Optimal Pilot Symbol Power Allocation in LTE

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***Abstract*—The UMTS Long Term Evolution (LTE) allows the pilot symbol power to be adjusted with respect to that of the data symbols. Such power increase at the pilot symbols results in a more accurate channel estimate, but in turn reduces the amount of power available for the data transmission. In this paper, we derive optimal pilot symbol power allocation based on maximization of the post-equalization Signal to Interference and Noise Ratio (SINR) under imperfect channel knowledge. Simulation validates our analytical mode for optimal pilot symbol power allocation.**

***Index Terms*—LTE, Channel Estimation, OFDM, MIMO.**

I. Introduction

Current systems for cellular wireless communication are designed for coherent detection. Therefore, channel estimator is a crucial part of a receiver. UMTS Long Term Evolution (LTE) provides a possibility to change the power radiated at the pilot subcarriers relative to the that at data subcarriers. Clearly, this additional degree of freedom in the system design provides potential for optimization.

A. Related Work

In order to optimize the pilot symbol power allocation a model that takes into account the pilot power adjusting, receiver structure and channel estimation error at the same time, is needed. It has been shown by simulation that pilot symbol power allocation has a strong impact on capacity [1]. Authors of [2] show by simulation the impact of different power allocations on the system's Bit Error Rate (BER). However, their analysis is based on Signal to Noise Ratio (SNR) so that they only approximate the impact of imperfect channel knowledge on BER for Binary Phase-shift Keying (BPSK) modulation. In [3], optimal pilot symbol allocation is derived analytically for Phase-shift Keying (PSK) modulation of order two and four, using BER as the optimization criterion. In [4] optimal pilot symbol power in Multiple Input Multiple Output (MIMO) system is derived based on lower bound for capacity. Authors of [5] investigate power allocation between pilot and data symbols for MIMO systems using post-equalization Signal to Interference and Noise Ratio (SINR) as the optimization function. However, they only approximate the SINR expression and their model is tightly connected with a Linear Minimum Mean Square Error (LMMSE) channel estimator.

B. Contribution

In this paper, we derive analytical expressions for optimal power allocation for a MIMO system with Zero Forcing (ZF) equalizer under imperfect channel state information. We utilize the post-equalization SINR, as the optimization function, which is analogous to the throughput maximization in a real system.

The main contributions of the paper are:

- By maximizing the post-equalization SINR, we deliver optimal values for the pilot symbol power adjustment in MIMO Orthogonal Frequency Division Multiplexing (OFDM) systems.
- The post-equalization SINR expression is derived for a ZF receiver under imperfect channel knowledge.
- We analytically derive the Mean Square Error (MSE) expression of the Least Squares (LS) channel estimator utilizing linear interpolation.
- Simulation results with an LTE compliant simulator [6, 7] confirm optimal values for pilot symbol power.
- As with our previous work, all data, tools, as well implementations needed to reproduce the results of this paper can be downloaded from our homepage [8].

The remainder of the paper is organized as follows. In Section II we describe the mathematical system model. In Section III, we derive the post-equalization SINR expression for ZF with imperfect channel knowledge. The channel estimators of this work are briefly discussed in Section IV and their MSEs are derived. We formulate the optimization problem for optimal pilot symbol power allocation in Section V. Finally, we present LTE simulation results in Section VI and conclude our paper in Section VII.

II. System Model

In this section, we briefly point out the key aspects of LTE relevant for this paper, as well as an system model.

In the time domain the LTE signal consists of frames with a duration of $10\,\mathrm{ms}$. Each frame is split into ten equally long subframes and each subframe into two equally long slots with a duration of $0.5\,\mathrm{ms}$. Depending on the cyclic prefix length, being either extended or normal, each slot consists of $N_\mathrm{s} = 6$ or $N_\mathrm{s} = 7$ OFDM symbols, respectively. In LTE, the subcarrier spacing is fixed to $15\,\mathrm{kHz}$. Twelve adjacent subcarriers of one slot are grouped into a so-called resource block. The number of resource blocks in an LTE slot ranges from 6 up to 100, corresponding to a bandwidth from $1.4\,\mathrm{MHz}$ up to $20\,\mathrm{MHz}$.

