ON PERFORMANCE BOUNDS FOR MIMO OFDM BASED WIRELESS COMMUNICATION SYSTEMS

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ABSTRACT

In this paper, we develop several increasingly tight and restrictive performance bounds for OFDM based MIMO wireless communication systems. Starting with channel capacity as the ultimate upper bound on the throughput, step by step we develop tighter bounds on the practically achievable throughput by incorporating typical design constraints. The presented bounds are applied to a Long Term Evolution system, allowing us to identify dominant sources of performance losses. The investigation reveals flaws of wireless communication systems thereby pointing at directions to efficiently improve performance in system design.

***Index Terms*—** Performance bounds, LTE, OFDMA, MIMO

1. INTRODUCTION

Since many years the channel capacity of multi-antenna Gaussian channels is well known [1], but still modern wireless communication systems are far from achieving throughputs that come close to capacity. Although modern turbo and LDPC codes are performing close to ideal Shannon codes [2], there are several other sources of performance loss that all together accumulate to a large fraction of the theoretically possible throughput. In this work, we systematically identify these sources and account for their corresponding throughput losses. The investigation shows that a large part of the performance degradation is caused by the system overhead required for timing and frequency synchronization as well as channel estimation.There are also other more fundamental losses, like the finite code block length or utilization of Bit Interleaved Coded Modulation (BICM) [3] instead of Gaussian signaling, which can be quantified theoretically. We do not consider losses due to sub-optimalities in e.g. the channel estimation process. We compare our bounds to practically achievable throughputs in a Long Term Evolution (LTE) system, utilizing the standard compliant Vienna LTE Link Level Simulator [4], [5]. The gap that is finally left between the tightest upper bound on throughput and the real achievable performance can be accounted to the channel code itself.

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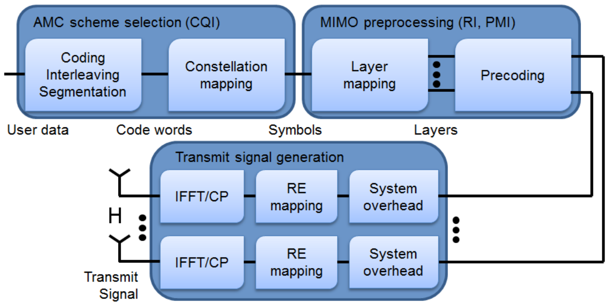

Fig. 1. System architecture of a modern wireless communication system.

We focus on Multiple Input Multiple Output (MIMO) Orthogonal Frequency Division Multiplexing (OFDM) communication systems, because practically all modern standardized systems are based on this architecture (e.g. 3GPP LTE, IEEE Worldwide Interoperability for Microwave Access (WiMAX)). Furthermore, these systems employ BICM which separates the coding and modulation mapping into two independent entities, thereby slightly sacrificing optimality but gaining in terms of decreased complexity. Finally, we assume closed loop linearly precoded MIMO systems that utilize feedback from the receiver to choose the appropriate precoder from a finite code book.

By taking into account a variety of practical design constraints, we develop increasingly tight bounds for the throughput of a single user MIMO OFDM system in Section 2. Afterwards, in Section 3, we apply the proposed bounds to an LTE system and evaluate the throughput loss caused by the individual constraints. We conclude in Section 4.

2. BOUNDS ON THE ACHIEVABLE THROUGHPUT

The amount of information that any communication system can transmit reliably over a given channel is upper bounded by the well known **channel capacity**. We consider a MIMO OFDM system with N_t transmit and N_r receive antennas, which splits the total system bandwidth into K orthogonal subcarriers. In such a system channel capacity can be achieved by employing Singular Value Decomposition (SVD)-based precoders and receive filters on each subcarrier at the transmitter and receiver side respectively. Furthermore, the total transmit power must be optimally distributed over the resulting parallel SISO channels, which can be achieved utilizing the well known water filling power allocation algorithm [4], [6].

