Spatial diversity impact on the local delay of homogeneous and clustered wireless networks

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***Abstract*—The law of the time to transmit a packet from a node to its intended receiver, in a wireless network with randomly deployed, multiple-antennas equipped nodes, whose communication is impaired by fading and interference, is investigated. SINR-based coverage at the physical layer is assumed, and the useful signal strength is shaped through either cooperative or non-cooperative beamforming. In particular, two rate-optimizing strategies are exploited, i.e. dominant eigenmode transmission (DET) and interference alignment (IA). The MAC is governed by spatial ALOHA with prescribed medium access probability. The rationale of the work stems from the fact that even in the case of observed individual (per-node) finite-mean random geometric delay (which implies a non-vanishing next-hop throughput) the large population average of such delay may be unbounded in several cases of practical interest. Both the case of homogeneous Poisson point process as well as of clustered Poisson are analyzed, providing monotonicity results on the average local delay achievable under a set of common assumptions on the communication scenario, and varying the number of spatial degrees of freedom available for transmit beamforming and/or interference suppression.**

***Index Terms*—Clustered Poisson; MIMO; Interference Alignment; Wireless Networks.**

I. Introduction

Interference characterization and management has a strong impact on the overall throughput and delay performance of a wireless network. Random placement/mobility of the users further impacts on the network, causing some nodes to be in persistent outage while others can have unfair favored access to the network resources. One way to investigate whether there is a considerable fraction of nodes experiencing unfavorable conditions in network resources sharing is to evaluate the average (over space and time) delay that the typical node experiences under prescribed physical and MAC layer assumptions, as done in [1]. An unbounded value of the average delay does not imply vanishingly small throughput, but it is indeed an indicator of non desirable working conditions for the network. Interestingly, [1] shows how the network can be put in a favorable working condition through a sudden phase transition phenomenon which occurs when some typical parameters (transmitter-receiver distance, Aloha medium access probability, thermal noise power) are kept below or above certain thresholds. The observed behavior is called the *wireless contention phase transition*. In [1], the authors propose an analysis which departs from the classical SINR-based success/failure model, through the introduction of adaptive coding. Furthermore, they also propose to resort to diversity (in terms of more variability in fading, the addition of more receivers, or again the exploitation of nodes mobility) to keep the delay value bounded, and analyze the impact of several forms of variability on the delay of the typical node in a random network, whose nodes are placed according to an Homogeneous Poisson Point Process (HPPP). The communication is impaired both by random fading and thermal noise, and the nodes access the channel following an ALOHA policy with fixed medium access probability. Among their main findings, they show that the assumption of node mobility, causing the whole network to be re-sampled from a spatial statistics point of view at each snapshot, leads to break dependence in certain cases. This yields to a decreasing value of the mean local delay of the typical node and then to a throughput improvement even if one does not use nodes mobility to transport packets. Based on the need for investigating delay-throughput improvements due to diversity, in this work, we extend some results of [1], focusing on the impact of spatial diversity on the local delay value. We embody in our analysis also multiple-antenna equipped nodes, and investigate also the case of non-homogeneously deployed nodes. In this last case, we assume the spatial process of nodes locations to be constituted by clusters of nodes. We provide the analysis for the impact of spatial diversity on the delay statistics, leaving the corresponding phase transition phenomenon discussion for future work. We investigate the relative performance of interference alignment (IA) applied among nodes belonging to the same cluster, following our previous work [2], and of Dominant Eigenmode Transmission (DET). Both transmission strategies exploit beamforming, and have been proven to be rate-optimizing in the context of Bayesian games [3]. In particular, DET is the optimum strategy to be exploited when egoistic utility is to be maximized (i.e. each user in the network aim at boosting its signal power strength without caring for the interference level so caused to other users). IA, in turn, is the high-SNR asymptotic equivalent of a distributed (hence cooperative) beamforming algorithm aimed at balancing egoistic versus altruistic utility [3] (i.e. a distributed beamforming strategy whose aim is to trade off useful signal boosting with minimization of the interference level caused to other users converges at IA precoding in the high-SNR regime). We show that IA performs better for short-range communication, while DET is to be preferred when the transmitter-receiver distance increases.