A received OFDM symbol in frequency domain can be written as

$$\mathbf{y}_{n_\mathrm{r}} = \sum_{n_\mathrm{t}=1}^{N_\mathrm{t}} \mathbf{X}_{n_\mathrm{t}} \mathbf{h}_{n_\mathrm{t},n_\mathrm{r}} + \mathbf{n}_{n_\mathrm{r}}, \tag{1}$$

where the vector $\mathbf{h}_{n_\mathrm{t},n_\mathrm{r}}$ contains the channel coefficients in the frequency domain between the n_t-th transmiter and n_r-th receiver antennas and $\mathbf{n}_{n_\mathrm{r}}$ is additive white zero mean Gaussian noise with variance σ_n^2 at the n_r-th receiver antenna. The diagonal matrix $\mathbf{X}_{n_\mathrm{t}} = \mathrm{diag}\left(\mathbf{x}_{n_\mathrm{t}}\right)$ comprises the precoded data symbols $\mathbf{x}_{\mathrm{d},n_\mathrm{t}}$ and the pilot symbols $\mathbf{x}_{\mathrm{p},n_\mathrm{t}}$ at the n_t-th transmit antenna placed by a suitable permutation matrix $\mathbf{P}$ on the main diagonal

$$\mathbf{x}_{n_\mathrm{t}} = \mathbf{P}\ \left[\mathbf{x}_{\mathrm{p},n_\mathrm{t}}^\mathrm{T}\ \mathbf{x}_{\mathrm{d},n_\mathrm{t}}^\mathrm{T}\right]^\mathrm{T}. \tag{2}$$

The length of the vector $\mathbf{x}_{n_\mathrm{t}}$ is N_sub corresponding to the number of subcarriers. Let us denote by N_p and N_d, the number of pilot symbols and the number of precoded data symbols, respectively. The precoded data symbols $\mathbf{x}_{\mathrm{d},n_\mathrm{t}}$ are obtained from the data symbols via precoding

$$\left[x_{\mathrm{d},1,k} \cdots x_{\mathrm{d},N_\mathrm{t},k}\right]^\mathrm{T} = \mathbf{W}_k \left[s_{1,k} s_{2,k} \cdots s_{N_\mathrm{l},k}\right]^\mathrm{T}, \tag{3}$$

where $x_{\mathrm{d},n_\mathrm{t},k}$ is a precoded data symbol at the n_t-th transmit antenna port and the k-th subcarrier, $\mathbf{W}_k$ is the precoding matrix at the k-th subcarrier and $s_{n_\mathrm{l},k}$ is the data symbol of the n_l-th layer at the k-th subcarrier.

Note that according to Equation (2), also the vectors $\mathbf{y}_{n_\mathrm{r}}$, $\mathbf{h}_{n_\mathrm{t},n_\mathrm{r}}$ and $\mathbf{w}_{n_\mathrm{r}}$ can be divided into two parts corresponding to the pilot symbol positions and to the data symbol positions.

For the derivation of the post-equalization SINR, we will use the input-output relation at the subcarrier level, given as:

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{W}_k \mathbf{s}_k + \mathbf{n}_k. \tag{4}$$

