Antenna Selection in Polarization Diverse MIMO Transmissions with Convex Optimization

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***Abstract*—A critical factor in the deployment of multiple input multiple-output systems is the cost of multiple analog transmit/receive chains. This problem can be mitigated by antenna subset selection at the transmitter/receiver. With antenna selection, a small number of analog radio frequency chains are multiplexed between a much larger number of transmit/receive antenna elements. In this paper, we present a low complexity approach to receive antenna selection for capacity maximization, based on the theory of convex optimization. By relaxing the antenna selection variables from discrete to continuous, we arrive at a convex optimization problem. We show via extensive Monte-Carlo simulations that the proposed algorithm provides performance very close to that of optimal selection based on exhaustive search. We consecutively optimize not only the selection of the best antennas but also the angular orientation of individual antenna elements in the array for a so-called true polarization diversity system.**

Keywords-Antenna Selection, True polarization diversity, spatial correlation, angular correlation, convex optimization.

I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) systems have received increased attention because they significantly improve wireless link performance through capacity and diversity gains [1]. A major limiting factor in the deployment of MIMO systems is the cost of multiple analog chains (such as low noise amplifiers, mixers and analog-to-digital converters) at the receiver end. Antenna selection at the transmitter/receiver is a powerful technique that reduces the number of analog chains required, yet preserving the diversity benefits obtained from the full MIMO system. With antenna selection, a limited number of transmit/receive chains are dynamically multiplexed between several transmit/receive antennas. MIMO antenna selection techniques have thus been extensively studied, and there are several antenna selection criteria. For full-diversity space-time codes, a subset of available antennas can be selected to maximize the channel norm [2]. For spatial-multiplexing systems, antennas can be selected to minimize the error ratios [3]. A useful tutorial paper on antenna selection can be found in [4]. Various selection algorithms applied to MIMO OFDM systems can be found in [5]. Exhaustive search based on maximum output SNR is proposed in [6], when the system uses linear receivers. Since exhaustive search is computationally expensive for large MIMO systems, several sub-optimal algorithms with lower complexity are derived at the expense of performance. A selection algorithm based on accurate approximation of the conditional error probability of quasi-static MIMO systems is derived in [7]. In [8], the authors formulate the receive antenna selection problem as a combinatorial optimization problem and relax it to a convex optimization problem. They employ an interior point algorithm, i.e., a barrier method, to solve a relaxed convex problem. However, they treat only the case of capacity maximization. An alternative approach to receive antenna selection for capacity maximization that offers near optimal performance at a complexity, significantly lower than the schemes in [9] but marginally greater than the schemes in [10], is described in [11]. In [12], [13] a new approach to antenna selection is proposed, based on the minimization of the union bound, which is the sum of the all Pairwise Error Probabilities (PEPs).

Our approach is based on formulating the selection problem as a combinatorial optimization problem and relaxing it to obtain a problem with a concave objective function and convex constraints. We follow the lines of [8] [11], and extend it to systems with both spatial and angular correlation, so-called True Polarization Diversity (TPD) [14]–[16] arrays. We optimize the performance of systems with such arrays of antennas which are both spatially separated and also inclined at a certain angle. A model for combined spatial and angular correlation functions is also given in [17], but we adhere to the work from Valenzuela [14]–[16]. We apply a simple norm based antenna selection method to polarization diverse array. Application of receive antenna selection on polarized array can be found in [18] [19].

The remainder of this paper is organized as follows: Section II details the generic model and the structure of polarization diverse arrays. We also describe the correlation models for both spatial and angular arrays in this section. In Section III, the performance metric to optimize with receive antenna selection, is outlined with details. Description of the performance as an optimization problem is presented in IV. Important simulation results for polarization diverse systems and comparison with the performance of Uniform Linear Arrays (ULA) are discussed in Section V. We conclude our work in Section VI.