We will now consider a practical wireless communication system with an architecture as shown in Figure 1. Such a system typically first codes the user data ("AMC scheme selection" in the figure), next the data is mapped onto the spatial transmission layers and appropriately precoded ("MIMO preprocessing") and afterwards the transmit signal is generated ("Transmit signal generation") by inserting the system overhead, mapping the data onto subcarriers and applying an Inverse Fast Fourier Transform (IFFT).

If the transmitter does not have any channel knowledge, the Mutual Information (MI) for a particular channel realization $\mathbf{H}_k \in \mathbb{C}^{N_r \times N_t}, k \in \{1, \ldots, K\}$, assuming zero mean complex Gaussian distributed transmit signals and receiver noise, is the appropriate performance measure instead of the channel capacity [7]:

$$I^{(\mathrm{OL})} = \sum_{k=1}^{K} \log_2 \det \left(\mathbf{I}_{N_r} + \frac{P}{\sigma_n^2 N_t} \mathbf{H}_k \mathbf{H}_k^H \right). \quad (1)$$

Here, σ_n^2 denotes the variance of the additive Gaussian receiver noise and P the transmit power. Precoding can be applied without channel knowledge to increase the diversity of the transmission (e.g. LTE utilizes cyclic delay diversity precoding [8]). Then the precoder is already contained in the effective channel matrix $\mathbf{H}_k$ in Equation (1). This bound is denoted **Open Loop Mutual Information (OLMI)**.

In a Frequency Division Duplex (FDD) system only by means of receiver feedback, knowledge about the channel from transmitter to receiver can be obtained. To limit the amount of feedback required, the possible precoders are restricted to a given unitary code book $\mathcal{W}$ (e.g. [8]). An optimum power allocation by means of water filling is not possible in that case as the effective channel matrix (precoder and channel) is not diagonal anymore, leading in most cases to uniform power allocation. The appropriate performance measure for such a system is the **Closed Loop Mutual Information (CLMI)** defined as:

$$I^{(\mathrm{CL})} = \sum_{k=1}^{K} \max_{\mathbf{W}_k \in \mathcal{W}} \log_2 \det \left(\mathbf{I}_{N_r} + \frac{P}{\sigma_n^2 N_t} \mathbf{H}_k \mathbf{W}_k \mathbf{W}_k^H \mathbf{H}_k^H \right). \quad (2)$$

To further reduce the feedback overhead, often the same precoder is used for the total system bandwidth which leads to the **Wideband Closed Loop Mutual Information (WB-CLMI)** bound:

$$I^{(\mathrm{CL,WB})} = \max_{\mathbf{W} \in \mathcal{W}} \sum_{k=1}^{K} \log_2 \det \left(\mathbf{I}_{N_r} + \frac{P}{\sigma_n^2 N_t} \mathbf{H}_k \mathbf{W} \mathbf{W}^H \mathbf{H}_k^H \right). \quad (3)$$

Until now we assumed an ideal Maximum Likelihood (ML) detector [6] at the receiver. Because such detectors have exponential complexity in the number of antennas they are seldom used in practice. Instead, simple linear receive filters (Zero Forcing (ZF), Minimum Mean Squared Error (MMSE)) are used to decouple the individual spatial data streams and decode them independently. Denoting with $\mathbf{F}_k$ the receive equalizer filter for subcarrier k, the post-equalization Signal to Interference and Noise Ratio (SINR) for spatial transmission layer l is given by [9]:

$$\mathrm{SINR}_{k,l} = \frac{P \cdot |\mathbf{K}_k[l,l]|^2}{P \cdot \sum_{i \neq l} |\mathbf{K}_k[l,i]|^2 + \sigma_n^2 \cdot \sum_i |\mathbf{F}_k[l,i]|^2} \quad (4)$$

$$\mathbf{K}_k = \mathbf{F}_k \mathbf{H}_k \mathbf{W}. \quad (5)$$

$\mathbf{K}_k[l,l]$ denotes the element in the lth row and lth column of the matrix $\mathbf{K}_k$. As we are assuming Gaussian transmit signals, the channel experienced on each layer after equalization corresponds to an AWGN channel with Signal to Noise Ratio (SNR) given by (4). Using AWGN channel capacity, we therefore define the **Wideband Closed Loop Mutual Information with Linear Receiver (WB-CLMI-LR)** bound as:

$$I^{(\mathrm{CL,WB,LR})} = \max_{\mathbf{W} \in \mathcal{W}} \sum_{k=1}^{K} \sum_{l=1}^{L(\mathbf{W})} \log_2 \det \left(1 + \mathrm{SINR}_{k,l}\right). \quad (6)$$