Matrix $\mathbf{H}_k$ is the MIMO channel matrix of size $N_\mathrm{r} \times N_\mathrm{t}$. Furthermore, matrix $\mathbf{W}_k$ is a unitary precoding matrix of size $N_\mathrm{t} \times N_\mathrm{l}$. In LTE the precoding matrix can be chosen from a finite set of precoding matrices [9]. The vector $\mathbf{s}_k$ comprises the data symbols of all layers at the k-th subcarrier. We denote the effective channel matrix that includes the effect of the channel and precoding by $\mathbf{G}_k$

$$\mathbf{G}_k = \mathbf{H}_k \mathbf{W}_k. \tag{5}$$

Furthermore, let us denote the average data power transmitted at one layer by σ_s^2, the total data power by σ_x^2, and the average pilot symbol power by σ_p^2

$$\sigma_s^2 = \mathbb{E}\left\{\|s_{l,k}\|_2^2\right\} = \frac{1}{N_\mathrm{l}}, \tag{6}$$

$$\sigma_x^2 = \sum_{n_\mathrm{t}=1}^{N_\mathrm{t}} \mathbb{E}\left\{\|\mathbf{x}_{\mathrm{d},n_\mathrm{t}}\|_2^2\right\} = 1, \tag{7}$$

$$\sigma_\mathrm{p}^2 = \sum_{n_\mathrm{t}=1}^{N_\mathrm{t}} \mathbb{E}\left\{\|\mathbf{x}_{\mathrm{p},n_\mathrm{t}}\|_2^2\right\} = 1. \tag{8}$$

III. Post-equalization SINR

In this section, we derive an analytical expression for the post-equalization SINR of a MIMO system under imperfect channel knowledge and ZF equalizer given by the system model in Equation (4). In the following section, we omit the subcarrier index k, since the concept we present is independent of it.

If the equalizer has perfect channel knowledge available, the ZF estimate of the data symbol $\mathbf{s}$ is given as

$$\hat{\mathbf{s}} = \left(\mathbf{G}^\mathrm{H}\mathbf{G}\right)^{-1}\mathbf{G}^\mathrm{H}\mathbf{y}. \tag{9}$$

The data estimate $\hat{\mathbf{s}}$ given by Equation (9) results in the post-equalization SINR of the l-th layer given as [10]

$$\gamma_l = \frac{\sigma_s^2}{\sigma_n^2 \mathbf{e}_l^\mathrm{H}\left(\mathbf{G}^\mathrm{H}\mathbf{G}\right)^{-1}\mathbf{e}_l}, \tag{10}$$

where the vector $\mathbf{e}_l$ is an $N_\mathrm{l} \times 1$ zero vector with the l-th element being 1. This vector serves to extract the corresponding layer power after the equalizer.

Let us proceed to the case of imperfect channel knowledge. We define the perfect channel as the channel estimate plus the error matrix due to the imperfect channel estimation

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{E}, \tag{11}$$

where the elements of the matrix $\mathbf{E}$ are independent of each other with variance σ_e^2. Inserting Equation (11) in Equation (4), the input output relation changes to

$$\mathbf{y} = \left(\hat{\mathbf{H}} + \mathbf{E}\right)\mathbf{W}\mathbf{s} + \mathbf{n}. \tag{12}$$

Since the channel estimation error matrix $\mathbf{E}$ is unknown at the receiver, the ZF solution is given again by Equation (9), but channel matrix $\mathbf{H}$ is replaced by its estimate $\hat{\mathbf{H}}$, which is known at the receiver

$$\hat{\mathbf{s}} = \left(\hat{\mathbf{G}}^\mathrm{H}\hat{\mathbf{G}}\right)^{-1}\hat{\mathbf{G}}^\mathrm{H}\mathbf{y}, \tag{13}$$

with matrix $\hat{\mathbf{G}}$ being equal to $\hat{\mathbf{H}}\mathbf{W}$. The error after the equalization process using ZF is given as

$$\hat{\mathbf{s}} - \mathbf{s} = \left(\hat{\mathbf{G}}^\mathrm{H}\hat{\mathbf{G}}\right)^{-1}\hat{\mathbf{G}}^\mathrm{H}\left(\mathbf{E}\mathbf{W}\mathbf{s} + \mathbf{n}\right). \tag{14}$$

