Efficiency of Partial Frequency Reuse in Power Used Depending on User's Selection for Cellular Networks

Bujar Krasniqi and Christoph F. Mecklenbräuker
Vienna University of Technology, Institute of Telecommunications
Christian Doppler Laboratory for Wireless Technologies for Sustainable Mobility
Gusshausstr. 25/389, A-1040 Vienna
email: {bujar.krasniqi,cfm}@nt.tuwien.ac.at

Abstract—**In this paper we apply constrained optimization techniques to optimally allocate bandwidth and transmit power to the users in a cellular network. We utilize partial frequency reuse as inter-cell interference mitigation technique considering multiple users uniformly located in the cell. Efficient algorithms are proposed to select the user in the cell regions. Moreover, based on user's selection the maximization of the minimum rate is used to optimally allocate the bandwidth and power to the users. We further demonstrate by simulations that using partial frequency reuse as inter-cell interference mitigation technique is more efficient in power use than using frequency reuse-1 or frequency reuse-3.**

I. Introduction

Next generation mobile communication systems use Orthogonal Frequency Division Multiple Access (OFDMA) as their multiple access scheme in the downlink [1], [2]. Since cell edge users may suffer severely from Inter-Cell Interference (ICI), several schemes have been proposed for ICI mitigation. One of those schemes is Partial Frequency Reuse (PFR), which is applied for example in [3], [4], [5].

The characteristics of the optimal power allocation for two base stations, employing also scheduling schemes, have been studied in [6] under frequency reuse-1. Additionally to the sum-rate maximization power control problem, in [7] the authors also investigate the maximization of the minimum rate for two users. An efficient algorithm for solving the sum-rate maximization problem in convex form for PFR under the assumption that all reuse-1 users are served with equal power is developed in [8]. Additionally to this the authors have shown that the maximization of the minimum rate and the minimization of the sum-power can be transformed in convex optimization problems. A study about the utilization of the cell edge (outer) bandwidth as cell center (inner) bandwidth in PFR for maximizing the cell capacity density is done in [5]. In [9], the authors have shown that almost all of the cell outer bandwidth can be re-allocated as cell inner bandwidth in PFR whenever we have only inner users active. A frequency reuse technique like combination of power allocation and interference-aware for attaining the coverage and high spectral efficiency is investigated by the authors in [10]. In [11] the authors have proposed differentiable spectrum partition where the reuse distance is used to find frequency reuse factors. They have shown that this is an effective scheme when the network is experiencing non-uniform traffic load. Differently from [11] we propose different selection of users between reuse-1 users and reuse-3 users and based on those selections we find frequency reuse factors. For each selection of users we investigate the efficiency of power used.

To the best of our knowledge, there are currently no studies which consider the efficiency of power used by partial frequency reuse. Our contributions can be summarized as follows. In Section II we show the realistic system model including the adaptive frequency reuse pattern scheme for PFR.

In Section III we study the allocation of the bandwidth and power to the users depending on the large-scale path-loss attenuations. Efficient algorithms are proposed to select the users in the belonging cell (sector) regions based on large-scale path-loss attenuation threshold. The maximization of the minimum rate is used by those algorithms to optimally allocate the bandwidth and power to the users. Furthermore, we present in Section IV simulation results which confirm the efficiency of partial frequency reuse in power used to satisfy the minimum rate criteria for the users, compared with reuse-1 and reuse-3. Also by simulation it is proven that the large-scale path-loss attenuation threshold defined as the mean over all large-scale path-loss attenuations of users is an optimal metric for power efficiency of partial frequency reuse. Conclusions are drawn in Section V.

II. System Model

In our realistic system we consider N^{in} users located uniformly in the inner region of the cell (the full frequency reuse region) and $\mathrm{M}^{\mathrm{out}}$ users located uniformly in the outer region of the cell (the partial frequency reuse region), as indicated in Fig. 1. Based on the user's large-scale path-loss attenuation a user is selected to be an inner user or an outer user. The adaptive frequency reuse pattern applied in our system model is shown Fig. 2. The frequency reuse pattern shows that the bandwidth and the power assignment to the inner users and the outer users depend on the amount of the users selected as inner and outer users. Frequency reuse-1 is used to serve the inner

