Soft-Information-Based Joint Network-Channel Coding for the Two-Way Relay Channel

Andreas Winkelbauer and Gerald Matz

Institute of Telecommunications, Vienna University of Technology
Gusshausstrasse 25/389, 1040 Vienna, Austria; email: {andreas.winkelbauer, gerald.matz}@tuwien.ac.at

***Abstract*—We consider network coding for the time-division two-way relay channel (TWRC). Based on achievable rates, we study the optimal transmission time allocation for a three-phase transmission protocol. While existing network coding schemes for the TWRC perform "hard" network coding, we propose a novel soft-information-based joint network-channel coding scheme. A detailed description of the operation of each network node in our scheme is given. Moreover, we discuss optimal log-likelihood ratio quantization as well as the transmission of quantized soft information. Finally, we assess the performance of the proposed scheme using numerical simulations, including a comparison to conventional "hard" network coding and to direct transmission without relay.**

***Keywords*—Network coding, cooperative communications, soft information processing, two-way relaying**

I. Introduction

In the *two-way relay channel* (TWRC) two users exchange independent messages with the help of one relay. This setting is relevant, e.g., for cooperative cellular networks. *Network coding* [1] at the relay was recognized as efficient means to improve spectral efficiency in this context. Our focus is on the half-duplex case with a three-phase transmission protocol, which is easy to implement in practice. In the first two phases, the users separately transmit their messages and in the third phase the relay broadcasts a network-coded combination of the user data. For such a scenario, [2] optimizes the transmission time allocation for fixed power allocation whereas [3] optimizes the power allocation for fixed time allocation.

We note that the TWRC with full-duplex transmission has been considered in [4]–[6]. Furthermore, for the half-duplex case with a two-phase protocol (i.e., simultaneous user transmissions), achievable rates have been discussed in [7]–[11] and practical schemes using network coding have been developed in [12]–[15]. The authors of [16] consider achievable rates of two-phase and three-phase schemes and give a comparison of relaying strategies for the TWRC.

In this paper we propose a novel transmission scheme for the three-phase time-division TWRC with network coding, based on soft information processing at the relay in the spirit of [17]. In our scheme, the relay re-encodes the received signals using the soft-encoder recently proposed in [18]. The network-encoded soft information is then quantized and forwarded to the users. We design optimal log-likelihood ratio (LLR) quantization at the relay [19], and we present a practical solution for the transmission of quantized LLRs. Simulation results confirm that the proposed scheme provides significant gains compared to existing schemes without soft information processing at the relay. Our scheme allows for low-complexity implementation with small decoding delay.

The remainder of this paper is organized as follows. Section II introduces the system model and deals with information-theoretic aspects of the three-phase TWRC. In Section III we describe the joint network-channel coding at the relay and in Section IV we explain the corresponding decoders. Section V focuses on the design of the LLR quantizer and the transmission of the quantizer indices. Simulation results are presented in Section VI and conclusions are provided in Section VII.

II. System Model

A. TWRC Model

We consider two nodes A and B that exchange independent messages with the help of the relay R. All transmissions happen in separate time slots. The total number of channel uses per transmission is $M = M_\mathsf{A} + M_\mathsf{B} + M_\mathsf{R}$, where M_i denotes the number of symbols transmitted by node $i \in \{\mathsf{A}, \mathsf{B}, \mathsf{R}\}$. The fraction of transmission time allocated to node i is denoted by $\Delta_i = M_i/M$.

At node A, a vector $\mathbf{u}_\mathsf{A}$ of K_A information bits is encoded by a terminated systematic rate-1/2 convolutional code with generator matrix $\mathbf{G} = (1 \; g_1/g_0)$, where $g_1 = 13_8$ and $g_0 = 15_8$ (this corresponds to the constituent code of the turbo code used in UMTS LTE [20]). Using the rate-matching strategy of UMTS LTE (cf. [20]), the encoder output is punctured to obtain a codeword $\mathbf{c}_\mathsf{A}$ of length $2M_\mathsf{A} = 2\Delta_\mathsf{A} M$, which is then mapped to a sequence $\mathbf{x}_\mathsf{A}$ of M_A QPSK symbols. The signal $\mathbf{x}_\mathsf{A}$ is broadcast to B and R in the first transmission time slot. In the second time slot, node B transmits to A and R the signal $\mathbf{x}_\mathsf{B}$, which is obtained in a completely analogous manner. The average information rate (in bits per channel use) of nodes A and B is given by $R_\mathsf{A} = K_\mathsf{A}/M$ and $R_\mathsf{B} = K_\mathsf{B}/M$, respectively. The sum-rate equals $R_S = R_\mathsf{A} + R_\mathsf{B}$.

