Low Complexity Approximate Maximum Throughput Scheduling for LTE

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***Abstract*—In this paper we address the challenge of multiuser scheduling in the downlink of 3GPP UMTS/LTE. Long Term Evolution (LTE) imposes the constraint of using the same code rate, modulation order and transmit power for all resources a User Equipment (UE) is scheduled onto. This, in addition to the lack of channel knowledge, prohibits theoretical concepts such as capacity maximization to be applied for resource allocation. Based on the Channel Quality Indicator (CQI) feedback we derive a linearized model for multiuser scheduling. In contrast to other proposals we use Mutual Information Effective SNR Mapping (MIESM) to calculate an average CQI value for all UE resources. This enables a rate increase while still guaranteeing an imposed Block Error Ratio (BLER) constraint. The proposed framework can also be applied to implement other scheduling strategies. This is demonstrated by comparing different standard schedulers in terms of achieved throughput and fairness.**

***Index Terms*—LTE, OFDMA, multiuser scheduling, adaptive resource allocation, linear programming**

I. Introduction

Orthogonal Frequency Division Multiple Access has been adopted in emerging broadband wireless access networks such as 3GPP UMTS/LTE [1] and IEEE 802.16x (WiMAX) [2] due to its inherent immunity to intersymbol interference and scheduling flexibility in resource allocation. This flexibility allows the exploitation of frequency, temporal and multiuser diversity offered by the wireless broadcast channel. By employing sophisticated multiuser scheduling algorithms, which transmit data to different users on favourable resources, a high system capacity can be achieved.

In our work we focus on the downlink of Long Term Evolution (LTE). In this system the time-frequency grid (resource grid [3]) spanned by OFDM is divided into several Resource Blocks (RBs). User Equipment (UE) resource allocation is carried out on a Resource Block (RB) or a subband basis (each subband consists of several contiguous RBs). Sophisticated scheduling requires the UEs to feed back for each RB (or subband) the supported Channel Quality Indicator (CQI) value, corresponding to a specific code rate - modulation order combination [4]. The standard specifies different feedback granularity possibilities, ranging from wideband (just a single CQI value for all considered RBs) to best M feedback (a distinct CQI value for the M best RBs). For our simulations we consider distinct CQI values for every RB. In our work in [5] and [6] we proposed a spatial preprocessing and link adaption (CQI) feedback scheme based on Mutual Information Effective SNR Mapping (MIESM). MIESM is a well know technique from link level abstraction [7], [8]. It allows to map the SINR experienced on several resources to an equivalent AWGN channel SNR. In [5] and [6] we have shown that MIESM-based wideband feedback achieves close to optimal performance in terms of throughput while fulfilling an imposed constraint on a maximum allowed Block Error Ratio (BLER). In this paper we use the same feedback strategy, but on an RB basis. The required MIESM averaging is carried out at the scheduler after resource allocation.

In this paper we first formulate the sum rate maximization resource allocation problem in Section II based on MIESM to calculate the supported CQI value. This leads to a nonlinear binary integer program. We then use a linear approximation in Section III to come up with a linearized model, which allows to solve the resource allocation problem very efficiently with a Linear Program (LP). We show that this LP is actually equivalent to a best CQI scheduler. Next we give simulation results in Section IV that compare our proposed sum rate maximizing scheduler with the one proposed in [9]. Afterwards in Section V we show that the linearized model is directly applicable to other scheduling strategies as well. In Section VI we compare the performance of different scheduling strategies in terms of throughput and fairness. Concluding remarks will be provided in Section VII.

II. Sum Rate Maximizing Scheduler

Consider the downlink of an OFDM single antenna (SISO) multiuser LTE system. Let N be the number of available RBs and K the number of users. Every user k feeds back a CQI vector $\mathbf{CQI}_k \in \{1, \ldots, \mathrm{CQI}^{(\max)}\}^{N\times 1}$ containing supported CQI values for the N RBs (in LTE $\mathrm{CQI}^{(\max)} = 15$).

These CQIs correspond to supported modulation order - code rate combinations (given in Table 7.2.3-1 in [4]) for the individual RBs. If a UE k is served on several RBs it is necessary to find an average supported CQI value $\overline{\mathrm{CQI}}_k$. We achieve this by first mapping the CQI values of the considered RBs to corresponding SNR values. Next we compute an equivalent AWGN channel SNR value $\mathrm{SNR}_{\mathrm{eq},k}$ by applying MIESM and from this value we arrive at $\overline{\mathrm{CQI}}_k$.

In [10] the author shows that, assuming a BLER target of ≤ 0.1, the mapping function from SNR to CQI is linear. Therefore $\mathbf{CQI}_k$ is linearly related to a corresponding quantized

SNR vector $\mathbf{SNR}_k^{[\mathrm{dB}]} \in \{\mathrm{SNR}^{(1)}, \ldots, \mathrm{SNR}^{(\max)}\}^{N\times 1}$

$$\mathbf{SNR}_k^{[\mathrm{dB}]} = s_1 \mathbf{CQI}_k + s_2 \mathbf{1}, \tag{1}$$

where s_1 and s_2 are coefficients obtained from the linear mapping function and $\mathrm{SNR}^{(i)}$ is the quantized SNR value corresponding to $\mathrm{CQI}^{(i)}$. The notation $(.)^{[\mathrm{dB}]}$ indicates that the value is given in dB.

