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Weighted Sum-Rate Maximization for Two Users in Partial Frequency Reuse Cellular Networks

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***Abstract*—In this paper we apply constrained optimization techniques to optimally allocate bandwidth and transmit power to the users in a cellular network. We utilize partial frequency reuse with multiple users in the full and partial frequency regions as inter-cell interference mitigation technique. We show that the non-convex sum-rate maximization problem becomes convex under some simplifying assumptions. Moreover, an efficient and problem specific algorithm is developed to solve the problem for a fixed bandwidth allocation. Our results show that in the optimum, the full amount of power is assigned to the users. We further demonstrate that re-allocating the initially outer bandwidth to the inner bandwidth while keeping the power allocation unchanged, results in an increased sum-rate.**

I. Introduction

In cellular networks, the sum-rate maximization of all users in all cells under variable power and bandwidth allocation to full and partial frequency reuse users is a non-convex optimization problem. However, in [1] the single-carrier power control problems of maximizing the minimum rate among users and minimizing the total sum-power were found to be transformable to geometric [2] and hence convex optimization problems, notably without any high-SINR approximation.

Next generation mobile communication systems use Orthogonal Frequency Division Multiple Access (OFDMA) as their modulation scheme in the downlink [3], [4]. Since cell edge users may suffer severely from Inter-Cell Interference (ICI), several schemes have been proposed for ICI mitigation. One of those schemes is Partial Frequency Reuse (PFR), which is applied for example in [5], [6], [7].

The characteristics of the optimal power allocation for two base stations, employing also scheduling schemes, has been studied in [8] under frequency reuse-1. Additionally to the sum-rate maximization power control problem, in [9] the authors also investigate the maximization of the minimum rate for two users. The maximization of the sum-rate, the maximization of the minimum rate, and the sum-power minimization for multiple users in PFR are studied in [10]. To the best of our knowledge, there are currently no studies which consider improvement in sum-rate maximization by partial bandwidth re-allocation.

Our contributions can be summarized as follows. In Section II we show the system model including the bandwidth allocation scheme for PFR. In Section III we study the sum-rate maximization problem for PFR systems considering inter-cell interference and both, power and bandwidth allocation between PFR and Full Frequency Reuse (FFR) users. Instead of the high-SINR approximation applied in [1] we utilize as simplifying assumption equal power allocations for the FFR users in all cells in order to arrive at a convex optimization problem. For a fixed bandwidth allocation we even derive a simple water-filling-like power allocation algorithm. In [5],[11] the authors have mentioned that the cell edge bandwidth can be re-used as cell center bandwidth whenever the cell edge user is idle. A study about the utilization of the cell edge (outer) bandwidth as cell center (inner) bandwidth considering the user density is done in [7]. In this study the authors have shown that only an amount of cell edge bandwidth can be reused as cell center bandwidth as the result of optimizing over the optimal frequency partitioning radius. In Section IV we show that almost all of the cell outer bandwidth can be re-allocated as cell inner bandwidth whenever the outer user is idle. Furthermore, we present in Section V simulation results which confirm the rate gains by applying the proposed suboptimal allocation scheme. The simulation results show that re-allocation of the outer bandwidth increases the rate of the inner user and also the total sum-rate. Conclusions are drawn in Section VI.

II. System Model

In our system we consider one user who is located in the inner region of the cell (the full frequency reuse region) and one user located in the outer region of the cell (the partial frequency reuse region), as indicated in Fig. 1. Up to a radius r, each cell utilizes frequency reuse-1 in all sectors. From that radius r to the border of the neighbor cell, frequency reuse-3 is utilized. Based on the users' received SINRs, a scheduler decides whether a user is considered an inner user or an outer user. The frequency pattern applied in our system model is shown Fig. 2. The user who is located in the inner region of the cell receives power from its own base station BS_0 and also interference from the six neighboring base stations BS_k, $k = 1 \ldots 6$. More distant base stations are not considered in our system model but all our results can be easily extended to