From Equation (14) we can compute the MSE matrix

$$\begin{aligned}\mathrm{MSE} &= \mathbb{E}\left\{\left(\hat{\mathbf{s}} - \mathbf{s}\right)\left(\hat{\mathbf{s}} - \mathbf{s}\right)^\mathrm{H}\right\} \\ &= \left(\sigma_n^2 + \sigma_x^2\sigma_e^2\right)\left(\hat{\mathbf{G}}^\mathrm{H}\hat{\mathbf{G}}\right)^{-1},\end{aligned} \tag{15}$$

with σ_e^2 being the MSE of the channel estimator. Equation (15) directly leads to the SINR of the individual layers

$$\gamma_l = \frac{\sigma_s^2}{\left(\sigma_n^2 + \sigma_e^2\sigma_x^2\right)\mathbf{e}_l^\mathrm{H}\left(\hat{\mathbf{G}}^\mathrm{H}\hat{\mathbf{G}}\right)^{-1}\mathbf{e}_l}. \tag{16}$$

Note, that in practice variables σ_s^2, σ_x^2 and σ_n^2 are replaced by their estimates.

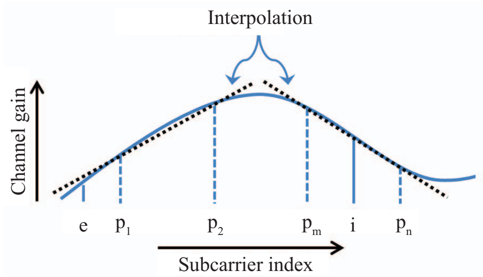

Fig. 1. Linear inter- and extrapolation of the channel on pilot subcarriers to intermediate subcarriers.

IV. Channel Estimation

In this section, we present state-of-the-art channel estimators and derive analytical expressions for their MSE. Due to the orthogonal pilot symbol pattern utilized in LTE, the MIMO channel can be estimated as $N_\mathrm{t}N_\mathrm{r}$ individual Single Input Single Output (SISO) channels. Therefore, in the following section, we omit the antenna indices.

A. LS Channel Estimation

The LS channel estimator [11, 12] for the pilot symbol positions is given as the solution to the minimization problem:

$$\hat{\mathbf{h}}_\mathrm{p}^\mathrm{LS} = \arg\min_{\hat{\mathbf{h}}_\mathrm{p}} \left\| \mathbf{y}_\mathrm{p} - \mathbf{X}_\mathrm{p}\,\hat{\mathbf{h}}_\mathrm{p} \right\|_2^2 = \mathbf{X}_\mathrm{p}^{-1}\,\mathbf{y}_\mathrm{p} \tag{17}$$

The remaining channel coefficients at the data subcarriers have to be obtained by means of interpolation. In this work, we utilize linear interpolation of the pilot subcarriers on each antenna to obtain the channel vector $\mathbf{h}$ containing the channel elements of all subcarriers in one OFDM symbol.

In order to derive the theoretical MSE, we distinguish three cases, as shown in Figure 1:

- MSE at a pilot subcarrier p_i: $\sigma_{e_p}^2$
- MSE at an interpolated subcarrier i: $\sigma_{e_i}^2$
- MSE at an extrapolated subcarrier e: $\sigma_{e_e}^2$

At a pilot position p_i the estimated channel element equals:

$$\hat{h}_{p_i} = \frac{y_{p_i}}{x_{p_i}} = h_{p_i} + \underbrace{\frac{n_{p_i}}{x_{p_i}}}_{\Delta h_{p_i}} \tag{18}$$

The MSE of the channel estimator is given by the noise variance $\sigma_{e_p}^2 = \sigma_n^2$. To obtain the variance at an interpolated position consider the following equation to compute the interpolated channel element $\hat{h}_i$ from the two neighboring pilot positions $\hat{h}_{p_m}$ and $\hat{h}_{p_n}$:

$$\hat{h}_i = \hat{h}_{p_m} + \underbrace{\frac{i - p_m}{p_n - p_m}}_{c_1} \cdot (\hat{h}_{p_n} - \hat{h}_{p_m}) \tag{19}$$

