DISTRIBUTED AVERAGING IN WIRELESS SENSOR NETWORKS UNDER AN ALOHA-LIKE COMMUNICATION PROTOCOL

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Abstract — Distributed averaging is a fundamental tool for inference in wireless sensor networks; it builds on the exchange of messages between nodes to achieve consensus. Practical implementations of distributed averaging require a communication protocol that manages the message exchange. We propose a simple asynchronous ALOHA-like protocol that requires no node coordination and applies to any averaging scheme that tolerates link failures. We analytically characterize the collisions occurring with this protocol and we provide numerical performance results for asynchronous distributed averaging under the proposed protocol.

1. INTRODUCTION

1.1. Background

There are mainly three classes of algorithms for distributed averaging: randomized gossip [1], average consensus (AC) [2], and consensus propagation (CP) [3]. A recent comparison between AC and gossiping can be found in [4]. In wireless sensor networks (WSN), it is desirable to exploit the broadcast nature of the medium. While AC implicitly allows for broadcast transmissions, broadcast versions of gossip and CP have been presented in [5] and [6], respectively. While most existing work deals with static scenarios (one measurement per node), we are interested in the dynamic case with repeated measurements of a time-dependent quantity. A modification of CP for this case has been studied in [5]. A discrete-time dynamic AC algorithm has been proposed in [7].

The issue of medium access has received rather little attention in the literature on distributed averaging. A collision-free transmission protocol for WSN was introduces in [8]. Other works [9, 10] study the impact of link failures on AC, but these link failures are not tied to the network topology (similar results for CP apparently do not exist). A different approach is to study the SINR in random geometric graphs [11] and ensure error-free messages with appropriate channel coding.

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1.2. Contributions

In this paper, we investigate an asynchronous communication protocol for static and dynamic distributed averaging. Our specific contributions are:

- We propose an ALOHA-like communication protocol that is very simple to implement and thus highly attractive for WSN.
- We derive closed-form expressions and bounds quantifying the occurrence of message collisions with the proposed protocol.
- We compare the averaging performance of dynamic CP and dynamic AC.
- We provide a numerical performance assessment of the performance of asynchronous CP under the proposed communication protocol and compare it with collision-free synchronous CP.

In Section 2, we describe and analyze our proposed protocol in detail. Numerical results are provided in Section 3 and Section 4 concludes the paper.

2. ALOHA-LIKE PROTOCOL FOR DISTRIBUTED AVERAGING

2.1. WSN Topology

We model the WSN via a random geometric graph [12] in which the nodes correspond to the sensors and the edges correspond to communication links. We assume that I nodes are randomly placed within a region $\mathcal{A}$ according to a uniform distribution. The positions of the nodes are denoted $\mathbf{x}_i$, $i = 1, \ldots, I$. Two nodes i and j can communicate with each other if they are located within a prescribed distance r, i.e., if $\|\mathbf{x}_i - \mathbf{x}_j\|_2 < r$. The (graph) neighbors of node i are thus given by $\mathcal{N}(i) = \{j \neq i : \ \|\mathbf{x}_i - \mathbf{x}_j\|_2 < r\}$. the degree of sensor node i equals the number of its neighbors, i.e., $d(i) = |\mathcal{N}(i)|$. If the area $|\mathcal{A}|$ and the number of nodes I tend to infinity such that the average node density $\eta = \frac{I-1}{|\mathcal{A}|}$ is fixed, the asymptotic degree distribution for this random geometric graph is given by a Poisson distribution with parameter $r^2\pi\eta$.