The paper is organized as follows: Section II contains the spatial system model description, i.e. no time-dimension is added yet to the picture. The nodes deployment statistics are given and the physical layer strategies are detailed in proper subsections. In Section III the time variability is introduced and the main concepts needed from the seminal analysis in [1] for further developments are recalled. Results for the cases of homogeneous as well as clustered network are given. Section IV concludes the paper.

II. Spatial system model

Let the random set $\Phi = \{x_1, x_2, \ldots, x_\infty\}$ be a stationary and isotropic Poisson point process, capturing the locations x_j's of the transmitting nodes on the infinite plane $\mathbb{R}^2$. Φ is chosen either as HPPP of given density λ, or, in the clustered case, as a Neyman-Scott model, where the potential transmitters spatial point process results from homogeneous independent clustering applied to a stationary Poisson process [4]. The cluster process then consists of a parent HPPP of intensity λ_p, whose points identify the cluster centers and serve as reference points for the daughter points (transmitters) that are scattered i.i.d around the parent point, according to an isotropic distribution, assumed throughout this paper to be a circularly symmetric normal distribution of variance $2\sigma^2$, i.e.

$$f(x) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{\|x\|^2}{2\sigma^2}\right). \tag{1}$$

Here, we assume that the number of points K in a representative cluster Ψ is fixed. The clusters form a partition of Φ. Since the scattering density of the representative cluster is assumed isotropic, the whole cluster process is isotropic. Therefore, the overall intensity of the cluster process is the product of the parent process intensity and the number of nodes per cluster $\lambda = \lambda_p K$. Denoting the set of indices of the points as $\phi = \{1, 2, \ldots, \infty\}$, we can refer to the set of indices of the points in a representative cluster as $\psi = \{k, k+1, \ldots, k+K-1\}$, which is a subset of ϕ containing the indices of Ψ.

Following the widely adopted *bipolar model*, the receivers are not considered a part of the spatial stochastic process. We assume that each transmitter node serves one receiver located at a deterministic distance from its associated transmitter, i.e. the distance $d_{ii} = \|x_i - z_i\|$ between transmitter $i \in \phi$ located at x and its intended receiver located at z_i is assumed constant. In our model, each receiver is associated with a single transmitter, hence we shall refer to receiver $i \in \phi$ as the receiver associated to node x_i. The analysis of other receiver models, like e.g. the nearest neighbor selection from a random or deterministic set of receivers, introduced and partly investigated in [1], is left for future work. Transmitters and receivers are equipped with $N_\mathrm{T} \geq 1$ and $N_\mathrm{R} \geq 1$ antennas, respectively.

At the physical layer, we adopt a SINR-coverage metric and neglect the effect of noise. Let us assume that all nodes transmit with unit power, and let $|\bar{h}_{ii}|^2$ denote the power of the channel fading process between node i and its intended receiver, after processing by a linear receive filter (see below). Communication is assumed successful if and only if the SIR exceeds a given threshold T. Therefore, the probability of successful transmission is given by

$$\mathbb{P}^s = \mathbb{P}\left(\frac{|\bar{h}_{ii}|^2\gamma_{ii}}{\sum_{j\in(\phi)} |\bar{h}_{ij}|^2\gamma_{ij}} \geq T\right), \tag{2}$$

where $\gamma_{ij} = d_{ij}^{-\alpha}$ is the path-loss function, and $I_\phi = \sum_{j\in(\phi)} |\bar{h}_{ij}|^2\gamma_{ij}$ the accumulated interference from the overall network. Assuming flat Rayleigh fading on the channel and linear (unit norm) receive filter, that is independent on the interfering links, $|\bar{h}_{ij}|^2$, namely the received power from the generic interfering node $j \neq i \in \phi$, is exponentially distributed. The MAC strategy is assumed throughout to be pure spatial ALOHA, with medium access probability p.