B. Channel Model

Assuming that all links are Gaussian, we have

$$\mathbf{y}_{ij} = d_{ij}^{-n/2} \mathbf{x}_i + \mathbf{w}_{ij}, \qquad (1)$$

Funding by FWF Grant N10606 (SISE-Infonet).

where $\mathbf{x}_i$ is the signal transmitted by node i, $\mathbf{y}_{ij}$ is the corresponding receive signal at node j, d_{ij} is the distance between nodes i and j, n is the path-loss exponent, and $\mathbf{w}_{ij}$ is zero-mean, circularly symmetric complex Gaussian noise with covariance matrix $N_0\mathbf{I}$. We impose an average transmit power constraint at each node i, i.e., $\mathrm{E}\{\|\mathbf{x}_i\|_2^2\}/M_i = 1$, where $\mathrm{E}\{\cdot\}$ denotes the expectation operator and $\|\cdot\|_2$ denotes the ℓ_2-norm. Thus, the signal-to-noise ratio (SNR) for the link between node i and node j is given by $\gamma_{ij} = d_{ij}^{-n}/N_0$. Note that $d_{ji} = d_{ij}$ and thus $\gamma_{ji} = \gamma_{ij}$. We assume that the SNR γ_{ij} is known at the receiving node j.

For simplicity we assume that the relay R is placed halfway between A and B so that $d_{\mathsf{AR}} = d_{\mathsf{BR}} = d_{\mathsf{AB}}/2$ and $\gamma_{\mathsf{AR}} = \gamma_{\mathsf{BR}} = 2^n\gamma_{\mathsf{AB}}$. Assuming a path-loss exponent of $n = 3.52$ according to the Okumura-Hata model [21], we have $[\gamma_{\mathsf{AR}}]_{\mathrm{dB}} = [\gamma_{\mathsf{BR}}]_{\mathrm{dB}} = [\gamma_{\mathsf{AB}}]_{\mathrm{dB}} + 10.6\mathrm{dB}$. Due to the higher SNR on the relay-to-user links, the relay uses 16-QAM.

C. Information-Theoretic Limits

We next use results on achievable rates to find the optimal transmit time allocation and the corresponding achievable sum-rate. Let $C_{ij}(\gamma_{ij})$ denote the system capacity of the point-to-point link between node i and node j under a given signaling scheme as a function of the channel SNR γ_{ij}. For our system, an achievable rate region is given by the closure of the set of all rate pairs $(R_{\mathsf{A}}, R_{\mathsf{B}})$ satisfying [16, Thm. 3]

$$R_{\mathsf{A}} < \min\{\Delta_{\mathsf{A}} C_{\mathsf{AR}}(\gamma_{\mathsf{AR}}), \Delta_{\mathsf{A}} C_{\mathsf{AB}}(\gamma_{\mathsf{AB}}) + \Delta_{\mathsf{R}} C_{\mathsf{RB}}(\gamma_{\mathsf{RB}})\},$$
$$R_{\mathsf{B}} < \min\{\Delta_{\mathsf{B}} C_{\mathsf{BR}}(\gamma_{\mathsf{BR}}), \Delta_{\mathsf{B}} C_{\mathsf{BA}}(\gamma_{\mathsf{BA}}) + \Delta_{\mathsf{R}} C_{\mathsf{RA}}(\gamma_{\mathsf{RA}})\}.$$

For this achievable rate region, the sum-rate-optimal time allocation can be obtained by solving the linear program

$$\mathbf{s}^\star = \arg\max_{\mathbf{s}} R_{\mathsf{A}} + R_{\mathsf{B}}, \quad \text{subject to} \;\; \mathbf{A}\mathbf{s} \succeq \mathbf{0}, \;\; \mathbf{s} \succeq \mathbf{0}, \;\; \Delta_{\mathsf{A}} + \Delta_{\mathsf{B}} + \Delta_{\mathsf{R}} = 1. \tag{2}$$

Here, $\mathbf{s} = (\Delta_{\mathsf{A}}\ \Delta_{\mathsf{B}}\ \Delta_{\mathsf{R}}\ R_{\mathsf{A}}\ R_{\mathsf{B}})^T$ and

$$\mathbf{A} = \begin{pmatrix} C_{\mathsf{AR}}(\gamma_{\mathsf{AR}}) & 0 & 0 & -1 & 0 \\ C_{\mathsf{AB}}(\gamma_{\mathsf{AB}}) & 0 & C_{\mathsf{RB}}(\gamma_{\mathsf{RB}}) & -1 & 0 \\ 0 & C_{\mathsf{BR}}(\gamma_{\mathsf{BR}}) & 0 & 0 & -1 \\ 0 & C_{\mathsf{BA}}(\gamma_{\mathsf{BA}}) & C_{\mathsf{RA}}(\gamma_{\mathsf{RA}}) & 0 & -1 \end{pmatrix}.$$