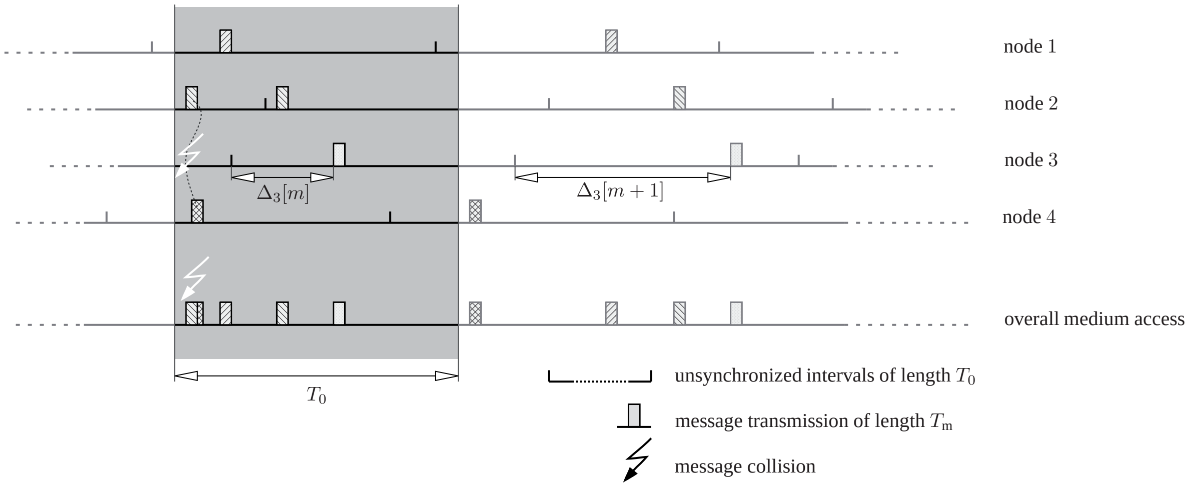

Fig. 1. *Illustration of proposed ALOHA-like protocol: medium access in the vicinity of node* 1 *with neighbor set* $\mathcal{N}(1) = \{2, 3, 4\}$.

2.2. Transmission Protocol

A schematic illustration of our transmission protocol is shown in Fig. 1. The transmission duration for all messages exchanged between nodes equals T_m. Each sensor has a local clock which defines transmission windows of duration T_0 (the transmission windows are not synchronized across sensors). The sensors do not use any kind of carrier sensing. Rather, a sensor broadcasts one message to its neighbors at a randomly chosen time instant $\Delta_i \sim \mathcal{U}\,(0, T_0 - T_\mathrm{m})$ within the transmission window. Hence, the transmission duty cycle (per node) equals $\tau = T_\mathrm{m}/(T_0 - T_\mathrm{m})$. The number of message collisions grows with increasing duty cycle τ (see, below); hence, typically $\tau \ll 1$. The proposed communication protocol is asynchronous and requires no coordination and thus is extremely simple to implement. In fact, it is similar to ALOHA [13], with the main difference that there is no collision resolution, i.e., sensors do not retransmit messages which have been lost due to a collision. We assume that receiving sensor nodes are able to detect collisions (e.g. using a CRC) and discard the corresponding messages. Avoiding retransmissions and discarding messages is possible since distributed averaging algorithms iteratively exchange messages and hence a limited loss of messages does not necessarily deteriorate the averaging performance.

2.3. Collision Analysis

We next analyze the occurrence of collisions for our protocol assuming a random geometric graph topology. To this end, we neglect the differences in message propagation time. We consider two types of collisions which correspond to two practical implementation constraints (see Fig. 2):

- We assume that the sensor nodes only have half-duplex capabilities and hence cannot transmit and receive simultaneously. If the messages sent by two neighboring nodes overlap in time, both nodes will not be able to decode the neighbor's message and we call this a *type 1 collision*.

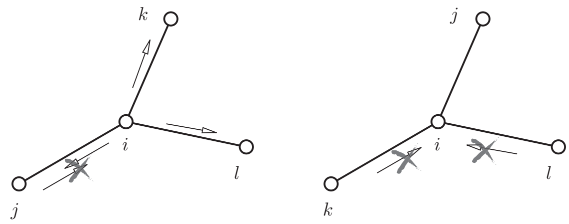

Fig. 2. *Possible collision types involving node i: type 1 collision (left) and type 2 collision (right).*

- The sensor nodes are considered to have only single-user detectors and hence are not able to decode multiple messages from different neighbors. Thus, if two neighbors of a given node broadcast messages that overlap in time, those messages will be lost and we have a *type 2 collision*.