II. SYSTEM MODEL

We consider a MIMO system with N_T transmit and M_R receive antennas. The channel is assumed to have frequency-flat Rayleigh fading with additive white Gaussian noise (AWGN) at the receiver. The received signal can thus be represented as

$$\mathbf{x}(k) = \sqrt{E_s}\mathbf{H}\mathbf{s}(k) + \mathbf{n}(k), \tag{1}$$

where $M_R \times 1$ vector $\mathbf{x}(k) = [x_1(k), \ldots, x_{M_R}(k)]^T$ represents the k^{th} sample of the signals collected at the M_R receive antennas, sampled at symbol rate. The $N_T \times 1$ vector $\mathbf{s}(k) = [s_1(k), \ldots, s_{N_T}(k)]^T$ is the k^{th} sample of the signal transmitted from the N_T transmit antennas. The symbol E_s denotes the average energy per receive antenna and per channel use, $\mathbf{n}(k) = [n_1(k), \ldots, n_{M_R}(k)]^T$ describes the noise of an AWGN channel with energy $N_0/2$ per complex dimension and $\mathbf{H}$ is the $M_R \times N_T$ channel matrix, where $H_{p,q}(p = 1, \ldots, M_R, q = 1, \ldots, N_T)$ is a scalar channel between the p^{th} receive antenna and q^{th} transmit antenna. The entries of $\mathbf{H}$ are assumed to be Zero-Mean Circularly Symmetric Complex Gaussian (ZMCSCG), such that the covariance matrix of any two columns of $\mathbf{H}$ is a scaled identity matrix. Perfect Channel State Information (CSI) is assumed at the receiver while performing antenna subset selection. No CSI is available at the transmitter. The correlation models are taken from the work of [14]–[17]. The array is with an aperture size of $L_r = \lambda/2$, the antennas in the array are randomly oriented in space and also separated by the spatial separation of d_r. Thus, we have $d_r = L_r/(M_R - 1)$. The inter element distance in ULA configuration depends on the radius. This limits the total number of antennas that can be stacked in a given area constraint. From [20] and [21], a practical measure for r is given to be 0.025λ. Thus, a maximum of nine antenna elements can be stacked in such configurations. The angles are represented by θ_r. The radiation patterns of all the elements in a ULA configuration are constant. But in an array of polarized antenna elements, different patterns exist due to the slant angles, hence introducing both, pattern and polarization diversity. Here, for the sake of simplicity we assume only polarization diversity and discard the effects produced by pattern diversity. The investigations of [15], [17], [22] describe the correlation models for structures with both angular as well as spatial diversity. We work on the modified model given in [17], which also is in agreement to the model presented in [15]. The spatial correlation between two consecutive identical antennas can be found in [14], given as

$$\varsigma_r = \sin(q_r)/q_r, \tag{2}$$

and is illustrated in Fig. 1, where $q_r = 2\pi d_r/\lambda$ and d_r is the inter-element distance as defined earlier. The correlation function between antenna elements, separated by an angular displacement is established by an equivalence between angular and spatial separation. This is called true polarization diversity [16] and shown below as

$$\varsigma_a = \sin(q_a)/q_a, \tag{3}$$

where $q_a = 2\pi\theta_r$. For a small number of receiving antennas and under Rayleigh fading scenarios the angular separation θ_r can be made equivalent to a spatial separation by

$$\theta_r = \varphi_{i-j}/180^\circ, \tag{4}$$

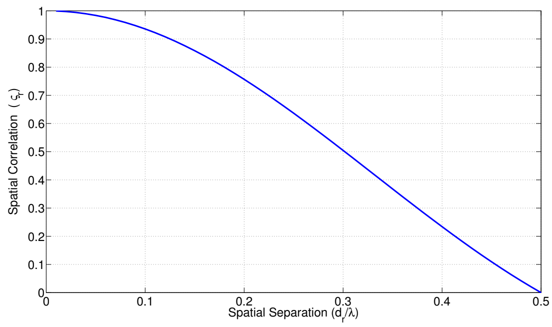

Fig. 1. Spatial correlation function.

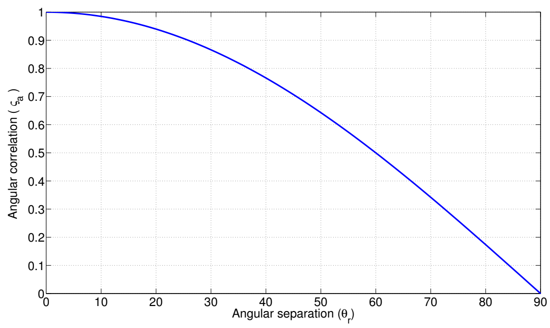

Fig. 2. Angular correlation function.

where $\varphi_{i-j} = \varphi_i - \varphi_j$ is the angular difference between two dipoles, and φ_i and φ_j are the orientation angles of dipoles, i and j with respect to vertical axis. The angular correlation function is shown in Fig. 2. The combined spatial-polarization correlation function as given in [17] is a separable function of space d_r and angle θ_r variables, shown below

$$\varsigma(d_r, \theta_r) = \mathrm{sinc}(kd_r)\cos\theta_r \tag{5}$$

If we have a ULA configuration, $\varsigma_r = \mathrm{sinc}(kd_r)$ and $\varsigma_a = \cos\theta_r$ for the angular separated configuration. We use these simple models in order to describe correlation values. It should be noted that effects of mutual coupling are ignored here for the sake of simplicity. We have shown a six element true polarization diversity antenna array in Fig. 3.

