Sum-Rate Maximization for Multiple Users in Partial Frequency Reuse Cellular Networks

Bujar Krasniqi[1], Martin Wolkerstorfer[2], Christian Mehlführer[1] and Christoph F. Mecklenbräuker[1]
[1]Vienna University of Technology, Institute of Communications and Radio-Frequency Engineering
[1]Christian Doppler Laboratory for Wireless Technologies for Sustainable Mobility
Gusshausstr. 25/389, A-1040 Vienna
[2]The Telecommunications Research Center Vienna (FTW)
Donau-City-Strasse 1, A-1220 Vienna
email: {bujar.krasniqi,chmehl,cfm}@nt.tuwien.ac.at,wolkerstorfer@ftw.at

***Abstract*—We apply constrained optimization techniques to optimally allocate the bandwidth and power to the users in a cellular network. We investigate partial frequency reuse with multiple users in the full and partial frequency regions for inter-cell interference mitigation. We show that the non-convex sum-rate maximization problem becomes convex under certain simplifying assumptions. Moreover, an efficient algorithm is developed to solve the problem for a fixed bandwidth allocation. The optimal solution assigns all available power to the users. Finally, we prove that *even without simplifications* the non-convex problems of maximizing the minimum rate among users and minimizing the sum-power are transformable into convex forms.**

I. Introduction

The sum-rate maximization of all users in all cells under variable power and bandwidth allocation to full and partial frequency reuse users is a non-convex optimization problem. However, in [1] the single-carrier power control problems of maximizing the minimum rate among users and minimizing the total sum-power were found to be transformable to geometric [2] and hence convex optimization problems, notably without any high-SINR approximation.

Next generation mobile communication systems use Orthogonal Frequency Division Multiple Access (OFDMA) as their modulation scheme in the downlink [3], [4]. Since cell edge users may suffer severely from Inter-Cell Interference (ICI), several schemes have been proposed for ICI mitigation. One of those is Partial Frequency Reuse (PFR), which is applied in [5], [6], [7].

The characteristics of the optimal power allocation for two base stations, employing also scheduling schemes, has been studied in [8] under frequency reuse-1. Additionally to the sum-rate maximization power control problem, in [9] the authors also investigate the maximization of the minimum rate for two users. To the best of our knowledge, there are currently no studies that consider the maximization of the sum-rate, the maximization of the minimum rate, or the sum-power minimization in PFR.

Our contributions can be summarized as follows. In Section II we show the system model including the bandwidth allocation scheme for PFR. In Section III we study the sum-rate maximization problem for PFR systems considering inter-cell interference and both, power and bandwidth allocation between PFR and Full Frequency Reuse (FFR) users. Instead of the high-SINR approximation applied in [1] we utilize as simplifying assumption equal power allocations for the FFR users in all cells in order to arrive at a convex optimization problem. For a fixed bandwidth allocation we even derive a simple water-filling-like power allocation algorithm. Furthermore, we present in Section IV simulation results, which confirm the rate gains by applying the proposed suboptimal allocation scheme. In Section V we also find without further simplifying assumptions that the optimization of the minimum rate among users and the minimization of the total sum-power in the network can be stated as convex optimization problems and hence solved efficiently. More specifically, these findings extend those in [1] as we additionally consider a power allocation problem between FFR and PFR users, and also optimize the bandwidth allocation between FFR and PFR users jointly with the users' transmit powers. Conclusions are drawn in Section VI.

II. System Model

In our system we consider N^{in} users that are located in the inner region of the cell (the full frequency reuse region) and M^{out} users located in the outer region of the cell (the partial frequency reuse region), as indicated in Fig. 1. Based on the users' received SINRs, a scheduler decides whether a user is considered an inner user or an outer user. The frequency pattern [5] applied in our system model is shown Fig. 2. The users located in the inner region of the cell, S_{01}, receive power from their own sector antenna of base station BS_0 and also interference from all other sectors of base stations BS_k, $k = 0 \dots 6$. The sum-rate achieved by the users in the inner region is given by

$$\sum_{n=1}^{N^{\mathrm{in}}} R_n^{\mathrm{in}} = \sum_{n=1}^{N^{\mathrm{in}}} B_n^{\mathrm{in}} \log_2 \left(1 + \frac{G_{0n}^{\mathrm{in}} p_0^{\mathrm{in}}}{N_0 B_n^{\mathrm{in}} + \sum_{k=0}^{6} G_{kn}^{\mathrm{in}} p_k^{\mathrm{in}}} \right) \quad (1)$$