Type 1 Collisions. A collision between two neighboring nodes i and $j \in \mathcal{N}(i)$ occurs if sensor j transmits within the interval $[\Delta_i - T_\mathrm{m}, \Delta_i + T_\mathrm{m}]$. Consider a given transmitting node with d neighbors. For each message of a neighbor, the probability for a type 1 collisions equals $p_1 = 2T_\mathrm{m}/(T_0 + T_\mathrm{m})$. Because the neighbors are unsynchronized it can happen that in the observed interval one neighbor transmits two messages and another one none; on average, however, we have d messages. Since all transmissions are independent, the number of type 1 collisions at the given node has a binomial distribution $\mathcal{B}(d, p_1)$ (for simplicity of exposition, we here neglect rarely occurring collisions at the boundary whose frequency tends to zero for $T_m \to \infty$).

The probability for at least one type 1 collision for a node with degree d thus reads

$$P_1(d) = \mathrm{P}\{C_1 > 0 | D = d\} = 1 - \mathrm{P}\{C_1 = 0 | D = d\}$$
$$= 1 - (1 - p_1)^d = 1 - \left(\frac{T_0 - T_\mathrm{m}}{T_0 + T_\mathrm{m}}\right)^d,$$

where C_1 denotes the number of type 1 collisions and D is a random variable characterizing the degree of a randomly picked node. It is seen that $\mathrm{P}\{C_1 > 0|D = d\}$ grows rapidly for increasing number of neighbors d and increasing relative message duration T_m/T_0. Furthermore, the expected number of type 1 collisions at a node of degree d equals

$$\mathrm{E}\{C_1|D = d\} = p_1 d = \frac{2T_\mathrm{m}d}{T_0 + T_\mathrm{m}}.$$

Type 2 Collisions. We next consider type 2, which occur if two nodes sharing a common neighbor transmit messages in overlapping time intervals (cf. Fig. 2). Here, the collision analysis is slightly more involved. Picking at random a node i with degree $d > 1$, we are interested if messages of the sensors $k, l \in \mathcal{N}(i)$ collide (i.e., node k starts its transmission within $[\Delta_l - T_\mathrm{m}, \Delta_l + T_\mathrm{m}]$). This is equivalent to calculating the type 1 collisions in a fully connected graph whose nodes are given by $\mathcal{N}(i)$ (we assume the same simplifications as for type 1 collisions). For the first node in $\mathcal{N}(i)$, the number of collisions thus has binomial distribution $\mathcal{B}(d-1, p_1)$; for the second node in $\mathcal{N}(i)$, collisions with the first node have already been accounted for and hence are excluded, leading to a binomial distribution $\mathcal{B}(d-2, p_1)$. Repeating this argument for all nodes in $\mathcal{N}(i)$ leads to a sum of binomially distributed variables with equal success probability p_1 and cardinalities $d-1, d-2, \ldots, 1$, thence a binomial distribution $\mathcal{B}(d_2, p_1)$ with $d_2 = \sum_{l=1}^{d-1} l = d(d-1)/2$. It follows that the probability of at least one type 2 collision at a randomly picked node of degree d equals

$$\begin{aligned} P_2(d) = \mathrm{P}\{C_2 > 0|D = d\} &= 1 - \mathrm{P}\{C_2 = 0|D = d\} \\ &= 1 - (1 - p_1)^{d_2}, \end{aligned}$$

and the expected number of type 2 collisions equals

$$\mathrm{E}\{C_2|D = d\} = p_1 d_2 = \frac{T_\mathrm{m}d(d-1)}{T_0 + T_\mathrm{m}}.$$