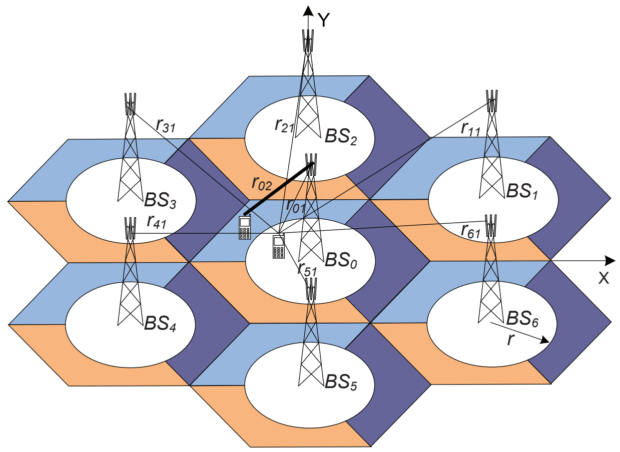

Fig. 1. Partial frequency reuse cell cluster

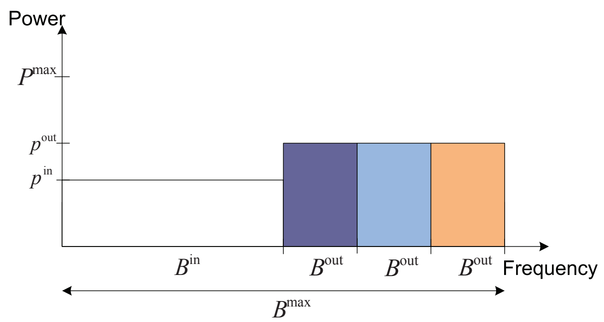

Fig. 2. Frequency reuse pattern

consider also interference from non-neighboring base stations. The sum-rate which can be achieved by the user in the inner region is given by

$$R^{\text{in}} = B^{\text{in}} \log_2 \left(1 + \frac{G_0^{\text{in}} p_0^{\text{in}}}{N_0 B^{\text{in}} + \sum_{k=1}^{6} G_k^{\text{in}} p_k^{\text{in}}} \right), \tag{1}$$

where B^{in} is the bandwidth utilized in the inner region and N_0 is the noise spectral density. The pathloss attenuations of direct and interference channels are defined by the pathloss exponent model [6] and given by G_{0n}^{in} and G_{kn}^{in}. The transmit power assigned to the user in the inner region is denoted by p_0^{in} and the interference power from the other base stations is denoted by p_k^{in}, $k = 1 \ldots 6$, with k denoting the index of the interfering base stations. The user who is located in the outer region of the cell is considered to be interference free because of the PFR employed in this region. The transmit power assigned to the user in the outer region is denoted by p_0^{out}. Thus, the sum-rate achieved by the user in the outer region is given by

$$R^{\text{out}} = B^{\text{out}} \log_2 \left(1 + \frac{G_0^{\text{out}} p_0^{\text{out}}}{N_0 B^{\text{out}}} \right) \tag{2}$$

where B^{out} denotes the bandwidth utilized in the outer region and G_0^{out} denotes the pathloss attenuation for the direct channel of the outer user.

III. Efficient Algorithms for Weighted Sum-Rate Maximization

The sum-rate maximization problem is non-convex as it contains the sum-rate maximization in standard power control as a special case [1]. Under unequal allocation of the interference power p_k^{in}, $k = 1 \ldots 6$ and the power p_0^{in}, but for a fixed bandwidth allocation B^{in} it can still be solved efficiently by geometric programming under a high-SINR approximation $\log(1+\text{SINR}) \approx \log(\text{SINR})$, or sequentially approximated by geometric programs, *cf.* [1]. Differently, under the simplifying assumption that all cells use equal power $p_k^{\text{in}} = p_0^{\text{in}}$, $k = 1 \ldots 6$ to serve the inner users we will show in this section that the sum-rate maximization problem becomes convex and is solvable in a water-filling-like manner. While for simplicity of notation we only consider users located in a single cell, all presented problems and algorithms can be extended to the case of multiple users and multiple cells. As a result of the simplification in terms of equal transmit power for all base stations in the inner region, the sum-rate of the user in this region is given by

$$R^{\text{in}} = B^{\text{in}} \log_2 \left(1 + \frac{G_0^{\text{in}} p_0^{\text{in}}}{N_0 B^{\text{in}} + \sum_{k=1}^{6} G_k^{\text{in}} p_0^{\text{in}}} \right) \tag{3}$$