where B_n^{in} is the bandwidth utilized in the inner region and N_0 is the noise spectral density. The large scale path-loss including antenna gain, penetration loss, shadowing and fast fading is expressed in the form [10],

$$G = 128.1 + 10\alpha \log_{10}(r) + A + L_p + X_\sigma + F \quad (2)$$

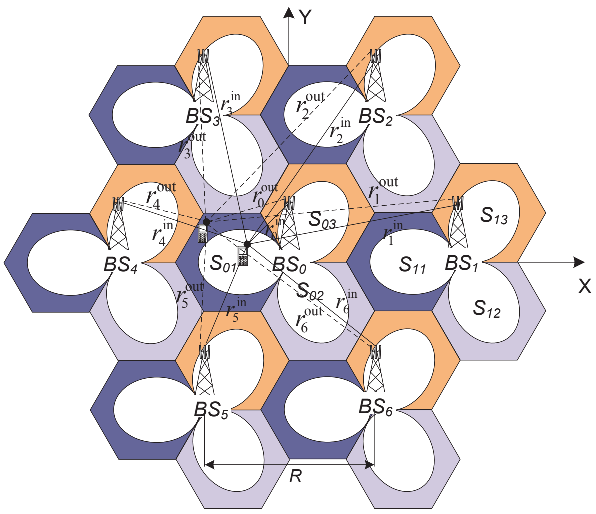

Fig. 1. Partial frequency reuse cell cluster

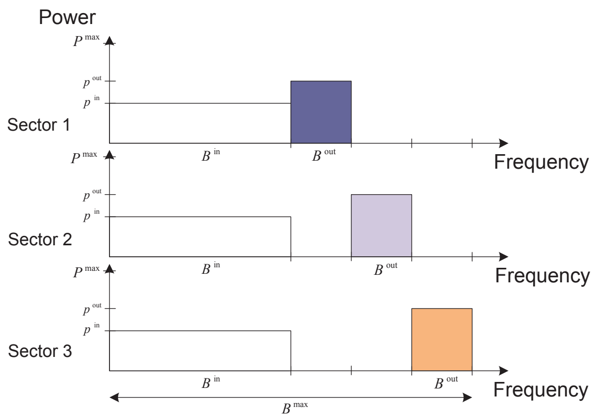

Fig. 2. Frequency reuse pattern for an arbitrary base station

where G is in dB, α the path-loss exponent, r the distance between the mobile station and base station in km, A the sum of user antenna gain and base station antenna gain in dB, L_p the penetration loss in dB, X_σ the log-normal shadowing in dB, and F the fast fading channel coefficient in dB. The antenna gain A is defined by horizontal antenna pattern [10]. The large scale path-loss attenuation of directed channels G_{0n}^{in} is defined by Equation (2). The large scale path-loss attenuation of interference channels G_{kn}^{in} is also defined by Equation (2) except of the fast fading, which is not taken into account here. The transmit power assigned to the users in the inner region is denoted by p_0^{in} and the interference power from the other base stations is denoted by p_k^{in}, $k = 1 \dots 6$, with k denoting the index of the interfering base stations. The users located in the outer region of the cell receive also interference from sectors of base stations that use the same frequency band. The transmit power assigned to the users in the outer region is denoted by p_0^{out} and the interference power from the other base stations is denoted by p_k^{out}, $k = 1 \dots 6$. Thus, the sum-rate achieved by all users in the outer region is given by

$$\sum_{m=1}^{M^{\text{out}}} R_m^{\text{out}} = \sum_{m=1}^{M^{\text{out}}} B_m^{\text{out}} \log_2 \left(1 + \frac{G_{0m}^{\text{out}} p_0^{\text{out}}}{N_0 B_m^{\text{out}} + \sum_{k=1}^{6} G_{km}^{\text{out}} p_k^{\text{out}}} \right) \tag{3}$$

where B_m^{out} denotes the bandwidth utilized in the outer region and G_{0m}^{out} and G_{km}^{out} denotes the large scale path-loss attenuation for the direct and interference channels of the m-th outer user.