The linear program (2) can be solved by standard methods (see, e.g., [22]). In the remainder of this paper, we restrict to the special case of the *symmetric TWRC* with $R \triangleq R_{\mathsf{A}} = R_{\mathsf{B}}$ and $\gamma_{\mathsf{AB}} = \gamma_{\mathsf{BA}}$, $\gamma_{\mathsf{AR}} = \gamma_{\mathsf{BR}}$, $\gamma_{\mathsf{RA}} = \gamma_{\mathsf{RB}}$. Here, $\Delta \triangleq \Delta_{\mathsf{A}} = \Delta_{\mathsf{B}} \in (0, 1/2]$ and $\Delta_{\mathsf{R}} = 1 - 2\Delta$. Hence, we can reformulate (2) as $\Delta^\star = \arg\max_\Delta R$. This simpler optimization problem admits the closed-form solution

$$\Delta^\star = \frac{C_{\mathsf{RB}}(\gamma_{\mathsf{RB}})}{C_{\mathsf{AR}}(\gamma_{\mathsf{AR}}) - C_{\mathsf{AB}}(\gamma_{\mathsf{AB}}) + 2C_{\mathsf{RB}}(\gamma_{\mathsf{RB}})}, \tag{3}$$

if $C_{\mathsf{AB}}(\gamma_{\mathsf{AB}}) < \min\{2C_{\mathsf{RB}}(\gamma_{\mathsf{RB}}), C_{\mathsf{AR}}(\gamma_{\mathsf{AR}})\}$; otherwise $\Delta^\star = 1/2$, i.e., the relay is inactive and the TWRC degenerates to two point-to-point channels. With (3), the maximum sum-rate equals $R_S^\star = 2\Delta^\star C_{\mathsf{AR}}(\gamma_{\mathsf{AR}})$. Fig. 1 shows $\Delta^\star$ and $R_S^\star$ versus γ_{AB} for the channel model described in the previous subsection. The case of Gaussian signaling is shown for comparison.

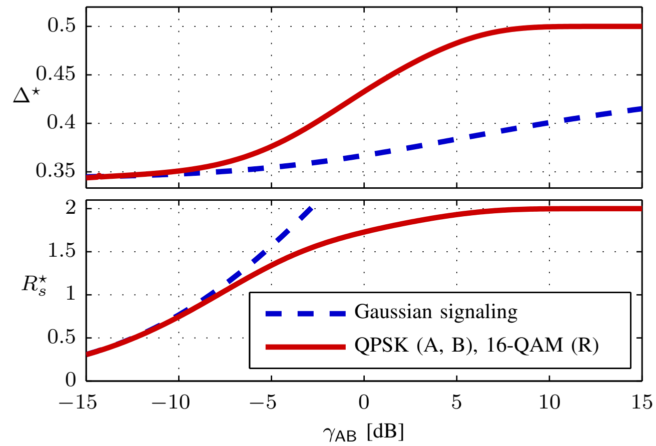

Figure 1: Optimal time allocation parameter $\Delta^\star$ and maximum achievable sum-rate $R_S^\star$ versus γ_{AB} for the symmetric TWRC.

III. Relay Operation

Fig. 2 shows a block diagram of the relay R, which processes the signals $\mathbf{y}_{\mathsf{AR}}$ and $\mathbf{y}_{\mathsf{BR}}$ (cf. (1)) received during the first two transmission phases. The relay performs (i) soft channel re-encoding of the observations, (ii) network encoding of the re-encoded observations, and (iii) broadcasting of the network-coded soft information to the users.

A. Soft Re-encoding

Since the processing of $\mathbf{y}_{\mathsf{AR}}$ and $\mathbf{y}_{\mathsf{BR}}$ is identical, we restrict to $\mathbf{y}_{\mathsf{AR}}$ in the following. First, $\mathbf{y}_{\mathsf{AR}}$ is passed through a soft demapper which delivers a block $\mathbf{L}_{\mathsf{A},\mathrm{ch}}$ of LLRs for the code bits $\mathbf{c}_{\mathsf{A}}$. These LLRs are then passed to a soft-input soft-output (SISO) BCJR channel decoder [23] which produces posterior LLRs $\mathbf{L}_{\mathsf{A}}$ for the information bits $\mathbf{u}_{\mathsf{A}}$. The interleaved information bit LLRs $\mathbf{L}'_{\mathsf{A}} = \Pi(\mathbf{L}_{\mathsf{A}})$ constitute the input of a SISO channel *en*coder [18], which computes LLRs for the codeword bits $\widetilde{\mathbf{c}}_{\mathsf{A}}$ corresponding to the interleaved information bits $\mathbf{u}'_{\mathsf{A}} = \Pi(\mathbf{u}_{\mathsf{A}})$. The SISO encoder uses the same generator matrix $\mathbf{G}$ as the encoders at A and B. Note that the SISO encoder is truncated, since exact termination of the SISO encoder is not possible. The output of the SISO encoder is punctured to match the length $M_{\mathsf{R}} = \Delta_{\mathsf{R}} M$ of the relay transmit signal $\mathbf{x}_{\mathsf{R}}$, resulting in a block of LLRs $\widetilde{\mathbf{L}}_{\mathsf{A}}$ for the punctured codeword $\widetilde{\mathbf{c}}_{\mathsf{A}}$. The same steps are performed for $\mathbf{y}_{\mathsf{BR}}$. Since $\mathbf{y}_{\mathsf{AR}}$ and $\mathbf{y}_{\mathsf{BR}}$ are received and processed in disjoint time slots, the processing chain needs to be implemented only once in hardware, thereby reducing silicon area.

