Sum-Rate Maximization by Bandwidth Re-allocation for Two Users in Partial Frequency Reuse Cellular Networks

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***Abstract*—In this paper we apply constrained optimization techniques to optimally allocate bandwidth and transmit power to the users in a cellular network. We utilize partial frequency reuse with one user in the full and one user in the partial frequency reuse regions as inter-cell interference mitigation technique. We show that the non-convex sum-rate maximization problem becomes convex under some simplifying assumptions. Moreover, an efficient algorithm is used to solve the problem for a fixed bandwidth allocation. Our results show that in the optimum, the full amount of power is assigned to the users. We further demonstrate by simulations that re-allocating the outer cell bandwidth to the inner user when no outer user is present, results in a significantly increased sum-rate.**

I. Introduction

In cellular networks, the sum-rate maximization of all users in all cells under variable power and bandwidth allocation to full and partial frequency reuse users is a non-convex optimization problem. However, in [1] the single-carrier power control problems of maximizing the minimum rate among users and minimizing the total sum-power were found to be transformable to geometric problems [2] and hence convex optimization problems, notably without any high-SINR approximation.

Next generation mobile communication systems use Orthogonal Frequency Division Multiple Access (OFDMA) as their modulation scheme in the downlink [3], [4]. Since cell edge users may suffer severely from Inter-Cell Interference (ICI), several schemes have been proposed for ICI mitigation. One of those schemes is Partial Frequency Reuse (PFR), which is applied for example in [5], [6], [7].

The characteristics of the optimal power allocation for two base stations, employing also scheduling schemes, has been studied in [8] under frequency reuse-1. Additionally to the sum-rate maximization power control problem, in [9] the authors also investigate the maximization of the minimum rate for two users. To the best of our knowledge, there are currently no studies considering the maximization of the sum-rate by bandwidth re-allocation for two users in PFR.

Our contributions can be summarized as follows. In Section II we show the system model including the bandwidth allocation scheme for PFR. In Section III we study the sum-rate maximization problem for PFR systems considering inter-cell interference and both, power and bandwidth allocation between PFR and Full Frequency Reuse (FFR) users. Instead of the high-SINR approximation applied in [1] we utilize as simplifying assumption equal power and interference allocations for the FFR and PFR users in all cells in order to arrive at a convex optimization problem. For a fixed bandwidth allocation we even use a simple water-filling-like power allocation algorithm [10]. In [5], [11] the authors have mentioned that the cell edge bandwidth can be re-used as cell center bandwidth whenever the cell edge user is idle. A study about the utilization of the cell edge (outer) bandwidth as cell center (inner) bandwidth considering the user density is done in [7]. In this study the authors have shown how the cell edge bandwidth can be reused as cell center bandwidth while optimizing over the optimal frequency partitioning radius. In Section IV we show that almost all of the cell outer bandwidth can be re-allocated as cell inner bandwidth whenever we have only inner users active. Furthermore, we present in Section V simulation results which confirm the rate gains by applying the proposed suboptimal allocation scheme. The simulation results show that re-allocation of the outer bandwidth increases the rates of the inner users and also the maximum sum-rate. Conclusions are drawn in Section VI.

II. System Model

In our system we consider one user located in the inner region of the cell (the full frequency reuse region) and one user located in the outer region of the cell (the partial frequency reuse region), as indicated in Fig. 1. Based on the users' received SINRs, a scheduler can decide whether a user is considered an inner user or an outer user. The frequency pattern [5] applied in our system model is shown Fig. 2. The user who is located in the inner region of the cell, S_{01} receives power from its own sector antenna of base station