The optimization problem is written in the following form

$$\underset{\mathbf{p},\mathbf{B}}{\text{maximize}} \quad w^{\text{in}} R^{\text{in}} + w^{\text{out}} R^{\text{out}} \tag{4a}$$

subject to

$$\mathbf{A} \cdot \begin{bmatrix} \mathbf{p} \\ \mathbf{b} \end{bmatrix} \preceq \mathbf{c}, \tag{4b}$$

$$\mathbf{p} \succeq \mathbf{0}, \tag{4c}$$

$$\mathbf{b} \succeq \mathbf{0} \tag{4d}$$

where $\preceq$ denotes a component-wise inequality and we define

$$\mathbf{p} = [p_0^{\text{in}}, p_0^{\text{out}}]^T,$$
$$\mathbf{b} = [B^{\text{in}}, B^{\text{out}}]^T,$$
$$\mathbf{c} = [P^{\max}, B^{\max}]^T,$$
$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \end{bmatrix}.$$

The individual weights for the inner user and the outer users are denoted by w^{in} and w^{out}. The maximum power and maximum bandwidth of the base station are denoted by $P^{\max}$ and $B^{\max}$. In the optimization problem (4), the constraints are linear and hence convex. In order to show concavity of the objective we investigate the second derivative of R^{in} with respect to p_0^{in}:

$$\frac{\partial^2 R^{\text{in}}}{\partial (p_0^{\text{in}})^2} = -\frac{B^{\text{in}}}{\log(2)} \frac{1}{\left(\frac{N_0 B^{\text{in}}}{G_0^{\text{in}} + \sum_{k=1}^{6} G_k^{\text{in}}} + p_0^{\text{in}} \right)^2} + \frac{B^{\text{in}}}{\log(2)} \frac{1}{\left(\frac{N_0 B^{\text{in}}}{\sum_{k=1}^{6} G_k^{\text{in}}} + p_0^{\text{in}} \right)^2}. \tag{5}$$

The concavity of R^{in} holds since

$$\frac{\partial^2 R^{\text{in}}}{\partial (p_0^{\text{in}})^2} < 0, \tag{6}$$

which is the case due to $G_0^{\text{in}} > 0$. As a consequence we find that $\tilde{R}^{\text{in}}(B^{\text{in}}, p_0^{\text{in}}) = B^{\text{in}} R^{\text{in}}(p_0^{\text{in}}/B^{\text{in}})$ is concave as it is the perspective of a concave function [12]. Furthermore, since R^{out} is concave, the sum of concave functions is concave as well and the optimization problem (4) is therefore concave.

A. Water-filling-like power allocation

Although the optimization problem (4) is concave, deriving an analytic solution was found to be intractable. However, for constant bandwidth allocation we derive in the following a power allocation algorithm based on the Karush-Kuhn-Tucker (KKT) optimality conditions [12]. For simplifying the written equations we are substituting $G_0^{\text{in}} = a$, $\sum_{k=1}^{6} G_k^{\text{in}} = b$ and $G_0^{\text{out}} = d$. The Lagrangian for problem (4) is written as

$$\begin{aligned} L(\mathbf{p}, \mu, \boldsymbol{\lambda}) =& w^{\text{in}} R^{\text{in}} + w^{\text{out}} R^{\text{out}} \\ & - \mu(\mathbf{1}^T \mathbf{p} - P^{\max}) + \boldsymbol{\lambda}^T \mathbf{p} \end{aligned} \tag{7}$$

where μ and $\boldsymbol{\lambda} = [\lambda^{\text{in}}, \lambda^{\text{out}}]$ are the Lagrange multipliers for the sum-power constraint and the positivity constraint, respectively. Applying the KKT conditions [12] we have:

$$\mathbf{p} \succeq \mathbf{0}, \tag{8a}$$

$$\mathbf{1}^T \mathbf{p} - P^{\max} \leq 0, \tag{8b}$$

$$\boldsymbol{\lambda} \succeq \mathbf{0}, \tag{8c}$$

$$\lambda^{\text{in}} p^{\text{in}} = 0, \tag{8d}$$

$$\lambda^{\text{out}} p^{\text{out}} = 0, \tag{8e}$$

$$\begin{aligned} \frac{\partial L}{\partial p_0^{\text{in}}} =& -w^{\text{in}} \frac{B^{\text{in}}}{\log(2)} \frac{a N_0 B^{\text{in}}}{\left[N_0 B^{\text{in}} + (a+b) p_0^{\text{in}}\right]} \\ & \cdot \frac{1}{(N_0 B^{\text{in}} + b_n p_0^{\text{in}})} + \mu - \lambda^{\text{in}} \end{aligned} \tag{8f}$$

$$\begin{aligned} \frac{\partial L}{\partial p_0^{\text{out}}} =& -w^{\text{out}} \frac{B^{\text{out}}}{\log(2)} \frac{d}{(N_0 B^{\text{out}} + d p_0^{\text{out}})} + \\ & + \mu - \lambda^{\text{out}} \end{aligned} \tag{8g}$$

The last two equations (8f) and (8g) in the KKT conditions stand for the first derivative of the Lagrangian given by Equation (7) with respect to p_0^{in} and p_0^{out}, respectively.