B. Soft Network Encoding

We perform soft network encoding of the punctured output of the two SISO channels encoders. This allows us to use a single broadcast transmission to convey information about both codewords $\widetilde{\mathbf{c}}_{\mathsf{A}}$ and $\widetilde{\mathbf{c}}_{\mathsf{B}}$ to both nodes simultaneously. The network encoder implements the “soft” version of the parity check equations $\mathbf{c}_{\mathsf{R}} = \widetilde{\mathbf{c}}_{\mathsf{A}} \oplus \widetilde{\mathbf{c}}_{\mathsf{B}}$, where “$\oplus$” denotes element-wise modulo-2 addition. The output of the soft network

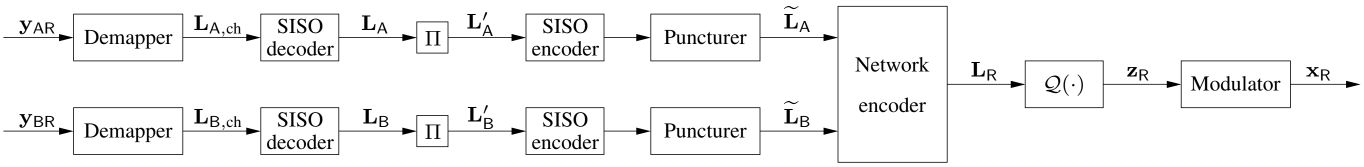

Figure 2: Block diagram for the relay node R.

encoder thus is $\mathbf{L}_{\mathsf{R}} = \widetilde{\mathbf{L}}_{\mathsf{A}} \boxplus \widetilde{\mathbf{L}}_{\mathsf{B}}$, where the boxplus operation "$\boxplus$" (see [24]) is performed element-wise.

C. Transmission of Soft Information

The LLRs $\mathbf{L}_{\mathsf{R}}$ computed by the soft network encoder are real numbers and hence must be quantized for digital transmission. We use a scalar quantizer $\mathcal{Q}(\cdot)$ with Q levels that maps each LLR to a corresponding quantizer index $z_{\mathsf{R}} \in \{0, 1, ..., Q-1\}$. The quantizer indices $\mathbf{z}_{\mathsf{R}}$ are then modulated onto 16-QAM symbols. The LLR quantizer and symbol mapping both turn out be critical and are discussed in more detail in Section V.

We note that in the case of two-level quantization ($Q = 2$), the SISO encoder and the network encoder can be replaced by a conventional encoder and an XOR operation, respectively. The output of the "hard" network encoder is then modulated directly. However, the decoders at A and B still use the reliability information provided by the (quantized) LLRs l_0 and l_1 (cf. (4)) about the transmitted bits.

IV. Receivers at Nodes A and B

The decoding at B (A) is based on viewing the encoder at A (B) and the re-encoding at the relay as a distributed parallel concatenated convolutional code (PCCC). In the following, we restrict to the receiver processing at node B (node A decodes in an analogous manner). Fig. 3 shows a block diagram of the joint network-channel decoder employed at node B, which processes the observations $\mathbf{y}_{\mathsf{AB}}$ and $\mathbf{y}_{\mathsf{RB}}$ (cf. (1)) received in the first and the third time slot, respectively. Both observations are passed through a soft demapper. The channel LLRs $\mathbf{L}_{\mathsf{R,ch}}$ corresponding to $\mathbf{y}_{\mathsf{RB}}$ are passed to the source decoder, which computes an estimate $\hat{\mathbf{L}}_{\mathsf{R}}$ of the network-coded LLRs $\mathbf{L}_{\mathsf{R}}$ (the source decoder is described in Section V). The network decoder then computes an estimate $\hat{\mathbf{L}}_{\mathsf{A}}$ of the punctured code bit LLRs $\widetilde{\mathbf{L}}_{\mathsf{A}}$ according to $\hat{\mathbf{L}}_{\mathsf{A}} = (\mathbf{1} - 2\widetilde{\mathbf{c}}_{\mathsf{B}}) \odot \hat{\mathbf{L}}_{\mathsf{R}}$ ("$\odot$" denotes element-wise multiplication). Here we used the fact that node B knows its own information bits $\mathbf{u}_{\mathsf{B}}$ and hence can exactly compute the punctured code bits $\widetilde{\mathbf{c}}_{\mathsf{B}}$ using the same encoder as the relay (cf. the upper-most branch in Fig. 3). It is worth noting that performing network decoding iteratively, i.e., in a turbo-like fashion, does *not* improve the performance of the system, since the receivers know their own message perfectly and hence can completely reverse the network encoding operation in a single step.