III. Receive Antenna Selection in MIMO

The earliest works on antenna selection have been in the context of Single-Input Multiple-Output (SIMO) systems. For example, selection diversity, where the receiver only selects the strongest antenna signal has long been used in SIMO systems [23]. Receive antenna selection in MIMO systems offer more degrees of freedom than in SIMO systems. We focus here on receive antenna selection for capacity maximization. The capacity of the MIMO system described in Section II is given by the well known formula

$$C(\mathbf{H}) = \log_2 \det\left(\mathbf{I}_{N_T} + \frac{\gamma}{N_T}\mathbf{R}_{ss}\mathbf{H}^H\mathbf{H}\right), \tag{6}$$

where $\gamma = E_s/N_0$, $\mathbf{R}_{ss} = E\left(\mathbf{s}(k)\mathbf{s}(k)^H\right)$ is the covariance matrix of the transmitted signals with $\mathrm{trace}(\mathbf{R}_{\mathrm{ss}}) = 1$. The

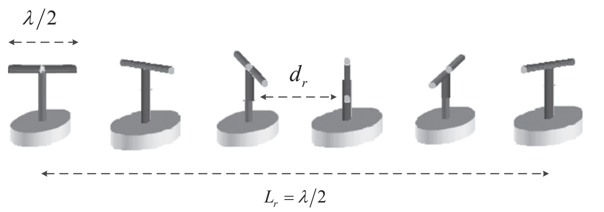

Fig. 3. True polarization diversity antenna array with $M_R = 6$ antenna elements.

determinant is denoted by $\det(\cdot)$ and $\mathbf{I}_{N_T}$ represents the $N_T \times N_T$ identity matrix. However, when only $M_R^{'} < M_R$ receive antennas are used, the capacity becomes a function of the antennas chosen. If we represent the indices of the selected antennas by $\mathbf{r} = [r_1, \ldots, r_{M_R^{'}}]$, the effective channel matrix is $\mathbf{H}$ with those rows only corresponding to these indices. Denoting the resulting $M_R^{'} \times N_T$ matrix by $\mathbf{H}_r$, the channel capacity with antenna selection is given by

$$C_r(\mathbf{H}_r) = \log_2\det\left(\mathbf{I}_{N_T} + \frac{\gamma}{N_T}\mathbf{R}_{ss}\mathbf{H}_r^H\mathbf{H}_r\right). \quad (7)$$

In the absence of CSI at the transmitter, $\mathbf{R}_{ss}$ is chosen as $\mathbf{I}_{N_T}$. Our goal is to chose the index set $\mathbf{r}$ such that the capacity in (7) is maximized. A closed form characterization of the optimal solution is difficult. We propose a possible selection scheme in the next section.

IV. Optimization algorithm for Antenna Selection

We formulate the problem of receive antenna selection as a constrained convex optimization problem [24] that can be solved efficiently using numerical methods such as interior-point algorithms [25]. Similar to [11], the $\Delta_i (i = 1, \ldots, M_R)$ is defined such that,

$$\Delta_i = \begin{cases} 1, & i^{th} \text{ receive antenna selected} \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

By definition, $\Delta_i = 1$ if $r_i \in \mathbf{r}$, and 0 else. Now, consider an $M_R \times M_R$ diagonal matrix $\mathbf{\Delta}$ that has Δ_i as its diagonal entries. Thus, the MIMO channel capacity with antenna selection can be re-written as

$$\begin{aligned} C_r(\mathbf{\Delta}) &= \log_2\det\left(\mathbf{I}_{N_T} + \frac{\gamma}{N_T}\mathbf{H}^H\mathbf{\Delta}\mathbf{H}\right) \\ &= \log_2\det\left(\mathbf{I}_{M_R} + \frac{\gamma}{N_T}\mathbf{\Delta}\mathbf{H}\mathbf{H}^H\right). \quad (9) \end{aligned}$$