III. Efficient Algorithms for Sum-Rate Maximization

The sum-rate maximization problem is non-convex as it contains the sum-rate maximization in standard power control as a special case [1]. Under unequal allocation of interference powers p_k^{in} and p_k^{out}, $k = 1 \dots 6$ with the power p_0^{in} and p_0^{out}, but for a fixed bandwidth allocation B^{in} and B^{out} it can still be solved efficiently by geometric programming under a high-SINR approximation $\log(1 + \text{SINR}) \approx \log(\text{SINR})$, or sequentially approximated by geometric programs, *cf.* [1]. Differently, under the simplifying assumption that all cells use equal powers $p_k^{\text{in}} = p_0^{\text{in}}$ and $p_k^{\text{out}} = p_0^{\text{out}}$, $k = 1 \dots 6$ to serve the inner and outer users we show in this section that the sum-rate maximization problem becomes convex and is solvable in a water-filling-like manner. While for simplicity of notation we only consider users located in a single cell, all presented problems and algorithms can be extended to the case of multiple users over multiple cells. Using a vector-matrix notation the optimization problem is compactly written

$$\underset{\mathbf{p},\mathbf{b}}{\text{maximize}} \quad \mathbf{1}^T \mathbf{R}^{\text{in}} + \mathbf{1}^T \mathbf{R}^{\text{out}} \tag{4a}$$

$$\text{subject to}$$

$$\mathbf{A} \cdot \begin{bmatrix} \mathbf{p} \\ \mathbf{b} \end{bmatrix} \preceq \mathbf{c}, \tag{4b}$$

$$\mathbf{p} \succeq \mathbf{0}, \tag{4c}$$

$$\mathbf{b} \succeq \mathbf{0}, \tag{4d}$$

where $\preceq$ denotes a component-wise inequality, $\mathbf{R}^{\text{in}}$ and $\mathbf{R}^{\text{out}}$ denotes the vector elements of inner and outer user rates. We define

$$\begin{aligned} \mathbf{R}^{\text{in}} &= [R_1^{\text{in}}, R_2^{\text{in}}, \dots, R_{N^{\text{in}}}^{\text{in}}], \\ \mathbf{R}^{\text{out}} &= [R_1^{\text{out}}, R_2^{\text{out}}, \dots, R_{M^{\text{out}}}^{\text{out}}], \\ \mathbf{c} &= [P^{\max}, B^{\max}]^T, \\ \mathbf{A} &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}. \end{aligned} \tag{5}$$

The maximum power and maximum bandwidth of the considered cell are denoted by $P^{\max}$ and $B^{\max}$. The power vector $\mathbf{p}$ and the bandwidth vector $\mathbf{b}$ are defined by

$$\begin{aligned} \mathbf{p} &= [p_0^{\text{in}}, p_0^{\text{out}}]^T, \\ \mathbf{b} &= [B^{\text{in}}, B^{\text{out}}]^T. \end{aligned} \tag{6}$$

In the optimization problem (4), the constraints (4b), (4c) and (4d) are linear and hence convex. It can be easily shown that the second derivative of R_n^{in} with respect to p_0^{in} is concave. As a consequence we find that $\tilde{R}^{\mathrm{in}}(B_n^{\mathrm{in}}, p_0^{\mathrm{in}}) = B_n^{\mathrm{in}} R_n^{\mathrm{in}}(p_0^{\mathrm{in}}/B_n^{\mathrm{in}})$ is concave as it is the perspective of a concave function [11]. Furthermore, since R_m^{out} is concave because it has a similar form as R_n^{in} and the sum of concave functions is concave as well, the optimization problem (4) is therefore concave.

A. Water-filling-like power allocation

Deriving an analytic solution for (4) was found to be intractable. However, for constant bandwidth allocation we derive in this section a power allocation algorithm based on the Karush-Kuhn-Tucker (KKT) optimality conditions [11]. For simplifying the written equations we are substituting $G_{0n}^{\mathrm{in}} = a_n$, $\sum_{k=1}^{6} G_{kn}^{\mathrm{in}} = b_n$, $G_{0m}^{\mathrm{out}} = d_m$ and $\sum_{k=1}^{6} G_{km}^{\mathrm{out}} = e_m$. The Lagrangian for problem (4) is written as

$$\begin{aligned} L(\mathbf{p}, \mu, \boldsymbol{\lambda}) =& \mathbf{1}^T \mathbf{R}^{\mathrm{in}} + \mathbf{1}^T \mathbf{R}^{\mathrm{out}} \\ & - \mu(\mathbf{1}^T \mathbf{p} - P^{\max}) + \boldsymbol{\lambda}^T \mathbf{p} \end{aligned} \tag{7}$$

where μ and $\boldsymbol{\lambda} = [\lambda^{\mathrm{in}}, \lambda^{\mathrm{out}}]$ are the Lagrange multipliers for the sum-power and positivity constraints, respectively. Applying the KKT conditions [11] we have