Finally, the output of the network decoder and the output of the demapper corresponding to $\mathbf{y}_{\mathsf{AB}}$ are processed by a conventional iterative (turbo) decoder. The distributed PCCC formed by the constituent codes at nodes A and R is decoded by alternatingly executing the BCJR algorithm on the constituent codes and exchanging extrinsic information

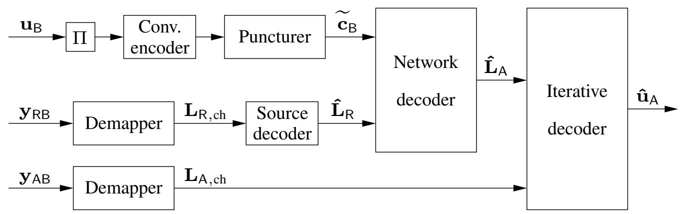

Figure 3: Receiver block diagram for node B.

about the information bits between the decoders. After a fixed number of iterations, the posterior LLRs for the information bits $\mathbf{u}_{\mathsf{A}}$ are sliced to obtain the decoding result $\hat{\mathbf{u}}_{\mathsf{A}}$. The decoding complexity of our scheme is moderate since the iterative decoder converges usually after only one iteration (cf. Section VI). Moreover, a small decoding delay can be achieved with short information block lengths K_{A}, K_{B} without noticeable performance loss.

V. Quantizer and Signal Design

We next discuss the design of the LLR quantizer and transmit signal at the relay and we propose a simple, yet effective, source decoder.

A. Quantizer Design

The relay uses digital modulation and hence needs to quantize the network coded LLRs. The scalar Q-level quantizer $\mathcal{Q}$ maps an LLR L_{R} to a reproducer value l_k, $k \in \mathcal{Z} = \{0, 1, \ldots, Q-1\}$, according to

$$\mathcal{Q}(L_{\mathsf{R}}) = l_k \quad \text{if} \;\; L_{\mathsf{R}} \in \mathcal{I}_k,$$

where $\mathcal{I}_k$ denotes the kth quantization interval. The relay then broadcasts the index $z_{\mathsf{R}} = k$ to A and B.

Our quantizer design is based on maximizing the mutual information $I(c_{\mathsf{R}}; z_{\mathsf{R}})$ between the network-coded bit $c_{\mathsf{R}} = \widetilde{c}_{\mathsf{A}} \oplus \widetilde{c}_{\mathsf{B}}$ and the quantizer index z_{R} for fixed Q. A local optimum of this non-convex problem can be found numerically using the *information bottleneck method* (IBM) [25]. This approach has first been used in [19], where, however, error-free transmission of the quantizer indices $\mathbf{z}_{\mathsf{R}}$ has been assumed. The iterative IBM algorithm requires the distribution $p(c_{\mathsf{R}}, L_{\mathsf{R}})$ as input and yields a $0/1$-valued assignment $p(z_{\mathsf{R}}|L_{\mathsf{R}})$ which in turn determines the quantization intervals $\mathcal{I}_k$ (see [19] for the details). However, since $I(c_{\mathsf{R}}; z_{\mathsf{R}})$ does not depend on the reproducer values l_k, the latter can still be freely chosen. Following [26], we use the LLRs of the equivalent discrete channel $p(z_{\mathsf{R}}|c_{\mathsf{R}})$, i.e.,

$$l_k = \log \frac{p(c_{\mathsf{R}} = 0 \,|\, z_{\mathsf{R}} = k)}{p(c_{\mathsf{R}} = 1 \,|\, z_{\mathsf{R}} = k)}, \quad k \in \mathcal{Z}. \tag{4}$$

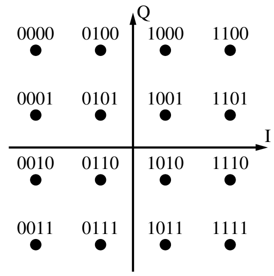

Figure 4: Optimized mapping for four-level quantization.