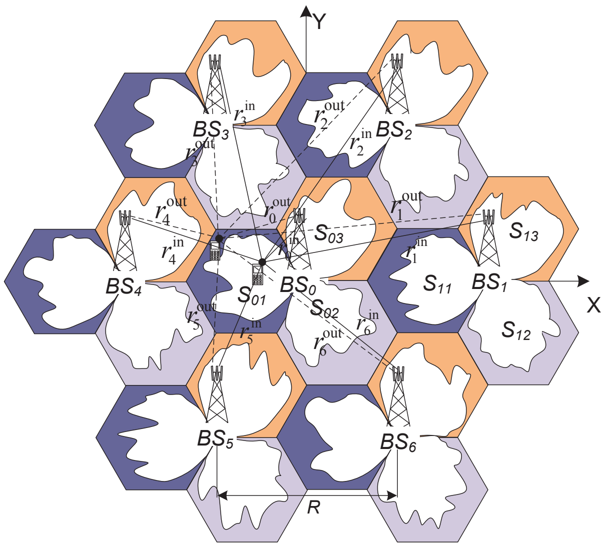

Fig. 1. Partial frequency reuse cell cluster

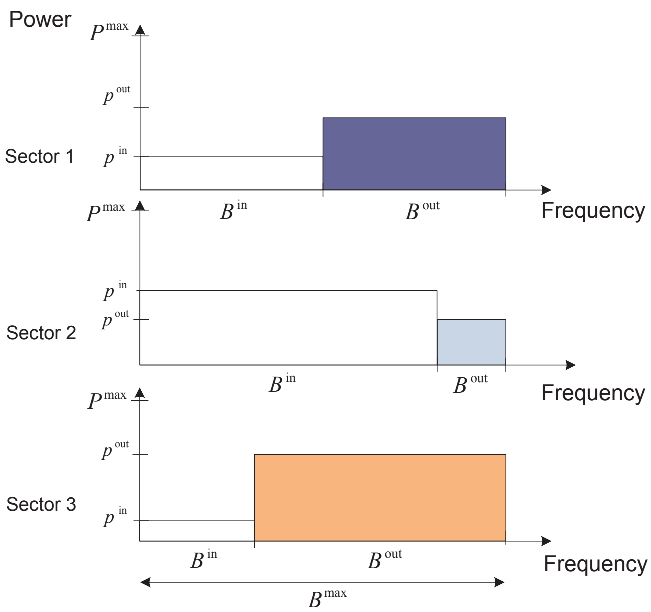

Fig. 2. Frequency reuse pattern

users and frequency reuse-3 is used to serve the outer users. The users who are located in the inner region of the cell, S_{01} receive power from their own sector antenna of base station BS_0 and also interference from all other antenna sectors of base stations BS_k, $k = 0 \ldots 6$. More distant base stations are not considered in our system model but all our results can be easily extended to consider also interference from more non-neighboring base stations. The rate achieved by the user n in the inner region of cell S_{01} is given by

$$R_n^{\mathrm{in}} = B_n^{\mathrm{in}} \log_2 \left(1 + \frac{G_{0n}^{\mathrm{in}} p_0^{\mathrm{in}}}{N_0 B_n^{\mathrm{in}} + \sum_{k=1}^{6} G_{kn}^{\mathrm{in}} p_k^{\mathrm{in}}} \right), \quad (1)$$

where B_n^{in} is the bandwidth assigned to the user n in the inner region of cell S_{01} and N_0 is the noise spectral density. The large-scale path-loss attenuation including antenna gain, penetration loss, shadowing and small-scale fading is expressed in the following form [12],

$$G_{ki}^{s} = -[128.1 + 10\alpha \log_{10}(r_k) + A_k + L_p + X_\sigma + F], \quad (2)$$

where G_{ki}^{s} is in dB, the superscript $s \in \{\mathrm{in}, \mathrm{out}\}$ denotes the inner or the outer users, the subscript $i \in \{n, m\}$ denotes the user's index, α the path-loss exponent, r_k the distance between the mobile station and the base station BS_k in m, A_k the sum of user antenna gain and base station antenna gain in dB, L_p the penetration loss in dB, X_σ the log-normal shadowing in dB. The small-scale fading F in dB is a chi-square distribution χ_2^2 with two degrees of freedom. The antenna gain A_k is defined by a horizontal antenna pattern [12]. The large-scale path-loss attenuation of *directed* channel G_{0n}^{in} is defined by Equation (2). Following [13], we neglect the effect of small-scale fading in Equation (2) for *interference* channels because the small-scale effects average out in the interference powers (Note that small-scale fading is included in the user's *directed* channel). The transmit power assigned to the users in the inner region is denoted by p_0^{in} and the interference power from the other base stations is denoted by p_k^{in}, $k = 1 \ldots 6$, with k denoting the index of the interfering base stations. The users located in the outer region of the cell receive also interference from all non-neighboring sectors that use the same frequency band. The transmit power assigned to the users in the outer region is denoted by p_0^{out} and the interference power from the other base stations is denoted by p_k^{out}, $k = 1 \ldots 6$. Thus, the rate achieved by the user m in the outer region is given by