A. Cooperative and non-cooperative beamforming

Under our assumptions, the signal at receiver i, before receive processing through a linear filter $\mathbf{u}_i$ can be written as

$$\mathbf{y}_i = \sum_{j \in \phi} \sqrt{\gamma_{ij}} \mathbf{H}_{ij} \mathbf{v}_j s_j + \mathbf{n}_i, \tag{3}$$

where $s_j \in \mathbb{C}$ represents the scalar signal transmitted by node j, and $\mathbf{v}_j \in \mathbb{C}^{N_{\mathrm{T}} \times 1}$ is the associated beamforming/precoding vector which will be further specified, according to different assumptions on the presence or absence of cooperation among (a subset of) the transmit nodes. $[\mathbf{H}_{ij}]_{i,j \in \phi}$ are complex $N_{\mathrm{R}} \times N_{\mathrm{T}}$ matrices representing the MIMO channels between transmitter j and receiver i, while $\mathbf{n}_i$ is a noise term, accounting for the thermal noise generated in the radio frequency front-end of the receiver and interference from sources other than the considered transmitters $j \in \phi$.

In the following, we will investigate the performance of different (cooperative or selfish) transmission strategies. In particular, we analyze two extreme (in terms of degree of cooperation and information sharing) cases of IA and Dominant Eigenmode Transmission (DET).

1) Dominant Eigenmode Transmission: In absence of any cooperation between nodes, every communicating pair aims at maximizing a selfish metric, the received signal power in our case, without paying any attention to the interference caused to unintended receivers eventually in range. To this end, given the useful channel matrix $[\mathbf{H}_{ii}]$, the receive filter will project the signal along the left eigenvector corresponding to the maximum singular value of $[\mathbf{H}_{ii}]$, say $\mathbf{u}_i$, while the transmitter will, in turn, beamform along the corresponding right eigenvector $\mathbf{v}_i$. This leads the term $\bar{h}_{ii} = \mathbf{u}_i^{\mathrm{H}} \mathbf{H}_{ii} \mathbf{v}_i$ to be distributed as the square of the maximum singular value of the channel matrix $\mathbf{H}_{ii}$ or, equivalently, to follow the law of the dominant eigenvalue λ_{max} of the square hermitian matrix $\mathbf{W} = \mathbf{H}_{ii} \mathbf{H}_{ii}^{\mathrm{H}}$, i.e. its complementary cumulative distribution function is given by [5, Formula 9]

$$G_{|\bar{h}_{ii}|^2}(\lambda_{max}) = 1 - \frac{\det(\mathbf{\Psi}(\lambda_{max}))}{\prod_{k=1}^{q} \Gamma(q-k+1)\Gamma(s-k+1)} \tag{4}$$

where $\Gamma(\cdot)$ is the usual Euler Gamma function, and the i,j-th entry of $\mathbf{\Psi}(\lambda_{max})$ can be written as $(s-q+i+j-2)!(1-e^{-\lambda_{max}} \sum_{k=0}^{s-q+i+j-2} \frac{\lambda_{max}^k}{k!})$. Herein, $q = \min\{N_{\mathrm{T}}, N_{\mathrm{R}}\}$ and $s = \max\{N_{\mathrm{T}}, N_{\mathrm{R}}\}$.

Since no interference suppression policy has been exploited, an equivalent scalar input-output relation of the system can be given as

$$\bar{y}_i = \sqrt{\gamma_{ii}} \bar{h}_{ii} s_i + \sum_{j \in \phi \setminus \{i\}} \sqrt{\gamma_{ij}} \bar{h}_{ij} s_j + \bar{n}_i, \tag{5}$$

with $\bar{n}_i = \mathbf{u}_i^{\mathrm{H}} \mathbf{n}_i$, and exponentially distributed coefficients $|\bar{h}_{ij}|^2$'s, given by $\bar{h}_{ij} = \mathbf{u}_i^{\mathrm{H}} \mathbf{H}_{ij} \mathbf{v}_j$.

2) Interference Alignment: To perform a cooperative precoding like interference alignment, the transmitters $i \in \psi$ share their knowledge about the frequency-flat channel matrices $[\mathbf{H}_{ij}]_{i,j \in \psi}$ and choose a precoding vector $\mathbf{v}_i$ in order to steer their transmitted signal into a subspace of minimum dimension at each unintended receiver, where it creates interference. The receiver uses a projection vector $\mathbf{u}_i$ on the subspace orthogonal to the intra-cluster interference in order to suppress unintended signals from transmitters $(j \neq i) \in \psi$. The remaining dimension of the received signal space is used for interference free communication with the intended transmitter.

