ reduction of computational efforts, which, however, are still too massive to be useful for the solution of engineering problems
- **RANS**: modelling of all scales $\Rightarrow$ engineering problems can be solved with an acceptable amount of computational efforts, which, however, increase with increasing values of $Re$
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Analytically-based methods

- full NS eqs: starting efforts, but still no complete theory exists

- non-dimensional Reynolds-averaged NS eqs
  (nominally 2D, curvature effects on BL flow of higher order):

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\frac{u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} - \frac{\partial \langle u'^2 \rangle}{\partial x} - \frac{\partial \langle u'v' \rangle}{\partial y} + \frac{1}{Re} \nabla^2 u \\
\frac{u}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial y} - \frac{\partial \langle u'v' \rangle}{\partial x} - \frac{\partial \langle v'^2 \rangle}{\partial y} + \frac{1}{Re} \nabla^2 v \\
y = 0: \quad u = v = u' = v' = 0, \quad y \sim \delta(x; Re): \quad u \sim u_e(x), \quad \tau \sim 0
\end{align*}
\]

\[\Rightarrow \text{asymptotic theory faces closure problem for } \tau\]
Analytically-based methods

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  Scheichl & Kluwick (JFS, 2008)

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⇒ asymptotic theory faces closure problem for $\tau$
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Classical theory of turbulent small-defect BLs: \[ u_\tau := \sqrt{\tau_w} \to 0 \]


\[ \tau_w = Re^{-1} (\partial u / \partial y)_{y=0} \]

\[ u / u_\tau \sim 1 / \kappa \ln y^+ + C_i \]

\{ outer predominately inviscid region \}

\{ overlap region: \[ y^+ := y u_\tau Re \to \infty \] \}

assumptions


(a) locally isotropic turbulence \[ \Rightarrow \langle u'^2 \rangle, \langle u' v' \rangle, \langle v'^2 \rangle \] of same magnitude

(b) wall layer: total shear stress essentially unaffected by pressure gradient

(c) direct match with outer fully turbulent region \[ \Rightarrow \] two-tiered BL
Classical theory of turbulent small-defect BLs:  \( u_\tau := \sqrt{\tau_w} \rightarrow 0 \)


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O(u_\tau) \begin{cases} \text{outer predominately inviscid region} \\ \text{overlap region: } y^+ := y u_\tau Re \to \infty \end{cases}
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Classical theory of turbulent small-defect BLs: \( u_\tau := \sqrt{\tau_w} \rightarrow 0 \)


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y = \begin{cases}
  \text{outer predominately inviscid region} & \text{overlap region: } y^+ := y u_\tau Re \rightarrow \infty \\
  \text{viscous wall layer} & \frac{u}{u_\tau} \sim \frac{1}{\kappa} \ln y^+ + C_i
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Distinguished limit \( \gamma := u_\tau/u_e \to 0, \ Re \to \infty \)

viscous wall layer : \( y^+ = y u_\tau Re \)
\[
\frac{u}{u_e} \sim \gamma(x, Re) u^+(x, y^+) + \cdots \\
\frac{\tau}{u_e^2} \sim \gamma^2(x, Re) t^+(x, y^+) + \cdots \\
p \sim p_0(x) + \cdots
\]

outer defect layer : \( \eta = y/\delta \)
\[
\frac{u}{u_e} \sim 1 - \gamma \frac{\partial F_1(x, \eta)}{\partial \eta} + O(\gamma^2) \\
\frac{\tau}{u_e^2} \sim \gamma^2 T_1(x, \eta) + O(\gamma^3) \\
p \sim p_e(x) + O(\gamma^2)
\]

experimental observation / result from first principles (?)


\[
u^+(y^+) \sim \kappa^{-1} \ln y^+ + C_i, \quad y^+ \to \infty, \quad \kappa \approx 0.384, \quad C_i \approx 4.1
\]

provides expansion in outer layer and matching with wall layer

\( \eta \to 0 : \quad \partial F_1/\partial \eta \sim -\kappa^{-1} \ln \eta + C_o(x), \quad T_1 \to 1; \quad p_0(x) = p_e(x) \)

skin-friction law \( \kappa/\gamma \sim \ln(Re \gamma \delta u_e) + \kappa(C_i + C_o) \sim \ln Re \)
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Substitution into Reynolds-averaged NS eqs