$$R_m^{\mathrm{out}} = B_m^{\mathrm{out}} \log_2 \left(1 + \frac{G_{0m}^{\mathrm{out}} p_0^{\mathrm{out}}}{N_0 B_m^{\mathrm{out}} + \sum_{k=1}^{6} G_{km}^{\mathrm{out}} p_k^{\mathrm{out}}} \right) \quad (3)$$

where B_m^{out} denotes the bandwidth assigned to the user m in the outer region. Similarly to the inner users, G_{0m}^{out} and G_{km}^{out} denote the large-scale path-loss attenuation for the *directed* and the *interference* channels of the m-th outer users.

III. Efficient Algorithms for Bandwidth and Power Allocation Depending on User's Selection

In this section we show the bandwidth and power utilization to the users. The maximization of the minimum rate [14] given by Equation (4) is used to assign the bandwidth and power to the users.

$$\underset{\beta_{cn}^{\text{in}},\beta_{cm}^{\text{out}}\in\mathcal{R}_+,\mathbf{p}\succeq\mathbf{0}}{\text{maximize}} \quad \min\{\beta_{c1}^{\text{in}}t^{\text{in}}\log(2),...,\beta_{c\text{N}^{\text{in}}}^{\text{in}}t^{\text{in}}\log(2), \beta_{c1}^{\text{out}}t^{\text{out}}\log(2),...,\beta_{c\text{M}^{\text{out}}}^{\text{out}}t^{\text{out}}\log(2)\} \tag{4a}$$

subject to

$$t^{\text{in}} \leq \log\left(1+\frac{p_c^{\text{in}}}{n_1^{\text{in}}\beta_{c1}^{\text{in}}+\sum_{k\in\mathcal{C}\backslash c}g_{k1}^{\text{in}}p_k^{\text{in}}}\right),\ \forall c\in\mathcal{C}, \tag{4b}$$

$$\vdots$$

$$t^{\text{in}} \leq \log\left(1+\frac{p_c^{\text{in}}}{n_{\text{N}^{\text{in}}}^{\text{in}}\beta_{c\text{N}^{\text{in}}}^{\text{in}}+\sum_{k\in\mathcal{C}\backslash c}g_{k\text{N}^{\text{in}}}^{\text{in}}p_k^{\text{in}}}\right),\ \forall c\in\mathcal{C}, \tag{4c}$$

$$t^{\text{out}} \leq \log\left(1+\frac{p_c^{\text{out}}}{n_1^{\text{out}}\beta_{c1}^{\text{out}}+\sum_{k\in\mathcal{C}\backslash c}g_{k1}^{\text{out}}p_k^{\text{out}}}\right),\ \forall c\in\mathcal{C}, \tag{4d}$$

$$\vdots$$

$$t^{\text{out}} \leq \log\left(1+\frac{p_c^{\text{out}}}{n_{\text{M}^{\text{out}}}^{\text{out}}\beta_{c\text{M}^{\text{out}}}^{\text{out}}+\sum_{k\in\mathcal{C}\backslash c}g_{k\text{M}^{\text{out}}}^{\text{out}}p_k^{\text{out}}}\right), \forall c\in\mathcal{C}, \tag{4e}$$

$$\sum_{n=1}^{\text{N}^{\text{in}}}\beta_{cn}^{\text{in}}+\sum_{m=1}^{\text{M}^{\text{out}}}\beta_{cm}^{\text{out}}\leq 1, \forall c\in\mathcal{C} \tag{4f}$$

$$p_c^{\text{in}}+p_c^{\text{out}}\leq P_c^{\max},\ \forall c\in\mathcal{C}, \tag{4g}$$