The goal of a sum rate maximizing scheduler is to allocate resources such that the sum of the user throughputs is maximized. Let $\mathbf{b}_k \in \{0,1\}^{N\times 1}$ be a binary vector indicating which RBs are allocated to user k

$$\mathbf{b}_k(n) = 1 \Longleftrightarrow \text{RB } n \text{ is allocated to user } k, \tag{2}$$

with $\mathbf{b}_k(n)$ corresponding to the nth value of the vector $\mathbf{b}_k$. In a SISO system an RB can only be used by a single UE, therefore RB allocations must not overlap

$$\mathbf{b}_j^T \cdot \mathbf{b}_i = 0 \quad \forall i \neq j. \tag{3}$$

Note that this assumption need not be true in a multiuser MIMO system, because different spatial layers may be allocated to different users on overlapping resources. The supported CQI value of user k, $\overline{\mathrm{CQI}}_k$, is computed by averaging $\mathbf{SNR}_k$ with the help of MIESM to obtain an equivalent AWGN channel SNR, $\mathrm{SNR}_{\mathrm{eq},k}$. This SNR is then mapped back to the CQI domain via the inverse of the linear mapping function. Because the CQI value must be an integer in the range $1, \ldots, \mathrm{CQI}^{(\max)}$ the result has to be rounded down:

$$\mathrm{SNR}_{\mathrm{eq},k} = \beta f^{-1}\left(\frac{1}{||\mathbf{b}_k||_1}\sum_{n=1}^{N} f\left(\frac{\mathbf{SNR}_k(n)\mathbf{b}_k(n)}{\beta}\right)\right) \tag{4}$$

$$\overline{\mathrm{CQI}}_k = \left\lfloor \frac{\mathrm{SNR}_{\mathrm{eq},k}^{[\mathrm{dB}]} - s_2}{s_1} \right\rfloor \tag{5}$$

The function f is given by the Bit Interleaved Coded Modulation (BICM) capacity [11]. The variable β is a calibration factor used to adjust the mapping to the different code rates and modulation alphabets. Therefore it depends on the CQI value. Theoretically it would be necessary to repeat the averaging for all different β values, but the calibration has shown that β is always close to one and therefore set equal to one for simplicity. The whole nonlinear averaging and mapping procedure of eqs. (1), (4) and (5) is condensed in the function R, which yields the spectral efficiency (in bits per channel use) by mapping the average CQI value $\overline{\mathrm{CQI}}_k$ to its corresponding spectral efficiency. The throughput of user k in bits/s, T_k, therefore equals

$$\mathrm{T}_k = c \cdot R(\mathbf{CQI}_k, \mathbf{b}_k) \cdot ||\mathbf{b}_k||_1, \tag{6}$$

where c is a constant that transforms from bits/channel use to bits/s.

Finally, the sum rate maximization problem can be formulated:

$$\{\mathbf{b}_1^\star, \ldots, \mathbf{b}_K^\star\} = \underset{\{\mathbf{b}_1,\ldots,\mathbf{b}_K\}}{\operatorname{argmax}} \sum_{k=1}^{K} \mathrm{T}_k \tag{7}$$
$$\text{subject to:}$$
$$\mathbf{b}_j^T \cdot \mathbf{b}_i = 0 \quad \forall i \neq j$$
$$\mathbf{b}_k(n) \in \{0,1\} \quad \forall n, k.$$

This is a highly nonlinear binary integer program, for which no efficient solution exists. It cannot be implemented in realtime because scheduling decisions must be carried out in every subframe, that is, every 1 ms. Therefore it is necessary to further simplify the model which is achieved in the following section.

III. Linearized Model

In LTE, the CQI feedback from a single UE can only span up to four values, as only 2 bits of feedback are allowed per resource [4]. These 2 bits per RB signal an offset value to an average CQI value, which is also fed back. This constraint can be utilized to simplify the nonlinear optimization problem by linearly approximating the BICM function f necessary for MIESM Eq. (4). Figure 1 shows linear MMSE fits of the envelope of the BICM curves. Normally the BICM functions are given over SNR, but due to the linear mapping between SNR and CQI, they are here directly given over CQI. The