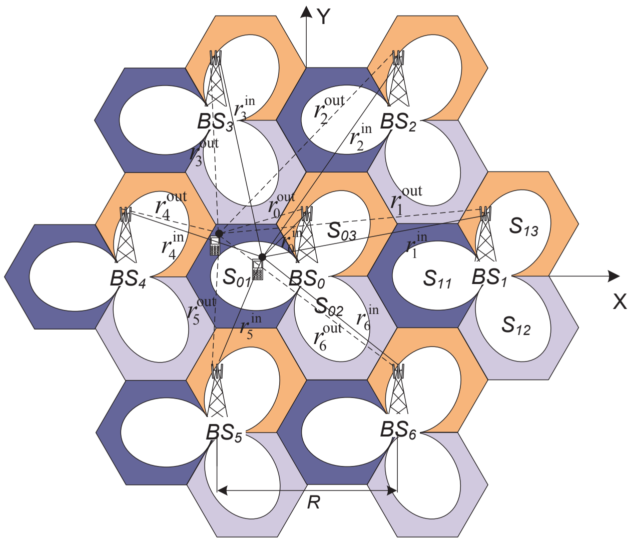

Fig. 1. Partial frequency reuse cell cluster

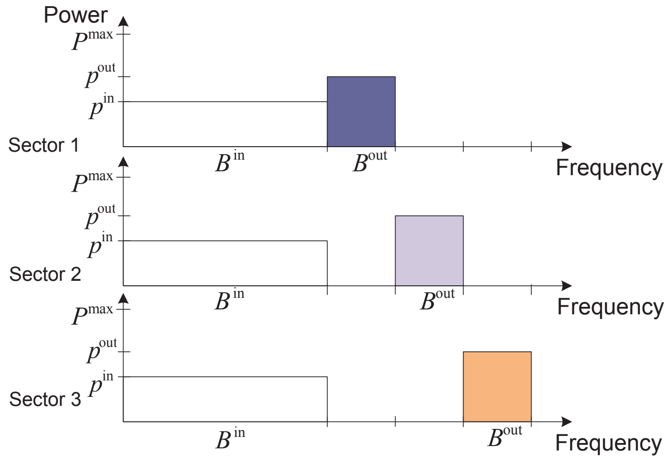

Fig. 2. Frequency reuse pattern

BS_0 and also interference from all other base stations BS_k, $k = 0 \ldots 6$. More distant base stations are not considered in our system model but all our results can be easily extended to consider also interference from non-neighboring base stations. The rate achieved by the user in the inner region is given by

$$R^{\text{in}} = B^{\text{in}} \log_2 \left(1 + \frac{G_0^{\text{in}} p_0^{\text{in}}}{N_0 B^{\text{in}} + \sum_{k=1}^{6} G_k^{\text{in}} p_k^{\text{in}}} \right), \quad (1)$$

where B^{in} is the bandwidth utilized in the inner region and N_0 is the noise spectral density. The large scale path-loss attenuation including antenna gain, penetration loss, shadowing and fast fading is expressed in the form [12],

$$G_k = 128.1 + 10\alpha \log_{10}(r_k) + A_k + L_p + X_\sigma + F \quad (2)$$

where G_k is in dB, α the path-loss exponent, r_k the distance between the mobile station and the base station BS_k in km, A_k the sum of user antenna gain and base station antenna gain in dB, L_p the penetration loss in dB, X_σ the log-normal shadowing in dB, and F the fast fading channel coefficient in dB. The antenna gain A_k is defined by a horizontal antenna pattern [12]. The large scale path-loss attenuation of directed channels G_0^{in} is defined by Equation (2). The large scale path-loss attenuation of interference channels G_k^{in} is also defined by Equation (2) except for the fast fading, which is not taken into account here. The transmit power assigned to the user in the inner region is denoted by p_0^{in} and the interference power from the other base stations is denoted by p_k^{in}, $k = 1 \ldots 6$, with k denoting the index of the interfering base stations. The user located in the outer region of the cell receive also interference from all non-neighboring sectors that use the same frequency band. The transmit power assigned to the user in the outer region is denoted by p_0^{out} and the interference power from the other base stations is denoted by p_k^{out}, $k = 1 \ldots 6$. Thus, the rate achieved by the user in the outer region is given by

$$R^{\text{out}} = B^{\text{out}} \log_2 \left(1 + \frac{G_0^{\text{out}} p_0^{\text{out}}}{N_0 B^{\text{out}} + \sum_{k=1}^{6} G_k^{\text{out}} p_k^{\text{out}}} \right) \quad (3)$$

where B^{out} denotes the bandwidth utilized in the outer region and G_0^{out} and G_k^{out} denotes the large scale path-loss attenuation for the direct and interference channels of the outer user.