$$\mathbf{p} \succeq \mathbf{0}, \tag{8a}$$
$$\mathbf{1}^T \mathbf{p} - P^{\max} \leq 0, \tag{8b}$$
$$\boldsymbol{\lambda} \succeq \mathbf{0}, \tag{8c}$$
$$\lambda^{\mathrm{in}} p^{\mathrm{in}} = 0, \tag{8d}$$
$$\lambda^{\mathrm{out}} p^{\mathrm{out}} = 0, \tag{8e}$$
$$\begin{aligned} \frac{\partial L}{\partial p_0^{\mathrm{in}}} &= -\sum_{n=1}^{N^{\mathrm{in}}} \frac{B_n^{\mathrm{in}}}{\log(2)} \frac{a_n N_0 B_n^{\mathrm{in}}}{\left[N_0 B_n^{\mathrm{in}} + (a_n + b_n) p_0^{\mathrm{in}}\right]} \\ &\quad \cdot \frac{1}{(N_0 B_n^{\mathrm{in}} + b_n p_0^{\mathrm{in}})} + \mu - \lambda^{\mathrm{in}} \\ &= f^{\mathrm{in}}\left(p_0^{\mathrm{in}}\right) - \lambda^{\mathrm{in}} = 0, \end{aligned} \tag{8f}$$
$$\begin{aligned} \frac{\partial L}{\partial p_0^{\mathrm{out}}} &= -\sum_{m=1}^{M^{\mathrm{out}}} \frac{B_m^{\mathrm{out}}}{\log(2)} \frac{d_m N_0 B_m^{\mathrm{out}}}{\left[N_0 B_m^{\mathrm{out}} + (d_m + e_m) p_0^{\mathrm{out}}\right]} \\ &\quad \cdot \frac{1}{(N_0 B_m^{\mathrm{out}} + e_m p_0^{\mathrm{out}})} + \mu - \lambda^{\mathrm{out}} \\ &= f^{\mathrm{out}}\left(p_0^{\mathrm{out}}\right) - \lambda^{\mathrm{out}} = 0. \end{aligned} \tag{8g}$$

The last two equations (8f) and (8g) in the KKT conditions are the first derivative of the Lagrangian given by Equation (7) with respect to p_0^{in} and p_0^{out}, respectively.

We continue to show that for a fixed variable μ, the optimal power allocation can be computed efficiently. Combining the positivity constraints (8c) with (8f) and the complementary slackness constraints (8d), we find the optimum $p_0^{\mathrm{in}}(\mu)$ as a function of μ after calculating the root of $f^{\mathrm{in}}\left(p_0^{\mathrm{in}}\right)$. Similarly, using (8c), (8g) and (8e) we find the optimum $p_0^{\mathrm{out}}(\mu)$ as a function of μ by finding the root of $f^{\mathrm{out}}\left(p_0^{\mathrm{out}}\right)$. An efficient algorithm for finding these two roots in the general case is the Newton-Raphson method [12]. Using $\bar{\mu}^{\mathrm{out}} = \sum_{m=1}^{M^{\mathrm{out}}} \frac{N_0 \log(2)}{d_m}$ and $\bar{\mu}^{\mathrm{in}} = \sum_{n=1}^{N^{\mathrm{in}}} \frac{N_0 \log(2)}{a_n}$ gives the optimum allocation

$$p_0^{\mathrm{in}} = \begin{cases} p_0^{\mathrm{in}}(\mu), & \text{if } \frac{1}{\mu} \geq \bar{\mu}^{\mathrm{in}}, \\ 0, & \text{otherwise}, \end{cases} \tag{9}$$

$$p_0^{\mathrm{out}} = \begin{cases} p_0^{\mathrm{out}}(\mu), & \text{if } \frac{1}{\mu} \geq \bar{\mu}^{\mathrm{out}}, \\ 0, & \text{otherwise}. \end{cases} \tag{10}$$

In the case $N^{\mathrm{in}} = M^{\mathrm{out}} = 1$, the above roots can be computed analytically, giving the explicit solution

$$p_0^{\mathrm{in}} = \begin{cases} \frac{-(a_1+2b_1) N_0 B_1^{\mathrm{in}} + \sqrt{\Delta^{\mathrm{in}}}}{2(a_1+b_1)b_1}, & \text{if } \frac{1}{\mu} \geq \bar{\mu}^{\mathrm{in}}, \\ 0, & \text{otherwise}, \end{cases} \tag{11}$$

where Δ^{in} under the square root in equation (11) is given by

$$\Delta^{\mathrm{in}} = (a_1 N_0 B_1^{\mathrm{in}})^2 + 4 a_1 b_1 \left(a_1 + b_1\right) \frac{N_0 (B_1^{\mathrm{in}})^2}{\mu \log(2)}.$$