The LLR quantizer parameters designed according to Algorithm 1 and (4) are stored at all nodes. The relay chooses the appropriate quantizer parameters depending on the SNR $\gamma_{\mathsf{AR}} = \gamma_{\mathsf{BR}}$ and signals this choice to the users. Throughout we assume that A and B are fully aware of the quantizer choice at the relay. We note that for $Q > 2$ the quantizer output distribution $p(z_{\mathsf{R}})$ is strongly non-uniform. The resulting non-uniform prior distribution of the transmit symbols $\mathbf{x}_{\mathsf{R}}$ should be taken into account by the demappers at the user nodes.

Finally, we note that the quantizer design described above is superior to other quantizer designs, e.g., mean-square error optimal (Lloyd-Max) quantization and uniform quantization.

B. Signal Design

We next discuss how to map the quantizer indices $\mathbf{z}_{\mathsf{R}}$ to 16-QAM symbols for two-level and four-level quantization. For $Q = 2$, we map four quantizer indices to one 16-QAM symbol using Gray labeling. For $Q = 4$, each index z_{R} is represented by its natural binary representation $\mathbf{b}_{\mathsf{R}} = (b_0 \; b_1)$ and the four bits $\mathbf{b}_{\mathsf{R}}^{(1)}$, $\mathbf{b}_{\mathsf{R}}^{(2)}$ of two quantizer indices $\big(z_{\mathsf{R}}^{(1)} \; z_{\mathsf{R}}^{(2)}\big)$ are mapped to one 16-QAM symbol. In this case, however, Gray labeling is not optimal because the quantizer bits are not equally significant, i.e., *it matters* which *bit is in error*. We optimized the symbol labeling by using the mutual information between $\mathbf{b}_{\mathsf{R}}$ and the corresponding channel output as score function [27] and by applying the *binary switching algorithm* (BSA) [28]. The BSA quickly finds a very good (if not the optimal) bit labeling without performing a brute-force search over all $\big(2^4\big)! \approx 2 \cdot 10^{13}$ possible labelings. The resulting mapping for the bits $\big(b_0^{(1)} \; b_1^{(1)} \; b_0^{(2)} \; b_1^{(2)}\big)$ is shown in Fig. 4.

C. Source Decoder

The source decoders at A and B aim at computing an estimate of the actual reproducer value $l_{z_{\mathsf{R}}}$ using the LLRs $\mathbf{L}_{\mathsf{R,ch}}$. A simple source decoder could, for example, detect the transmitted quantizer index z_{R}, yielding $\hat{z}_{\mathsf{R}} \in \mathcal{Z}$, and output the corresponding reproducer value $l_{\hat{z}_{\mathsf{R}}}$. This is optimal iff $\hat{z}_{\mathsf{R}} \equiv z_{\mathsf{R}}$, i.e., iff the transmission of the quantizer index is error-free. In our system, the quantizer indices are transmitted over a noisy channel and hence transmission errors will occur. In this case source decoding should not be performed as described above.

Instead, we propose a source decoder which computes a convex combination of the reproducer values l_k, where the coefficients are the posterior probabilities of the Q quantizer indices z_{R}. Thus, the output of the source decoder is given by

$$\hat{L}_{\mathsf{R}} = \sum_{k=0}^{Q-1} l_k \, p\big(z_{\mathsf{R}} = k \,|\, \widetilde{\mathbf{L}}\big). \tag{5}$$

Here, $\widetilde{\mathbf{L}}$ is the sub-block of channel LLRs in $\mathbf{L}_{\mathsf{R,ch}}$ that correspond to z_{R}. Note that $|\hat{L}_{\mathsf{R}}| \leq \max_k |l_k|$. The posterior distribution of the quantizer index in (5) can be expressed as

$$p\big(z_{\mathsf{R}} = k \,|\, \widetilde{\mathbf{L}}\big) = C \, p(z_{\mathsf{R}} = k) \prod_{j=0}^{J-1} \frac{\exp\big(-b_j^{(k)} \widetilde{L}_j\big)}{1 + \exp\big(-\widetilde{L}_j\big)},$$

where C is a normalization factor, $\widetilde{L}_j$ is the jth element of $\widetilde{\mathbf{L}}$ and $\big(b_0^{(k)} \; b_1^{(k)} \ldots \; b_{J-1}^{(k)}\big)$ is the binary representation of the quantizer index $z_{\mathsf{R}} = k$ (here $J = \lceil \log_2 Q \rceil$ denotes the number of bits per quantizer index). The effectiveness of the proposed source decoder (5) is confirmed by the simulation results in Section VI.

VI. Simulation Results

In this section we compare the bit error ratio (BER) performance of the proposed scheme to three reference schemes. The first two reference systems are point-to-point systems using either the convolutional code with generator matrix $\mathbf{G}$ or the UMTS LTE turbo code as channel code. The third reference scheme is a conventional joint network-channel coding scheme for the TWRC with “hard” network coding (XOR) at the relay. In this case the relay performs network coding only if both observations were decoded successfully, otherwise it does not transmit at all. For our scheme we show the BER performance for two-level and four-level quantization with and without the source decoder (SD) described in Section V-C. The BER performance comparison is fair since all schemes transmit at the same overall rate, the same average transmit power constraint is imposed at all nodes, and the total power per transmission is the same for all schemes.