viscous wall layer: \( du^+/dy^+ + t^+ = 1 \) \ldots constant total shear stress

outer defect layer: \( \delta \sim \gamma \Delta_1(x) + O(\gamma^2) \)

leading-order equation

\[
(E + 2\beta_0)\eta F_1' - EF_1 - \Delta_1 F_{1,e} \partial F_1 / \partial x = F_{1,e} (T_1 - 1), \quad F_{1,e} = F_1(x, 1)
\]

\[
E := 1 - \Delta_1 dF_{1,e} / dx, \quad \beta_0 := - (\Delta_1 F_{1,e} du_e / dx) / u_e
\]

layer thickness ratio (Kármán number) \( \delta^+ \) exponentially small

\[
\delta^+ = \left( \frac{u_\tau Re}{\delta} \right)^{-1} \sim \frac{1}{Re \gamma^2 \Delta_1 u_e} \sim \frac{1}{\Delta_1 u_e} \exp(-\kappa/\gamma)
\]

consequences

classical theory not capable of describing BL separation - Sykes (JFM, 1980)

"manipulate" velocity defect \( 1 - u/u_e = O(\gamma) \), BL thickness \( \Delta_1 \)
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Routes to separation

\[ \frac{\delta^+}{Re} = \frac{1}{\delta^2} \]

\[ \delta^+ = \frac{1}{Re \delta^2} \]

\[ \eta \]

\[ \delta^+ = (\delta^+ Re)^{-1} = O(\delta^+) \Rightarrow \text{locally } \frac{\partial p}{\partial x} \gg 1 \Rightarrow \text{massive separation} \]

\[ \delta^+ = \frac{1}{Re \delta^2} \Rightarrow \text{marginal separation} \]

\[ O(\epsilon) \]

\[ \text{log law} \]

\[ \text{defect layer} \]

\[ \text{viscous wall layer} \]

consider BLs having…

\[ \epsilon \gg \gamma \Rightarrow \frac{\partial p}{\partial x} = O(1) \Rightarrow \text{marginal separation} \]

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\[ \{ \text{defect layer} \} \]

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Scheichl & Kluwick (AIAA J, 2007)

Scheichl, Kluwick & Smith
Routes to separation

\[ \eta \]

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\[ \log \text{law} \]

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Problem formulation – motivation
Local description of turbulent break-away separation . . .

. . . requires answers to three basic questions:

(1) Global topology of flow for ‘vanishing viscosity’?
(2) Characteristics of incident boundary layer flow?
(3) How do these issues interdepend?

How turbulent are the BL and the separated SL ?
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basic assumptions

- flow incompressible, nominally steady, 2D
- free-stream turbulence disregarded

Re := \tilde{U} \tilde{L} / \tilde{\nu} \to \infty

canonical example: circular-cylinder flow

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Global flow picture

\( Re \gtrapprox 3 \times 10^6 \): postcritical regime, \( Re \to \infty \): transition point approximates \( F \)

\( Re^{-1} = 0 \): ultimate or \( T \)-state of flow, transition in \( F \)

\( u_\infty = 1 \)

transition

Neish & Smith (JFM, 1992)

unlikely Scheichl & Kluwick (JFS, 2008)

corroborated experimentally up to \( Re \approx 4 \times 10^7 \)

Global flow picture

\( Re \gtrsim 3 \times 10^6 : \) *postcritical* regime, \( Re \rightarrow \infty : \) transition point approximates \( F \)

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\[ \theta_S \approx 115^\circ \]

\( u_S \lesssim u_\infty \)

\( p_S \lesssim p_\infty \)

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$R \sim S$?

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Separation

Is there really a fully developed TBL as $Re \to \infty$?

Oil-flow measurements:

$$Re_C := \tilde{U}\tilde{C}/\tilde{\nu}, \quad \tilde{C} = \text{airfoil chord length}$$

by courtesy of G. Schewe (Göttingen, 2001)
Separation

Is there really a fully developed TBL as $Re \to \infty$?