Total Collisions. Since the type 1 collision event and the type 2 collision event are not mutually exclusive the total number C of collisions is upper bounded as $C \le C_1 + C_2$. The probability of at least one collision ($C > 0$) at a randomly picked node with degree d is upper bounded as

$$\mathrm{P}\{C > 0|D = d\} \le P_1(d) + P_2(d).$$

This further implies the following bound for the unconditional collision probability:

$$\begin{aligned} \mathrm{P}\{C > 0\} &= \sum_d \mathrm{P}\{C > 0|D = d\}\ \mathrm{P}\{D = d\} \\ &\le \sum_d (P_1(d) + P_2(d))\, \mathrm{P}\{D = d\}, \end{aligned}$$

where $\mathrm{P}\{D = d\}$ denotes the graph's degree distribution (Poisson in the case of random geometric graphs).

The expected number of collisions can be bounded as

$$\begin{aligned} \mathrm{E}\{C\} &\le \sum_d (\mathrm{E}\{C_1|D = d\} + \mathrm{E}\{C_2|D = d\})\, \mathrm{P}\{D = d\} \\ &= \mathrm{E}\left\{p_1 D + p_1 \frac{D(D-1)}{2}\right\} = \frac{p_1}{2}\, \mathrm{E}\{D(D+1)\}. \end{aligned}$$

For random geometric graphs, we thus obtain

$$\mathrm{E}\{C\} \le \frac{T_\mathrm{m}}{T_0 + T_\mathrm{m}} \left((r^2\pi\eta + 1)^2 - 1\right),$$

where η denotes the average node density and r is the communication range (see Subsection 2.1).

2.4. Averaging Algorithms

Consensus Propagation. CP is a distributed averaging algorithm which solves a convex optimization problem via belief propagation and achieves convergence to the global average at each node [3]. The algorithm consists of two types of messages, the μ-messages and the K-messages (which are independent of the μ-messages and are attenuated through a positive constant β; see the simulation section). In dynamic scenarios, it was shown that CP requires small β in order to be able to track temporal signal evolutions (cf. [5]); as a consequence, the K-messages in this case typically converge after a few iterations and therefore eventually only the μ-messages need to be transmitted. In contrast to AC, the CP message updates are non-linear and hence difficult to analyze. We note that CP also allows for a weight matrix for messages which can be used to tune the performance. However, to date there is no existing work regarding the weight optimization and therefore we assume identical weights. The interpretation of CP as distributed belief propagation suggests that the scheduling of the message updates can play an important role, specifically in view of the fact that most WSN are characterized by a communication graph with numerous, possible very short, cycles. In the next section, we thus study the performance of the dynamic broadcast version of CP developed in [5] under (i) the proposed ALOHA-like protocol (this version of CP will be referred to as CPA), (ii) the usually considered unrealistic synchronous protocol, and (iii) a completely asynchronous (random) protocol.

Average Consensus. AC is easier to analyze than CP due to the linearity of the message updates. Therefore many papers deal with AC, specifically with weight optimization to enhance the convergence speed. Setting some weights to zero implies no transmission over the corresponding link; this is useful for enforcing asynchronous operation. For the comparison with dynamic CP, we use the first-order dynamic AC algorithm proposed in [7]. This algorithm, which has received rather little attention in the signal processing community, includes temporal changes of the desired and measured signals into the update equations. For simplicity and fairness, we assume constant averaging weights as in [2].