The second Equality (9) follows from the matrix identity

$$\det(\mathbf{I}_m + \mathbf{AB}) = \det(\mathbf{I}_n + \mathbf{BA}).$$

The capacity expression given by $C_r(\mathbf{\Delta})$ is concave in $\mathbf{\Delta}$. The proof follows from the following facts: The function $f(\mathbf{X}) = \log_2\det(\mathbf{X})$ is concave in the entries of $\mathbf{X}$ if $\mathbf{X}$ is a positive definite matrix, and the concavity of a function is preserved under an affine transformation [24]. We transform (9) into another form that includes the correlation matrices,

$$C_r(\mathbf{\Delta}) = \log_2\det\left(\mathbf{I}_{M_R} + \frac{\gamma}{N_T}\mathbf{\Delta}\mathbf{R}_R^{1/2}\mathbf{H}\mathbf{R}_T^{1/2}\mathbf{R}_T^{H/2}\mathbf{H}^H\mathbf{R}_R^{H/2}\right), \quad (10)$$

where $\mathbf{R}_T^{1/2}$ and $\mathbf{R}_R^{1/2}$ are the normalized correlation matrices at the transmit and receive side. We assume that antennas at the transmit side are well separated to avoid any correlation. The matrix $\mathbf{R}_T^{1/2}$ would then be an identity matrix and can be ignored in the above equation. After applying rotation and simplification, (10) can be written as,

$$C_r(\mathbf{\Delta}) = \log_2\det\left(\mathbf{I}_{M_R} + \frac{\gamma}{N_T}\mathbf{R}_R^{H/2}\mathbf{\Delta}\mathbf{R}_R^{1/2}\mathbf{H}\mathbf{H}^H\right). \quad (11)$$

We split the correlation matrix $\mathbf{R}_R^{1/2}$ into two parts: the spatial separation and the polarization of individual antenna elements and obtain,

$$C_r(\mathbf{\Delta}) = \log_2\det\left(\mathbf{I}_{M_R} + \frac{\gamma}{N_T}\mathbf{R}_S^{H/2} \cdot \mathbf{R}_P^{H/2}\mathbf{\Delta}\mathbf{R}_P^{1/2} \cdot \mathbf{R}_S^{1/2}\mathbf{H}\mathbf{H}^H\right), \quad (12)$$

where $\mathbf{R}_S^{1/2}$ is the normalized correlation matrix due to the spatial separation and $\mathbf{R}_P^{1/2}$ is the additional correlation matrix due the polarization of antenna elements. The elements of these matrices are found from (2) and (3), respectively. The variables Δ_i are binary valued (0 or 1) integer variables, thereby rendering the selection problem NP-hard. We seek a simplification by relaxing the binary integer constraints and allowing $\Delta_i \in [0, 1]$. To make things easily tractable we divide the optimization problem into two parts. We first find the optimum $\mathbf{R}_P^{1/2}$ and then find the optimum $\mathbf{\Delta}$ as a separate optimization problem. Thus, the problem of receive antenna subset selection for capacity maximization is approximated by the constrained convex relaxation plus rounding schemes:

$$\text{maximize} \quad \log_2\det\left(\mathbf{I}_{M_R} + \frac{\gamma}{N_T}\mathbf{R}_S^{H/2} \cdot \mathbf{R}_P^{H/2}\mathbf{R}_P^{1/2} \cdot \mathbf{R}_S^{1/2}\mathbf{H}\mathbf{H}^H\right) \quad (13a)$$

subject to

$$r_P(m, m) = 1, \quad m = 1, \ldots, M_R \quad (13b)$$

$$|r_P(m, n)| \leq 1, \quad m, n = 1, \ldots, M_R; m \neq n \quad (13c)$$

$$\mathbf{R}_S^{1/2} \cdot \mathbf{R}_P^{1/2} \leq [1]_{M_R \times M_R}, \quad (13d)$$

where $[1]_{M_R \times M_R}$ is a matrix of all the elements equal to one. We now suppose that $\mathbf{R}_{PD}^{1/2} = \mathbf{R}_S^{1/2} \cdot \mathbf{R}_P^{1/2}$, where $\mathbf{R}_P^{1/2}$ is the optimum correlation matrix. We use this matrix $\mathbf{R}_P^{1/2}$ obtained from (13d), to obtain the optimum $\mathbf{\Delta}$,

$$\text{maximize} \quad \log_2\det\left(\mathbf{I}_{M_R} + \frac{\gamma}{N_T}\mathbf{R}_{PD}^{H/2}\mathbf{\Delta}\mathbf{R}_{PD}^{1/2}\mathbf{H}\mathbf{H}^H\right) \quad (14a)$$

subject to

$$0 \leq \Delta_i \leq 1, \quad i = 1, \ldots, M_R \quad (14b)$$

$$\text{trace}(\mathbf{\Delta}) = \sum_{i=1}^{M_R} \Delta_i = M_R^{'}. \quad (14c)$$

