ON TURBULENCE IN HYDRODYNAMIC LUBRICATION

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Summary The contribution deals with a self-consistent description of time-mean turbulent lubricant flow, i.e. flow through a wedge-shaped gap confined by counter-sliding solid surfaces. The methods adopted are matched asymptotic expansions, where the slenderness or aspect ratio of the gap and the accordingly defined Reynolds number represent the perturbation parameters. As a remarkable finding, a load-bearing capacity as a consequence of the resultant pressure distribution can only be maintained for fully developed turbulent flow provided its asymptotic structure flow differs distinctly to that known from other turbulent internal flows as pipe flows or channel flows or classical turbulent boundary layers. The basic analysis is carried out without resorting to a specific Reynolds shear stress closure. However, the resulting requirements for asymptotically correct turbulence models are discussed. The theoretical study is accompanied by a numerical study of the boundary layer equations governing the fully turbulent core flow. Finally, the impact of cavitation, a phenomenon highly relevant in lubrication theory, on the novel flow structure is addressed.

PATHWAY TO TURBULENCE STARTING AT CLASSICAL LUBRICATION THEORY

The classical long-standing theory of lubrication is essentially traced back to the following five assumptions: (i) a Newtonian lubricant filling (ii) a relatively slender gap separating the counter-sliding rigid smooth surfaces, i.e. with (iii) a surface roughness being insignificantly small compared to the gap height (lubrication in the fully hydrodynamic regime), (iv) the neglect of inertia terms in the Navier–Stokes equations, and (v) strictly laminar flow. Then the continuity equation can be integrated across the clearance subject to the kinematic boundary conditions provided the well-known Reynolds equation (in its most general form) governs the pressure distribution between the wetted surfaces.

Classical lubrication theory in an asymptotic framework

In this classical theory the streamwise velocity profile represents a superposition of Poiseuille and Couette flow. It is noteworthy that for impervious surfaces as considered in the following the special configuration of a constant gap height is well-understood, albeit in different context: in absence of a counter-sliding motion such a flow is commonly referred to as Hele–Shaw flow (i.e. pure Poiseuille flow in the case of plane flow), and in absence of an imposed pressure difference between the inlet and outlet boundaries of the clearance it reduces to (in general three-dimensional unsteady) pure Couette flow.

As further assumptions adopted in the present study, the specific gap geometry, i.e. the distribution of its height, and hence the flow observed in a suitably adopted frame of reference are stationary, and mass forces due to the motion of the latter relative to an inertial frame of reference can be neglected. Then the basic prerequisites (ii) and (iv) are formally expressed as

(ii) \( \epsilon = \tilde{H} / \bar{L} \ll 1 \), (iv) \( \tilde{R} = \tilde{U} \bar{L} / \tilde{\nu} \ll 1 / \epsilon^2 \),

where the slenderness parameter \( \epsilon \) and the Reynolds number \( \tilde{R} e \) are formed with a typical gap height \( \tilde{H} \), a length \( \bar{L} \) characteristic of the lubricated region in the main direction of the sliding motion, a reference value \( \tilde{U} \) of the sliding speed, and the (constant) kinematic viscosity \( \tilde{\nu} \).

Extension of the classical theory for moderate to large Reynolds numbers

Interestingly, the relaxation of assumption (iv) has attracted little attention from theoreticians, despite the undeniable importance of convective effects in engineering applications as e.g. thrust bearings for high-speed rotors. In those cases \( \tilde{R} e = \epsilon^2 \tilde{R} e \) can no longer be regarded as a small parameter but rather as a quantity of \( O(1) \). On condition (v) the lubricant flow then is of boundary-layer type, where the pressure is not known in advance but adjusts as part of the solution of the underlying boundary layer equations, replacing the conventional Reynolds equation. Without doubt, a systematic treatment of such “internal” boundary layers for a most comprehensive variety of gap geometries and inlet and outlet conditions is rather involved as dealing with the three-dimensionality of the flow requires advanced numerical methods. Here we restrict the analysis to incompressible lubricants of uniform density \( \tilde{\rho} \) and plane gaps: then the resulting two-dimensional flow is essentially governed by the inflow and outlet conditions and the slope of the gap height, which fix the constant volume flux. Such flows have been studied e.g. in [1]. It seems expedient to briefly discuss some interesting questions associated with such types of internal boundary layer problems as these are also highly relevant in the turbulent case considered subsequently.

At first, the ellipticity of the boundary layer problem due to the unknown pressure distribution demands particular attention. In [1] it is accounted for by prescribing the (uniform) inflow velocity in dependence of the inlet and the outlet pressure. However, in any well-posed formulation of the problem different combinations of the inlet and outlet values of velocity and pressure may likely serve as upstream and downstream boundary conditions, but the last word is not yet spoken. Here we refer to the close resemblance of such internal flows to thin shear layers with free surfaces under the action of gravity, see e.g. [2], and boundary layers in mixed convection, see e.g. [3]. In these cases a well-posed formulation of the
boundary layer problem is crucially associated with appropriate downstream conditions: these suppress the generation of eigensolutions of unbounded growth, which (under certain conditions) eventually enforces termination of the downstream integration in form of a singularity. Secondly, we note that classical lubrication theory predicts flow reversal if the pressure gradient is adverse (negative contribution of the Poiseuille flow) and the local (negative) ratio \( r \) of the volume fluxes due to the Couette and the Poiseuille flow is greater than \(-3\): then \((3 + r)/2\) and \((3 + r)/3\) denote the fractions of the gap height occupied by the reverse-flow zone attached at the fixed boundary and that of upstream flow, respectively. It is interesting how the prediction of reverse flow based on boundary layer calculations as in [1] affects this criterion.