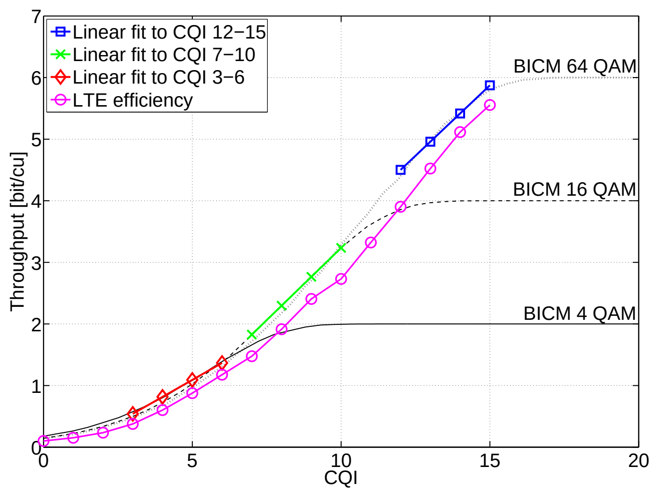

Fig. 1. Linear MMSE fits to the envelope of the BICM functions.

figure shows that for a range of four CQI values linear approximations are reasonable. By applying this approximation, SNR averaging boils down to computing the arithmetic mean. Still, to calculate the supported CQI value $\overline{\mathrm{CQI}}_k$ according to (5), a nonlinear operation (rounding) is necessary. In order to achieve linearity, this operation is ignored for the resource allocation process and noninteger CQI values are allowed. Note that these approximations are just applied during resource allocation. As soon as this process is completed, MIESM is used for every

UE and its RBs to come up with an integer supported CQI value.

One possibility to compute the efficiency corresponding to a noninteger CQI value is to linearly interpolate the efficiencies corresponding to integer CQI values defined in the LTE standard [4] (cf. the magenta circle marked line in Fig. 1). Another possibility is using the theoretical BICM functions. We consider BICM for this purpose (as both are almost parallel, simulation results differed only marginally using either of the two). In order to avoid nonlinearities it is necessary to make use of the linear fits again. Which linear fit has to be applied depends on the actual value of $\overline{\mathrm{CQI}}_k$. Since $\overline{\mathrm{CQI}}_k$ has to be in the range $[\min(\mathbf{CQI}_k), \ldots, \max(\mathbf{CQI}_k)]$ the appropriate fit can be chosen in advance.

Using the above assumptions, the rate of user k, $\overline{\mathrm{R}}_k$, in bits per channel use becomes

$$\overline{\mathrm{R}}_k = d_k \frac{\mathbf{CQI}_k^T \mathbf{b}_k}{||\mathbf{b}_k||_1} + e_k, \tag{8}$$

where d_k and e_k are the coefficients from the linear fit. The throughput equals

$$\begin{aligned} \mathrm{T}_k &= c \cdot \overline{\mathrm{R}}_k ||\mathbf{b}_k||_1 = \\ &= c \cdot d_k \mathbf{CQI}_k^T \mathbf{b}_k + c \cdot \underbrace{e_k ||\mathbf{b}_k||_1}_{\mathbf{e}_k^T \mathbf{b}_k} = \\ &= (\underbrace{c \cdot d_k \mathbf{CQI}_k^T + c \cdot \mathbf{e}_k^T}_{\mathbf{c}_k^T}) \mathbf{b}_k. \end{aligned} \tag{9}$$

The step from the second to the third line is possible as $\mathbf{b}_k$ is binary and $\mathbf{e}_k = e_k \mathbf{1}$. In (9) the user throughput finally is linear in the RB allocation $\mathbf{b}_k$.

For convenience let us introduce the following vector notation

$$\mathbf{b} = \mathrm{vec}\left(\begin{bmatrix} \mathbf{b}_1^T \\ \mathbf{b}_2^T \\ \vdots \\ \mathbf{b}_K^T \end{bmatrix} \right) \in \{0,1\}^{N \cdot K \times 1} \tag{10}$$

$$\mathbf{c} = \mathrm{vec}\left(\begin{bmatrix} \mathbf{c}_1^T \\ \mathbf{c}_2^T \\ \vdots \\ \mathbf{c}_K^T \end{bmatrix} \right) \in \mathbb{R}^{N \cdot K \times 1}. \tag{11}$$

$\mathbf{b}$ contains the RB allocation for all UEs in the form that the first K rows correspond to the first RB of users $1, \ldots, K$, the next K rows to the second RB of all users and so on. Similarly the vector $\mathbf{c}$ contains the corresponding rates.

With this notation the sum rate maximization problem can be written as following binary linear program

$$\begin{aligned} \mathbf{b}^\star = \underset{\mathbf{b}}{\mathrm{argmax}}\, &\mathbf{c}^T \mathbf{b} \\ \text{subject to:}& \\ \mathbf{A} \cdot \mathbf{b} &\leq \mathbf{1}_N \\ \mathbf{b}(n) \in \{0,1\}& \;\; \forall n. \end{aligned} \tag{12}$$

The matrix $\mathbf{A} \in \{0,1\}^{N \times KN}$ ensures that every RB is used at most by a single UE:

$$\mathbf{A} = \overbrace{\begin{bmatrix} \overbrace{11\ldots1}^{K} & 0\ldots & & 0 \\ 0\ldots & 11\ldots1 & 0\ldots & 0 \\ \vdots & & \ddots & \vdots \\ & & & \\ 0\ldots & & & 11\ldots1 \end{bmatrix}}^{K \cdot N} \tag{13}$$

Problem (12) can be solved efficiently by a binary solving method such as branch and bound [12].