Then, the MSE can be computed as:

$$\sigma_i^2 = \mathbb{E}\left\{ \left\| h_i - \hat{h}_i \right\|_2^2 \right\} \tag{20}$$

$$= \mathbb{E}\left\{ \left\| h_i - \left(\hat{h}_{p_m} + c_1 \cdot (\hat{h}_{p_n} - \hat{h}_{p_m}) \right) \right\|_2^2 \right\} \tag{21}$$

By plugging (18) into (20), expanding the square and using the identities

$$\mathbb{E}\left\{ \Delta h_{p_i}^* h_{p_i} \right\} = \mathbb{E}\left\{ \frac{n_{p_i}}{x_{p_i}} h_{p_i} \right\} = 0, \tag{22}$$

$$c_2 = 1 - c_1, \tag{23}$$

we obtain for the LS MSE at an interpolated position:

$$\sigma_{e_i}^2 = \mathbf{R}_{\mathbf{h},\mathbf{h}}^{(i,i)} + c_1^2(\mathbf{R}_{\mathbf{h},\mathbf{h}}^{(p_n,p_n)} + \sigma_n^2) + c_2^2(\mathbf{R}_{\mathbf{h},\mathbf{h}}^{(p_m,p_m)} + \sigma_n^2)$$
$$+2c_1c_2\Re\left(\mathbf{R}_{\mathbf{h},\mathbf{h}}^{(p_m,p_n)}\right) - 2c_1\Re\left(\mathbf{R}_{\mathbf{h},\mathbf{h}}^{(i,p_n)}\right) - 2c_2\Re\left(\mathbf{R}_{\mathbf{h},\mathbf{h}}^{(p_m,i)}\right) \tag{24}$$

Here, $\mathbf{R}_{\mathbf{h},\mathbf{h}}^{(i,j)}$ denotes the element in the i-th row and j-th column of matrix $\mathbf{R}_{\mathbf{h},\mathbf{h}} = \mathbb{E}\left\{\mathbf{h}\mathbf{h}^\mathrm{H}\right\}$. For an extrapolated position, we utilize the two consecutive neighboring pilot positions, p_1 and p_2, to obtain the estimated channel element according to:

$$\hat{h}_e = \hat{h}_{p_1} + \underbrace{\frac{e - p_1}{p_2 - p_1}}_{c_1}(\hat{h}_{p_2} - \hat{h}_{p_1}) \tag{25}$$

Then, the same Formula (24), with appropriately modified indices, applies for the extrapolated MSE $\sigma_{e_e}^2$ as well. The MSE at an extrapolated position is generally larger than at an interpolated position, because the cross correlation between the channels at position e and p_2 is smaller than those between the channels at position i and p_n.

The total MSE of LS channel estimator is given as mean over all subcarriers. Due to the dense pilot symbol pattern and thus strong correlation over frequency, the total MSE can be expressed as

$$\sigma_e^2 = c_e \sigma_n^2, \tag{26}$$

where c_e is a constant. Equation (26) follows from Equation (23) and Equation (24).

B. LMMSE Channel Estimation

The LMMSE channel estimator requires the second order statistics of the channel and the noise. It can be shown that the LMMSE channel estimate is obtained by multiplying the LS estimate with a filtering matrix $\mathbf{A}_\mathrm{LMMSE}$ [13–15]:

$$\hat{\mathbf{h}}_\mathrm{LMMSE} = \mathbf{A}_\mathrm{LMMSE}\hat{\mathbf{h}}_\mathrm{p}^\mathrm{LS} \tag{27}$$