Oil-flow measurements: $Re_C := \bar{U} \bar{C} / \bar{v}$, $\bar{C}$ = airfoil chord length

$Re_C \approx 6 \times \bar{C}$

$Re_C = 7.4 \times 10^5$: $c_D \approx 0.11$, $c_L \approx 1.1$

$Re_C = 7.7 \times 10^6$: $c_D \approx 0.14$, $c_L \approx 0.65$

By courtesy of G. Schewe (Göttingen, 2001)
Prandtl (Göttingen, 1914)
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‘Ideal-fluid limit’

Hierarchy of external flow – shear layer: potential flow approached as $Re \to \infty$

Oseen (1927), Lamb (1932), Batchelor (1956), Prandtl (1961), Sychev et al. (1998)

$Re^{-1} = 0$: external potential flows

Imai (J Phys Soc Japan, 1953),
Birkhoff & Zarantonello (1957), Gurevich (1966)

- free streamlines confine (open) dead-water region
- class of flows / $\theta_S$ controlled by
  Brillouin–Villat parameter $k \geq 0$, free-stream velocity $u_S \leq u_\infty$

\[
\begin{align*}
  u_e & \sim b(k)\theta + O(\theta^2), & \theta \to 0^+ \\
  & \sim u_S[1 + 2k(-s)^{1/2} + O(-s)], & s \to 0^- \\
  & \equiv u_S, & s \geq 0 \\
  \kappa & = \kappa_B(s), & s < 0 \\
  & \sim -k s^{-1/2} + \kappa_B(0) + O(s^{1/2}), & s \to 0^+
\end{align*}
\]
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Kirchhoff-type flows – numerical solutions

Open cavity: \( u_S \equiv u_\infty = 1 \)

surface velocity \( u_e(\theta, k) \)

\[ k = 0, 0.05, 0.1, \ldots, 0.5 \]

\[ 55^\circ 2' 30'' \leq \theta_S \lesssim 126^\circ \]

\( k = 0: \)

Brillouin–Villat condition

\[ 126^\circ \lesssim \theta_S \leq 180^\circ \]

\[ 0.5 \lesssim k < \infty \]

\[ 1 \geq u_S \geq 0 \]
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Question: overall flow structure as $Re \to \infty$

(1) Global topology of flow for ‘vanishing viscosity’?
(2) Characteristics of incident boundary layer flow?
(3) How do these issues interdepend?
Question: overall flow structure as $Re \to \infty$

1. **Global** topology of flow for ‘vanishing viscosity’?
2. Characteristics of **incident** boundary layer flow?
3. How do these issues **interdepend**?
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Asymptotic framework – turbulence intensity level


Prandtl-type BL: \( k, T, \text{Re} \gg 1 \)

\[
N = \text{Re}^{1/2} n
\]

\[
\left\{ \text{Re}^{1/2} \left[ \psi, -\langle u'v' \rangle \right], p \right\} \sim \left\{ \left[ \psi, Tr \right](\theta, N; k, T), p_F - u_e^2(\theta; k)/2 \right\} + O(\text{Re}^{-1/2})
\]

\[
\partial_N \psi \partial_{N\theta} \psi - \partial_\theta \psi \partial_{NN} \psi = u_e \partial_\theta u_e + \partial_N [Tr + \partial_{NN} \psi], \quad 0 \leq T < \infty
\]

\[
N \to 0: \quad \psi \to 0, \quad \psi_N \to 0, \quad r = O(N^3), \quad N \to \infty: \quad \psi_N \to u_e, \quad r \to 0
\]

solution as \( s = \theta - \theta_S \to 0 \_ \_ \_ \)

\( T \gg 1: \)

Goldstein singularity at

\[
s = s_G = O[k^6 T^{-8} (\ln T)^{16}]
\]

\( \Rightarrow \) interaction length scale \( \delta_{TD} \)
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\partial_N \Psi \partial_N \theta \psi - \partial_\theta \psi \partial_N \Psi = u_e \partial_\theta u_e + \partial_N [Tr + \partial_N \Psi], \quad 0 \leq T < \infty
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Concept of ‘underdeveloped’ TBL
BL originating in $F$ parametrised by $T \gg 1$

small defect: \[ u_e - u = O(\epsilon), \quad r = O(\epsilon^2) \]

BL eq: \[ -\langle u'v' \rangle = Re^{-1/2}Tr = O(\sigma\epsilon), \quad \delta = O(\sigma) \]

\[
\begin{aligned}
\left\{ \begin{array}{l}
\frac{u_en - \psi}{u_e \delta\epsilon}, \\ -\frac{\langle u'v' \rangle}{u_e^2 \delta\epsilon}, \\ \frac{\delta}{\sigma}
\end{array} \right\} & \sim \left\{ \begin{array}{l}
[F, \Sigma](\theta, \eta; k), \\ \Delta(\theta; k)
\end{array} \right\} + O(\epsilon), \\
\eta = \frac{n}{\delta}
\end{aligned}
\]

fully developed TBL: \[ -\langle u'v' \rangle = O(\epsilon^2), \quad \sigma \propto \epsilon \sim \kappa / \ln Re \]
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$\Rightarrow T = Re^{1/2}\sigma/\epsilon$