We continue to show that for a fixed variable μ, the optimal power allocation can be computed efficiently. Combining the positivity constraints (8c) with (8f) and the complementary slackness constraints (8d), we find the optimum p_0^{in} as a function of μ. Similarly, using (8c), (8g) and (8e) we find the optimum p_0^{out} as a function of μ. Using $\bar{\mu}^{\text{out}} = \frac{N_0 \log(2)}{d w^{\text{out}}}$ and $\bar{\mu}^{\text{in}} = \frac{N_0 \log(2)}{a w^{\text{in}}}$, the analytical solution for the power in the inner region is given by equation

$$p_0^{\text{in}} = \begin{cases} \frac{-(a+2b) N_0 B^{\text{in}} + \sqrt{\Delta}}{2(a+b)b} & ; \text{if } \frac{1}{\mu} \geq \bar{\mu}^{\text{in}}, \\ 0 & ; \text{otherwise}, \end{cases} \tag{9}$$

where Δ under the square root in Equation (9) is given by

$$\Delta = (a N_0 B^{\text{in}})^2 + 4ab\,(a+b)\,\frac{w^{\text{in}} N_0 (B^{\text{in}})^2}{\mu \log(2)}.$$

The optimal assigned power from a user in the outer region is analogously given by

$$p_0^{\text{out}} = \begin{cases} \frac{w^{\text{out}} B^{\text{out}}}{\mu \log(2)} - \frac{N_0 B^{\text{out}}}{d} & ; \text{if } \frac{1}{\mu} \geq \bar{\mu}^{\text{out}}, \\ 0 & ; \text{otherwise}, \end{cases} \tag{10}$$

We search for the optimal water-level $1/\mu$ by using a simple bisection search due to the nondifferentiability of the Lagrangian. Theoretically we know that in the optimum, Equation (8b) holds with equality. This follows as the rate of the PFR-users R^{out} is monotonously increasing in p_0^{out}.

IV. Bandwidth Re-allocation Scheme

In this section we explain the bandwidth re-allocation scheme utilized. The bandwidth re-allocation scheme consists of using the outer bandwidth as inner bandwidth when the outer user is idle. The way of re-allocation the outer bandwidth and using it as inner bandwidth is shown in Fig. 3. The

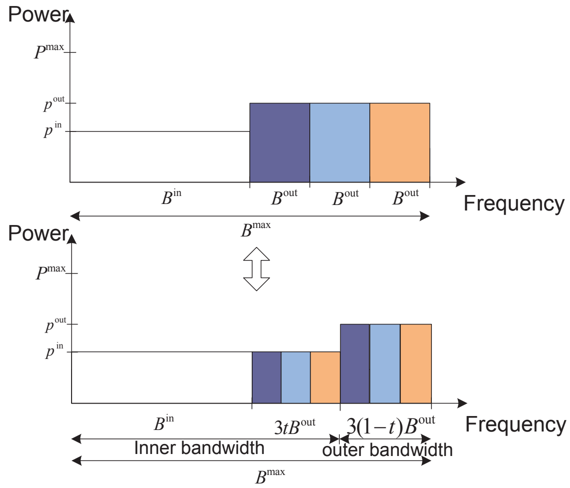

Fig. 3. Partial bandwidth re-allocation for inner user

parameter t describes how much of outer bandwidth is re-allocated to be used as inner bandwidth. In order to account for the bandwidth re-allocation we modify Equation (3) for the inner user rate as follows:

$$\begin{aligned} R^{\text{in}} =& B^{\text{in}} \log_2 \left(1 + \frac{G_0^{\text{in}} p_0^{\text{in}}}{N_0 B^{\text{in}} + \sum_{k=1}^{6} G_k^{\text{in}} p_0^{\text{in}}}\right) + \\ & + t B^{\text{out}} \log_2 \left(1 + \frac{G_0^{\text{in}} p_0^{\text{in}}}{N_0 t B^{\text{out}}}\right) \end{aligned} \tag{11}$$

Equation (2) for the rate of the outer user is modified accordingly:

$$R^{\text{out}} = (1-t) B^{\text{out}} \log_2 \left(1 + \frac{G_0^{\text{out}} p_0^{\text{out}}}{N_0 (1-t) B^{\text{out}}}\right) \tag{12}$$

From Equation (11), we see that all the outer bandwidth can be re-used as inner bandwidth. Note that if $t = 1$ than $R^{out} = 0$.