Therefore, for every cluster Ψ, IA is achieved in the spatial domain with degree of freedom one iff there exists $N_{\mathrm{T}} \times 1$ unit-norm vectors (precoding vectors) $\mathbf{v}_i$ and $N_{\mathrm{R}} \times 1$ unit-norm vectors (interference suppression vectors) $\mathbf{u}_i$ such that, for all $i \in \psi$,

$$\mathbf{u}_i^{\mathrm{H}} \mathbf{H}_{ij} \mathbf{v}_j = 0, \forall (j \neq i) \in \psi, \quad \text{and} \tag{6}$$

$$\operatorname{rank}\left(\mathbf{u}_i^{\mathrm{H}} \mathbf{H}_{ii} \mathbf{v}_i\right) = 1. \tag{7}$$

Under our assumptions for the channel matrices $\mathbf{H}_{ij}$, whose entries are drawn independently and identically from a continuous distribution, an IA solution with multiplexing gain one exists (with probability one) [6], [7] for the K-users interference channel given by the nodes belonging to the same cluster iff

$$N_{\mathrm{R}} + N_{\mathrm{T}} - 1 \geq K. \tag{8}$$

Perfect suppression of the intra-cluster interference from the receive signal $\mathbf{y}_i$ (3) is then achieved by projecting it onto the orthogonal subspace of the interference, so as to obtain

$$\bar{y}_i = \mathbf{u}_i^{\mathrm{H}} \mathbf{y}_i = \sqrt{\gamma_{ii}} \mathbf{u}_i^{\mathrm{H}} \mathbf{H}_{ii} \mathbf{v}_i s_i + \sum_{j \in (\phi \setminus \psi)} \sqrt{\gamma_{ij}} \mathbf{u}_i^{\mathrm{H}} \mathbf{H}_{ij} \mathbf{v}_j s_j + \mathbf{u}_i^{\mathrm{H}} \mathbf{n}_i. \tag{9}$$

where only the inter-cluster interference remains, and where both the effective channel coefficient $\bar{h}_{ii} = \mathbf{u}_i^{\mathrm{H}} \mathbf{H}_{ii} \mathbf{v}_i$ as well as the effective noise term $\bar{n}_i$ keep to have the same variance as the components in $\mathbf{H}_{ii}$ and, respectively, the same variance σ_n^2 as the noise vector $\mathbf{n}_i$.

The equivalent scalar input-output relation of the system after interference suppression yields [2]

$$\bar{y}_i = \sqrt{\gamma_{ii}}\bar{h}_{ii}s_i + \sum_{j\in(\phi\backslash\psi)} \sqrt{\gamma_{ij}}\bar{h}_{ij}s_j + \bar{n}_i, \quad (10)$$

which can be interpreted as a system with single antenna terminals where the intra-cluster interference has been completely suppressed.

III. Delay analysis

In order to study the delay and evaluate its statistics, we have to add a time-dimension to the Poisson bipolar model described above. Borrowing the framework from the reference paper [1], we assume that all the nodes are perfectly synchronized, over a slotted time indexed by the discrete index n. A snapshot of the network can be represented through a marked Poisson process $\widetilde{\Phi}$, either clustered or homogeneous, whose multidimensional mark contains information regarding the MAC decision for the typical node and other parameters of interest at a given time instant, i.e. $\widetilde{\Phi} = \{x_i, (e_i(n), y_i, \bar{h}_{ij}(n), \bar{n}_i(n))\}$, where $e_i(n)$ are i.i.d. (both in i and n) Bernoulli random variables with parameter p given by the ALOhA medium access probability, y_i is the location of the receiver of node i (satisfying the bipolar model constraint $||x_i - y_i|| = r$) and $\bar{h}_{ij}(n)$, $\bar{n}_i(n)$ are defined in the previous Section. Since we focus on the interference-limited regime, from now on we will set, without loss of generality, $\bar{n}_i(n) = 0$.