where $\beta_{cn}^{\text{in}} = B_n^{\text{in}}/B_c^{\max}$ and $\beta_{cm}^{\text{out}} = B_m^{\text{out}}/B_c^{\max}$ are the normalized inner and the outer bandwidths, t^{in} and t^{out} are the normalized [2 p.99] inner and outer user rates, respectively. The subscript c denote the cell and the calligraphic $\mathcal{C}$ denote the set of cells. Furthermore, $n_n^{\text{in}} = N_0/G_{0n}^{\text{in}}$ and $g_{kn}^{\text{in}} = G_{kn}^{\text{in}}/G_{0n}^{\text{in}}$ are the normalized noise and the normalized interference channel large scale path-loss attenuation for the inner users, respectively. Similar normalization is considered for the outer users. In the Generalized Geometric Problem (GGP) optimization problem formulated in Equation (4), the constraints (4b)-(4c) show that the normalized inner user rates are constrained by the normalized minimum requirement inner user rate. Similarly for the outer users are formulated the constraints (4d)-(4e). The last two constraints (4f) and (4g) are the bandwidth and the power constraints. The maximization of the minimum rate in Equation (4) can be transformed in a Geometric Problem (GP) optimization problem [14] and solved efficiently using the Disciplined Convex Programming (DCP) [15] where CVX is used to get optimal bandwidth and power. For selection the users in which cell regions they are belonging, we use efficient algorithms. Those algorithms select the users as multiple inner and multiple outer users, only inner users, only outer users.

A. Multiple users selected inner users and multiple users selected outer users

To distinguish for multiple inner and multiple outer users, we compare the user's large-scale path-loss attenuation with threshold G_{tgt} which can be any value between minimum and maximum over all large-scale path-loss attenuations of users. If the user's large-scale path-loss attenuation is higher than G_{tgt}, than those users are considered to be the inner users, otherwise outer users. The selection of users and their bandwidth and power assignment are done using the Algorithm 1.

Algorithm 1 Bandwidth and Power Assignment Case III-A

Require: $\text{G}_{\text{tgt}} \in (\min(\mathbf{G}), \max(\mathbf{G})), (\mathbf{r}, \theta)$.

1: **if** $G > \text{G}_{\text{tgt}}$ **then**
2: $\quad(\mathbf{r}^{\text{in}}, \theta^{\text{in}}) \leftarrow (\mathbf{r}, \theta)$
3: **else**
4: $\quad(\mathbf{r}^{\text{out}}, \theta^{\text{out}}) \leftarrow (\mathbf{r}, \theta)$
5: **end if**
6: ***Calculate the values of*** p_0^{in}, p_k^{in}, p_0^{out}, p_k^{out}, B_n^{in}, B_m^{out} using Equation (4).

where $\mathbf{G}$ denotes the vector elements of large-scale path-loss attenuation, $(\mathbf{r}, \theta)$ denote the vector elements of polar coordinates of users. Similarly $(\mathbf{r}^{\text{in}}, \theta^{\text{in}})$ denote the vector elements of polar coordinates for the inner users and $(\mathbf{r}^{\text{out}}, \theta^{\text{out}})$ denote the vector elements of polar coordinates for the outer users. The polar coordinates are included in all algorithms since they show the locations of mobile users. The transmission rates of the inner users can be calculated using Equation (1) and for outer users using Equation (3).

B. All users selected as inner users

To distinguish only for inner users, we define the threshold G_{tgt} to be smaller than all large-scale path-loss attenuations of all users as it is shown in Algorithm 2.

Algorithm 2 Bandwidth and Power Assignment Case III-B

Require: $\text{G}_{\text{tgt}} < \min(\mathbf{G})$.

1: **if** $G > \text{G}_{\text{tgt}}$ **then**
2: $\quad(\mathbf{r}^{\text{in}}, \theta^{\text{in}}) \leftarrow (\mathbf{r}, \theta)$
3: **end if**
4: ***Calculate the values of*** p_0^{in}, p_k^{in}, B_n^{in} using Equation (4).

The Algorithm 2 from the first to third step compare all user's large-scale path-loss attenuations with threshold G_{tgt} and selects all users as inner users. In the last step runs the Equation (4) to calculate the power and bandwidth assignment to the inner users. The transmission rate of inner users are calculated using Equation (1).

C. All users selected as outer users

To distinguish only for outer users, we define the threshold G_{tgt} to be greater than all large-scale path-loss attenuations of users as it is shown in Algorithm 3.