Investigating the matrix $\mathbf{A}$ shows that it is not even necessary to solve a binary program. Under certain conditions it is possible to apply integer relaxation to a binary linear program without loss of optimality and ensurance that the solution is integer valued. This is possible whenever the constraint matrix $\mathbf{A}$ is totally unimodular[1] and the right hand side of the constraints is integer valued [12]. It is easily verified that these conditions are fulfilled for the given constraint matrix. Therefore the problem can be solved as an LP (e.g. with the simplex method) with additional constraints $0 \leq \mathbf{b}(n) \leq 1 \;\; \forall n$.

The solution can also be obtained in a different way by examining the structure of the problem. Each RB can at most be assigned to a single UE. Due to the linearity of the problem each RB will be assigned. No RB will be left out, although this might be the case if the problem was nonlinear; that is, the CQIs were allowed to vary over a larger range. As a consequence the UE with the largest corresponding value in $\mathbf{c}_k$ will obtain the RB revealing the scheduler identical to a best CQI scheduler that schedules the UE with the highest CQI value. This intuition is proven analytically in Appendix A. We conclude that the best CQI scheduler is sum rate maximizing for LTE under the given approximations.

The presented approach may also be applied in multiuser scenarios with many UEs even if the CQIs are not guaranteed to lie in a range of four values. In these cases multiuser diversity will enforce that UEs are scheduled on their best RBs, automatically leading to low CQI variation (see Section VI).

IV. Simulation Results for two Users

In this section we compare our proposed scheduler (Approximate Max. Throughput (AMT)), based on the LP relaxation, with the Best CQI (BCQI) scheduler and a sum rate maximizing scheduler proposed by R. Kwan et.al. in [9] (Kwan Max. Throughput (KMT)). In [9] the authors are tackling the challenge of resource allocation under the conditions given by the LTE standard (limited channel knowledge, single CQI value per UE). In order to solve the optimization problem, they assume that a UE can only support the lowest CQI value of all

[1] every square non-singular submatrix is unimodular; that is, it has determinant ± 1 and integer entries

RBs it is assigned to. This will be shown to entail a rate loss compared to our solution although the imposed constraint on a maximum allowed BLER is fulfilled by both approaches. In [13] the authors also deal with the problem of scheduling under the constraint of a single adaptive modulation and coding scheme per user, but in a more abstract way. This fact in addition to the complex structure of the solution prevents it from direct application in a realtime system.

In order to compare different schedulers we use a standard compliant LTE physical layer simulator [14] that is publicly available [15]. We implement the slightly suboptimal ($\sim 5\%$ rate loss) scheduling strategy proposed in [9] due to the high complexity of the optimal integer linear program solution.

In the first simulation we consider a scenario with two UEs and a difference in the mean SNR of the two UEs of $\Delta\text{SNR} = 3\,\text{dB}$. The parameters of the simulation are summarized in Table I. We assume a blockfading channel model with a constant channel during one subframe duration (1 ms) and channel realizations independent between subframes. The CQI feedback from the UEs arrives with zero delay, meaning that the scheduler knows the feedback before the actual transmission. We assume distinct feedback values for all RBs, but we do not explicitly enforce the constraint that the CQI feedback must lie in a range of four values (in most of the cases this is anyway fulfilled). We use a full transmit buffer assumption for our simulations; that is, users fully utilize all resources they get.

TABLE I
SIMULATION PARAMETERS

Parameter	Value
System bandwidth	1.4 MHz
Number of subcarriers	72
Number of RBs N	12
Number of users K	2
Channel Model	ITU-T VehA [16]
Antenna configuration	1 transmit, 1 receive (1×1)
Receiver	Zero Forcing ZF
Schedulers	Best CQI (BCQI)
	Approx. Max. Throughput (AMT)
	Kwan. Max. Throughput (KMT)

Figure 2 shows the sum throughput of both UEs for the three different schedulers over the SNR. BCQI and AMT perform similar. Our proposed scheduler gains about $1.8-2\,\text{dB}$ compared to the proposal of Kwan et.al.

Figure 3 shows a comparison of the BLERs of the two UEs when different schedulers are employed. There is a slight difference in the BLER performance of BCQI and AMT, because the RB assignment is not unique if both users feed back the same CQI value for a resource. The user with the higher SNR (UE1) needs about $-2\,\text{dB}$ SNR to achieve the imposed target BLER for all schedulers, while the worse user (UE2) requires about $-1\,\text{dB}$ SNR. The reason for this is that UE2 in general gets less resources than UE1 and therefore the codeblocksize is smaller which impairs the code performance. The BLER of KMT is lower than that of our proposed method because it underestimates the channel and uses a too conservative CQI value.