In order to find the LMMSE filtering matrix, the MSE

$$\sigma_e^2 = \mathbb{E}\left\{ \left\| \mathbf{h} - \mathbf{A}_\mathrm{LMMSE}\hat{\mathbf{h}}_\mathrm{p}^\mathrm{LS} \right\|_2^2 \right\}, \tag{28}$$

has to be minimized, leading to

$$\mathbf{A}_\mathrm{LMMSE} = \mathbf{R}_{\mathbf{h},\mathbf{h}_\mathrm{p}}\left(\mathbf{R}_{\mathbf{h}_\mathrm{p},\mathbf{h}_\mathrm{p}} + \sigma_n^2\mathbf{I}\right)^{-1}, \tag{29}$$

where the matrix $\mathbf{R}_{\mathbf{h}_\mathrm{p},\mathbf{h}_\mathrm{p}} = \mathbb{E}\left\{\mathbf{h}_\mathrm{p}\mathbf{h}_\mathrm{p}^\mathrm{H}\right\}$ is the channel autocorrelation matrix at the pilot symbol positions, and the matrix $\mathbf{R}_{\mathbf{h},\mathbf{h}_\mathrm{p}} = \mathbb{E}\left\{\mathbf{h}\mathbf{h}_\mathrm{p}^\mathrm{H}\right\}$ is the channel crosscorrelation matrix.

To derive the theoretical MSE, we plug Equation (29) into Equation (28):

$$\sigma_e^2 = \mathbb{E}\left\{ \left(\mathbf{h} - (\mathbf{R}_{\mathbf{h},\mathbf{h}_\mathrm{p}}(\mathbf{R}_{\mathbf{h}_\mathrm{p},\mathbf{h}_\mathrm{p}} - \sigma_n^2\mathbf{I})^{-1}\hat{\mathbf{h}}_\mathrm{p}^{\mathrm{LS}}) \right) \cdot \left(\mathbf{h} - (\mathbf{R}_{\mathbf{h},\mathbf{h}_\mathrm{p}}(\mathbf{R}_{\mathbf{h}_\mathrm{p},\mathbf{h}_\mathrm{p}} - \sigma_n^2\mathbf{I})^{-1}\hat{\mathbf{h}}_\mathrm{p}^{\mathrm{LS}}) \right)^{\mathrm{H}} \right\} \tag{30}$$

After a straightforward manipulation, the average MSE at each subcarrier is expressed as

$$\sigma_e^2 = \frac{1}{N_{\mathrm{sub}}} \mathrm{tr}\left\{ \mathbf{R}_{\mathbf{h},\mathbf{h}} - \mathbf{R}_{\mathbf{h},\mathbf{h}_\mathrm{p}} \left(\mathbf{R}_{\mathbf{h}_\mathrm{p},\mathbf{h}_\mathrm{p}} + \sigma_n^2 \mathbf{I} \right)^{-1} \mathbf{R}_{\mathbf{h}_\mathrm{p},\mathbf{h}} \right\}.$$

V. Power Allocation

In this section, we describe the problem of optimal pilot power allocation in LTE based on the maximization of the post-equalization SINR under imperfect channel knowledge. Although the shown results are from the application to LTE, the presented concept can be applied to any MIMO OFDM system.

If we increase the power at the pilot symbol by a factor c_p^2, the MSE of the channel estimator improves by the factor c_p^2

$$\tilde{\sigma}_e^2 = \frac{\sigma_e^2}{c_\mathrm{p}^2}. \tag{31}$$

However, the power of the data symbols has to be decreased by a factor c_d^2 in order to keep the total transmit power constant. Obviously, the two factors c_p^2 and c_d^2 are connected. For this purpose, we define a variable p_off, expressing the power offset between the mean energy of the pilot symbols and the data symbols, and refer to it as pilot offset. The variables c_p^2, c_d^2 and p_off are interconnected as follows:

$$c_\mathrm{p} = \frac{N_\mathrm{p} + N_\mathrm{d}}{p_\mathrm{off} N_\mathrm{d} + N_\mathrm{p}} \tag{32}$$

$$c_\mathrm{d} = \frac{N_\mathrm{p} + N_\mathrm{d}}{N_\mathrm{d} + \frac{N_\mathrm{p}}{p_\mathrm{off}}} = p_\mathrm{off} c_\mathrm{p} \tag{33}$$