Algorithm 3 Bandwidth and Power Assignment Case III-C

Require: $\mathrm{G_{tgt}} > \max(\mathbf{G})$.

1: **if** $G < \mathrm{G_{tgt}}$ **then**
2: $(\mathbf{r}^{\mathrm{out}}, \theta^{\mathrm{out}}) \leftarrow (\mathbf{r}, \theta)$
3: **end if**
4: ***Calculate the values of*** p_0^{out}, p_k^{out}, B_m^{out} using Equation (4).

Similarly to Algorithm 2, the Algorithm 3 in the 1-3 step selects all users as outer users by comparing the user's large-scale path-loss attenuations with threshold $\mathrm{G_{tgt}}$. In the last step run the Equation (4) to calculate the bandwidth and power assignment to the outer users. The transmission rates for the outer users are calculated using Equation (3).

IV. Simulation Results

In this simulation we consider uniform distance between users. A realistic urban scenario is considered with its parameters shown in Table I.

TABLE I
SIMULATION PARAMETERS

parameters	value
Maximum base station power $P_c^{\max}$	40 W
Maximum base station bandwidth $B_c^{\max}$	20 MHz
Noise spectral density N_0	-174 dBm/Hz
Center frequency f	2.0 GHz
Path-loss exponent α	3.75
Penetration loss L_p	20 dB
Shadowing X_σ	$\mathcal{N}(0, 8)$ dB
Small-scale fading F	χ_2^2 dB,
Inter base station distance R	600 m
Maximum cell range r	$(2/3)R$ m
Minimum requirement inner user rate t^{in}	2.5 Mbit/s
Minimum requirement outer user rate t^{out}	2.5 Mbit/s
Number of users within cell	75

During simulations we have considered 100 realizations, where per each realizations all users have experienced different channels due to shadowing and small-scale fading. The efficiency of used power in median values over those 100 realizations versus frequency reuse is shown in Fig. 3. Frequency re-use is a direct mapping of large scale path-loss attenuation threshold $\mathrm{G_{tgt}}$. Frequency reuse-1 is considered only when the threshold $\mathrm{G_{tgt}}$ is chosen to be smaller than the minimum large-scale path-loss attenuations from all over the users within the cell. In this case inner user's selection and their power and bandwidth assignment are calculated using Algorithm 2. Similarly the frequency re-use 3 is considered when the threshold is chosen to be larger than the maximum large-scale path-loss attenuations over all users within considered cell. Outer user selection and their power and bandwidth assignment is done using Algorithm 3. All the other thresholds for $\mathrm{G_{tgt}}$ between the minimum and maximum large-scale path-loss attenuations of users are mapped in frequency re-use between 1 and 3. For inner and outer user's selection and their bandwidth and power assignment the Algorithm 1 is used. From the simulation results shown in Fig. 3, we see that the