In order to introduce and discuss our findings we give some definitions from [1]:

***Definition* 1** *The local delay of a given node is the number of time slots needed for a node $x_i \in \Phi$ to successfully transmit, i.e.*

$$\boldsymbol{L} = \inf\{n \geq 1 : \delta_i(n) = 1\}\ , \quad (11)$$

where $\delta_i(n)$ is the indicator of the event $\frac{|\bar{h}_{ii}|^2(n)\gamma_{ii}}{I_\Phi(n)} \geq T$ (see eq. (2)).

***Definition* 2** *The average local delay can be defined as*

$$\boldsymbol{D} = \mathbb{E}[\boldsymbol{L}]\ , \quad (12)$$

and, following [1, Lemma 3] it can be equivalently expressed as

$$\boldsymbol{D} = \mathbb{E}\left[\frac{1}{\pi_t}\right]\ , \quad (13)$$

with $\pi_t = p\mathbb{P}^s$, where $\mathbb{P}^s$ is the success probability (2) for the typical transmitter-receiver pair.

From Def. 2 we can see that π_t is the temporal rate of successful packet transmission from node i, i.e. the throughput of this node.

A. Homogeneous Poisson Point Process

In the interference-limited case assumed here, the average local delay in the homogeneous case can be written as [1, Prop. 3.5]

$$\mathbf{D} = \frac{1}{p}\exp\left\{2p\pi\lambda\int_0^\infty \frac{vT/\gamma_{ii}}{\gamma_{ii} + (1-p)T/\gamma_{ii}}dv\right\}. \quad (14)$$

In presence of spatial diversity at both ends of the links, the CCDF of the useful signal assumes the general form

$$F_{|\bar{h}_{ii}|^2}(h) = \sum_{n\in\mathcal{N}} e^{-nh} \sum_{k\in\mathcal{K}} a_{n,k}h^k \quad (15)$$

for proper[1] finite sets $\mathcal{N}, \mathcal{K} \subset \mathbb{N}_0$ and specific values of the normalizing coefficients $a_{n,k}$, depending on N_T, N_R, and on the adopted transmit beamforming technique. It is worth to remark that the analysis in [8, Appendix I] refers to a system with spatial diversity at only one end of the link, while just providing a conjecture on the applicability of the technique to a Multiple-Input-Multiple-Output (MIMO) channel, based on the joint eigenvalues density expression for the Wishart matrix given in [9]. However, from the results in [10, Corollary 3] and in [11, Theorem I] it follows that under Rayleigh fading the received power density for bipolar networks still can be expressed as (15). In particular, for a 2×2 MIMO system (i.e. when $N_\mathrm{R} = N_\mathrm{T} = 2$), (4) coincides with (15) if $\mathcal{N} = \{1,2\}$, $\mathcal{K} = \{0,1,2\}$ and the coefficients are such that $a_{1,1} = a_{2,1} = a_{2,2} = 0$, $a_{1,0} = 2$, $a_{2,0} = -1$ and $a_{1,2} = 1$. Notice that this holds whenever the received signal power distribution can be expressed through the (joint or marginal) statistics of one or more squared singular values of the channel matrix $\mathbf{H}_{ii}$, either ordered or not, and the DET is just a particular case of the analysis.

From this observation, we can establish the following proposition, which establishes a performance hierarchy between single-antenna nodes and multi-antennas nodes with DET.

[1] Notice that, as already stressed in [8], (15) is not a suitable CCDF unless the sets $\mathcal{N}$ and $\mathcal{K}$ are properly chosen and the coefficients $a_{n,k}$'s adequately normalize the function's values.