In a next step, assumptions (iii) and (v) are reviewed critically. It is well-known that steady Couette and typical Poiseuille flows as pipe and channel flows become (globally) unstable when the values of the characteristic Reynolds numbers exceed critical thresholds of an order of magnitude of \(10^4\). In the present context this Reynolds number is given by \( \tilde{U} H / \nu \), equal to \( Re_\kappa / \epsilon \). Simultaneously, in engineering applications \( \epsilon \) is typically of the order of \(10^{-4}\), so that assuming \( Re_\kappa = O(1) \), in connection with strictly laminar flow seems questionable. The herewith indicated inclusion of the Reynolds shear stress in the boundary layer equations coins the notion of superlaminar flow (in a state of laminar–turbulent transition). Eventually, for \( Re_\kappa \gg 1 \), the contribution of the viscous stress can be neglected and the fully turbulent regime is reached. This situation has been tackled on a semi-empirical basis, see e.g. [4]. Unfortunately, for developed turbulence this approach impedes a match of the bulk region with the (extremely thin) viscous wall layers adjacent to the surfaces, where both stresses are still of equal importance. This inconsistency ties in with the presence of the full inertia terms in leading order as it contradicts the classical notion of an asymptotically small streamwise velocity deficit with respect to the characteristic flow speed in the fully turbulent part of boundary layers or internal flows. However, here this classical description would predict a predominantly inviscid “lubricant” flow prevailing in most of the gap and thus a breakdown of the load-bearing capacity.

**RIGOROUS FLOW DESCRIPTION: A DISTINGUISHED LIMIT**

A rational treatment of the time-mean turbulent flow for \( Re_\kappa \gg 1 \) when the inertia terms are fully retained as suggested above leads to a multi-structured flow picture, in interesting analogy to that put forward in [5]. It consists of the core (bulk) region comprising most of the gap, the two wall layers, and two intermediate layers separating the first from the latter. The intermediate regions are reminiscent of the small velocity defect typical for wall-bounded turbulent shear flows, so that matching the fully turbulent flow with that in the viscous sublayers is governed by the logarithmic law of the wall. In the case of fully developed turbulent flow, one finds by discarding both the prerequisites (iv) and (v) that the bounds of validity (1) of classical lubrication theory are violated in favour of the characterisation

\[
\text{ii) } \epsilon \ll 1, \quad (\text{fully developed turbulence}) \quad \kappa K = \sqrt{\epsilon} \ln Re = O(1),
\]

Here \( \kappa \) denotes the von Kármán constant and \( K \) represents a similarity parameter. It describes a distinguished (least-degenerate) limit that introduces a formal coupling between \( \epsilon \) and \( Re_\kappa \).

Let \( x, y, h, u, v, \tau \) and \( p \) represent normal coordinates fixed to the particular sliding sheet that is steady in the reference frame, respectively, along and perpendicular to its wetted surface, the local gap height, the components of the flow velocity in \(-\) (streamwise) and \(y\)-direction, the Reynolds shear stress, and the pressure. Here \( x \) is made non-dimensional by \( L, y \) and \( h \) by \( H \), the flow velocity by the constant sliding speed \( U \), and \( \tau \) and \( p \) by \( \tilde{p}U^2 \). In the core region the flow quantities are appropriately expanded in the form \([u, v, \tau, \epsilon] \sim [U, V, T](x, y) + \cdots \) and \( p \sim P(x) + \cdots \) as \( \epsilon \to 0 \). We then arrive at the full boundary layers equations for turbulent flow, subject to the conditions of matching with the intermediate layers:

\[
\begin{align*}
\partial_x U + \partial_y V &= 0, \quad \partial_x U + \partial_y V = -dP/dx + \partial_y T, \\
y &= 0: \quad V = 0, \quad K \sqrt{|T|} \text{sgn}(T) = U, \quad y = h(x): \quad V = U dh/dx, \quad K \sqrt{|T|} \text{sgn}(T) = 1 - U.
\end{align*}
\]

The relationships between the wall shear stress \( T \) and the slip velocity relative to the motion of the respective sliding sheet in (4) provide the skin-friction law. In order to study (3), (4) numerically, an asymptotically correct shear stress closure is supplied, i.e. one consistent with the skin-friction law and the multi-layered structure of the flow (cf. [5]), and appropriate inlet and outlet conditions. Here we recall the impact of the latter in the context of laminar flow as addressed above.

Two phenomena (and their interplay) are of specific interest: (a) the occurrence of flow reversal, so that both the slip velocity and the wall shear stress in (4) change sign close to the positions of separation and reattachment which renders the proposed skin-friction locally invalid; (b) the (in applications undesired) onset of cavitation when the value of \( P \) falls below that referring to the vapour pressure, so that the lubricant undergoes a phase change. Both events require an extension of the flow description presented here, and efforts in this direction yield first encouraging results.

**References**