III. Sum-Rate Maximization by Using an Efficient Algorithm

The sum-rate maximization problem is non-convex as it contains the sum-rate maximization in standard power control as a special case [1]. Under unequal allocation of the interference power p_k^{in}, $k = 1 \ldots 6$ and the power p_0^{in}, but for a fixed bandwidth allocation B^{in} it can still be solved efficiently by geometric programming under a high-SINR approximation $\log(1 + \mathrm{SINR}) \approx \log(\mathrm{SINR})$, or sequentially approximated by geometric programs, *cf.* [1]. Differently, under the simplifying assumption that all cells use equal power $p_k^{\text{in}} = p_0^{\text{in}}$ and $p_k^{\text{out}} = p_0^{\text{out}}$, $k = 1 \ldots 6$ to serve the inner user and the outer user, we will show in this section that the sum-rate maximization problem becomes convex and is solvable in a water-filling-like manner. As a result of the simplification in terms of equal transmit power for all base stations in the inner region, the rate of the user in this region is given by

$$R^{\text{in}} = B^{\text{in}} \log_2 \left(1 + \frac{G_0^{\text{in}} p_0^{\text{in}}}{N_0 B^{\text{in}} + \sum_{k=1}^{6} G_k^{\text{in}} p_0^{\text{in}}} \right) \quad (4)$$

Similarly the rate of outer user is simplified:

$$R^{\text{out}} = B^{\text{out}} \log_2 \left(1 + \frac{G_0^{\text{out}} p_0^{\text{out}}}{N_0 B^{\text{out}} + \sum_{k=1}^{6} G_k^{\text{out}} p_0^{\text{out}}} \right) \quad (5)$$

The optimization problem is written in the following form

$$\begin{aligned} &\underset{\mathbf{p},\mathbf{b}}{\text{maximize}} \quad R^{\text{in}} + R^{\text{out}} && \text{(6a)}\\ &\text{subject to} \\ &\qquad B^{\text{in}} + B^{\text{out}} \leq B^{\max}, && \text{(6b)}\\ &\qquad p_0^{\text{in}} + p_0^{\text{out}} = P^{\max}, && \text{(6c)}\\ &\qquad \mathbf{p} \succeq \mathbf{0} && \text{(6d)}\\ &\qquad \mathbf{b} \succeq \mathbf{0} && \text{(6e)} \end{aligned}$$

where $\succeq$ denotes a component-wise inequality and we define

$$\begin{aligned} \mathbf{p} &= [p_0^{\text{in}}, p_0^{\text{out}}]^T, \\ \mathbf{b} &= [B^{\text{in}}, B^{\text{out}}]^T. \end{aligned} \quad (7)$$

The maximum power and maximum bandwidth of the base station are denoted by $P^{\max}$ and $B^{\max}$. In the optimization problem (6), the constraints are linear and hence convex. In order to show concavity of the objective we investigate the second derivative of $R^{\text{in}}(p_0^{\text{in}})$ with respect to p_0^{in} for constant bandwidth $B^{\text{in}} = 1$:

$$\begin{aligned} \frac{\partial^2 R^{\text{in}}(p_0^{\text{in}})}{\partial (p_0^{\text{in}})^2} =& -\frac{1}{\log(2)} \frac{1}{\left(\frac{N_0}{G_0^{\text{in}} + \sum_{k=1}^{6} G_k^{\text{in}}} + p_0^{\text{in}}\right)^2} \\ &+ \frac{1}{\log(2)} \frac{1}{\left(\frac{N_0}{\sum_{k=1}^{6} G_k^{\text{in}}} + p_0^{\text{in}}\right)^2}. \end{aligned} \quad (8)$$

The concavity of $R^{\text{in}}(p_0^{\text{in}})$ holds since

$$\frac{\partial^2 R^{\text{in}}(p_0^{\text{in}})}{\partial (p_0^{\text{in}})^2} < 0, \quad (9)$$

which is the case due to $G_k^{\text{in}} > 0,\ k = 0...6$. As a consequence we find that $R^{\text{in}}(B^{\text{in}}, p_0^{\text{in}}) = B^{\text{in}} R^{\text{in}}(p_0^{\text{in}}/B^{\text{in}})$ is concave as it is the perspective of a concave function [13]. Furthermore, since R^{out} is concave because it has a similar form as R^{in}, the sum of concave functions is concave as well and the optimization problem (6) is therefore concave.