***Proposition* 1** *The average local delay in a HPPP network with multiple-antenna equipped nodes employing DET is lower than the average local delay achievable in the same network with single-antenna equipped nodes, i.e.*

$$\boldsymbol{D}_{N_{\mathrm{R}}=N_{\mathrm{T}}=1} \geq \boldsymbol{D}_{N_{\mathrm{T}}>1,N_{\mathrm{R}}>1}. \quad (16)$$

Proof: While the useful signal density in presence of diversity assumes the expression (15), due to the hypotheses of linear receive filter independent on the signal on the interfering links, the aggregate interference keeps the same expression as for the case of absence of spatial diversity, so as to give the known expression for the success probability (see, e.g. [8, Formula 10])

$$\mathbb{P}^s = \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}}\left[a_{n,k}\left(-\frac{\zeta}{n}\right)^k \frac{d^k}{d\zeta^k}\mathcal{L}_{I_\phi}(\zeta)\right]_{\zeta=nT/\gamma_{ii}}, \quad (17)$$

with

$$\mathcal{L}_{I_\phi}(\zeta) = \exp\left\{-2\pi\lambda\int_0^\infty \frac{t}{1+\frac{|t|^\alpha}{\zeta}}dt\right\} = e^{\lambda C_\alpha \zeta} \quad (18)$$

the Laplace transform of the interference in a HPPP network [12], and $C_\alpha = \frac{2\pi}{\alpha}\Gamma\left(\frac{2}{\alpha}\right)\Gamma\left(1-\frac{2}{\alpha}\right)$.

Following the same steps as in [8, Eqs. (52)-(53)] we can then write

$$\mathbb{P}^s \approx 1 - \lambda\zeta^{2/\alpha}\frac{C_\alpha}{K_{N_{\mathrm{T}}N_{\mathrm{R}},\alpha}} \quad (19)$$

where $K_{N_{\mathrm{T}}N_{\mathrm{R}},\alpha}$, whose explicit expression is given again in [8, Formula (12)], grows as $(N_{\mathrm{T}}N_{\mathrm{R}})^{2/\alpha}$, hence leading to $\mathbb{P}^s_{N_{\mathrm{T}}>1,N_{\mathrm{R}}>1} \geq \mathbb{P}^s_{N_{\mathrm{R}}=N_{\mathrm{T}}=1}$ and, as a consequence, proving (16).

B. Clustered Poisson Process

In the clustered scenario with fixed number K of nodes per cluster, the success probability under Rayleigh fading and no spatial diversity is known to be [4, Appendix II]

$$\begin{aligned}\mathbb{P}_s &= \exp\left\{-\lambda_p\int_{\mathbb{R}^2} 1-\tilde{\beta}(z,y)^K \mathrm{d}y\right\} \\ &\quad \cdot \int_{\mathbb{R}^2}\tilde{\beta}(z,y)^{K-1}\mathrm{d}y, \end{aligned} \quad (20)$$

where

$$\tilde{\beta}(z,y) = \int_{\mathbb{R}^2}\frac{f(x)}{1+\frac{T||x-y-z||^{-\alpha}}{\gamma_{ii}}}\mathrm{d}x, \quad (21)$$

f is the scattering function of the daughter points around the parent one, and z the location of the intended receiver for the transmitter located at the origin. In (20), the first factor accounts for the interference coming from the clusters other than the one the transmitter belongs to, while the second accounts for the transmitting cluster interference. As shown in [4, Sec.III.B], the first term dominates the behavior of long-range transmissions success probability, while the second mainly impacts short range ones. The main parameter impacting the delay is indeed the fixed hop length of the bipolar model in the clustered case. As a consequence, we formulate the following

***Proposition* 2** *In a clustered Poisson network with spatial model given in Section II, in the case of short-range, IA feasible MIMO transmissions, i.e. for $d_{ii} \leq \Delta$, and $N_{\mathrm{R}}+N_{\mathrm{T}}-1 \geq K$,*

$$\boldsymbol{D}^c_{N_{\mathrm{R}}=N_{\mathrm{T}}=1} \geq \boldsymbol{D}^c_{N_{\mathrm{R}}>N_{\mathrm{T}}>1,DET} \geq \boldsymbol{D}^c_{IA}, \quad (22)$$

where $\boldsymbol{D}^c$ stands for the average delay in the clustered case and Δ is a threshold value for the transmitter-receiver distance. Otherwise, for $d_{ii} \leq \Delta$,

$$\boldsymbol{D}^c_{N_{\mathrm{R}}=N_{\mathrm{T}}=1} \geq \boldsymbol{D}^c_{IA} \geq \boldsymbol{D}^c_{N_{\mathrm{R}}>N_{\mathrm{T}}>1,DET}. \quad (23)$$

Proof.