The optimal assigned power to an outer user is analogously given by

$$p_0^{\mathrm{out}} = \begin{cases} \frac{-(d_1+2e_1) N_0 B_1^{\mathrm{out}} + \sqrt{\Delta^{\mathrm{out}}}}{2(d_1+e_1)e_1}, & \text{if } \frac{1}{\mu} \geq \bar{\mu}^{\mathrm{in}}, \\ 0, & \text{otherwise}, \end{cases} \tag{12}$$

where Δ^{out} under the square root in equation (12) is given by

$$\Delta^{\mathrm{out}} = (d_1 N_0 B_1^{\mathrm{out}})^2 + 4 d_1 e_1 \left(d_1 + e_1\right) \frac{N_0 (B_1^{\mathrm{out}})^2}{\mu \log(2)}.$$

For searching the optimal water-level $1/\mu$ we use a simple bisection search due to the nondifferentiability of the Lagrangian.

IV. Simulation Results

In this section, we show simulation results carried out for two users, one located in the inner region of the cell and the other one located in the outer region of the cell. During the simulations we have considered hundred channel realizations over which we have calculated the average rate. The parameters used for the simulations are shown in Table I. Using Equations (11) and (12) while searching for the optimal water-level $\frac{1}{\mu}$ through bisection search, we have simulated the optimal power assignment for the inner user and the outer user. The optimal power assignment for the inner user and the outer user is shown in Fig. 3. When the maximum base

TABLE I
SIMULATION PARAMETERS

parameters	**value**
Maximum base station power $P^{\max}$	5 W
Maximum base station bandwidth $B^{\max}$	20 MHz
Noise spectral density N_0	-174 dBm/Hz
Center frequency f	2.0 GHz
Pathloss exponent α	3.75
Penetration loss L_p	20 dB
Shadowing X_σ	$\mathcal{N}(0, 8)$dB
Fast Fading F	$\mathcal{CN}(0, 1)$dB,
Inter base station distance R	700 m
Position of inner user in polar coordinates	$(120 \text{ m}, 160^0)$
Position of outer user in polar coordinates	$(370 \text{ m}, 160^0)$

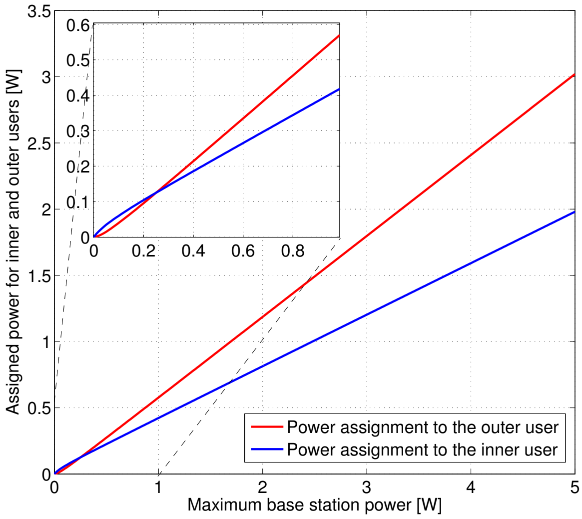

Fig. 3. Optimal power assignment to the inner and outer users

station power is very low, all the power is assigned to the inner user. Until a maximum base station power of 0.25 W, more power is assigned to the inner user than to the outer user. For maximum base station power higher than 0.25 W, more power is assigned to the outer user than to the inner user. Assigning more power to the outer user when the maximum base station power is higher, contributes in reducing the ICI and increasing the maximum sum-rate, since the outer user is interfered only from the non-neighboring sectors that use the same frequency band. The maximum sum-rate achieved by the inner and the outer user as a function of the maximum base station power is shown in Fig. 4. From Fig. 4 we see

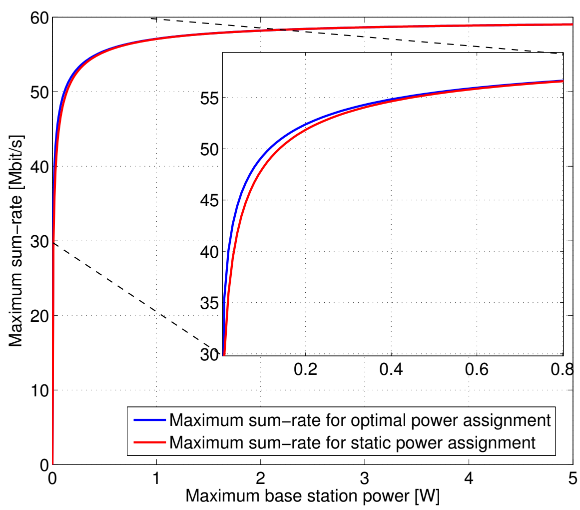

Fig. 4. Maximum sum-rate for the inner and the outer users

the sum-rate gains compared with a static power allocation of $p_0^{\text{in}} = 40\% P^{\max}, p_0^{\text{out}} = 60\% P^{\max}$ (corresponding to the optimum power allocation for $P^{\max} = 5\,\text{W}$).