The objective function in (13a) is concave because the correlation matrices defined by $\mathbf{R}_P^{1/2}$ and $\mathbf{R}_S^{1/2}$ are positive definite and hermitian. Since the constraints (13b)-(13d) are linear and affine, the whole optimization algorithm (13) is concave and can be solved efficiently using disciplined convex programming [26]. Similarly, the constraints (14b)-(14c) are linear and affine, so the optimization problem (14) is concave and can be solved using disciplined convex programming [26]. Also the diagonal matrix $\mathbf{\Delta}$ is positive semi-definite. From the optimum values of $\mathbf{R}_P^{1/2}$ found, we can proceed to find the optimum angles of polarization or orientation. From the (possibly) fractional solution obtained by solving the above problem, the $M_R^{'}$ largest Δ_i's are chosen and the corresponding indices represent the receive antennas to be selected. The optimum capacity (7) is then calculated by using only the selected subset r, which is found through (13) and (14). The ergodic capacity after selection now reads,

$$C(\tilde{\mathbf{\Delta}}) = \log_2\det\left(\mathbf{I}_{M_R'} + \frac{\gamma}{N_T}\tilde{\mathbf{R}}_{PD}^{H/2}\tilde{\mathbf{\Delta}}\tilde{\mathbf{R}}_{PD}^{1/2}\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H\right), \quad (15)$$

where $(\tilde{\cdot})$ denotes a matrix, composed only of the selected r. In summary we try to find the optimum angles θ_r's, which optimize the ergodic capacity with receive antenna selection. Practically this system is only realizable, if all the antenna elements in an array can be independently rotated around their axes. Physically realizing such system is not easy, but methods to emulate the rotating effect through the use of parasitic elements has been investigated in [27].

V. Results

In this section, we evaluate the performance of the proposed antenna selection algorithm via Monte-Carlo simulations [26]. We solve the optimization algorithm using the MATLAB based tool for convex optimization called CVX [26]. We use ergodic capacity as a metric for performance evaluation, which is obtained by averaging over results, obtained from 1000 independent realizations of the channel matrix $\mathbf{H}$. For each realization, the entries of the channel matrix are uncorrelated ZMCSCG random variables. We take the example of real valued correlation matrices calculated from (5). In Fig. 4 we show the results for $M_R'/6$ selection. In Fig. 5 we show the results for capacity against M_R' for values of N_T. In Fig. 4 and 5 we also show the simulation results for systems with only vertical oriented antenna elements i.e, only separated spatially (ULA). We see clearly that the performance of these systems is substantially less than the systems which contain both spatial and angular separation. The optimization problem similar to (16) for only spatially separated systems is given by,

$$\text{maximize} \quad \log_2\det\left(\mathbf{I}_{M_R} + \frac{\gamma}{N_T}\mathbf{R}_S^{H/2}\mathbf{\Delta}\mathbf{R}_S^{1/2}\mathbf{H}\mathbf{H}^H\right) \quad (16a)$$

subject to

$$0 \le \Delta_i \le 1, \quad i = 1, \ldots, M_R \quad (16b)$$

$$\text{trace}(\mathbf{\Delta}) = \sum_{i=1}^{M_R} \Delta_i = M_R^{'}. \quad (16c)$$

The above stated optimization problem is now simpler because of only one matrix $\mathbf{\Delta}$ to be optimized with two constraints. As an example for a polarization diverse system, we show in (17), the diagonal matrix $\mathbf{\Delta}$ for a $2/6$ selection. We see that $\text{trace}(\mathbf{\Delta}) = \sum_{i=1}^{M_R} \Delta_i = 2$. We take the two largest elements of the vector $\text{trace}(\mathbf{\Delta})$ and find the ergodic capacity with the respective indices $(r = 2, 3)$ of the rows of the channel matrix $\mathbf{H}$. Now we show an optimum correlation matrix in (19) $\mathbf{R}_P^{1/2}$ for a given $\mathbf{R}_S^{1/2}$, calculated for the optimum $\mathbf{\Delta}$, as an example. The $\tilde{\mathbf{\Delta}}$ matrix formed after selection, is given in (18). We use the same indices $(r = 2, 3)$ again to select the rows and columns of correlation matrix $\mathbf{R}_P^{1/2}$. The selected correlation matrix is shown in (20). From this matrix the corresponding angles are $\theta_r = 0, 71°$. We show more examples of selection systems with the corresponding optimum angles in Table I at 20dB SNR.