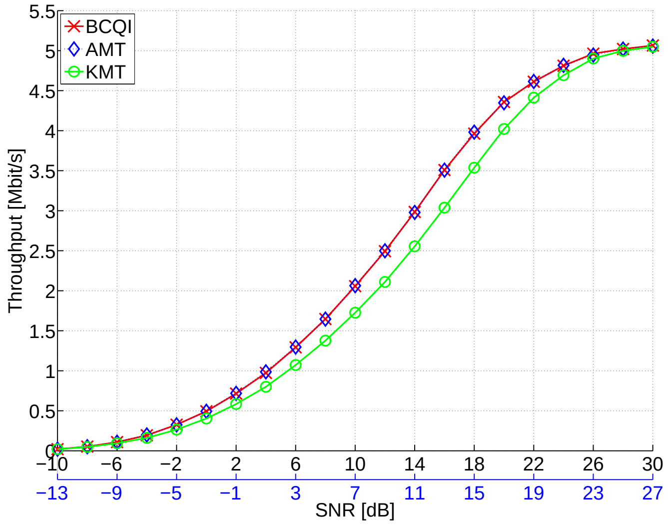

Fig. 2. Sum throughput obtained with different schedulers plotted over SNR for UE 1 and UE 2.

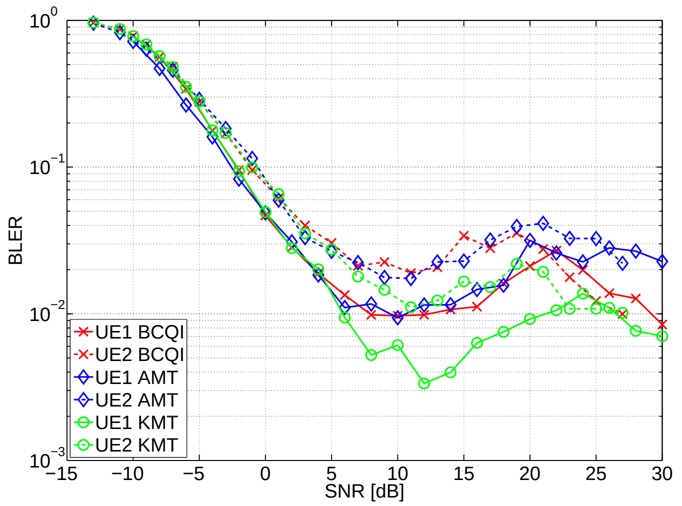

Fig. 3. Block error ratio obtained with different schedulers plotted over SNR for UE 1 and UE 2.

V. Application to Other Scheduling Strategies

In this section we show how the proposed framework can be used to implement other scheduling strategies. We consider some fair schedulers namely the Resource Fair (RF) scheduler, the MaxMin. scheduler and the Proportional Fair (PF) scheduler.

A. *Resource Fair Scheduler*

The RF scheduler tries to maximize the sum rate of all UEs while guaranteeing fairness with respect to the number of RBs a UE gets. This can be easily achieved by imposing the additional constraint

$$||\mathbf{b}_k||_1 = \frac{N}{K} \quad \forall k, \tag{14}$$

if $\frac{N}{K}$ is integer, otherwise some UEs will get $\lfloor \frac{N}{K} \rfloor$ while others get $\lceil \frac{N}{K} \rceil$ to make up for the total number of RBs (to guarantee fairness one should randomize this decision). This can be easily achieved by including an additional row for each UE in the matrix $\mathbf{A}$ (13) that sums up all RBs of a UE. This does not harm the unimodularity of the matrix, so the problem can be solved as an LP.

B. MaxMin. Scheduler

The task of a MaxMin. scheduler is to maximize the minimum of the user throughputs. This scheduler is Pareto optimal, meaning that the rate of one UE cannot be increased without decreasing the rate of another UE that has a lower rate than the one considered [17]. The optimization problem can be formulated as

$$\{\mathbf{b}_1^\star, \ldots, \mathbf{b}_K^\star\} = \underset{\{\mathbf{b}_1,\ldots,\mathbf{b}_K\}}{\operatorname{argmax}} \min_k \mathbf{c}_k^T \mathbf{b}_k \quad (15)$$

$$\text{subject to:}$$

$$\mathbf{b}_j^T \cdot \mathbf{b}_i = 0 \ \ \forall i \neq j$$

$$\mathbf{b}_k(n) \in \{0,1\} \ \ \forall n, k.$$

Introducing the variable ϵ allows to recast the problem into a linear integer program

$$\{\mathbf{b}_1^\star, \ldots, \mathbf{b}_K^\star, \epsilon^\star\} = \underset{\{\mathbf{b}_1,\ldots,\mathbf{b}_K,\epsilon\}}{\operatorname{argmax}} \ \epsilon \quad (16)$$