We choose $K_{\mathsf{A}} = K_{\mathsf{B}} = 256$ information bits and the sum-rate $R_S = 1$. This yields a total of $M = 512$ channel uses per transmission. In our system, R_S is achievable at $\gamma_{\mathsf{AB}} = -7.8\,\text{dB}$ and the corresponding optimal transmit time allocation is $\Delta_{\mathsf{A}} = \Delta_{\mathsf{B}} = 0.358$ (cf. Fig. 1). Thus we have $M_{\mathsf{A}} = M_{\mathsf{B}} = 183$ and $M_{\mathsf{R}} = 146$. For the point-to-point systems we have $M_{\mathsf{A}} = M_{\mathsf{B}} = 256$. The BER results shown in Fig. 5 have been obtained by Monte Carlo simulation of 60 000 transmissions for each scheme at each marker position. We use random interleavers, where the interleaver depth is equal to the block length and a different interleaver realization is used for each transmission. All relaying schemes perform a single decoder iteration. The turbo code uses five iterations.

From Fig. 5 we observe that the point-to-point schemes are outperformed by all relaying schemes. The turbo code does not perform significantly better than the convolutional code due to the small block length. Our scheme with two-level quantization outperforms the hard network coding scheme, since the relay transmits LLRs l_0 and l_1 even if it could not decode

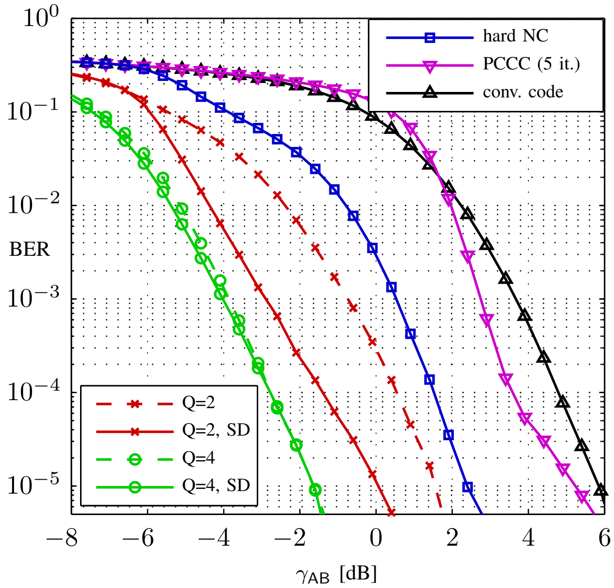

Figure 5: BER performance comparison.

both observations completely. The source decoder improves the performance of our scheme with $Q = 2$ significantly, e.g., by more than 2 dB at a BER of 10^{-3}. However, four-level quantization gains another one to two decibels compared to the case of $Q = 2$ with source decoding. For $Q = 4$, the source decoder improves the performance only slightly. In terms of BER performance, our scheme does not benefit from a larger block length and additional decoder iterations. We conjecture that this is because the BER is dominated by the error floor of the decoder which is reached after only one iteration. (The error floor of the decoder decays as the SNRs increase.)

VII. Conclusions

We have proposed a novel transmission scheme for the TWRC based on soft network coding at the relay. For this scheme, we have discussed the optimal allocation of transmission time to the network nodes. Soft re-encoding is performed by the relay based on a recently proposed SISO channel encoder. We have considered optimal LLR quantization together with a practical scheme for the transmission of quantized LLRs. Simulation results show that the proposed transmission scheme features excellent performance and offers a gain of up to 4.5 dB at a BER of 10^{-3} compared to a conventional "hard" network coding scheme. Low decoding complexity and a small decoding delay make this scheme perfectly suited for next generation cooperative wireless networks.

References

[1] R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network information flow," *IEEE Trans. Inf. Theory*, vol. 46, pp. 1204–1216, July 2000.

[2] C. Hausl and J. Hagenauer, "Iterative network and channel decoding for the two-way relay channel," in *Proc. IEEE Int. Conf. on Communications*, vol. 4, June 2006, pp. 1568–1573.

[3] P. Larsson, N. Johansson, and K.-E. Sunell, "Coded bi-directional relaying," in *Proc. IEEE Vehicular Technology Conf.*, vol. 2, May 2006, pp. 851–855.

[4] B. Rankov and A. Wittneben, "Achievable rate regions for the two-way relay channel," in *Proc. IEEE Int. Symposium on Inf. Theory*, July 2006, pp. 1668–1672.