V. Convex Joint Power and Bandwidth Allocation Problems

In this section we study two different types of optimization problems for systems employing partial frequency reuse and performing power and bandwidth allocation. Without relying on any assumptions on power allocations or SINR as in Section III, we proof that these problems can be transformed into convex ones and hence solved efficiently using state-of-the-art convex optimization methods [11].

A. Maximization of the Minimum Rate

We formulate the problem of maximizing the minimum rate among all users in all cells in the following form:

$$\underset{\beta^{\text{in}},\beta^{\text{out}},t^{\text{in}},t^{\text{out}}\in\mathcal{R}_+,\mathbf{p}\succeq\mathbf{0}}{\text{maximize}} \quad \min\{\beta^{\text{in}}t^{\text{in}}\log(2), \beta^{\text{out}}t^{\text{out}}\log(2)\} \tag{13a}$$

subject to

$$t^{\text{in}} \leq \log\left(1 + \frac{p_c^{\text{in}}}{n_u^{\text{in}}\beta^{\text{in}} + \sum_{k\in\mathcal{C}\backslash c} g_{ku}^{\text{in}} p_k^{\text{in}}}\right), \ \forall u \in \mathcal{U}_c^{\text{in}}, \ \forall c \in \mathcal{C}, \tag{13b}$$

$$t^{\text{out}} \leq \log\left(1 + \frac{p_c^{\text{out}}}{n_u^{\text{out}}\beta^{\text{out}} + \sum_{k\in\mathcal{C}\backslash c} g_{ku}^{\text{out}} p_k^{\text{out}}}\right), \ \forall u \in \mathcal{U}_c^{\text{out}}, \ \forall c \in \mathcal{C}, \tag{13c}$$

$$\beta^{\text{in}} + \beta^{\text{out}} \leq 1, \tag{13d}$$

$$p_c^{\text{in}} + p_c^{\text{out}} \leq P_c^{\max}, \ \forall c \in \mathcal{C}, \tag{13e}$$

where β^{in}, β^{out}, t^{in} and t^{out} are the normalized bandwidths and minimum rates allocated to inner and outer users, respectively. The subscripts u and c denote the user and cell, the calligraphies $\mathcal{U}$ and $\mathcal{C}$ denote the set of users and the set of cells. Furthermore, $n_u^{\text{in}} = N_{0u}/G_u^{\text{in}}$ and $g_{ku}^{\text{in}} = G_{ku}^{\text{in}}/G_u^{\text{in}}$ are the normalized noise and the normalized interference channel large scale path-loss attenuation for the inner users, respectively. Similar normalization is considered for the outer users.

Proposition 1: The max-min-rate problem (13) can be transformed into a convex optimization problem.

Proof: We begin by exchanging the objective in (13a) by its logarithm which notably does not change the optimal variables. Introducing several variable transformations $\tilde{t}^{\text{in}} = \log(e^{t^{\text{in}}} - 1)$, $\tilde{t}^{\text{out}} = \log(e^{t^{\text{out}}} - 1)$, $\tilde{\beta}^{\text{in}} = \log(\beta^{\text{in}})$, $\tilde{\beta}^{\text{out}} = \log(\beta^{\text{out}})$, $\tilde{p}_c^{\text{in}} = \log(p_c^{\text{in}})$ and $\tilde{p}_c^{\text{out}} = \log(p_c^{\text{out}})$, problem (13) can be written in the form

$$\underset{\tilde{\mathbf{p}},\tilde{\beta}^{\text{in}},\tilde{\beta}^{\text{out}},\tilde{t}^{\text{in}},\tilde{t}^{\text{out}}}{\text{maximize}} \quad \min\{\log(\log(e^{\tilde{t}^{\text{in}}}+1)) + \tilde{\beta}^{\text{in}} + \log(\log(2)), \log(\log(e^{\tilde{t}^{\text{out}}}+1)) + \tilde{\beta}^{\text{out}} + \log(\log(2))\} \tag{14a}$$