$$\text{subject to:}$$

$$\epsilon \leq \mathbf{c}_k^T \mathbf{b}_k \ \ \forall k$$

$$\mathbf{b}_j^T \cdot \mathbf{b}_i = 0 \ \ \forall i \neq j$$

$$\mathbf{b}_k(n) \in \{0,1\} \ \ \forall n, k.$$

Next we combine the different RB vectors $\mathbf{b}_k$ into a single vector $\mathbf{b}$ as in (10), append the variable ϵ at the end of the vector to get $\widetilde{\mathbf{b}} \in \mathbb{R}^{(KN+1)\times 1}$ and put the $\mathbf{c}_k$ into a matrix $\mathbf{C} \in \mathbb{R}^{K\times KN+1}$

$$\widetilde{\mathbf{b}} = \begin{bmatrix} \mathbf{b} \\ \epsilon \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} \overbrace{\overbrace{c_{1,1} & 0 & 0 & 0\ldots0}^{K} & c_{1,2} & 0 & 0 & 0\ldots1}^{K\cdot N+1} \\ 0 & c_{2,1} & 0 & 0\ldots0 & 0 & c_{2,2} & 0 & 0\ldots1 \\ 0 & 0 & c_{3,1} & 0\ldots0 & 0 & 0 & c_{3,2} & 0\ldots1 \\ \vdots & & & \ddots & & & & \ddots\vdots \end{bmatrix}$$

where $c_{i,j} = -\mathbf{c}_i(j)$. Using this notation the optimization problem can be written as

$$\widetilde{\mathbf{b}}^\star = \underset{\widetilde{\mathbf{b}}}{\operatorname{argmax}} \, [\mathbf{0}_{KN}, 1] \cdot \widetilde{\mathbf{b}} \quad (17)$$

$$\text{subject to:}$$

$$\mathbf{C} \cdot \widetilde{\mathbf{b}} \leq \mathbf{0}_K$$

$$\mathbf{A} \cdot \mathbf{b} \leq \mathbf{1}_N$$

$$\mathbf{b}(n) \in \{0,1\} \ \ \forall n.$$

This linear binary integer program cannot be relaxed to an LP without sacrificing optimality, as the constraint matrices are not totally unimodular. Nevertheless, we apply the relaxation here and round the solution simply to the nearest integer. Simulations have shown that the relaxation only entails a minimal rate loss.

C. Proportional Fair Scheduler

A scheduling P is proportionally fair if and only if, for any feasible scheduling S, it satisfies:

$$\sum_k \frac{\overline{\mathrm{T}}_k^{(S)} - \overline{\mathrm{T}}_k^{(P)}}{\overline{\mathrm{T}}_k^{(P)}} \leq 0 \quad (18)$$

where $\overline{\mathrm{T}}_k^{(S)}$ is the temporal average rate of user k by scheduler S [17]. In [18] necessary and sufficient conditions for a multicarrier scheduler to be proportionally fair are derived. Based on these conditions a slightly suboptimal reduced-complexity algorithm is derived in [19] which can directly be applied with the proposed framework. We use this algorithm with a window size of 10 subframes for the exponential window that is used for averaging the user throughput.

VI. Simulation Results for Multiple Users

In this section we compare different schedulers in terms of their achieved throughput and fairness. Additionally to the schedulers presented in previous sections we use a round robin (RR) scheduler, that schedules users with a fixed pattern, such that every UE gets the same number of contiguous resources. We quantify fairness using Jain's fairness index [20]

$$J(\mathbf{T}) = \frac{\left(\sum_{k=1}^K \mathbf{T}(k)\right)^2}{K \sum_{k=1}^K \mathbf{T}(k)^2}, \quad (19)$$

where $\mathbf{T}$ is a vector of measured (simulated) user throughputs. Jain's fairness index equals one if all throughputs are the same and perfect fairness is achieved. With decreasing fairness, Jain's fairness index approaches zero. We consider here absolute fairness, meaning that we don't take into account the SNR differences in our fairness measure.

The simulation setup consists of a single cell SISO scenario with 25 UEs having average SNRs ranging from 1 to 25 dB in 1 dB steps. Our simulation parameters are summarized in Table II. Figure 4 shows the throughput achieved by the different UEs when applying several resource allocation schemes. The max. throughput schedulers (AMT, KMT, BCQI) achieve high throughputs for UEs with good SNR but users with low SNR are never severed. The figure also shows, that the scheduler proposed by Kwan et.al. performs as good as our proposal in this case. This is due to multiuser diversity, which causes UEs to be scheduled only on their best RBs, where the CQI values are hardly varying. Figure 5 shows the sum throughput achieved in the cell. The rate maximizing schedulers behave similar and outperform the others in terms of throughput, because they only serve UEs with good channel conditions. The round robin scheduler performs worst, as it does not take into account the channel state for resource allocation. In terms of fairness, the situation more or less reverses, as Figure 6

TABLE II
SIMULATION PARAMETERS

Parameter	Value
System bandwidth	10 MHz
Number of subcarriers	600
Number of RBs N	100
Number of users K	25
Channel Model	3GPP TU [21]
Antenna configuration	1 transmit, 1 receive (1 × 1)
Receiver	Zero Forcing ZF
Schedulers	Best CQI (BCQI)
	Approx. Max. Throughput (AMT)
	Kwan. Max. Throughput (KMT)
	Round Robin (RR)
	Proportional Fair (PF)
	Resource Fair (RF)
	MaxMin.