V. Simulation Results

In this section, we show simulation results carried out for two users, one located in the inner region of cell and the other one located in the outer region of cell. The parameters used for the simulations are shown in Table I. Using Equations (9) and (10) while searching for the optimal water-level $\frac{1}{\mu}$ through bisection search, we have simulated the optimal power assignment for the inner user and the outer user. The optimal power assignment for the inner user and the outer user is shown in Fig. 4. When the maximum base station power is very low, all the power is assigned to the inner user. Until a maximum base station power of 5.6 W, more power is assigned to the inner user than to the outer user. For maximum base station power higher than 5.6 W, more power is assigned to the outer user than to the inner user. Assigning more power to the outer user when the maximum base station power is higher, contributes in reducing the ICI and increasing the maximum sum-rate. The individual transmission rates of the inner and the outer user for different values of maximum base station power are shown in Fig. 5. From the simulation results shown in Fig. 5, we see that by increasing the maximum transmit power of the base station, also the rates of the users are increased. We also see that the rate-regions for different values of maximum base station power are concave.

TABLE I
Simulation Parameters

parameters	**value**
Maximum base station power $P^{\max}$	20 W
Maximum base station bandwidth $B^{\max}$	20 MHz
Noise spectral density N_0	-174 dBm/Hz
Center frequency f	2.6 GHz
Pathloss exponent α	3.0
Bandwidth reallocation parameter t	$0 < t < 99\%$
Distance between two base stations	3000 m
Position of inner user in polar coordinates	$(900\text{ m}, 180^0)$
Position of outer user in polar coordinates	$(1400\text{ m}, 160^0)$

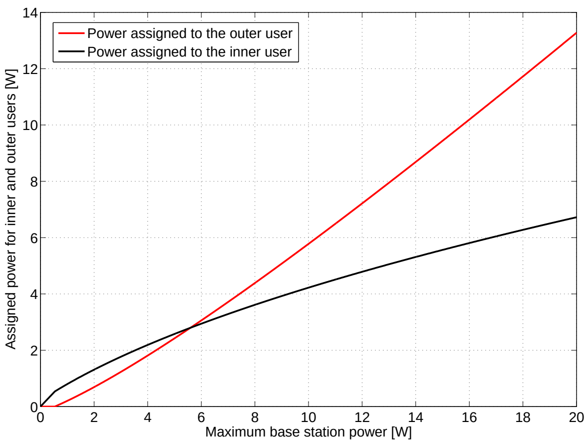

Fig. 4. Optimal power assignment to the inner and outer users

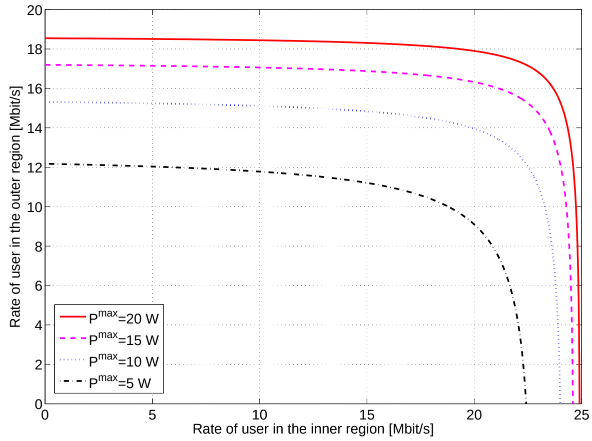

Fig. 5. Rate-regions for the inner and the outer users

This result also follows theoretically from the concavity of the rate functions in (2) and (3), respectively. As proved in Section III, the sum-rate optimization problem (4) is concave. A consequence of this is the concavity of the maximum sum-rate achieved by the inner and the outer user as a function of the maximum base station power, cf. Fig. 6. From Fig. 6 we see the sum-rate gains compared with a static power allocation of $p_0^{\text{in}} = 33\% P^{\max}, p_0^{\text{out}} = 67\% P^{\max}$ (corresponding to the optimum power allocation for $P^{\max} = 20\,\text{W}$). All the simulation results until now are done without considering outer bandwidth re-allocation, hence $t = 0$. In the following we keep the optimal power assigned to the inner user and outer user as it is shown in Fig. 4, and re-allocate the outer bandwith to the inner user by increasing the parameter t. The rate regions for the inner user and the outer user, for different percentage of outer bandwidth re-allocation are shown in Fig. 7.