subject to

$$\tag{14b}$$

$$\log\Big(e^{\tilde{t}^{\text{in}}+\tilde{\beta}^{\text{in}}+\log(n_u^{\text{in}})-\tilde{p}_c^{\text{in}}}+\sum_{k\in\mathcal{C}\setminus c}e^{\tilde{t}^{\text{in}}+\log(g_{ku}^{\text{in}})+\tilde{p}_k^{\text{in}}-\tilde{p}_c^{\text{in}}}\Big)\leq 0,\quad \forall u\in\mathcal{U}_c^{\text{in}},\forall c\in\mathcal{C},\tag{15a}$$

$$\log\Big(e^{\tilde{t}^{\text{out}}+\tilde{\beta}^{\text{out}}+\log(n_u^{\text{out}})-\tilde{p}_c^{\text{out}}}+\sum_{k\in\mathcal{C}\setminus c}e^{\tilde{t}^{\text{out}}+\log(g_{ku}^{\text{out}})+\tilde{p}_k^{\text{out}}-\tilde{p}_c^{\text{out}}}\Big)\leq 0,\quad \forall u\in\mathcal{U}_c^{\text{in}},\forall c\in\mathcal{C},\tag{15b}$$

$$\log\left(e^{\tilde{\beta}^{\text{in}}}+e^{\tilde{\beta}^{\text{out}}}\right)\leq 0,\tag{15c}$$

$$\log\left(e^{\tilde{p}_c^{\text{in}}}+e^{\tilde{p}_c^{\text{out}}}\right)-\log(P_c^{\max})\leq 0,\quad \forall c\in\mathcal{C},\tag{15d}$$

where in constraints (14b)-(15d) we additionally took the logarithm of both sides of the inequalities. Convexity of all constraints follows from the convexity of the log-sum-exp function [11, p. 74]. Concavity of the objective (14a) follows from $\partial^2/\partial x^2(\log(\log(e^x+1)))\geq 0$ and the fact that the point-wise minimum of concave functions is concave [11, p. 81]. □

B. Sum-Power Minimization

The problem of minimizing the sum-power used by all cells in the network can be written as

$$\underset{\beta^{\text{in}},\beta^{\text{out}},t_u^{\text{in}},t_u^{\text{out}}\in\mathcal{R}_+,\forall u\in\mathcal{U}^{\text{in}}\cup\mathcal{U}^{\text{out}},c\in\mathcal{C},\mathbf{p}\succeq\mathbf{0}}{\text{minimize}}\quad \sum_{c\in\mathcal{C}}p_c^{\text{in}}+\sum_{c\in\mathcal{C}}p_c^{\text{out}}\tag{16a}$$

subject to

$$t_u^{\text{in}}\geq R_u^{\text{tgt}}/(\beta^{\text{in}}\log(2)),\ \forall u\in\mathcal{U}_c^{\text{in}},\forall c\in\mathcal{C},\tag{16b}$$

$$t_u^{\text{out}}\geq R_u^{\text{tgt}}/(\beta^{\text{out}}\log(2)),\ \forall u\in\mathcal{U}_c^{\text{out}},\forall c\in\mathcal{C},\tag{16c}$$

$$t_u^{\text{in}}\leq\log\left(1+\frac{p_c^{\text{in}}}{n_u^{\text{in}}\beta^{\text{in}}+\sum_{k\in\mathcal{C}\setminus c}g_{ku}^{\text{in}}p_k^{\text{in}}}\right),\ \forall u\in\mathcal{U}_c^{\text{in}},\ \forall c\in\mathcal{C},\tag{16d}$$

$$t_u^{\text{out}}\leq\log\left(1+\frac{p_c^{\text{out}}}{n_u^{\text{out}}\beta^{\text{out}}+\sum_{k\in\mathcal{C}\setminus c}g_{ku}^{\text{out}}p_k^{\text{out}}}\right),\ \forall u\in\mathcal{U}_c^{\text{out}},\ \forall c\in\mathcal{C},\tag{16e}$$

$$\beta^{\text{in}}+\beta^{\text{out}}\leq 1,\tag{16f}$$

$$p_c^{\text{in}}+p_c^{\text{out}}\leq P_c^{\max},\ \forall c\in\mathcal{C},\tag{16g}$$

where R_u^{tgt} is the minimum target rate of a user.

Proposition 2: The sum-power minimization problem (16) can be transformed into a convex optimization problem.