Here, $L(\mathbf{W})$ denotes the number of parallel spatial data streams (transmission rank), which depends on the precoder. The precoder determines whether a spatial multiplexing or a beam forming gain (or a mixture of both) is obtained by the MIMO system.

Another throughput loss is caused by the design of the channel coding and modulation mapping. To achieve channel capacity, coding with a Gaussian code book is required [10]. In practical systems, only a finite modulation alphabet is supported (e.g. 4/16/64 QAM). Depending on whether the code is jointly designed with the modulation mapping or the two are independently designed, the scheme is denoted Coded Modulation (CM) or BICM. The capacity of such systems is well known, albeit not in closed form [11]. At low SNR CM performs better than BICM. Nevertheless, BICM is preferred in practice because it allows to combine any channel code with any arbitrary modulation alphabet via a bit interleaver. For a Single Input Single Output (SISO) AWGN channel the BICM capacity for arbitrary modulation alphabets can easily be evaluated by means of Monte Carlo simulations (see [11]). This defines a mapping $B(\mathrm{SNR})$ from SNR to spectral efficiency in bits per channel use. If multiple modulation alphabets are defined, the mapping $B(\mathrm{SNR})$ chooses the maximum BICM capacity with respect to the alphabets. Although in case of finite modulation alphabets the inter-layer interference experienced after equalization is not Gaussian anymore, we employ $B(\mathrm{SNR})$ instead of the AWGN capacity in Equation (6) to take into account the effect of utilizing a BICM system[1]. We therefore define the **BICM bound** according to:

$$BI^{(\mathrm{CL,WB,LR})} = \max_{\mathbf{W}\in\mathcal{W}} \sum_{k=1}^{K} \sum_{l=1}^{L(\mathbf{W})} B\left(\mathrm{SINR}_{k,l}(\mathbf{W})\right). \quad (7)$$

The necessarily finite block length of the code also causes a performance degradation. It prevents an optimal Shannon code (a code that achieves the Shannon capacity limit for infinite block length) from achieving channel capacity. Consider an ideal Shannon code of given rate R, in information bits per channel symbol, with infinite block length. The Block Error Probability (BLEP) of such a code corresponds to a step function over SNR dropping from one to zero at exactly the SNR that corresponds to channel capacity $C = R$. The finite block length N (in channel symbols) causes the BLEP to decrease continuously over SNR. The BLEP P_B of the "best" code is upper bounded by the Gallager bound given by [2]:

$$P_B < 2^{-N\cdot E(R)};\;\; E(R) = \max_{q} \max_{0\le\rho\le 1} \left[E_0(\rho, q) - \rho R\right] \quad (8)$$

$$E_0(\rho, q) = -\log_2 \int_y \left[\sum_x q(x)p(y|x)^{\frac{1}{1+\rho}}\right]^{1+\rho}. \quad (9)$$

In these equations, x denotes the channel input signal, $q(x)$ is the channel input distribution, y is the channel output signal, $p(y|x)$ is the conditional probability of the channel output given the channel input and ρ is an auxiliary optimization variable. $E(R)$ is called the Gallager exponent. Similarly, the BLEP of the "best" code is lower bounded by the Sphere Packing bound [13] whose formulation is equal to the Gallager bound except ρ is just lower bounded by zero and not upper bounded. If the optimizing ρ lies between zero and one, the two bounds coincide and define the BLEP of the best code. In our calculations this was always the case.