[5] L.-L. Xie, "Network coding and random binning for multi-user channels," in *Proc. 10th Canadian Workshop on Inf. Theory*, June 2007, pp. 85–88.

[6] W. Nam, S. Chung, and Y. Lee, "Capacity bounds for two-way relay channels," in *Proc. IEEE Int. Zurich Seminar on Communications*, March 2008, pp. 144–147.

[7] B. Rankov and A. Wittneben, "Spectral efficient signaling for half-duplex relay channels," in *Proc. Thirty-Ninth Asilomar Conf. on Signals, Systems and Computers*, Oct. 2005, pp. 1066–1071.

[8] R. Knopp, "Two-way radio networks with a star topology," in *Proc. Int. Zurich Seminar on Communications*, Feb. 2006, pp. 154–157.

[9] P. Popovski and H. Yomo, "Bi-directional amplification of throughput in a wireless multi-hop network," in *Proc. IEEE Vehicular Technology Conf.*, vol. 2, May 2006, pp. 588–593.

[10] ——, "The anti-packets can increase the achievable throughput of a wireless multi-hop network," in *Proc. IEEE Int. Conf. on Communications*, vol. 9, June 2006, pp. 3885–3890.

[11] T. Oechtering, C. Schnurr, I. Bjelakovic, and H. Boche, "Achievable rate region of a two phase bidirectional relay channel," in *Proc. 41st Annual Conf. on Information Sciences and Systems*, March 2007, pp. 408 –413.

[12] S. Zhang, S. Liew, and P. Lam, "Physical-layer network coding," in *Proc. 12th Annual Int. Conf. on Mobile Computing and Networking*, Sept. 2006, pp. 358–365.

[13] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," in *Proc. Conf. on Applications, Technologies, Architectures, and Protocols for Computer Communications*, vol. 37, no. 4, Oct. 2007, pp. 397–408.

[14] J. Zhao, M. Kuhn, A. Wittneben, and G. Bauch, "Asymmetric data rate transmission in two-way relaying systems with network coding," in *Proc. IEEE Int. Conf. on Communications*, May 2010, pp. 1–6.

[15] D. Wübben and Y. Lang, "Generalized sum-product algorithm for joint channel decoding and physical-layer network coding in two-way relay systems," in *Proc. IEEE Global Telecommunications Conf.*, Dec. 2010, pp. 1–5.

[16] S. J. Kim, P. Mitran, and V. Tarokh, "Performance bounds for bidirectional coded cooperation protocols," *IEEE Trans. Inf. Theory*, vol. 54, no. 11, pp. 5235 –5241, Nov. 2008.

[17] S. Yang and R. Kötter, "Network coding over a noisy relay: a belief propagation approach," in *Proc. IEEE Int. Symposium on Inf. Theory*, June 2007, pp. 801–804.

[18] A. Winkelbauer and G. Matz, "On efficient soft-input soft-output encoding of convolutional codes," *accepted for publication at the IEEE Int. Conf. on Acoustics, Speech and Signal Processing*, May 2011.

[19] G. Zeitler, R. Koetter, G. Bauch, and J. Widmer, "Design of network coding functions in multihop relay networks," in *5th Int. Symposium on Turbo Codes and Related Topics*, Sept. 2008, pp. 249–254.

[20] 3GPP, "TS 25.212 Multiplexing and channel coding (FDD)," www.3gpp.org, TS 25.212 v. 10.1.0, Dec. 2010.

[21] J. Laiho, A. Wacker, and T. Novosad, *Radio network planning and optimisation for UMTS*, 2nd ed. John Wiley & Sons, Dec. 2005.

[22] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge Univ. Press, Dec. 2004.

[23] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inf. Theory*, vol. 20, pp. 284–287, March 1974.

[24] J. Hagenauer, E. Offer, and L. Papke, "Iterative decoding of binary block and convolutional codes," *IEEE Trans. Inf. Theory*, vol. 42, no. 2, pp. 429–445, March 1996.

[25] N. Tishby, F. Pereira, and W. Bialek, "The information bottleneck method," in *Proc. 37th Allerton Conf. on Communication and Computation*, Sept. 1999, pp. 368–377.

[26] C. Novak, P. Fertl, and G. Matz, "Quantization for soft-output demodulators in bit-interleaved coded modulation systems," in *Proc. IEEE Int. Symposium on Inf. Theory*, July 2009, pp. 1070–1074.

[27] F. Schreckenbach, N. Görtz, J. Hagenauer, and G. Bauch, "Optimized symbol mappings for bit-interleaved coded modulation with iterative decoding," in *Proc. IEEE Global Telecommunications Conf.*, vol. 6, Dec. 2003, pp. 3316–3320.

[28] K. Zeger and A. Gersho, "Pseudo-gray coding," *IEEE Trans. Comm.*, vol. 38, no. 12, pp. 2147–2158, Dec. 1990.