Proof: Similarly to above we make the variable transformations $\tilde{t}_u^{\text{in}}=\log(e^{t_u^{\text{in}}}-1)$, $\forall u\in\mathcal{U}^{\text{in}}$, $\tilde{t}_u^{\text{out}}=\log(e^{t_u^{\text{out}}}-1)$, $\forall u\in\mathcal{U}^{\text{out}}$, $\tilde{\beta}^{\text{in}}=\log(\beta^{\text{in}})$, $\tilde{\beta}^{\text{in}}=\log(\beta^{\text{out}})$, $\tilde{p}_c^{\text{in}}=\log(p_c^{\text{in}})$ and $\tilde{p}_c^{\text{out}}=\log(p_c^{\text{out}})$, $\forall c\in\mathcal{C}$. Again taking the logarithm on both sides of all constraints, constraints (16d) - (16g) become convex similarly as in the max-min-rate problem above. Constraint (16b), similarly as constraint (16c), becomes

$$\log(\log(e^{\tilde{t}_u^{\text{in}}}+1))\geq\log(R_u^{\text{tgt}})-\tilde{\beta}^{\text{in}}-\log(2),\quad \forall u\in\mathcal{U}_c^{\text{in}},\forall c\in\mathcal{C},\tag{17}$$

which is convex as $\log(\log(e^x+1))$ is a convex function. The convexity of the transformed problem follows then together with the convexity of the transformed objective. □

VI. Conclusions

In this paper we formulated the sum-rate maximization problem for partial frequency reuse networks. By applying dual decomposition techniques, we efficiently solved this constrained optimization problem when assuming that all cells use equal power to serve the inner users. From our simulation results we see that the proposed algorithm allocates the power to the users dynamically and additionally reduces inter-cell interference. Furthermore, we demonstrated that the minimum rate maximization and sum-power minimization problems under joint power and bandwidth allocation can be transformed to convex optimization problems and solved efficiently using state-of-the-art convex optimization methods.

Acknowledgments

The authors would like to thank Markus Rupp and Martin Wrulich for their fruitful comments. This work has been partially funded by the Christian Doppler Laboratory for Wireless Technologies for Sustainable Mobility, the Austrian Government and the City of Vienna within the competence center program COMET. The financial support by the Federal Ministry of Economy, Family and Youth and the National Foundation for Research, Technology and Development is gratefully acknowledged.

References

[1] M. Chiang, C. W. Tan, D. Palomar, D. O'Neill, and D. Julian, "Power control by geometric programming," *IEEE Transactions on Wireless Communications*, vol. 6, no. 7, pp. 2640 – 2651, July 2007.

[2] S. Boyd, S.-J. Kim, L. Vandenberghe, and A. Hassibi, "A tutorial on geometric programming," *Optimization and Engineering*, vol. 8, no. 1, pp. 67–127, March 2007.

[3] H. Holma and A. Toskala, *LTE for UMTS OFDMA and SC-FDMA Based Radio Access*. John Wiley & Sons, Ltd, 2009.

[4] E. Dahlman, S. Parval, J. Skold, and P. Beming, *3G Evolution, HSDPA and LTE for Mobile Broadband*. Elsevier, 2007.

[5] Y. Xiang, J. Luo, and C. Hartman, "Inter-cell interference mitigation through flexible resource reuse in OFDMA based communication networks," in *Proc. European Wireless 2007*, 2007.

[6] A. Alsawah and I. Fialkow, "Optimal frequency-reuse partitioning for ubiquitous coverage in cellular systems," in *16th European Signal Processing Conference - publi-etis.ensea.fr*, 2008.

[7] B. Krasniqi, M. Wrulich, and C. Mecklenbräuker, "Network-load dependent partial frequency reuse for LTE," in *Proc.9th International Symposium on Communications and Information Technology, 2009. ISCIT*, Sept. 2009, pp. 672 –676.

[8] A. Gjendemsjo, D. Gesbert, G. Oien, and S. Kiani, "Optimal power allocation and scheduling for two-cell capacity maximization," in *Proc. 4th International Symposium on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks*, april 2006, pp. 1 – 6.

[9] M. Charafeddine and A. Paulraj, "2-sector interference channel communication for sum rates and minimum rate maximization," in *Proc. 43rd Annual Conference on Information Sciences and Systems, 2009. CISS 2009.*, March 2009, pp. 951 –956.

[10] M. Rahman and H. Yanikomeroglu, "Inter-cell interference coordination in ofdma networks: A novel approach based on integer programming," in *Proc. IEEE 71st Vehicular Technology Conf. (VTC 2010-Spring)*, 2010, pp. 1–5.

[11] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[12] S. C. Chapra and R. P. Canale, *Numerical Methods for Engineers*. McGraw-Hill, 2009.