A. Water-filling-like power allocation

Although the optimization problem (6) is concave, deriving an analytic solution was found to be intractable. However, for constant bandwidth allocation a derivation of the power allocation algorithm based on the Karush-Kuhn-Tucker (KKT) optimality conditions [13] is done in [10]. For simplifying the written equations we are substituting $G_0^{\text{in}} = a$, $\sum_{k=1}^{6} G_k^{\text{in}} = b$, $G_0^{\text{out}} = d$ and $\sum_{k=1}^{6} G_k^{\text{out}} = e$. Based on the results for the Lagrangian and KKT conditions derived in [10] we have the following analytical expression for inner user power assignment

$$p_0^{\text{in}} = \begin{cases} \frac{-(a+2b)N_0 B^{\text{in}} + \sqrt{\Delta^{\text{in}}}}{2(a+b)b}, & \text{if } \frac{1}{\mu} \geq \frac{N_0 \log(2)}{a}, \\ 0, & \text{otherwise}, \end{cases} \quad (10)$$

where Δ^{in} under the square root in Equation (10) is given by

$$\Delta^{\text{in}} = (aN_0 B^{\text{in}})^2 + 4ab\,(a+b)\,\frac{N_0 (B^{\text{in}})^2}{\mu \log(2)}.$$

The optimal assigned power to the outer user is analogously given by

$$p_0^{\text{out}} = \begin{cases} \frac{-(d+2e)N_0 B^{\text{out}} + \sqrt{\Delta^{\text{out}}}}{2(d+e)e}, & \text{if } \frac{1}{\mu} \geq \frac{N_0 \log(2)}{d}, \\ 0, & \text{otherwise}, \end{cases} \quad (11)$$

where Δ^{out} under the square root in Equation (11) is given by

$$\Delta^{\text{out}} = (dN_0 B^{\text{out}})^2 + 4de\,(d+e)\,\frac{N_0 (B^{\text{out}})^2}{\mu \log(2)}.$$

where μ is the Lagrange multiplier [13]. For searching the optimal water-level $1/\mu$ we use a simple bisection search [14] due to the non-differentiability of the Lagrangian.

IV. Bandwidth Allocation Scheme

In this section we explain the bandwidth allocation scheme utilized. Considering the two user's position we have three cases of allocating the bandwidth among the users.

A. One user located in the inner cell region and one user located in the outer cell region

When one user is located in the inner region and the other user is located in the outer region, the bandwidth assignment to them is based on the frequency reuse pattern shown for sector S_{01} in Fig. 3. The inner bandwidth is denoted by B^{in} and the outer bandwidth is denoted by B^{out}. The optimal assigned power assignment to the inner user is calculated by using Equation (10) and for the outer user by Equation (11).

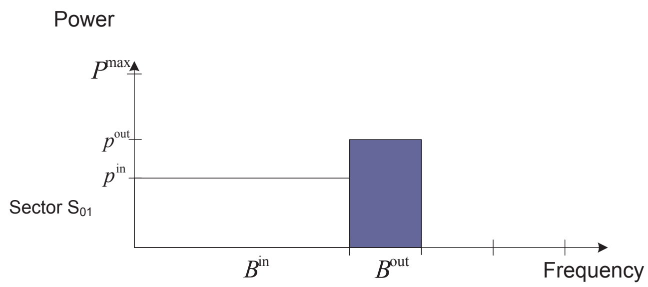

Fig. 3. Frequency reuse pattern for sector S_{01}

B. Both users are located in the inner cell region

When both users are located in the inner region of the cell, they have to share the inner bandwidth. The maximum base station power is assigned to both inner users. The transmission rate for User 1 is given as follows:

$$R_1^{\text{in}} = \frac{B^{\text{in}}}{2} \log_2 \left(1 + \frac{G_{01}^{\text{in}} P^{\max}}{N_0 \frac{B^{\text{in}}}{2} + \sum_{k=1}^{6} G_{k1}^{\text{in}} P^{\max}}\right) \quad (12)$$

and the transmission rate of the inner User 2 is:

$$R_2^{\text{in}} = \frac{B^{\text{in}}}{2} \log_2 \left(1 + \frac{G_{02}^{\text{in}} P^{\max}}{N_0 \frac{B^{\text{in}}}{2} + \sum_{k=1}^{6} G_{k2}^{\text{in}} P^{\max}}\right) \quad (13)$$