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small-defect layer: $\sigma \ll 1, \quad \epsilon = \kappa/(\sigma^2 \ln Re) \ll 1$

\[
\left\{\begin{array}{l}
\frac{u_e n - \psi}{u_e \delta \epsilon}, \quad -\langle u'v' \rangle, \quad \frac{\delta}{\sigma}
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\[
\left\{ \begin{array}{c}
    u_e n - \psi u_e \\
    -\langle u' v' \rangle \\
    u_e^2 \delta \epsilon \\
    u_e^2 \delta \epsilon
\end{array} \right\}, \quad \frac{\delta}{\sigma} \sim \left\{ [F, \Sigma](\theta, \eta; k), \quad \Delta(\theta; k) \right\} + O(\epsilon), \quad \eta = \frac{n}{\delta}
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Solutions of outer-defect eqs for $0 \leq \theta < \theta_S \doteq 113.5^\circ$ \quad $k = 0.45$

\begin{equation}
 u_e^2 \partial_\theta (u_e \Delta) \eta F' - \partial_\theta (u_e^3 \Delta F) = u_e^3 (\Sigma - 1), \quad \Sigma = |^2 F''| F''|,
 F(\theta, \eta; k), \quad \Delta(\theta; k)
\end{equation}

\begin{align*}
 \eta \to 0 : & \quad F' \sim -\kappa^{-1} \ln \eta + B(\theta) \iff \Sigma \sim 1, \quad \eta = 1 : \quad F' = F'' = \Sigma = 0 \\
 \theta \to 0 : & \quad F \sim F_0(\eta), \quad \Delta \sim \Delta_0 \theta, \quad u_e \sim b(k) \theta \quad \text{stagnant-flow}
\end{align*}

- algebraic closure for $\ell = \delta l$, Klebanoff’s intermittency factor
  Michel, Quémard & Durant (1969), Klebanoff (1955)

- Keller–Box scheme / method of lines, adaptive grid remeshing
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$\theta \to 0 : F \sim F_0(\eta), \quad \Delta \sim \Delta_0 \theta, \quad u_e \sim b(k) \theta$ stagnant-flow

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Solutions of outer-defect eqs, cont’d

Comparison with leading-order theory as \( s = \theta - \theta_S \to 0 \)_

\[
[F, \Delta] \sim [F_S(\eta; k), \Delta_S(k)] - \sum_{i=1}^{\infty} [f_i(s, k)F_i(\eta; k), g_i(s, k)\Delta_i(k)]
\]

\[
[F, \Delta] \sim [F_S(1 - 2g_1), \Delta_S(1 - g_1)] + O(s), \quad g_1 = 2k(-s)^{1/2}
\]

\( F'_i = O[\ln(\eta)], \quad \eta \to 0, \quad i = 1, 2, \ldots \) \Rightarrow \text{sublayer for } \eta = O(-s)

indicates onset of viscous/inviscid interaction
Solutions of outer-defect eqs, cont’d

Comparison with leading-order theory as \( s = \theta - \theta S \to 0_- \)

\[
\frac{k}{0.45}
\]

\[
\Delta \sim \Delta_0 \theta
\]

\[
c_p = 1 - u_e^2
\]

\[
F, \Delta \sim \left[ F_S(\eta; k), \Delta_S(k) \right] - \sum_{i=1}^{\infty} \left[ f_i(s, k)F_i(\eta; k), g_i(s, k)\Delta_i(k) \right]
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status quo of research

- increasing turbulence intensity shifts separation downstream, increases $k$
- BL never attains fully developed turbulent state

interactive BL $\Rightarrow$ scaling in terms of $k$, $Re \gg 1$

$$
\epsilon \sim \kappa / \ln Re, \quad \delta_{TD} = k^{8/9} Re^{-4/9}, \quad \delta = O(\delta_{TD}/\epsilon), \quad -\langle u'_i u'_j \rangle = O(\epsilon \delta)
$$

ongoing research

- TD problem (akin to laminar counterpart)
- self-induced separation $\Leftrightarrow$ (maximum) values of $k$, $\theta_S(k)$?
- structure of (large-scale) separated flow
status quo of research

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Thank you for your attention!