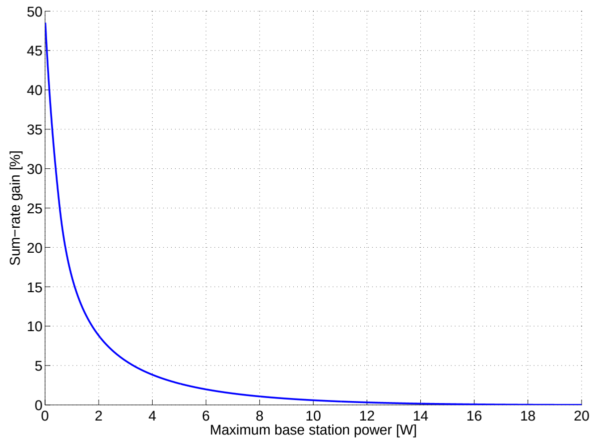

Fig. 6. Maximum sum-rate gain achieved by optimal power assignment

From the results shown in Fig. 7, we see that the rate of inner user is increased as the result of re-allocation of the outer bandwidth which is ICI-free to the inner user even keeping the

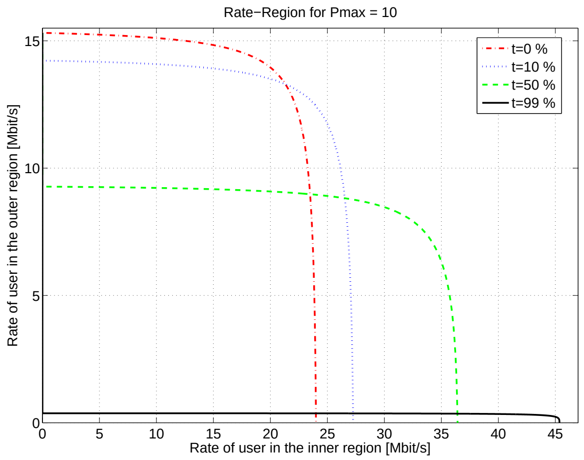

Fig. 7. Rate regions for constant base station power and bandwidth re-allocation

maximum base station power constant. In Fig. 8 is shown the maximum sum-rate versus the maximum base station power.

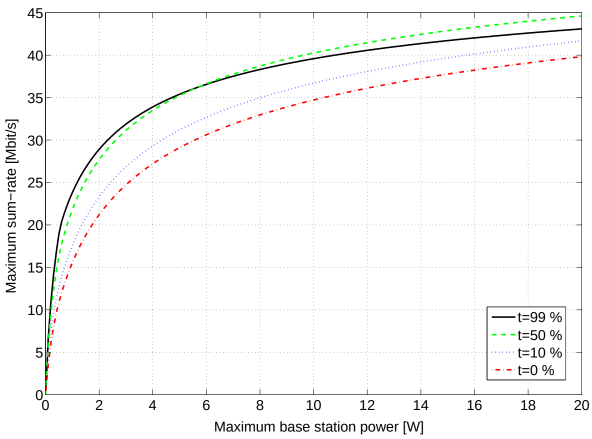

Fig. 8. Maximum sum-rate for the inner and the outer users

From the results shown in Fig. 8 we see the gain in the maximum sum-rate by outer bandwidth re-allocation.

VI. Conclusions

In this paper we formulated the sum-rate maximization problem for partial frequency reuse networks. By applying dual decomposition techniques, we efficiently solved this constrained optimization problem when assuming that all cells use equal power to serve the inner users. From our simulation results we see that the proposed algorithm allocates the power to the users dynamically and additionally reduces inter-cell interference. Furthermore, we demonstrated that it is possible that the outer bandwidth to be used as the inner bandwidth whenever the cell edge user is idle. This consists of increasing the transmission rate of the inner user and increasing also the total sum-rate.

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