The large scale path-loss attenuations of direct channels G_{01}^{in}, G_{02}^{in} are defined by Equation (2). The large scale path-loss attenuation of interference channels G_{k1}^{in}, G_{k2}^{in} are defined also by Equation (2) with F=0 dB. The outer bandwidth is less interfered than the inner bandwidth because it is interfered only by non-neighboring sectors. We therefore re-allocate that bandwidth to the inner users. The bandwidth re-allocation scheme consists of using the outer bandwidth as inner bandwidth. The way of re-allocating the outer bandwidth and using it as inner bandwidth is shown in Fig. 4. The

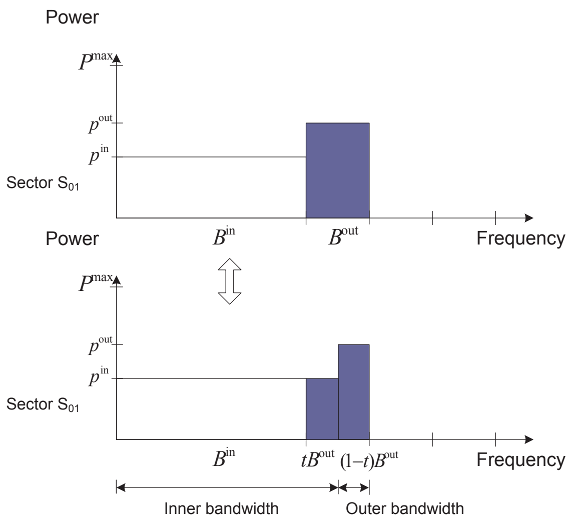

Fig. 4. Partial bandwidth re-allocation for inner user

parameter t describes how much of outer bandwidth is re-allocated to be used as inner bandwidth. In order to account for the bandwidth re-allocation we modify Equation (12) for the inner User 1 as follows:

$$R_1^{\text{in}} = \frac{B^{\text{in}}}{2}\log_2\left(1+\frac{G_{01}^{\text{in}}P^{\max}}{N_0\frac{B^{\text{in}}}{2}+\sum_{k=1}^{6}G_{k1}^{\text{in}}P^{\max}}\right)+ + t\frac{B^{\text{out}}}{2}\log_2\left(1+\frac{G_{01}^{\text{out}}P^{\max}}{N_0 t\frac{B^{\text{out}}}{2}+\sum_{k=1}^{6}G_{k1}^{\text{out}}P^{\max}}\right) \tag{14}$$

Equation (13) for the rate of the inner User 2 is modified accordingly:

$$R_2^{\text{in}} = \frac{B^{\text{in}}}{2}\log_2\left(1+\frac{G_{02}^{\text{in}}P^{\max}}{N_0\frac{B^{\text{in}}}{2}+\sum_{k=1}^{6}G_{k2}^{\text{in}}P^{\max}}\right)+ + t\frac{B^{\text{out}}}{2}\log_2\left(1+\frac{G_{02}^{\text{out}}P^{\max}}{N_0 t\frac{B^{\text{out}}}{2}+\sum_{k=1}^{6}G_{k2}^{\text{in}}P^{\max}}\right) \tag{15}$$

C. Both users located in the outer cell region

If both users are located in the outer region of the cell, they have to share the outer bandwidth from that sector. In this case, the inner bandwidth is not used at all and can not be re-used since this bandwidth experiences high interference at the cell edge. The transmission rate of the first outer user is:

$$R_1^{\text{out}} = \frac{B^{\text{out}}}{2}\log_2\left(1+\frac{G_{01}^{\text{out}}P^{\max}}{N_0\frac{B^{\text{out}}}{2}+\sum_{k=1}^{6}G_{k1}^{\text{out}}P^{\max}}\right) \tag{16}$$

The Equation for the transmission rate of the outer User 2 is:

$$R_2^{\text{out}} = \frac{B^{\text{out}}}{2}\log_2\left(1+\frac{G_{02}^{\text{out}}P^{\max}}{N_0\frac{B^{\text{out}}}{2}+\sum_{k=1}^{6}G_{k2}^{\text{out}}P^{\max}}\right) \tag{17}$$

The large scale path-loss attenuations of direct channels G_{01}^{out}, G_{02}^{out} are defined by Equation (2). The large scale path-loss attenuation of interference channels G_{k1}^{out}, G_{k2}^{out} are defined also by Equation (2).

V. Simulation Results

In this simulation we consider random user's positions. A realistic urban scenario is considered with its parameters shown in Table I.