$$\mathbf{\Delta} = \begin{bmatrix} 0.3957 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3847 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2196 \end{bmatrix}. \quad (17)$$

$$\tilde{\mathbf{\Delta}} = \begin{bmatrix} 1 & 0 \\ 0 & 0.3957 \end{bmatrix}. \quad (18)$$

$$\mathbf{R}_P^{1/2} = \begin{bmatrix} 1.000 & 0.189 & 0.174 & 0.033 & 0.000 & 0.229 \\ 0.189 & 1.000 & 0.000 & 0.139 & 0.297 & 0.951 \\ 0.174 & 0.000 & 1.000 & 0.081 & 0.050 & 0.210 \\ 0.033 & 0.139 & 0.081 & 1.000 & 0.000 & 0.000 \\ 0.000 & 0.297 & 0.050 & 0.000 & 1.000 & 0.143 \\ 0.229 & 0.951 & 0.210 & 0.000 & 0.143 & 1.000 \end{bmatrix}. \quad (19)$$

$$\tilde{\mathbf{R}}_P^{1/2} = \begin{bmatrix} 1.000 & 0.189 \\ 0.189 & 1.000 \end{bmatrix}. \quad (20)$$

TABLE I
Optimum Angles with $M_R'/9$ Selection at 20dB SNR for $M_R = 1, \cdots, 5$

M_R'	Indices (r)	Angles (θ_r°)
1	7	0
2	2,6	0,62
3	3,7,9	0,56,76
4	2,5,7,9	0,73,78,90
5	1,4,5,6,7	0,55,90,65,70

VI. Conclusions

In this work we investigated an polarization diverse antenna array with compact aperture, to optimize the performance in terms of ergodic capacity. We used the relaxation of a binary integer constraint to have a convex optimization algorithm and solved it, using disciplined convex programming. The optimization algorithm not only finds the best antennas for selection but also finds the optimum orientation angles for antenna elements within an array. We also compared the results with an array consisting of linear uniform elements. We found that by using an optimization algorithm, the performance of polarization diverse systems can be significantly enhanced.

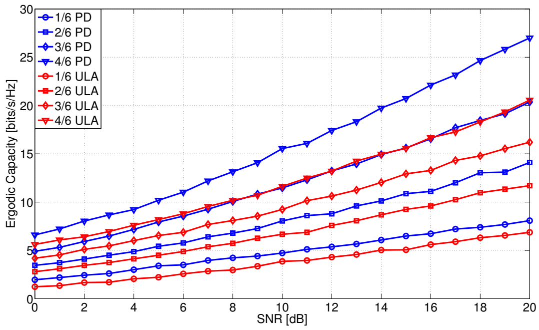

Fig. 4. Ergodic capacity v/s SNR, $M_R = 6$, $N_T = 1, 2, 3, 4$, $M_R' = N_T$, for Polarization Diverse (PD) and Uniform Linear Array (ULA) systems.

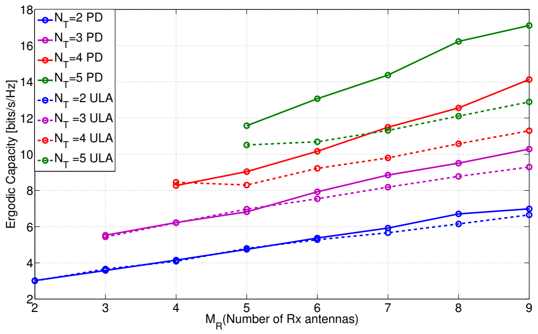

Fig. 5. Ergodic capacity v/s M_R, SNR=10dB, $N_T = 2, 3, 4, 5$, $M_R' = N_T$, for Polarization Diverse (PD) and Uniform Linear Array (ULA) systems.

ACKNOWLEDGMENTS

This work has been funded by the Christian Doppler Laboratory for Wireless Technologies for Sustainable Mobility, KATHREIN-Werke KG, and A1 Telekom Austria AG. The financial support by the Federal Ministry of Economy, Family and Youth and the National Foundation for Research, Technology and Development is gratefully acknowledged. The authors would like to thank Christoph F. Mecklenbräuker for his valuable comments and fruitful discussions. This work has also been funded by the Higher Education Commission, Islamabad, Pakistan.

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