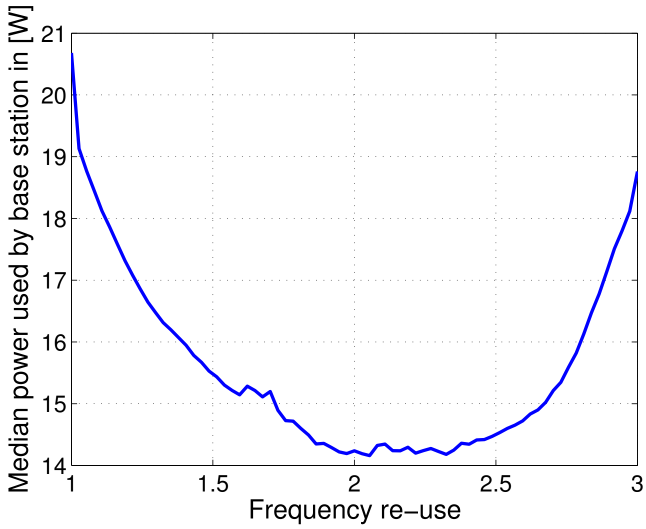

Fig. 3. Power efficiency depending on frequency re-use

highest power is used by base station when frequency reuse-1 is considered. The reason for such high power use is that the Algorithm 2 needs to adapt the base station power to serve all users such that user's achieved rate to be higher than the minimum requirement rate. In this case the cell edge users which are far from the base station and possibly with poor channels need more power. Increasing the frequency re-use results in the median power used decrease until the frequency re-use is 1.9. From the values of frequency re-use $1.9 - 2.4$ there is only a small variation in median power used. This is the region when PFR is the most efficient in terms of power used. Comparing the thresholds G_{tgt} selected in this region with the mean over all large-scale path-loss attenuations we saw that the threshold $\mathrm{G_{tgt}}$ is the same as the mean threshold. Our conclusion is that defining the threshold $\mathrm{G_{tgt}}$ as the mean over all large-scale path-loss attenuation of users is a good metric for selecting the users as inner and outer users. By increasing the frequency re-use more than 2.9 also the median power used increases. However, the median power used in frequency reuse-3 is smaller compared to frequency reuse-1 because the users in frequency reuse-3 are interfered only by non-neighboring cells. The uniform distribution of users within the cell and their large-scale path-loss attenuations for one realization scenario is shown in Fig. 4. From the simulation results shown in Fig. 4 we see that the users experience different channels due to shadowing and small-scale fading. Some users at the cell edge have better large-scale path-loss attenuations than some other users that are near base stations. For the same realization in Fig. 5 we have shown the user's selection and their achieved transmission rates. For selecting the users and their power and bandwidth assignment the Algorithm 1 is used. The calculated mean threshold has the value $\mathrm{G_{tgt}} = -110.27\,\mathrm{dB}$. Using the Algorithm 1 in this realization from 75 users located uniformly in cell $\mathrm{S_{01}}$, 35

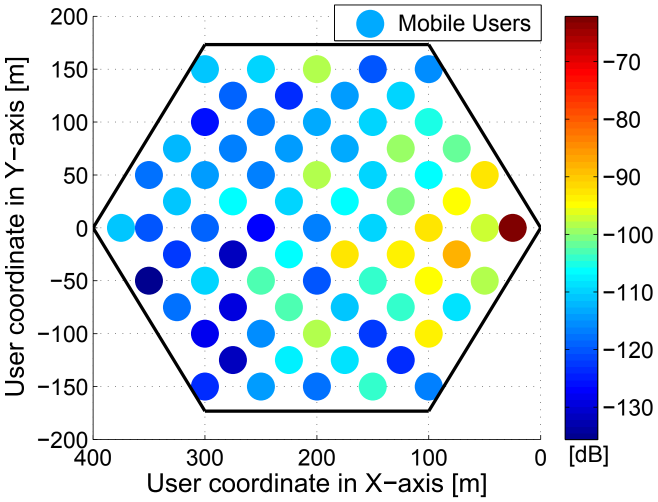

Fig. 4. Large-scale path-loss attenuations for uniform users distribution

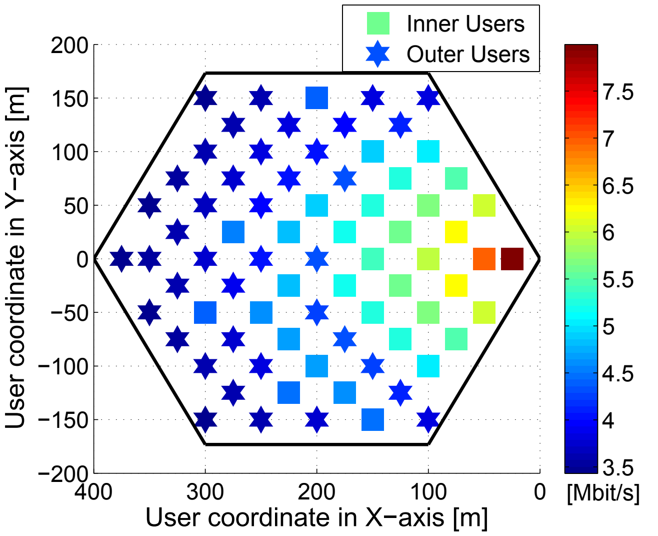

Fig. 5. Individual users rate for large-scale path-loss attenuation mean threshold $\mathrm{G_{tgt}} = -110.27\,\mathrm{dB}$

users are selected as inner users and 40 users are selected as outer users. From the total bandwidth allocated to cell S_{01}, around $47\,\%$ of bandwidth is assigned to the inner users, while $53\,\%$ of bandwidth is assigned to the outer users. From the maximum possible base station power $P_c^{\max}$, only $p_0^{\text{in}} = 5.04\,\mathrm{W}$ is assigned to the inner users and $p_0^{\text{out}} = 9.12\,\mathrm{W}$ is assigned to the users in the outer region.