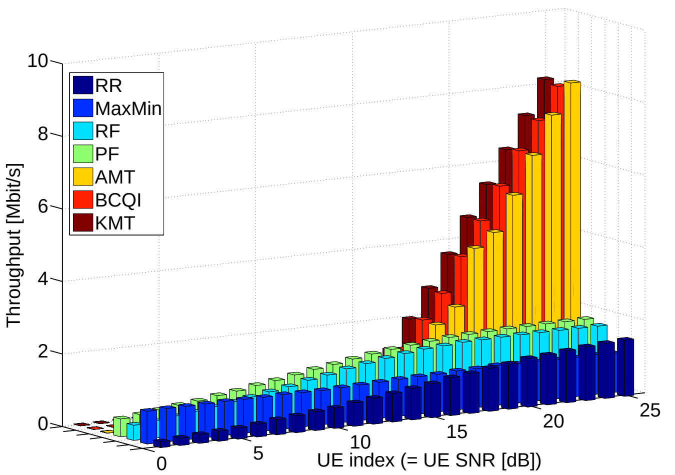

Fig. 4. Throughputs simultaneously achieved by UEs with different average SNRs for several schedulers.

shows. In accordance to Figure 4 pure rate maximization is not compatible with fairness. The best fairness is achieved with the maxmin. scheduler, which conforms to it's design goal. Good fairness is also achieved with the PF and RF schedulers. These two also deliver high sum throughput and therefore seem to be a good compromise. Not taking into account the channel conditions for resource allocation, as the RR scheduler does, clearly is a bad choice, because neither high fairness nor high throughput can be achieved.

All simulation results as well as the corresponding MATLAB code will be made available online in the next release (v.1.5) of our physical layer LTE simulator [15].

VII. CONCLUSION

In this paper we formulate the sum rate maximization resource allocation problem in the framework of Long Term Evolution. We develop a linearized model to simplify the nonlinear combinatorial optimization problem to a simple linear program. We show that solving this linear program is equivalent to allocating resources to the users with the best channel conditions (best CQI). A comparison to a rate

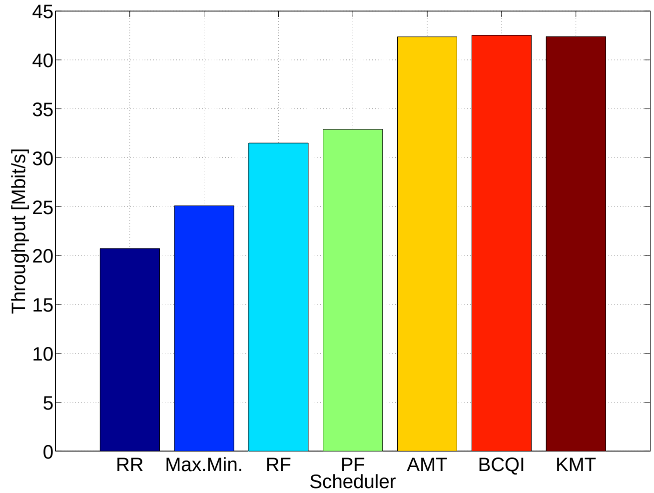

Fig. 5. Sum of user throughputs achieved with different schedulers.

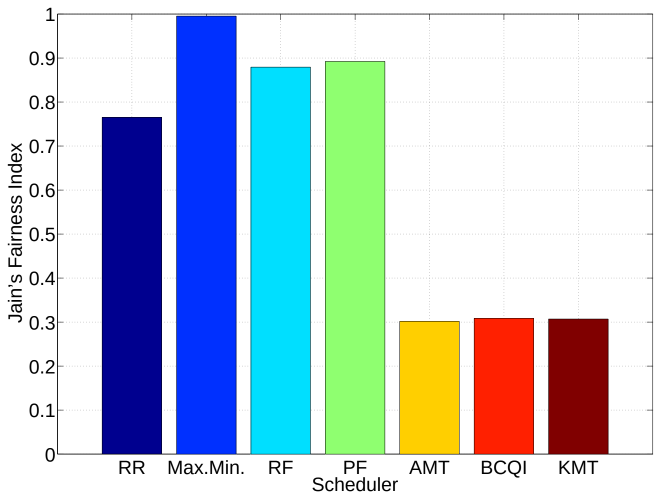

Fig. 6. Fairness achieved with different schedulers.

maximizing scheduling strategy proposed by Kwan et.al. in [9] is carried out by simulations. Our scheduler achieves better results for small user numbers, while for large user numbers the performance is similar. Next we show how the proposed framework can be used to implement other schedulers. We compare several schedulers in terms of achieved throughput and fairness by simulations. These show that proportional fair and resource fair schedulers deliver a good compromise between fairness and throughput.