Employing BICM renders the channel input signal uniformly distributed over the QAM alphabet. Therefore, the maximization with respect to $q(x)$ can be skipped. Assuming a linear receiver, the MIMO OFDM channel decomposes into parallel SISO AWGN channels with SNR given by Equation (4). Therefore one needs to evaluate the two bounds on BLEP for an AWGN channel with uniform input distribution over a given QAM constellation (e.g. numerically).

Because the code block length is finite, in general a BLEP of zero cannot be obtained anymore. A typical operating point for wireless communication systems is a target Block Error Ratio (BLER) of 10^{-1} [9]. Fixing the BLEP to the target BLER, the code rate of the best code that achieves the target BLER can be computed. This defines a mapping from SNR to code rate or spectral efficiency. An example is shown in Figure 2 for a block length of $N = 1\,000$ and 64 QAM. The figure compares the obtained spectral efficiency to the corresponding BICM capacity. The obtained spectral efficiency is

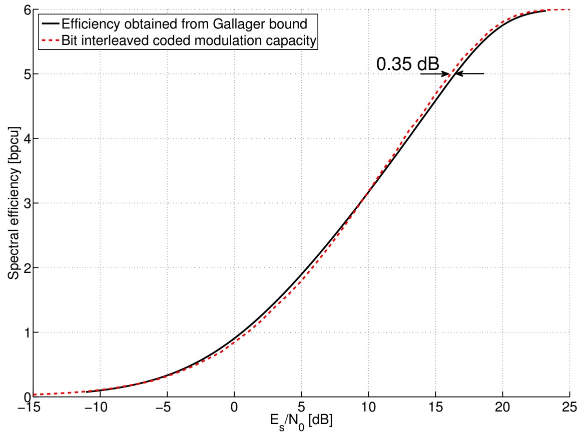

Fig. 2. Spectral efficiency obtained via the Gallager bound compared to BICM capacity of a 64QAM system.

below BICM capacity at high SNR (≥ 10 dB). At low SNR the obtained efficiency is larger than BICM capacity. The reason is that the Gallager and Sphere Packing bounds hold for a CM and not a BICM system. CM achieves higher spectral efficiency than BICM at low SNR [3]. Nevertheless, because it is known that CM and BICM perform equally good at high SNR [11], one can compute how much the BICM capacity needs to be shifted to take into account the finite code block length. This shift is obtained by matching the BICM capacity and the obtained spectral efficiency at high SNR. In Figure 2 the shift equals 0.35 dB. Using this shifted BICM efficiency $S(\mathrm{SNR})$ instead of the BICM capacity $B(\mathrm{SNR})$ in (7), we arrive at the **Shifted BICM (SBICM) bound**, denoted $SBI^{(\mathrm{CL,WB,LR})}$.

To achieve the SBICM bound, the communication system must support any possible code rate. In practical systems, typically just a small set of possible Adaptive Modulation and Coding (AMC) schemes is employed (15 in LTE/LTE-A). To still guarantee the desired BLER constraint, the supported AMC scheme with largest spectral efficiency less than or equal to the SBICM bound has to be utilized. Mathematically this can be formulated as "quantizing" the SBICM bound to the nearest AMC scheme with lower or equal spectral efficiency:

$$QSBI^{(\mathrm{CL,WB,LR})} = \lfloor SBI^{(\mathrm{CL,WB,LR})} \rfloor_{\mathcal{C}}. \quad (10)$$

The operator $\lfloor . \rfloor_{\mathcal{C}}$ means flooring with respect to the AMC schemes defined in the code book $\mathcal{C}$ of the considered system. We denote this bound **Quantized SBICM (QSBICM) bound**.

In general the MIMO channel is varying over time. Assuming ergodicity we obtain ergodic values for the bounds (e.g. ergodic capacity [6]) by means of Monte Carlo simulations for a chosen channel model in Section 3. During these Monte Carlo simulations we take into account the most obvious source of effective user throughput loss, namely the insertion of system overhead (e.g. for synchronization, channel estimation), by skipping the appropriate subcarriers, as

[1]For ZF receivers the post-equalization interference vanishes, validating the assumption of Gaussian noise, for MMSE receivers we resort to the Gaussian approximation of post-equalization interference [12].