The first inequality in (22) follows from the very same arguments as for Proposition 1. From (20), instead, it follows that for the cluster Poisson case, with single-antenna equipped nodes,

$$\mathbf{D}^c_{N_{\mathrm{R}}=N_{\mathrm{T}}=1} = \mathbb{E}\frac{\exp\left\{\lambda_p\int_{\mathbb{R}^2}1-\tilde{\beta}(z,y)^K\mathrm{d}y\right\}}{p\int_{\mathbb{R}^2}\tilde{\beta}(z,y)^{K-1}\mathrm{d}y}. \quad (24)$$

Applying IA between nodes belonging to the same cluster, as advocated in [2], enables to get rid of the interference coming from other members of the cluster. This cooperative interference-suppression is possible, provided that the feasibility constraint $N_{\mathrm{R}}+N_{\mathrm{T}}-1 \geq K$ is satisfied. In that case, the success probability reduces to [2]

$$\mathbb{P}^s_{IA} = \exp\left\{-\lambda_p\int_{\mathbb{R}^2}1-\tilde{\beta}(z,y)^K\mathrm{d}y\right\}. \quad (25)$$

Since for short transmitter-receiver distance the dominating impairment is the transmit cluster interference, we have

$$\mathbb{P}^s_{IA} \geq \mathbb{P}^s_{N_{\mathrm{T}}>1,N_{\mathrm{R}}>1} \geq \mathbb{P}^s_{N_{\mathrm{T}}=N_{\mathrm{R}}=1} \quad (26)$$

up to a certain value of d_{ii} and, as a consequence,

$$\mathbf{D}^c_{N_{\mathrm{R}}=N_{\mathrm{T}}=1} \geq \mathbf{D}^c_{N_{\mathrm{R}}>N_{\mathrm{T}}>1} \geq \mathbf{D}^c_{IA}. \quad (27)$$

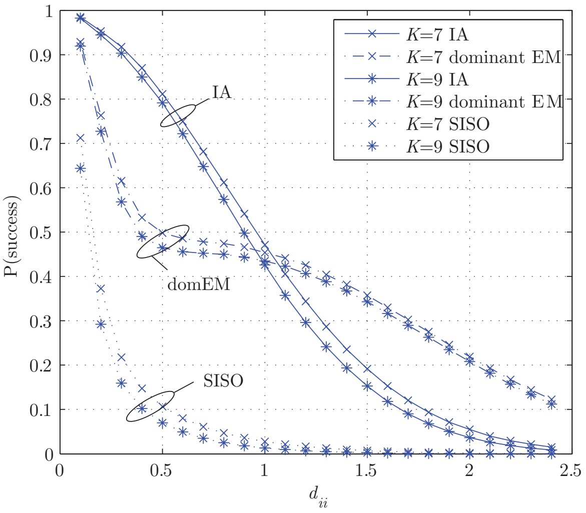

Fig. 1. $\mathbb{P}^s$ versus d_{ii} for $\lambda_p = 0.05$, $\sigma = 0.25$

On the other hand, when long hops are required, while (25) is strictly bounded away from one for large values of d_{ii}, this is not the case for the DET success probability expression, as it has been already verified numerically (see e.g. Fig. 1) and can be further confirmed analytically, putting the corresponding expression for the Laplace transform of a clustered Poisson process (see [4]) in (17).

IV. Conclusion

The average local delay for the next-hop transmission in wireless ALOHA networks with fixed medium access probability, Rayleigh fading and transmit and/or receive diversity has been analyzed both for the case of (spatially) homogeneously deployed, as well as clustered nodes. Monotonicity results are given, that provide, on one hand, a further proof of the benefits of spatial diversity employed at physical layer of wireless systems, and on the other hand enhance the need for adapting the choice of the proper transmit/receive diversity strategy to the local communication regime in non-homogeneous networks.

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