3. NUMERICAL RESULTS

3.1. Simulation Setup

We consider WSN whose topology is modeled by random geometric graphs over the region $\mathcal{A} = [0, 1] \times [0, 1]$. Unless stated otherwise, the number of sensor nodes is given by $I = 100$, the communication range equals $r = 0.2$ and we have a

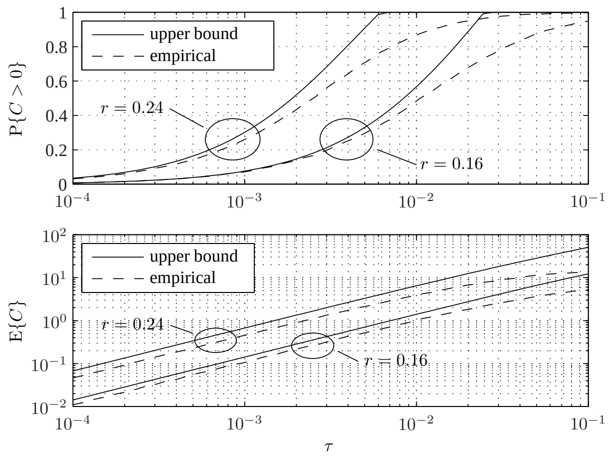

Fig. 3. *Analytical and empirical results for collisions with our proposed protocol for different duty cycles τ.*

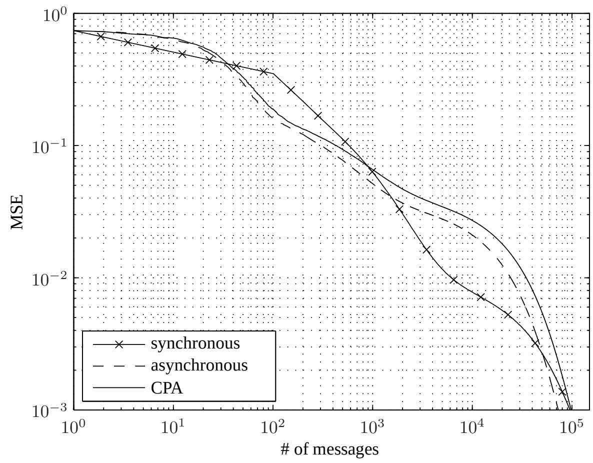

Fig. 4. *Convergence behavior of CP for different protocols ($\beta = 90$ and $\tau = 5 \cdot 10^{-3}$).*