Plugging in the variables c_d^2 and c_p^2 in Equation (16), we obtain the SINR expression with adjusted power of the pilot symbols

$$\gamma_l = \frac{\sigma_s^2 c_\mathrm{d}^2}{\left(\sigma_n^2 + \frac{\sigma_e^2}{c_\mathrm{p}^2} \sigma_x^2 c_\mathrm{d}^2 \right) \mathbf{e}_l^{\mathrm{H}} \left(\mathbf{G}^{\mathrm{H}} \mathbf{G} \right)^{-1} \mathbf{e}_l}. \tag{34}$$

If we insert Equation (32) and Equation (33) into Equation (34) and simplify the expression, we obtain:

$$\gamma_l = \frac{1}{\frac{N_l \sigma_n^2 \mathbf{e}_l^{\mathrm{H}} (\mathbf{G}^{\mathrm{H}}\mathbf{G})^{-1} \mathbf{e}_l}{(N_\mathrm{d} + N_\mathrm{p})^2} f(p_\mathrm{off})}, \tag{35}$$

for which the function $f(p_\mathrm{off})$ is given as

$$f(p_\mathrm{off}) = (p_\mathrm{off} N_\mathrm{d} + N_\mathrm{p})^2 \left(\frac{1}{p_\mathrm{off}^2} + c_e \right). \tag{36}$$

We refer to it as power allocation function. Note, that it is independent of channel realization and noise power. Therefore, it is possible to find an optimum value for pilot symbol power allocation independent of the SNR and channel realization.

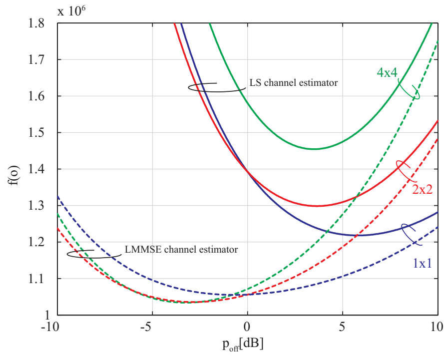

Fig. 2. Power allocation function $f(p_\mathrm{off})$ for different numbers of transmit antennas and LS (solid line) and LMMSE (dashed line) channel estimators

TABLE I
Values of the parameters of $f(p_\mathrm{off})$ for different number of transmit antennas for 1.4 MHz bandwidth, ITU PedA [17] channel model, LS and LMMSE channel estimators

Parameter	Tx = 1	Tx = 2	Tx = 4
N_d	960	912	864
N_p	48	96	144
LS			
c_e	0.3704	0.3704	0.5556
$p_\mathrm{off,opt}$ [dB]	≈5.8	≈3.6	≈3.5
LMMSE			
c_e	0.0394	0.0394	0.0544
$p_\mathrm{off,opt}$ [dB]	≈-0.7	≈-2.8	≈-3.2

The target is to find an optimal value of $p_\mathrm{off,opt}$ that maximizes the post-equalization SINR while keeping the overall transmit power constant

$$\underset{p_\mathrm{off}}{\text{maximize}} \quad \gamma_l \qquad \text{subject to} \quad N_\mathrm{d}\sigma_x^2 + N_\mathrm{p}\sigma_\mathrm{p}^2 = \text{const} \tag{37}$$

In order to maximize the post-equalization SINR, the power allocation function $f(p_\mathrm{off})$ in the denominator of Equation (35) has to be minimized. The minimum of the power allocation function $f(p_\mathrm{off})$ can be found by simply differentiating it and solving for 0. By these means, the optimal value of the variable p_off is given as solution to the following expression:

$$-2\left(p_\mathrm{off} N_\mathrm{d} + N_\mathrm{p}\right)^2 p_\mathrm{off}^{-3} + 2\left(p_\mathrm{off} N_\mathrm{d} + N_\mathrm{p}\right) N_\mathrm{d} \left(p_\mathrm{off}^{-2} + c_e\right) = 0. \tag{38}$$