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APPENDIX A

We show analytically that the AMT scheduler is equivalent to the BCQI scheduler by considering the Karush-Kuhn-Tucker (KKT) optimality conditions [22]. For this purpose we reformulate the linear relaxation of the optimization problem (12) in scalar notation. We set the first constraint equal to one, because every resource is used due to the linearity of the problem.

$$\{b^\star_{1,1}, \dots, b^\star_{N,K}\} = \underset{\{b_{1,1},\dots,b_{N,K}\}}{\operatorname{argmax}} \sum_{i=1}^{N}\sum_{j=1}^{K} c_{i,j} b_{i,j} \quad (20)$$

subject to:

$$\sum_{j=1}^{K} b_{i,j} = 1 \quad \forall i \in \{1,\dots,N\}$$

$$0 \le b_{i,j} \le 1 \quad \forall i \in \{1,\dots,N\},\ j \in \{1,\dots,K\}$$

The Lagrangian $L(\mathbf{b}, \lambda, \nu)$ is given by

$$L(\mathbf{b}, \lambda, \mu) = \sum_{i=1}^{N}\sum_{j=1}^{K} c_{i,j} b_{i,j} + \sum_{i=1}^{N}\sum_{j=1}^{K} \lambda_{i,j}(b_{i,j} - 1) + \dots$$

$$+ \sum_{i=1}^{N}\sum_{j=1}^{K} \lambda_{i+N,j}(-b_{i,j}) + \sum_{i=1}^{N} \nu_i \left(\sum_{j=1}^{K} b_{i,j} - 1 \right) \quad (21)$$

$$\mathbf{b} = [b_{1,1}, b_{1,2}, \dots, b_{N,K}]^T \in \mathbb{R}^{NK\times 1}$$
$$\lambda = [\lambda_{1,1}, \lambda_{1,2}, \dots, \lambda_{2N,K}]^T \in \mathbb{R}^{2NK\times 1}$$
$$\nu = [\nu_1, \nu_2, \dots, \nu_N]^T \in \mathbb{R}^{N\times 1}.$$

The KKT conditions result in the following system of equations:

(i) $b_{i,j} \le 1 \ \forall i, j$ $\quad (ii)$ $b_{i,j} \ge 0 \ \forall i, j$
(iii) $\sum_{j=1}^{K} b_{i,j} = 1 \ \forall i$ $\quad (iv)$ $\lambda_{i,j} \ge 0 \ \forall i, j$
(v) $\lambda_{i,j} \cdot (b_{i,j} - 1) = 0 \ \forall i, j$ $\quad (vi)$ $\lambda_{i+N,j} \cdot b_{i,j} = 0 \ \forall i, j$
(vii) $\lambda_{i,j} - \lambda_{i+N,j} = c_{i,j} - \nu_i \ \forall i, j$

Consider next the case $c_{i,j} - \nu_i > 0$.

$$c_{i,j} - \nu_i > 0 \overset{(vii)}{\Longrightarrow} \lambda_{i,j} > \lambda_{i+N,j} \overset{(iv)}{\Longrightarrow} \lambda_{i,j} > 0 \overset{(v)}{\Longrightarrow} b_{i,j} = 1$$
$$\overset{(vi)}{\Longrightarrow} \lambda_{i+N,j} = 0 \overset{(iii)}{\Longrightarrow} b_{i,n} = 0 \ \forall n \ne j \overset{(iv,vi)}{\Longrightarrow} \lambda_{i+N,n} \ge 0$$
$$\overset{(v)}{\Longrightarrow} \lambda_{i,n} = 0 \overset{(vii)}{\Longrightarrow} c_{i,n} - \nu_i \le 0 \Longrightarrow c_{i,j} > c_{i,n} \ \forall n.$$

In this case the user j with the largest CQI value on resource i is served. Considering the case $c_{i,j} - \nu_i < 0$ leads to $b_{i,j} = 0$ following a similar argumentation as above. This means that the user j is not served on resource i. The last possibility to be investigated is $c_{i,j} - \nu_i = 0$.

$$c_{i,j} - \nu_i = 0 \overset{(vii)}{\Longrightarrow} \lambda_{i,j} = \lambda_{i+N,j} \overset{(v,vi)}{\Longrightarrow} \lambda_{i,j} = \lambda_{i+N,j} = 0$$

If the user j is the only one for which $c_{i,j} - \nu_i = 0$ holds and there is no user k with $c_{i,k} - \nu_i > 0$ than (iii) necessitates $b_{i,j} = 1$ and the user gets the resource i. If there are more users $\{j, k, l, \dots\}$ that have $c_{i,n} - \nu_i = 0 \quad n \in \{j, k, l, \dots\}$ than time sharing of the resource i between the users, such that $\sum_{n\in\{j,k,l,\dots\}} b_{i,n} = 1$ is fulfilled, is an optimal solution. Employing the simplex method for solving the LP, a time sharing solution will not be produced, as this does not correspond to a corner point of the feasible set, but rather to a point on a connecting surface between corner points [23]. Therefore, in the case of equal CQI values for several users, a single user will get the resource.

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