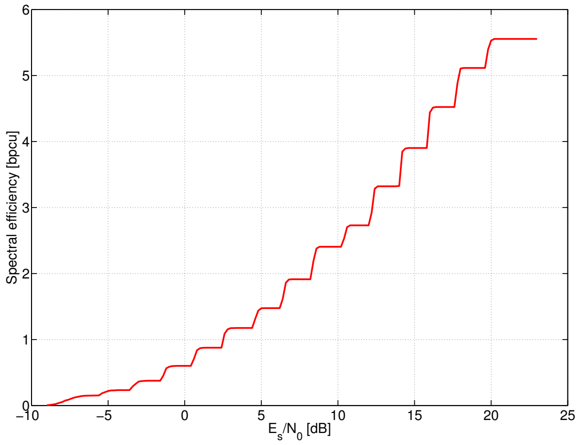

Fig. 3. Spectral efficiency obtained via link level simulations of a SISO LTE system over an AWGN channel.

defined by the considered standard. We refer to the bounds obtained in that way as **achievable** bounds, e.g. achievable mutual information [14].

The shifted BICM efficiency $S(\mathrm{SNR})$ is useful for predicting the optimal performance of a practical system as demonstrated in Section 3. This is achieved by computing the average efficiency over all subcarriers for all combinations of standard defined precoders $\mathcal{W}$ and modulation alphabets $\mathcal{A}$:

$$E(\mathbf{W}, A) = \frac{1}{K} \sum_{k=1}^{K} \sum_{l=1}^{L(\mathbf{W})} S_A\left(\mathrm{SINR}_{k,l}(\mathbf{W})\right). \qquad (11)$$

$S_A(\mathrm{SNR})$ denotes the shifted BICM efficiency corresponding to the modulation alphabet $A \in \mathcal{A}$. The SNR value of an AWGN channel that achieves the average efficiency $E(\mathbf{W}, A)$ can be computed via the inverse function of $S_A(\mathrm{SNR})$. This is very similar to Mutual Information Effective SNR Mapping (MIESM) [15]. Then lookup tables, obtained from link level simulations of a SISO AWGN channel, can be used to find the AMC scheme that achieves the highest spectral efficiency for the given SNR. Figure 3 shows an example of such a mapping for an LTE system. Note that for each AMC scheme the corresponding modulation alphabet dependent SNR value must be used. Knowing the best AMC scheme, the throughput can be computed by multiplying with the number of channel uses.

3. SIMULATION RESULTS FOR AN LTE SYSTEM

In this section, we apply the proposed bounds to an LTE compliant system. We compare the bounds to the optimal system performance obtained by exhaustive search simulations, utilizing the Vienna LTE Link Level Simulator [4], [5]. This means that we generate a channel and noise realization and simulate data transmission over this channel with all possible combinations of transmission ranks, precoders and AMC schemes. The optimum is the maximum throughput obtained over all feasible combinations. The results are averaged over 1 000 independent channel and noise realizations.

Channel model	VehA [16]
System bandwidth	1.4 MHz
Receiver equalizer	ZF
Antenna configuration	4×4
CQI feedback	✓
RI feedback	✓
PMI feedback	✓

Table 1. Settings for the comparison of the proposed throughput bounds to the simulated performance of an LTE system.

In our simulations we consider a 4×4 Closed Loop Spatial Multiplexing (CLSM) LTE system, whose settings are summarized in Table 1. All antennas are assumed to be spatially uncorrelated. In CLSM transmission mode, LTE enables the adaptation of the AMC scheme, transmission rank and spatial precoding matrix, by means of User Equipment (UE) feedback, in order to optimize the system performance. Thereby, the Channel Quality Indicator (CQI) chooses the appropriate AMC scheme to obtain a given target BLER (typically ≤ 0.1), the Rank Indicator (RI) signals the preferred number of parallel spatial data streams and the Precoding Matrix Indicator (PMI) informs the eNodeB about the currently best precoder, chosen from a finite code book [8].