TABLE I
SIMULATION PARAMETERS

parameters	**value**
Maximum base station power $P^{\max}$	5 W
Maximum base station bandwidth $B^{\max}$	20 MHz
Noise spectral density N_0	-174 dBm/Hz
Center frequency f	2.0 GHz
Pathloss exponent α	3.75
Penetration loss L_p	20 dB
Shadowing X_σ	$\mathcal{N}(0,8)$dB
Fast Fading F	$\mathcal{CN}(0,1)$dB,
Inter base station distance R	700 m
Number of user positions	100

The maximum average sum-rate simulation results for two users depending on their random locations are shown in Fig. 5. We have used the bandwidth allocation scheme explained in

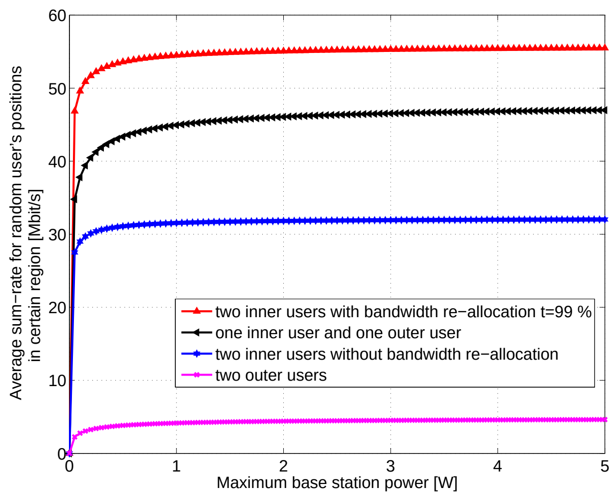

Fig. 5. Maximum sum-rate for random users positions

Section IV depending on the user positions. For each specific realization of user positions we have simulated 1000 channel realizations. For the specific simulation setup considered, the case that both users are located in the inner region occurs with a probability of 59%. The case when one user is located in the inner region and the other one located in the outer region occurs with a probability of 35%. The last case when both users are located in the outer region occurs with a probability of 6%. From the simulation results shown in Fig. 5 we see that the highest performance is achieved when we consider the bandwidth re-allocation to the inner users. The lowest performance is achieved when both users are located in the outer region since they are far from base stations and they have to share a smaller portion of bandwidth compared with the inner users. Without considering the re-allocation of the outer bandwidth to the inner users, the best performance is achieved by optimal power assignment when one user is located in the inner region and one user is located in the outer region. In Fig. 6 we show the simulation results for average sum-rate taken over random user's positions in the inner and outer regions. The lower curve represents the average of all sum-

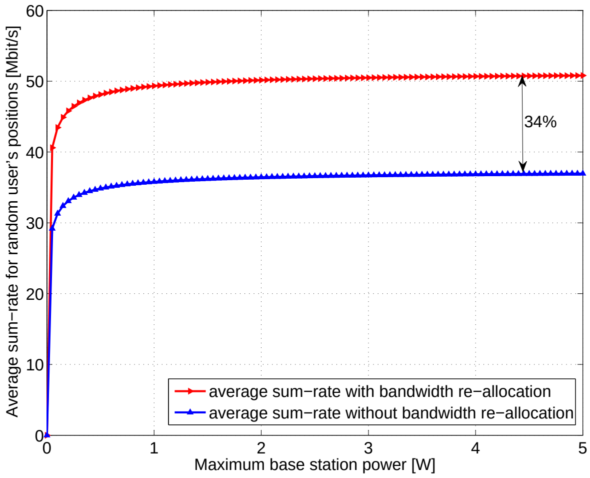

Fig. 6. Maximum average sum-rate for random users positions

rates when no re-allocation of the outer bandwidth to the inner users is carried out. A better performance in terms of average sum-rate is achieved when we consider the re-allocation of the outer bandwidth to the inner user. This is shown by the upper curve in Fig. 6. A performance increase of approximately 34% is achieved.

VI. Conclusions

In this paper we formulated the sum-rate maximization problem for two users in partial frequency reuse cellular networks. We showed the analytical expressions for power assignment to the inner and outer users under the assumption that all cells use equal power to serve the inner and outer users. Furthermore, we demonstrated that it is possible to re-use the outer bandwidth as the inner bandwidth whenever the cell edge user is idle. This results in an increase of transmission rates of the inner users and also of the total sum-rate by approximately 34%.

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