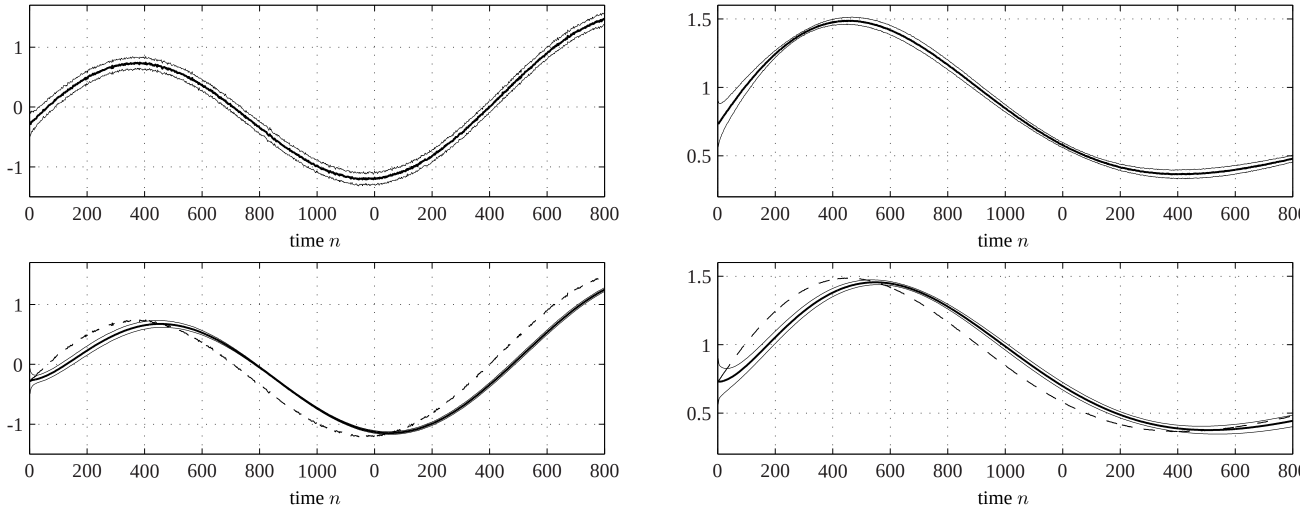

Fig. 5. *Mean (solid line) and standard deviation (thin solid line) of the spatial average obtained with AC (top) and CP with $\beta = 5$ (bottom) for an SNR of 2 dB (left panels) and 20 dB (right panels; note the different axis range). The true spatial average is shown as dashed line.*

transmission duty cycle of $\tau = 5 \cdot 10^{-3}$. We use the broadcast version of CP throughout. Furthermore, we assume perfect detection of a collision.

3.2. Collision Probability

We first verify our analytical results for the (type 1 and type 2) collisions occurring with our ALOHA-like protocol by empirically determining the collisions for duty cycles τ ranging from 10^{-4} to 10^{-1}. The empirical results were obtained by counting collisions at each node, which were then averaged over the nodes and over 100 scenarios. Fig. 3 shows the analytical and empirical results for the collision probability and the average number of collisions versus τ for $r = 0.24$ and $r = 0.16$. For the analytical curves we used refined versions of the results in Section 2.3.

It is seen that the analytical results match the empirical results well, except that they are slightly too pessimistic. This is due to the fact that the actual average node degree is smaller than the theoretical one since close to the boundary of $\mathcal{A}$ there are fewer neighbors. For better connected WSN (larger r), collisions become more frequent which is intuitively expected.

3.3. Averaging Algorithms

In the following simulations, we apply dynamic CP and dynamic AC to noisy sensor measurements of a smooth spatio-temporal field (see [14] for details).

We first illustrate the convergence behavior of synchronous CP, asynchronous CP, and CPA in Fig. 4; here, the CP parameter is chosen as $\beta = 90$. It is seen that there are only minor differences in performance between the three CP protocols. We note, however, that synchronous. In contrast, CPA is very simple to implement and yields virtually the same performance in spite of the fact that there is a significant number of collisions.

In Fig. 5 we illustrate the tracking behavior of dynamic CP and dynamic AC for SNRs of 2dB and 20dB. The temporal dynamics of the field are characterized by a bandwidth Θ which here equals $1.2 \cdot 10^{-3}$. Here, the step-size for AC was approximately equal to the reciprocal of the largest node

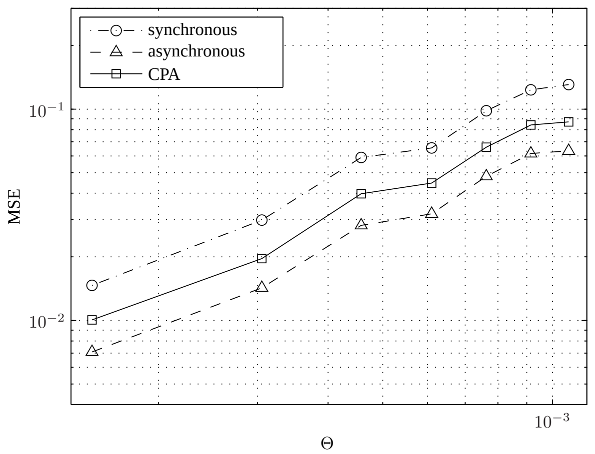

Fig. 6. *MSE versus Θ achieved with CP under different protocols.*

degree. While dynamic CP results in better noise suppression (smaller fluctuations of the estimates), it lags behind the true spatial average due to its inherent memory (this lag can be compensated using per-sensor predictors, see [5]). In contrast, dynamic AC suffers no temporal delay but is much more susceptible to noise (larger fluctuations).