V. Conclusions

In this paper, we formulated the efficient algorithms for selecting the users based on the criteria for large-scale path-loss attenuation. Those algorithms use the maximization of the minimum rate to optimally allocate the bandwidth and power to the users such that each user achieve a higher rate than the minimum requirement rate. By the simulation results we have proof that using partial frequency reuse which is a combinations of frequency reuse-1 and frequency reuse-3 as inter-cell interference mitigation scheme, is more efficient in terms of power used than using only frequency reuse-1 or frequency reuse-3. About $6\,\mathrm{W}$ is saved when partial frequency reuse is used compared with frequency reuse-1 and about $4\,\mathrm{W}$ is saved compared with frequency reuse-3.

Acknowledgments

The authors would like to thank Michal Šimko for his fruitful comments. This work has been partially funded by the Christian Doppler Laboratory for Wireless Technologies for Sustainable Mobility. The financial support by the Federal Ministry of Economy, Family and Youth and the National Foundation for Research, Technology and Development is gratefully acknowledged.

References

[1] H. Holma and A. Toskala, *LTE for UMTS OFDMA and SC-FDMA Based Radio Access*. John Wiley & Sons, Ltd, 2009.

[2] E. Dahlman, S. Parval, J. Skold, and P. Beming, *3G Evolution, HSDPA and LTE for Mobile Broadband*. Elsevier, 2007.

[3] Y. Xiang, J. Luo, and C. Hartman, "Inter-cell interference mitigation through flexible resource reuse in OFDMA based communication networks," in *Proc. European Wireless 2007*, 2007.

[4] A. Alsawah and I. Fialkow, "Optimal frequency-reuse partitioning for ubiquitous coverage in cellular systems," in *16th European Signal Processing Conference - publi-etis.ensea.fr*, 2008.

[5] B. Krasniqi, M. Wrulich, and C. Mecklenbräuker, "Network-load dependent partial frequency reuse for LTE," in *Proc.9th International Symposium on Communications and Information Technology, 2009. ISCIT*, Sept. 2009, pp. 672 –676.

[6] A. Gjendemsjo, D. Gesbert, G. Oien, and S. Kiani, "Optimal power allocation and scheduling for two-cell capacity maximization," in *Proc. 4th International Symposium on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks*, april 2006, pp. 1 – 6.

[7] M. Charafeddine and A. Paulraj, "2-sector interference channel communication for sum rates and minimum rate maximization," in *Proc. 43rd Annual Conference on Information Sciences and Systems, 2009. CISS 2009.*, March 2009, pp. 951 –956.

[8] B. Krasniqi, M. Wolkerstorfer, C. Mehlführer, and C. Mecklenbräuker, "Sum-rate maximization for multiple users in partial frequency reuse cellular networks," in *Proc. IEEE Broadband Wireless Access Workshop, Globecom 2010*, 2010.

[9] B. Krasniqi, M. Wolkerstorfer, C. Mehlführer, and C. Mecklenbräuker, "Sum-rate maximization by bandwidth re-allocation for two users in partial frequency reuse cellular networks," in *44th Asilomar Conference on Signals, Systems and Computers*, Pacific Grove (CA), USA, November 2010.

[10] Z. Xie and B. Walke, "Frequency reuse techniques for attaining both coverage and high spectral efficiency in ofdma cellular systems," in *Proc. WCNC 2010*, 2010.

[11] W. Fu, Z. Tao, J. Zhang, and D. Agrawal, "Differentiable spectrum partition for fractional frequency reuse in multi-cell ofdma networks," in *Proc. WCNC 2010*, 2010.

[12] M. Rahman and H. Yanikomeroglu, "Inter-cell interference coordination in ofdma networks: A novel approach based on integer programming," in *Proc. IEEE 71st Vehicular Technology Conf. (VTC 2010-Spring)*, 2010, pp. 1–5.

[13] J. M. Kelif and M. Coupechoux, "Joint impact of pathloss shadowing and fast fading - an outage formula for wireless networks," *CoRR*, vol. abs/1001.1110, 2010.

[14] B. Krasniqi and C. Mecklenbräuker, "Maximization of the minimum rate by geometric programming for multiple users in partial frequency reuse cellular networks," in *Proc . IEEE 74st Vehicular Technology Conf. (VTC 2011-Fall)*, 2011, pp. 1–5.

[15] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 1.21," http://cvxr.com/cvx, Feb. 2011.