An analytical solution for Equation (38) can be obtained by means of Ferrari's solution [16]. Figure 2 depicts $f(p_\mathrm{off})$ for different numbers of transmit antennas for LTE. Typical values of parameters N_d, N_p and c_e are given in Table I. Note, that although N_d and N_p depend on the utilized bandwidth, the minimum of $f(p_\mathrm{off})$ is independent of it, since N_d and N_p scale with the same constant with increasing bandwidth. The value of c_e is different for four transmit antennas due to the lower number of pilot symbols at the third and fourth antenna. The last row of Table I gives the optimal values of $p_\mathrm{off,opt}$ for different numbers of transmit antennas and an ITU PedA [17] type channel model. While utilizing the LMMSE channel estimator, one obtains negative values for $p_\mathrm{off,opt}$. This means, that optimally, the pilot symbol power has to be reduced in order to maximize the post-equalization SINR.

TABLE II
SIMULATOR SETTINGS FOR FAST FADING SIMULATIONS

Parameter	Value
Bandwidth	1.4 MHz
Number of transmit antennas	1, 2, 4
Number of receive antennas	1, 2, 4
Receiver type	ZF
Transmission mode	Open-loop spatial multiplexing
Channel type	ITU PedA [17]
MCS	9
coding rate	616/1 024 ≈ 0.602
symbol alphabet	16 QAM

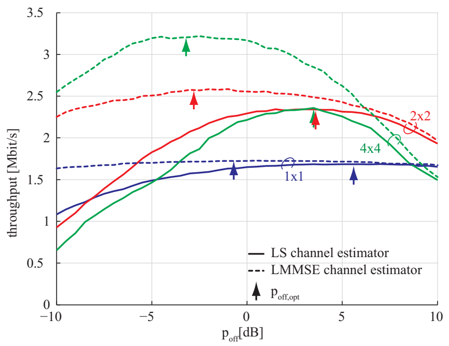

Fig. 3. Throughput of LTE downlink over pilot symbol power

Such result is intuitively clear, with improving quality of the channel estimator, it is sufficient to use less power on the pilot symbols to obtain the same quality of the channel estimate.

VI. SIMULATION RESULTS

In this section, we present simulation results and discuss the performance of LTE system using different pilot symbol powers. All results are obtained with the LTE Link Level Simulator version "r811" [6], which can be downloaded from www.nt.tuwien.ac.at/ltesimulator. All data, tools and scripts are available online in order to allow other researchers to reproduce the results shown in the paper [8].

Table II presents the most important simulator settings. Since what is to be observed is the optimal value of pilot symbol power adjustment, an SNR of 14 dB has been chosen, at which the system is not saturated for this specific Modulation and Coding Scheme (MCS).

Simulation results showing throughput performance for 1×1, 2×2, and 4×4 antenna configurations are shown in Figure 3 for LS and LMMSE channel estimators.

The maximum throughput values correspond excellently with $p_{\mathrm{off,opt}}$ shown in Table I, thus the simulation results match with the analytical results. Note, that although we show throughput results for specific SNR value, the optimal point is independent of it.

VII. CONCLUSION

In this paper, we have analytically derived the optimal pilot symbol power adjustment for MIMO OFDM systems. As optimization criterion we utilized the post-equalization SINR under imperfect channel state information, that is connected to the system throughout. Furthermore, we derive analytical expression for the post-equalization SINR under imperfect channel knowledge using ZF and we provide analytical expression for the MSE of the LS channel estimator utilizing linear interpolation. Throughput simulation results validate the accuracy of our analytical model for the optimum pilot power adjustment. All data, tools and scripts are available online in order to allow other researchers to reproduce the results shown in the paper [8].

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