In our last simulation we compare the mean square error (MSE) achieved with synchronous CP, asynchronous CP, and CPA in a dynamic scenario (the MSE is averaged over all sensors, all time instants, and 100 network realizations). Fig. 6 shows the MSE of the estimated spatial average versus the temporal bandwidth Θ of the field under consideration. It is seen that asynchronous CP outperforms synchronous CP (both impractical), while the proposed CPA schemes performs in between those two cases. Our interpretation of why asynchronous CP and CPA perform better than synchronous CP is that with synchronous CP a larger number of message updates happens in parallel and hence any given message needs a larger number of updates to propagate through the network.

4. CONCLUSIONS

We proposed a simple ALOHA-like protocol which can be used for distributed averaging, especially for consensus propagation. With this protocol, messages that cannot be decoded are dropped rather than retransmitted. A quantitative analytical assessment of the occurrence of collisions revealed that the message duty cycle and average number of neighbors per node are the parameters determining performance. Our simulation results showed that in spite of its simplicity and the fact that a significant number of messages are dropped, consensus propagation under the ALOHA-like protocol features excellent tracking and averaging performance.

5. REFERENCES

[1] S. Boyd, A. Ghosh, B. Prabhakar, and D. Shah, "Randomized gossip algorithms," *IEEE Trans. Inf. Theory*, vol. 52, pp. 2508–2530, June 2006.

[2] R. Olfati-Saber, J. Fax, and R. Murray, "Consensus and cooperation in networked multi-agent systems," *Proc. IEEE*, vol. 95, pp. 215–233, Jan. 2007.

[3] C. C. Moallemi and B. Van Roy, "Consensus propagation," *IEEE Trans. Inf. Theory*, vol. 52, pp. 4753–4766, Nov. 2006.

[4] P. Denantes, F. Benezit, P. Thiran, and M. Vetterli, "Which distributed averaging algorithm should I choose for my sensor network?," in *Proc. IEEE INFOCOM-2008*, (Phoenix, AZ), pp. 986–994, April 2008.

[5] V. Schwarz, C. Novak, and G. Matz, "Broadcast-based dynamic consensus propagation in wireless sensor networks," in *Proc. 43th Asilomar Conf. Signals, Systems, Computers*, (Pacific Grove, CA), Nov. 2009.

[6] T. Aysal, M. Yildiz, and A. Scaglione, "Broadcast gossip algorithms," in *Proc. IEEE ITW-2008*, (Porto, Portugal), pp. 343–347, May 2008.

[7] M. Zhu and S. Martínez, "Discrete-time dynamic average consensus," *Automatica*, vol. 46, issue 2, pp. 322–329, Feb. 2009.

[8] V. Rajendran, K. Obraczka, and J. Garcia-Luna-Aceves, "Energy-efficient, collision-free medium access control for wireless sensor networks," *Wireless Networks*, vol. 12, no. 1, pp. 63–78, 2006.

[9] S. Kar and J. Moura, "Sensor networks with random links: Topology design for distributed consensus," *IEEE Trans. Signal Processing*, vol. 56, pp. 3315–3326, July 2008.

[10] S. Patterson, B. Bamieh, and A. El Abbadi, "Distributed average consensus with stochastic communication failures," in *Proc. 46th IEEE Conf. Decision and Control*, (New Orleans, LA), pp. 4215–4220, Dec. 2007.

[11] M. Haenggi, J. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE J. Sel. Areas Comm.*, vol. 27, pp. 1029–1046, Sept. 2009.

[12] M. Penrose, *Random Geometric Graphs*. Oxford University Press, 2003.

[13] N. Abramson, "THE ALOHA SYSTEM: another alternative for computer communications," in *Proc. Fall Joint Computer Conference*, pp. 281–285, ACM, Nov. 1970.

[14] V. Schwarz and G. Matz, "Distributed reconstruction of time-varying spatial fields based on consensus propagation," in *Proc. IEEE ICASSP-2010*, (Dallas, TX), pp. 2926–2929, March 2010.