Figure 4 shows the results for the bounds and simulated throughput obtained for the 4×4 system. Considering an operating point of SNR $= 10.4$ dB the following is observed:

- Channel capacity (the leftmost curve) equals 12 Mbit/s.
- An LTE system, employing realistic feedback algorithms to compute the feedback indicators [9], achieves a throughput of 5.9 Mbit/s corresponding to 49% of channel capacity.
- The optimal LTE performance, assuming perfect knowledge of the channel and noise realization at the transmitter, equals 50.6% of channel capacity.

By means of the derived throughput bounds we next analyze the reasons for the 50% throughput gap between the practical system and channel capacity:

- The signaling overhead (achiev. channel capacity) causes a loss of 2 Mbit/s (16.7% of channel capacity).
- The loss caused by the finite set of precoders and equal power allocation equals 0.4 Mbit/s (3.3%). The loss is rather insignificant because at the considered SNR the achievable mutual information (without any precoding) already closes up to the achievable channel capacity.
- With linear receiver (CLMI-LR bound) the throughput decreases to 8.7 Mbit/s corresponding to a loss of 7.5%.
- Utilizing the same precoder (for feedback reduction) for the total system bandwidth (WB-CLMI-LR bound) leads to a loss of 0.5 Mbit/s or 4.2%. Note that this loss depends on the coherence bandwidth of the considered channel model (1.35 MHz for VehA [17]).
- The BICM bound, taking into account the modulation alphabets, lies 0.6 Mbit/s or 5% of channel capacity below the WB-CLMI-LR bound.

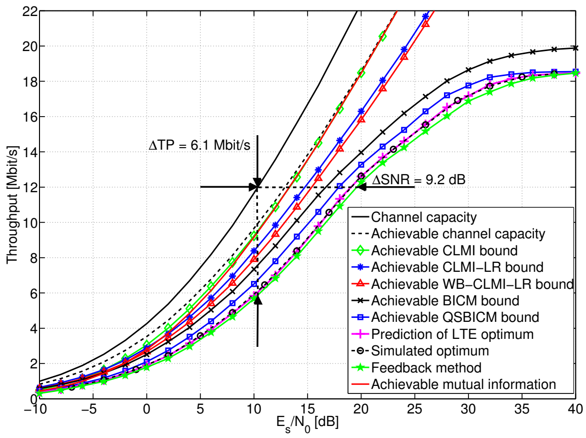

Fig. 4. Comparison of throughput bounds and simulated system performance for a 4×4 CLSM system.

- The finite code block length and finite set of supported code rates decrease the performance by 6.7% of channel capacity to 6.75 Mbit/s (QSBICM) corresponding to 56% of channel capacity.
- The method to predict the optimal LTE performance, described at the end of Section 2, overlaps with the simulated optimal performance, obtained by link level simulations.

Our tightest throughput bound is 6% off the simulated optimal performance. This difference is caused by the actual channel code performance. Note that we do not include additional losses caused by guard bands and a like.

4. CONCLUSION

In this paper, we analyze the performance of MIMO OFDM based wireless communication systems by means of throughput bounds. Taking into account typical practical design constraints like the structure of spatial precoding, finite sets of supported AMC schemes, finite code block length, system overhead and receiver design we arrive at increasingly tight bounds. The loss caused by each of these constraints is quantified. We compare the obtained bounds to the performance of a practical LTE system by means of link level simulations. This shows that LTE, utilizing ZF equalizers at the receivers, achieves around 50% of channel capacity, assuming perfect channel estimation and frequency synchronication, and ignoring losses caused by guard bands. The investigation reveals that a major loss of user throughput is caused by system overhead, especially for large antenna configurations. Also ZF equalizers, to separate the spatial streams, are not a useful choice. The limited number of supported AMC schemes (15 in LTE) also causes a strong throughput degradation. Finally, the performance of the channel code itself is far from what Shannon promises.

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